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ON $(gg)^*$-SEPARATION AXIOMS

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Abstract. In this paper, we introduce a new class of separation axioms namely $(gg)^*$- $T_k, k = 0, 1, 2$ spaces. We investigated some of their properties using $(gg)^*$-continuous functions, $(gg)^*$-irresolute functions, $(gg)^*$-closed maps, $(gg)^*$-open maps.

Keywords: $(gg)^*$- T_0; $(gg)^*$- T_1; $(gg)^*$- T_2.

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1. Introduction

In 1975, S. N. Maheshwari and R. Prasad [1], used semi-open sets to define and investigate new separation axiom namely Semi-T_0, Semi-T_1,Semi-T_2. Following them many topologist defined new separation axioms namely gpr-separation axioms[2], gsp-separation axioms[3], gg-separation axioms[4] etc. In this paper we used $(gg)^*$-closed sets in topological spaces[5], to define a new seperation axioms namely $(gg)^*$- T_0, $(gg)^*$- T_1, $(gg)^*$- T_2 spaces and the characterizations are studied.

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2. Preliminaries

Throughout this paper X or (X,τ) and Y or (Y,σ) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, the closure of A and the interior of A are denoted by $cl(A)$, $int(A)$ and A^c denotes the complement of A in X. We recall some of the basic definitions and results.

Definition 2.1. *A subset A of a topological space (X,τ) is called a*

(i) regular open set [4] if $A = int(cl(A))$ and a regular closed set if $cl(int(A)) = A$.

(ii) regular semi open [5] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

(iii)generalized - closed set (briefly g - closed) [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(iv) generalization of generalized closed set (briefly gg-closed) [4] if $gcl(A) \subseteq U$ whenever $A \subseteq U$and U is regular semi - open in X.

(v) generalization of generalized star closed sets (briefly $(gg)^*$- closed) [5] if $rcl(A) \subseteq U$ whenever$A \subseteq U$and U is gg - open in X.

The complements of the above sets are their respective open sets and vice versa.

Definition 2.2. *A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is called $(gg)^*$- continuous [6] if $f^{-1}(V)$ is $(gg)^*$ - closed in X for every closed set$V$ of Y.*

Definition 2.3. *A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is called $(gg)^*$- irresolute[6] if $f^{-1}(V)$ is $(gg)^*$ - closed in (X,τ) for every $(gg)^*$- closed set V in (Y,σ).*

Definition 2.4. *A map $f : (X,\tau) \rightarrow (Y,\sigma)$ is called generalization of generalized star closed map (briefly $(gg)^*$- closed map) if the image of every closed set in (X,τ) is $(gg)^*$- closed in (Y,σ).*

Definition 2.5. *A map $f : (X,\tau) \rightarrow (Y,\sigma)$ is called generalization of generalized star open map (briefly $(gg)^*$- open map) if the image of every open set in (X,τ) is $(gg)^*$- open in (Y,σ).*

Definition 2.6. *For a subset A of a space X , $(gg)^* - cl(A) = \bigcap\{B \subseteq X : B$ is $(gg)^*$ -closed and $A \subseteq B\}$.*

3. $(gg)^*$-T_k SPACES, $k \in \{0,1,2\}$

Definition 3.1. *A topological space (X,τ) said to be $(gg)^*$- T_0 if for each pair of distinct points $x, y \in X$, there exists a $(gg)^*$-open set U such that $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.*

Definition 3.2. *A topological space (X,τ) said to be $(gg)^*$- T_1 if for each pair of distinct points $x, y \in X$, there exists $(gg)^*$-open sets U and V containing x and y such that $x \in U$ and $y \notin U$ or $x \notin V$ and $y \in V$.*

Definition 3.3. *A topological space (X,τ) said to be $(gg)^*$- T_2 if for each pair of distinct points $x, y \in X$, there exists disjoint $(gg)^*$-open sets U and V such that $x \in U$ and $y \in V$.*

Theorem 3.4. *(i) Every $(gg)^*$- T_1 space is $(gg)^*$- T_0 space.*
(ii) Every $(gg)^*$-T_2 space is $(gg)^*$- T_1 space.

Proof. (i) Let X be a $(gg)^*$- T_1 space. Let $x, y \in X$ with $x \neq y$. Since X is a $(gg)^*$- T_1 space, there exists $(gg)^*$-open sets U and V containing x and y such that $x \in U$ and $y \notin U$ or $x \notin V$ and $y \in V$. Therefore X is a $(gg)^*$- T_0 space.
(ii)Let X be a $(gg)^*$- T_2 space. Let $x, y \in X$ with $x \neq y$. Since X is a $(gg)^*$- T_2 space, there exists disjoint $(gg)^*$-open sets U and V containing x and y such that $x \in U$ and $y \in V$. That is $x \notin V$ and $y \notin U$. Therefore X is a $(gg)^*$- T_1 space. □

Theorem 3.5. *A topological space X is $(gg)^*$- T_0 iff $(gg)^*$-closure of distinct points are distinct.*

Proof. Let X be a $(gg)^*$- T_0 space. Let $x, y \in X$ be such that $x \neq y$. Since X is a $(gg)^*$- T_0 space, there exists a $(gg)^*$-open set U such that $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.
Let us consider $x \in U$ and $y \notin U$. Then $x \notin X - U$ and $y \in X - U$. Since U is a $(gg)^*$-open set, $X - U$ is a $(gg)^*$- closed set in X containing y but not x. But $(gg)^*$- $cl(y)$ is the intersection of all $(gg)^*$- closed set in X containing y. Therefore $(gg)^*$-$cl(y) \subseteq X - U$. Since $x \notin X - U$, $x \notin (gg)^*$- $cl(y)$. But $y \in (gg)^*$- $cl(y)$. Hence $(gg)^*$- $cl(x) \neq (gg)^*$- $cl(y)$. Similarly we can prove the other case. Hence $(gg)^*$-closure of distinct points are distinct. Conversely suppose that $(gg)^*$- $cl(x) \neq (gg)^*$- $cl(y)$. If $x \neq y$ with $x, y \in X$. Then there exists atleast one point $z \in X$ such that $z \in (gg)^*$- $cl(x)$ and $z \notin (gg)^*$- $cl(y)$ or $z \notin (gg)^*$- $cl(x)$ and $z \in (gg)^*$- $cl(y)$. Now

let us consider $z \in (gg)^*$- $cl(x)$ and $z \notin (gg)^*$- $cl(y)$. Suppose $x \in (gg)^*$- $cl(y)$. Then $(gg)^*$- $cl(x) \subseteq (gg)^*$- $cl(y)$. Hence $z \in (gg)^*$- $cl(x) \subseteq (gg)^*$- $cl(y)$ and so $z \in (gg)^*$- $cl(y)$. Which is a contradiction. Hence $x \notin (gg)^*$- $cl(y)$. This implies $x \in X$- $(gg)^*$- $cl(y)$, which is a $(gg)^*$-open set in X containing x but not y. Hence X is a $(gg)^*$- T_0 space. □

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be an injective, $(gg)^*$-irresolute map. If Y is a $(gg)^*$- T_0 then X is a $(gg)^*$- T_0 space.*

Proof. Let $x, y \in X$ be such that $x \neq y$. Since f is injective, $f(x) \neq f(y)$. As Y is a $(gg)^*$- T_0 space, there exists a $(gg)^*$-open set U of Y such that $f(x) \in U$ and $f(y) \notin U$ or $f(x) \notin U$ and $f(y) \in U$. Since f is $(gg)^*$-irresolute, $f^{-1}(U)$ is $(gg)^*$-open set in X such that $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$ or $x \notin f^{-1}(U)$ and $y \in f^{-1}(U)$. Hence X is a $(gg)^*$- T_0 space. □

Theorem 3.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be an injective, $(gg)^*$-irresolute map. If Y is a $(gg)^*$- T_2 then X is a $(gg)^*$- T_2 space.*

Proof. Let $x, y \in X$ be such that $x \neq y$. Since f is injective, $f(x) \neq f(y)$. As Y is $(gg)^*$- T_2 space, there exists $(gg)^*$-open sets U, V of Y such that $f(x) \in U$ and $f(y) \in V$ and $U \cap V = \phi$. Since f is $(gg)^*$-irresolute, $f^{-1}(U)$, $f^{-1}(V)$ are $(gg)^*$-open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is a $(gg)^*$- T_2 space. □

Theorem 3.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, $(gg)^*$-continuous map. If Y is a T_1 space then X is a $(gg)^*$- T_1 space.*

Proof. Let $x_1, x_2 \in X$ be such that $x_1 \neq x_2$. Since f is bijective, there exists y_1, y_2 in Y with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since Y is a T_1-space, there exists open sets U, V of Y such that $y_1 \in U$ and $y_1 \notin V$ or $y_2 \in V$ and $y_1 \notin U$. Since f is $(gg)^*$- continuous, there exists $(gg)^*$-open sets $f^{-1}(U)$, $f^{-1}(V)$ in X such that $x_1 \in f^{-1}(U)$ and $x_1 \notin f^{-1}(V)$ or $x_2 \in f^{-1}(V)$ and $x_2 \notin f^{-1}(U)$. Hence X is a $(gg)^*$- T_1 space. □

Theorem 3.9. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, $(gg)^*$- open map. If X is a T_1 then Y is a $(gg)^*$- T_1 space.*

Proof. Let $y_1, y_2 \in Y$ be such that $y_1 \neq y_2$. Since f is bijective, there exists x_1, x_2 in X with $x_1 \neq x_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since X is a T_1-space, there exists open sets U, V of X such that $x_1 \in U$ and $x_2 \notin V$ or $x_2 \in V$ and $x_1 \notin U$. Since f is $(gg)^*$- open map, there exists $(gg)^*$-open sets $f(U)$, $f(V)$ in Y such that $f(x_1) \in f(U)$ and $f(x_1) \notin f(V)$ or $f(x_2) \in f(V)$ and $f(x_2) \notin f(U)$. That is there exists $(gg)^*$-open sets $f(U)$, $f(V)$ in Y such that $y_1 \in f(U)$ and $y_1 \notin f(V)$ or $y_2 \in f(V)$ and $y_2 \notin f(U)$. Hence Y is a $(gg)^*$- T_1 space. □

Theorem 3.10. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, $(gg)^*$- continuous function. If Y is a T_2 space then X is a $(gg)^*$- T_2 space.*

Proof. Let $x_1, x_2 \in X$ be such that $x_1 \neq x_2$. Since f is bijective, there exists y_1, y_2 in Y with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since Y is a T_2-space, there exists disjoint open sets U, V of Y such that $y_1 \in U$ and $y_2 \in V$. Since f is $(gg)^*$- continuous, there exists disjoint $(gg)^*$-open sets $f^{-1}(U)$, $f^{-1}(V)$ in X such that $x_1 = f^{-1}(y_1) \in f^{-1}(U)$ and $x_2 = f^{-1}(y_2) \in f^{-1}(V)$. Hence X is a $(gg)^*$- T_2 space. □

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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