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TRANSCENDENTAL-HYPERBOLIC FUNCTIONS AND THEIR ADOMIAN POLYNOMIALS WITH NUMERICAL RESULTS FACILITATED BY NONLINEAR SHANKS TRANSFORM

E.U. AGOM[1*], F.O. OGUNFIDITIMI[2], M. MAYAKI[3]

[1]Department of Mathematics, University of Calabar, Calabar, Nigeria

[2]Department of Mathematics, University of Abuja, Abuja, Nigeria

[3]Department of Mathematics and Computer Science, Ibrahim Babangida University, Lapie, Nigeria

Abstract. In this paper, we provide explicitly the Adomian polynomials (AP) for transcendental-hyperbolic functions in a linear functional and forced the convergence of inconsistent solution series when Adomian decomposition method (ADM) is deployed in related problems by nonlinear Shanks transform (NST). These were achievable by developing a theoretical background of AP for transcendental-hyperbolic functions based upon a thorough examination of the historical preceding of ADM. Application of the presented polynomials resulted to unreliable series solutions which was, however, upturned on using NST in the problems considered. This paper has unified the notion of modified AP for transcendental-hyperbolic nonlinear functions and its application to similar equations. It further presented a reliable technique that forced convergence in unpredictable and alternating series solutions that are obtain by ADM.

Keywords: transcendental-hyperbolic functions; Adomian polynomials; nonlinear Shanks transformation; Adomian decomposition method.

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*Corresponding author

E-mail address: agomeunan@gmail.com

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1. INTRODUCTION

Over the years ADM [1] and its improvement has become a very powerful technique for obtaining analytical and approximate analytical solutions to a generalised nonlinear equations.

$$L^n(\vartheta)+N(\vartheta)=\zeta, \qquad \vartheta=\vartheta(t) \tag{1}$$

Where L^n is the nth order derivative of ϑ which is a combination of highest order derivative and other differential operator. Which is corresponding to L^{-n} operator given as

$$L^{-n}=\int_0^{t_n}\int_0^{t_{n-1}}\int_0^{t_{n-2}}...\int_0^{t_1}(.)dt_1dt_2dt_3...dt_n \tag{2}$$

$N=N(\vartheta)$ is the nonlinear term which is transcendental-hyperbolic in this article and is to be decomposed into AP. And, $\zeta=\zeta(t)$ is the source term. The ADM by [1], has been widely reported in [2] - [15], [17] and [18]. It is a systematic analytical and approximation method applied to a wide class of equations. The method provided solutions in convergent series form under physically appropriate conditions. Nonetheless, it successful application, especially on the nonlinear problems, requires the right AP to be used for ultimate desired results, see [1, 8, 9] and the literatures therein. However, the nonlinearity in a linear functional varies; polynomial nonlinearity has been vividly reported in [8], other are [17, 11, 6, 12, 9, 10, 2, 13]. Trigonometric nolinearity has extensively been investigated in [9], see also [14] and the literature therein. Exponential and logarithmic nonlinearity can be seen in [18], transcendental hyperbolic sine and cosine has been reported in [13].

Reported investigation of AP in the are of transcendental hyperbolic nonlinear terms are still minimal in literature. The fundamental goal in this paper is to decompose the transcendental-hyperbolic nonlinear term $N(\vartheta)$ in a linear functional (1) into series of polynomials $\Sigma_{n=0}^{\infty}A_n$ using the modified AP as contain in [8]. Where the A_n are the AP. These polynomials are in hyperbolic form and equivalently in their respective exponential form for each $N(\vartheta)$. Due to the presence of noise term in each polynomial, convergence to a solution in the illustrative problems considered were far fetched by the traditional ADM. However, this was reversed on application of NST.

2. THE ADOMIAN POLYNOMIAL IN ADOMIAN DECOMPOSITION METHOD

These polynomials were introduced by [1] and it is given as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\Sigma_{i=0}^{\infty}\lambda^i \vartheta_i)]_{\lambda=0} \tag{3}$$

λ is a parameter introduced for convenient. A_n depends on $\vartheta_0, \vartheta_1, \vartheta_2$ $\ldots\vartheta_{n-2}, \vartheta_{n-1}, \vartheta_n$, where $n \in \mathbb{Z}^+$. Implementation of (3) on (1) basically gives

$$\begin{aligned}
A_0 &= A(\vartheta_0)\\
A_1 &= A(\vartheta_0, \vartheta_1)\\
A_2 &= A(\vartheta_0, \vartheta_1, \vartheta_2)\\
A_3 &= A(\vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3)\\
A_4 &= A(\vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4)\\
A_5 &= A(\vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5)\\
&\ldots\\
A_n &= A(\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_{n-2}, \vartheta_{n-1}, \vartheta_n)
\end{aligned}$$

From its inception till date, several modifications has been carried out on how these polynomials can be generated. These somewhat feasible form calculates the AP in a simple way using any known computer algebra software like Maple, Mathematica, etc. All these other forms of AP has been implemented and reported in [2] - [15], [17] and [18].

3. THE ADOMIAN POLYNOMIALS OF MAJOR TRANSCENDENTAL-HYPERBOLIC NON-LINEAR TERMS

In this section, we apply equation (3) to (1) to obtain the AP of the first five terms for each major hyperbolic functions; $\sinh\vartheta$, $\cosh\vartheta$, $\tanh\vartheta$, $\operatorname{sech}\vartheta$, $\operatorname{csch}\vartheta$ and $\coth\vartheta$. And, to avoid excessively long expressions, we denote $e^{\vartheta_0} - e^{-\vartheta_0}$ as ϑ_- and $e^{\vartheta_0} + e^{-\vartheta_0}$ as ϑ_+.

3.1: For $N(\vartheta) = \sinh\vartheta$

$$A_0 = \sinh\vartheta_0$$

$$A_1 = \vartheta_1 \cosh \vartheta_0$$
$$A_2 = \frac{1}{2!}\vartheta_1^2 \sinh \vartheta_0 + \vartheta_2 \cosh \vartheta_0$$
$$A_3 = \frac{1}{3!}\vartheta_1^3 \cosh \vartheta_0 + \vartheta_1 \vartheta_2 \sinh \vartheta_0 + \vartheta_3 \cosh \vartheta_0$$
$$A_4 = \frac{1}{4!}\vartheta_1^4 \sinh \vartheta_0 + \frac{1}{2!}\vartheta_1^2 \vartheta_2 \cosh \vartheta_0 + \frac{1}{2!}\vartheta_2^2 \sinh \vartheta_0 + \vartheta_3 \vartheta_1 \sinh \vartheta_0 + \vartheta_4 \cosh \vartheta_0$$

Equivalently,

$$A_0 = \frac{1}{2}\vartheta_-$$
$$A_1 = \frac{1}{2}\vartheta_1 \vartheta_+$$
$$A_2 = \frac{1}{2}\vartheta_2 \vartheta_+ + \frac{1}{4}\vartheta_1^2 \vartheta_-$$
$$A_3 = \frac{1}{2}\vartheta_3 \vartheta_+ + \frac{1}{2}\vartheta_1 \vartheta_2 \vartheta_- + \frac{1}{12}\vartheta_1^3 \vartheta_+$$
$$A_4 = \frac{1}{2}\vartheta_4 \vartheta_+ + \frac{1}{2}\vartheta_1 \vartheta_3 \vartheta_- + \frac{1}{4}\vartheta_1^2 \vartheta_2 \vartheta_+ + \frac{1}{4}\vartheta_2^2 \vartheta_- + \frac{1}{48}\vartheta_1^4 \vartheta_-$$

3.2: For $N(\vartheta) = \cosh \vartheta$

$$A_0 = \cosh \vartheta_0$$
$$A_1 = \vartheta_1 \sinh \vartheta_0$$
$$A_2 = \frac{1}{2!}\vartheta_1^2 \cosh \vartheta_0 + \vartheta_2 \sinh \vartheta_0$$
$$A_3 = \frac{1}{3!}\vartheta_1^3 \sinh \vartheta_0 + \vartheta_1 \vartheta_2 \cosh \vartheta_0 + \vartheta_3 \sinh \vartheta_0$$
$$A_4 = \frac{1}{4!}\vartheta_1^4 \cosh \vartheta_0 + \frac{1}{2}\vartheta_1^2 \vartheta_2 \sinh \vartheta_0 + \frac{1}{2}\vartheta_2^2 \cosh \vartheta_0 + \vartheta_3 \vartheta_1 \cosh \vartheta_0 + \vartheta_4 \sinh \vartheta_0$$

Alternatively,

$$A_0 = \frac{1}{2}\vartheta_+$$
$$A_1 = \frac{1}{2}\vartheta_1 \vartheta_-$$
$$A_2 = \frac{1}{2}\vartheta_2 \vartheta_- + \frac{1}{4}\vartheta_1^2 \vartheta_+$$
$$A_3 = \frac{1}{2}\vartheta_3 \vartheta_- + \frac{1}{2}\vartheta_1 \vartheta_2 \vartheta_+ + \frac{1}{12}\vartheta_1^3 \vartheta_-$$

$$A_4 = \frac{1}{2}\vartheta_4\vartheta_- + \frac{1}{2}\vartheta_1\vartheta_3\vartheta_+ + \frac{1}{4}\vartheta_1^2\vartheta_2\vartheta_- + \frac{1}{4}\vartheta_2^2\vartheta_+ + \frac{1}{48}\vartheta_1^4\vartheta_+$$

3.3: For $N(\vartheta) = \tanh\vartheta$

$$A_0 = \tanh\vartheta_0$$

$$A_1 = \vartheta_1 \operatorname{sech}^2\vartheta_0$$

$$A_2 = \vartheta_2 \operatorname{sech}^2\vartheta_0 - \vartheta_1^2 \operatorname{sech}^2\vartheta_0 \tanh\vartheta_0$$

$$A_3 = \vartheta_3 \operatorname{sech}^2\vartheta_0 - 2\vartheta_1\vartheta_2 \tanh\vartheta_0 \operatorname{sech}^2\vartheta_0 + \frac{2}{3}\vartheta_1^3 \tanh^2\vartheta_0 \operatorname{sech}^2\vartheta_0 - \frac{1}{3}\vartheta_1^3 \operatorname{sech}^4\vartheta_0$$

$$A_4 = \frac{2}{3}\vartheta_1^4 \tanh\vartheta_0 \operatorname{sech}^4\vartheta_0 - \vartheta_1^2\vartheta_2 \operatorname{sech}^4\vartheta_0 - \frac{1}{3}\vartheta_1^4 \tanh^3\vartheta_0 \operatorname{sech}^2\vartheta_0$$
$$+ 2\vartheta_1^2\vartheta_2 \tanh^2\vartheta_0 \operatorname{sech}^2\vartheta_0 - \vartheta_2^2 \tanh\vartheta_0 \operatorname{sech}^2\vartheta_0 - 2\vartheta_1\vartheta_3 \tanh\vartheta_0 \operatorname{sech}^2\vartheta_0$$
$$+ \vartheta_4 \operatorname{sech}^2\vartheta_0$$

Alternatively,

$$A_0 = \frac{\vartheta_-}{\vartheta_+}$$

$$A_1 = \vartheta_1 - \frac{\vartheta_1\vartheta_-^2}{\vartheta_+^2}$$

$$A_2 = \vartheta_2 - \frac{\vartheta_1\vartheta_-^2}{\vartheta_+} + \frac{\vartheta_1\vartheta_-^2}{\vartheta_+^3} - \frac{\vartheta_2\vartheta_-^2}{\vartheta_+^2}$$

$$A_3 = -\frac{1}{3\vartheta_+^4}(-3\vartheta_3\vartheta_+^4 + \vartheta_1^3\vartheta_+^4 - 2\vartheta_1^3\vartheta_+^2\vartheta_-^2 - 3\vartheta_1^2\vartheta_+^2\vartheta_- + 3\vartheta_1\vartheta_-^4 - 6\vartheta_1\vartheta_2\vartheta_+\vartheta_-^3$$
$$- 3\vartheta_3\vartheta_+^2\vartheta_-^2)$$

$$A_4 = \frac{1}{24\vartheta_+}(24\vartheta_4\vartheta_+ + 24\vartheta_1\vartheta_3\vartheta_- + 12\vartheta_2^2\vartheta_- + 12\vartheta_1^2\vartheta_2\vartheta_+ + \vartheta_1^4\vartheta_-)$$
$$- \frac{1}{6\vartheta_+^2}(6\vartheta_3\vartheta_+ + 6\vartheta_1\vartheta_2\vartheta_- + \vartheta_1^3\vartheta_+)(\vartheta_1\vartheta_-)$$
$$+ \frac{1}{2\vartheta_+^3}(2\vartheta_2\vartheta_+ + \vartheta_1^2\vartheta_-)(\vartheta_1\vartheta_-)^2 - \frac{1}{4\vartheta_+^2}(\vartheta_1\vartheta_+ + \vartheta_1^2\vartheta_-)(2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+)$$
$$+ \frac{\vartheta_1^2\vartheta_-}{\vartheta_+^3} + \frac{1}{\vartheta_+^3}(\vartheta_1^2\vartheta_+\vartheta_-)(\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+) - \frac{\vartheta_1\vartheta_+}{6\vartheta_+^2}(6\vartheta_3\vartheta_- + 6\vartheta_1\vartheta_2\vartheta_+ + \vartheta_1^3\vartheta_-)$$
$$+ \frac{\vartheta_1^4\vartheta_-^5}{\vartheta_+^5} - \frac{3}{2\vartheta_+^4}(\vartheta_1^2\vartheta_-^3)(2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+) + \frac{\vartheta_-}{4\vartheta_+^3}(2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+)^2$$

$$+\frac{\vartheta_1\vartheta_-^2}{3\vartheta_+^3}(6\vartheta_3\vartheta_- + 6\vartheta_1\vartheta_2\vartheta_+ + \vartheta_1^3\vartheta_-) + \frac{\vartheta_-}{\vartheta_+^2}(24\vartheta_4\vartheta_- + 24\vartheta_1\vartheta_3\vartheta_3 + 12\vartheta_2^2\vartheta_+$$
$$+ 12\vartheta_1^2\vartheta_2\vartheta_- + \vartheta_1^4\vartheta_+)$$

3.4: For $N(\vartheta) = \operatorname{csch}\vartheta$

$$A_0 = \operatorname{csch}\vartheta_0$$
$$A_1 = -\vartheta_1\operatorname{csch}\vartheta_0\coth\vartheta_0$$
$$A_2 = \frac{1}{2}\vartheta_1^2\operatorname{csch}\vartheta_0\coth^2\vartheta_0 + \frac{1}{2}\vartheta_1^2\operatorname{csch}^3\vartheta_0 - \vartheta_2\operatorname{csch}\vartheta_0\coth\vartheta_0$$
$$A_3 = -\frac{1}{6}\vartheta_1^3\operatorname{csch}\vartheta_0\coth\vartheta_0 + \frac{5}{6}\vartheta_1^3\operatorname{csch}^3\vartheta_0\coth\vartheta_0 + \vartheta_1\vartheta_2\operatorname{csch}\vartheta_0\coth^2\vartheta_0$$
$$+ \vartheta_1\vartheta_2\operatorname{csch}^3\vartheta_0 - \vartheta_3\operatorname{csch}\vartheta_0\coth\vartheta_0$$
$$A_4 = \frac{1}{24}\vartheta_1^4\operatorname{csch}\vartheta_0\coth^4\vartheta_0 - \frac{3}{4}\vartheta_1^4\operatorname{csch}^3\vartheta_0\coth^2\vartheta_0 - \frac{1}{2}\vartheta_1^2\vartheta_2\operatorname{csch}\vartheta_0\coth^3\vartheta_0$$
$$+ \frac{5}{24}\vartheta_1^4\operatorname{csch}^5\vartheta_0 + \frac{5}{2}\vartheta_2^2\vartheta_2\operatorname{csch}^3\vartheta_0\coth\vartheta_0 + \frac{1}{2}\vartheta_2^2\operatorname{csch}\vartheta_0\coth^2\vartheta_0$$
$$+ \vartheta_1\vartheta 3\operatorname{csch}\vartheta_0\coth^2\vartheta_0 - \frac{1}{2}\vartheta_2^2\operatorname{csch}^3\vartheta_0 - \vartheta_1\vartheta_3\operatorname{csch}^3\vartheta_0 - \vartheta_4\operatorname{csch}\vartheta_0\coth\vartheta_0$$

Alternatively,

$$A_0 = \frac{2}{\vartheta_-}$$
$$A_1 = \frac{2\vartheta_1\vartheta_+}{\vartheta_-}$$
$$A_2 = \frac{2\vartheta_1^2\vartheta_+}{\vartheta_-} - \frac{2\vartheta_1\vartheta_+ + \vartheta_1^2\vartheta_-}{\vartheta_-^2}$$
$$A_3 = -\frac{2\vartheta_1^3\vartheta_+^3}{\vartheta_-^4} + \frac{2\vartheta_1\vartheta_+(2\vartheta_2\vartheta_+ + \vartheta_1^2\vartheta_-)}{\vartheta_-^3} + \frac{6\vartheta_3\vartheta_+ + 6\vartheta_1\vartheta_2\vartheta_- + \vartheta_1^3\vartheta_+}{3\vartheta_-^2}$$
$$A_4 = \frac{2\vartheta_1^4\vartheta_+^4}{\vartheta_-^5} - \frac{3\vartheta_1^2\vartheta_+^2(2\vartheta_2\vartheta_+ + \vartheta_1^2\vartheta_-)}{\vartheta_-^4} + \frac{(2\vartheta_2\vartheta_+ + \vartheta_1^2\vartheta_-^2)^2}{2\vartheta_+^3}$$
$$+ \frac{2\vartheta_1\vartheta_+(6\vartheta_3\vartheta_+ + 6\vartheta_2\vartheta_1\vartheta_- + \vartheta_1^3\vartheta_+)}{3\vartheta_-^3}$$
$$+ \frac{24\vartheta_4\vartheta_+ + 24\vartheta_3\vartheta_1\vartheta_- + 12\vartheta_2^2\vartheta_- + 12\vartheta_2\vartheta_1^2\vartheta_+ + \vartheta_1^4\vartheta_-}{12\vartheta_-^2}$$

3.5: For $N(\vartheta) = \operatorname{sech}\vartheta$

$$A_0 = \operatorname{sech}\vartheta_0$$

$$A_1 = -\vartheta_1 \operatorname{sech}\vartheta_0 \tanh\vartheta_0$$

$$A_2 = \frac{1}{2}\vartheta_1^2 \operatorname{sech}\vartheta_0 \tanh^2\vartheta_0 - \frac{1}{2}\vartheta_1^2 \operatorname{sech}^3\vartheta_0 - \vartheta_2 \operatorname{sech}\vartheta_0 \tanh\vartheta_0$$

$$A_3 = -\frac{1}{6}\vartheta_1^3 \operatorname{sech}\vartheta_0 \tanh\vartheta_0 + \frac{5}{6}\vartheta_1^3 \operatorname{sech}^3\vartheta_0 \tanh\vartheta_0 + \vartheta_1\vartheta_2 \operatorname{sech}\vartheta_0 \tanh\vartheta_0 - \vartheta_1\vartheta_2 \operatorname{sech}^3\vartheta_0 - \vartheta_3 \operatorname{sech}\vartheta_0 \tanh\vartheta_0$$

$$A_4 = \frac{1}{24}\vartheta_1^4 \operatorname{sech}\vartheta_0 \tanh\vartheta_0 - \frac{3}{4}\vartheta_1^4 \operatorname{sech}^3\vartheta_0 \tanh^2\vartheta_0 - \frac{1}{2}\vartheta_1^2\vartheta_2 \operatorname{sech}\vartheta_0 \tanh\vartheta_0 + \frac{5}{24}\vartheta_1^4 \operatorname{sech}^5\vartheta_0 + \frac{5}{2}\vartheta_1^2\vartheta_2 \operatorname{sech}^3\vartheta_0 \tanh\vartheta_0 + \frac{1}{2}\vartheta_2^2 \operatorname{sech}\vartheta_0 \tanh\vartheta_0 + \vartheta_1\vartheta 3 \operatorname{sech}\vartheta_0 \tanh^2\vartheta_0 - \frac{1}{2}\vartheta_2^2 \operatorname{sech}^3\vartheta_0 - \vartheta_1\vartheta_3 \operatorname{sech}^3\vartheta_0 - \vartheta_4 \operatorname{sech}\vartheta_0 \tanh\vartheta_0$$

Equivalently,

$$A_0 = \frac{2}{\vartheta_+}$$

$$A_1 = \frac{2\vartheta_1\vartheta_-}{\vartheta_+^2}$$

$$A_2 = \frac{2\vartheta_1^2\vartheta_-^2}{\vartheta_+^3} - \frac{2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+}{\vartheta_+^2}$$

$$A_3 = -\frac{2\vartheta_1^3\vartheta_-^3}{\vartheta_-^4} + \frac{2\vartheta_1\vartheta_-(2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+)}{\vartheta_+^3} - \frac{6\vartheta_3\vartheta_- + 6\vartheta_1\vartheta_2\vartheta_+ + \vartheta_1^3\vartheta_-}{3\vartheta_+^2}$$

$$A_4 = \frac{2\vartheta_1^4\vartheta_-^4}{\vartheta_+^5} - \frac{3\vartheta_1^2\vartheta_-^2(2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+)}{\vartheta_+^4} + \frac{(2\vartheta_2\vartheta_- + \vartheta_1^2\vartheta_+)^2}{2\vartheta_+^3} + \frac{2\vartheta_1\vartheta_-(6\vartheta_3\vartheta_- + 6\vartheta_2\vartheta_1\vartheta_+ + \vartheta_1^3\vartheta_-)}{3\vartheta_+^3} + \frac{24\vartheta_1\vartheta_3\vartheta_+ + 12\vartheta_2^2\vartheta_+ + 12\vartheta_1\vartheta_2^2\vartheta_- + \vartheta_1^4\vartheta_+}{12\vartheta_+^2}$$

3.6: For $N(\vartheta) = \coth\vartheta$

$$A_0 = \coth\vartheta_0$$

$$A_1 = \vartheta_1(1 - \coth^2\vartheta_0)$$

$$A_2 = \vartheta_2(1 - \coth^2\vartheta_0) - \vartheta_1^2 \coth\vartheta_0(1 - \coth^2\vartheta_0)$$

$$A_3 = -\frac{1}{3}\vartheta_1^3(1-\coth^2\vartheta_0)^2 + \frac{2}{3}\vartheta_1^3\coth^2\vartheta_0(1-\coth^2\vartheta_0)$$
$$-2\vartheta_1\vartheta_2\coth\vartheta_0(1-\coth^2\vartheta_0) + \vartheta_3(1-\coth^2\vartheta_0)$$
$$A_4 = \frac{2}{3}\vartheta_1^4\coth\vartheta_0(1-\coth^2\vartheta_0)^2 - \vartheta_1^2\vartheta_2(1-\coth^2\vartheta_0)^2 - \frac{1}{3}\vartheta_1^4\coth^3\vartheta_0(1-\coth^2\vartheta_0)$$
$$+2\vartheta_1^2\vartheta_2\coth^2\vartheta_0(1-\coth^2\vartheta_0) - \vartheta_2^2\coth\vartheta_0(1-\coth^2\vartheta_0)$$
$$-2\vartheta_1\vartheta_3\coth\vartheta_0(1-\coth^2\vartheta_0) + \vartheta_4(1-\coth^2\vartheta_0)$$

Equivalently, with $e^{2\vartheta_0}+1=\vartheta_{2+}$ and $e^{2\vartheta_0}-1=\vartheta_{2-}$, we have

$$A_0 = \frac{\vartheta_{2+}}{\vartheta_{2-}}$$
$$A_1 = \frac{2\vartheta_1 e^{2\vartheta_0}}{\vartheta_{2-}} - \frac{2\vartheta_1\vartheta_{2+}e^{2\vartheta_0}}{\vartheta_{2+}^2}$$
$$A_2 = \frac{2\vartheta_2 e^{2\vartheta_0}}{\vartheta_{2-}} + \frac{2\vartheta_1^2 e^{2\vartheta_0}}{\vartheta_{2-}} - \frac{4\vartheta_1^2 e^{4\vartheta_0}}{\vartheta_{2-}^2} + \frac{4\vartheta_1^2\vartheta_{2+}e^{4\vartheta_0}}{\vartheta_{2+}^3} - \frac{2\vartheta_2\vartheta_{2+}e^{2\vartheta_0}}{\vartheta_{2-}^2} - \frac{2\vartheta_1^2\vartheta_{2+}e^{2\vartheta_0}}{\vartheta_{2-}^2}$$
$$A_3 = \frac{2\vartheta_3 e^{2\vartheta_0}}{\vartheta_{2-}} + \frac{4\vartheta_1\vartheta_2 e_2 e^{2\vartheta_0}}{\vartheta_{2-}} - \frac{8\vartheta_1^2 e^{4\vartheta_0}}{\vartheta_{2-}^2} + \frac{4\vartheta_1^3 e^{2\vartheta_0}}{3\vartheta_{2-}^2} + \frac{2\vartheta_2\vartheta_{2+}e^{2\vartheta_0}}{\vartheta_{2-}} - \frac{8\vartheta_1^3 e^{4\vartheta_0}}{\vartheta_{2-}^2}$$
$$+\frac{8\vartheta_1^3 e^{6\vartheta_0}}{\vartheta_{2-}^3} - \frac{8\vartheta_1^3\vartheta_1^3 e^{6\vartheta_0}}{\vartheta_{2-}^4} + \frac{8\vartheta_1\vartheta_2\vartheta_{2+}e^{4\vartheta_0}}{\vartheta_{2-}^3} + \frac{8\vartheta_1^3\vartheta_{2+}e^{4\vartheta_0}}{\vartheta_{2-}} - \frac{2\vartheta_2\vartheta_3 e^{2\vartheta_0}}{\vartheta_{2-}^2}$$
$$+\frac{4\vartheta_1\vartheta_2\vartheta_{2-}e^{2\vartheta_0}}{2\vartheta_{2-}^2} - \frac{4\vartheta_1^3\vartheta_{2+}e^{4\vartheta_0}}{3\vartheta_{2-}^2}$$

4. Theory of Adomian Decomposition Method and Nonlinear Shanks Transform on Transcendental-Hyperbolic Equations

The nonlinear hyperbolic-trigonometric equation

$$f(\vartheta) = 0 \tag{4}$$

can be expressed as

$$\vartheta = c + N(\vartheta), \qquad \vartheta \in \mathbb{R} \tag{5}$$

where $N(\vartheta)$ is a nonlinear function and c is a constant. ADM considers the solution equation (5) as

$$\vartheta = \sum_{n=0}^{\infty}\vartheta_n \tag{6}$$

And, ϑ_n is calculated recursively. The nonlinear term is decomposed as

$$N(\vartheta) = \sum_{n=0}^{\infty} A_n \tag{7}$$

where A_n is as defined in equation (3). See [15], [3], [4], [5], [7] and the literature therein. Substituting equations (6) and (7) in equation (5), we have a recurrence relation

$$\vartheta_0 = c \tag{8}$$

$$\vartheta_{n+1} = A_n(\vartheta) \tag{9}$$

Suppose $\{\vartheta_n\}$ is a sequence of partial sum of the series in equation (6), then the Shanks nonlinear transform by [16], denoted T$\{\vartheta_n\}$, is given as

$$T\{\vartheta_n\} = \frac{\{\vartheta_{n+1}\}\{\vartheta_{n-1}\} - \{\vartheta_n\}^2}{\{\vartheta_{n+1}\} + \{\vartheta_{n-1}\} - 2\{\vartheta_n\}} \tag{10}$$

which the first order iteration is given as

$$\iota_n = T\{\vartheta_n\} \tag{11}$$

Subsequent Shanks nonlinear iterated transform are

$$\kappa_n = T\{\iota_n\}$$

$$\mu_n = T\{\kappa_n\}$$

$$\cdots$$

These iterations, according to [16], often leads to the reasonable results. The more the iterations the better the results, this can be seen in the numerical illustrations in the following section.

5. Main Results

In this section we give examples by adopting the technique stated in the previous section on the theory of ADM and NST. The numerical calculations were made using Maple Mathematical software to ensure double precision arithmetic in order to reduce the round-off errors to the barest minimum.

Numerical Example 1. Consider the equation

$x + \sinh x = \frac{1}{2}$

The approximate analytical solution is $x = 0.2487139369$. Applying the Adomian polynomial, the theory of ADM and NST, we have the result as presented in Table 1. Its easy to see from Table 1 that $\vartheta = \sum_{n=0}^{\infty} \vartheta_n$ failed to yield reasonable converging result in column ϑ_n. This was quickly upturn on application of NST in column ι_n, κ_n, μ_n and finally through column ρ_n.

TABLE 1. Nonlinear Shanks Transform of Numerical Example 1

n	ϑ_n	ι_n	κ_n	μ_n	ρ_n
0	0.5000000000	0.2550813376	0.2484650317	0.2487523931	0.2468039040
1	−0.0216953055	0.2402894297	0.2488573072	0.2448934649	0.2487321902
2	0.5665052912	0.2585674172	0.2487140775	0.2486770372	
3	−0.1668375910	0.2378526863	0.2485760245	0.2487313987	
4	0.846290120	0.2606866501	0.2489524909		
5	−0.6767877430	0.2348986366	0.2484167479		
6	1.761565625	0.2657846151			
7	−2.321585858	0.2261027720			
8	4.741363540				
9	−7.776280080				

Numerical Example 2. Consider the equation

$x + \sinh x = \frac{1}{2} + \cosh x$

The approximate analytical solution is $x = 0.9046738485$. Also, applying the Adomian polynomial, the theory of ADM and NST, we have the result as presented in Table 2. Its obvious from Table 2 that $\vartheta = \sum_{n=0}^{\infty} \vartheta_n$ failed to yield reasonable converging result in column ϑ_n. However, this was overturn on application of NST in column ι_n, κ_n, μ_n and finally through column ρ_n

6. CONCLUSION

Modified Adomian decomposition method (ADM) has been successfully adapted to obtain Adomian polynomials (AP) of frequently occuring transcendental-hyperbolic nonlinear terms in a linear functional. We demonstrated with two test problems of transcendental hyperbolic

TABLE 2. Nonlinear Shanks Transform of Numerical Example 2

n	ϑ_n	ι_n	κ_n	μ_n	ρ_n
0	0.5000000000	0.8775406695	0.9036872640	0.9039309631	0.9039260151
1	1.106530660	0.9139030481	0.9049958618	0.9048670639	
2	0.7386512191	0.8996958171	0.8992782428	1.082847107	
3	1.073346459	0.9081495808	1.147379091		
4	0.7124523708	1.087748017	1.060161471		
5	1.139978405	1.132515271			
6	1.080479044	1.249930499			
7	1.495334687				
8	0.8945290538				

form to show the reliability of the AP so obtained. As a result of the noise term occurrence, the solution series convergence were contradictory as shown in column ϑ_n of Tables 1 and 2. These we facilitated to convergence using the nonlinear Shanks transform (NST), although the results accuracy depended on the order of nonlinear Shanks iteration used. In all, the result obtained were in excellent agreement with those obtained via analytic method with maximum absolute error less than 1%.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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