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SOME NEW RESULTS ON PROPER COLOURING OF EDGE-SET GRAPHS

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Abstract. In this paper, we present a foundation study for proper colouring of edge-set graphs. The authors consider that a detailed study of the colouring of edge-set graphs corresponding to the family of paths is best suitable for such foundation study. The main result is deriving the chromatic number of the edge-set graph of a path, P_{n+1}, $n \geq 1$. It is also shown that edge-set graphs for paths are perfect graphs.

Keywords: chromatic colouring; rainbow neighbourhood; rainbow neighbourhood number; edge-set graph.

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1. Introduction

For general notation and concepts in graphs and digraphs see [1, 2, 12]. Unless mentioned otherwise, all graphs we consider in this paper are finite, simple, connected and undirected graphs.

For a set of distinct colours $\mathscr{C} = \{c_1, c_2, c_3, \ldots, c_\ell\}$, a *vertex colouring* of a graph G is an assignment $\varphi : V(G) \mapsto \mathscr{C}$. A vertex colouring is said to be a *proper vertex colouring* of a graph G if no two distinct adjacent vertices have the same colour. The cardinality of a minimum set of colours in a proper vertex colouring of G is called the *chromatic number* of G and is denoted

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$\chi(G)$. A colouring of G with exactly $\chi(G)$ colours may be called a χ-colouring or a *chromatic colouring* of G.

A *minimum parameter colouring* of a graph G is a proper colouring of G which consists of the colours c_i; $1 \leq i \leq \ell$, with minimum possible values for the subscripts i. Unless stated otherwise, we consider minimum parameter colouring throughout this paper.

The set of vertices of G having the colour c_i is said to be the *colour class* of c_i in G and is denoted by $\mathscr{C}_i$. The cardinality of the colour class $\mathscr{C}_i$ is said to be the weight of the colour c_i, denoted by $\theta(c_i)$. Note that $\sum\limits_{i=1}^{\ell} \theta(c_i) = \nu(G)$.

Unless mentioned otherwise, we colour the vertices of a graph G in such a way that $\mathscr{C}_1 = I_1$, the maximal independent set in G, $\mathscr{C}_2 = I_2$, the maximal independent set in $G_1 = G - \mathscr{C}_1$ and proceed like this until all vertices are coloured. This convention is called *rainbow neighbourhood convention* (see [5]. The number of vertices in G which yield rainbow neighbourhoods, denoted by $r_\chi(G)$, is called the *rainbow neighbourhood number* of G.

In [5], the bounds on $r_\chi(G)$ corresponding to of minimum proper colouring, denoted by $r^-_\chi(G)$ and $r^+_\chi(G)$, have been defined as the minimum value and maximum value of $r_\chi(G)$ over all permissible colour allocations. If we relax connectedness, it follows that the null graph $\mathfrak{N}_n$ of order $n \geq 1$ has $r^-(\mathfrak{N}_n) = r^+(\mathfrak{N}_n) = n$. For bipartite graphs and complete graphs, K_n it follows that, $r^-(G) = r^+(G) = n$ and $r^-(K_n) = r^+(K_n) = n$.

We observe that if it is possible to permit a chromatic colouring of any graph G of order n such that the star subgraph obtained from vertex v as center and its open neighbourhood $N(v)$ the pendant vertices, has at least one coloured vertex from each colour for all $v \in V(G)$ then $r_\chi(G) = n$. Certainly, examining this property for any given graph is complex.

Lemma 1.1. [5] *For any graph G the graph $G' = K_1 + G$ has $r_\chi(G') = 1 + r_\chi(G)$.*

2. Rainbow Neighbourhood Number of Edge-set Graphs

Edge-set graphs were introduced in [4]. As the notion of an edge-set graph seems to be largely unknown. Therefore, the main definition and some important observations from [4] will be presented in this section.

Let A be a non-empty finite set. Let the set of all s-element subsets of A (arranged in some order), where $1 \leq s \leq |A|$, be denoted by $\mathcal{S}$ and the i-th element of $\mathcal{S}$ by, $A_{i,s}$.

Definition 2.1. [4] Let $G(V,E)$ be a non-empty finite graph with $|E| = \varepsilon \geq 1$ and $\mathcal{E} = \mathcal{P}(E) - \{\emptyset\}$, where $\mathcal{P}(E)$ is the power set of the edge set $E(G)$. For $1 \leq s \leq \varepsilon$, let $\mathcal{S}$ be the collection of all s-element subsets of $E(G)$ and $E_{s,i}$ be the i-th element of $\mathcal{S}$. Then, the *edge-set graph* corresponding to G, denoted by $\mathscr{G}_G$, is the graph with the following properties.

(i) $|V(\mathscr{G}_G)| = 2^{\varepsilon} - 1$ so that there exists a one to one correspondence between $V(\mathscr{G}_G)$ and $\mathcal{E}$;

(ii) Two vertices, say $v_{s,i}$ and $v_{t,j}$, in $\mathscr{G}_G$ are adjacent if some elements (edges of G) in $E_{s,i}$ is adjacent to some elements of $E_{t,j}$ in G.

From the above definition, it can be seen that the edge-set graph $\mathscr{G}_G$ of a given graph G is dependent not only on the number of edges ε, but the structure of G also. Note that it was erroneously remarked in [4] that non-isomorphic graphs of the same size have distinct edge-set graphs. Figure 2 illustrates one contradictory case.

Note that an edge-set graph $\mathscr{G}_G$ has an odd number of vertices. If G is a trivial graph, then $\mathscr{G}_G$ is an empty graph (since $\varepsilon = 0$). Also, $\mathscr{G}_{P_2} = K_1$ and $\mathscr{G}_{P_3} = C_3$. In [4] the following conventions were used.

(i) If an edge e_j is incident with vertex v_k, then we write it as $(e_j \rightarrow v_k)$.

(ii) If the edges e_i and e_j of a graph G are adjacent, then we write it as $e_i \sim e_j$.

(iii) The n vertices of the path P_n are positioned horizontally and the vertices and edges are labeled from left to right as $v_1, v_2, v_3, \ldots, v_n$ and $e_1, e_2, e_3, \ldots, e_{n-1}$, respectively.

(iv) The n vertices of the cycle C_n are seated on the circumference of a circle and the vertices and edges are labeled clockwise as $v_1, v_2, v_3, \ldots, v_n$ and $e_1, e_2, e_3, \ldots, e_n$, respectively such that $e_i = v_i v_{i+1}$, in the sense that $v_{n+1} = v_1$.

Invoking the definition and observations given above, it is noticed that both $d^t_{G(e)}(G)$ and $d_{G(e)}(v_i)$ are single values, while $d_{G(v_k)}(e_j) \leq d_{G(v_m)}(e_j), (e_j \rightarrow v_k), (e_j \rightarrow v_m)$. The graphs having three edges e_1, e_2, e_3 are graphs P_4, C_3, and $K_{1,3}$. The corresponding edge-set graphs on the vertices $v_{1,1} = \{e_1\}, v_{1,2} = \{e_2\}, v_{1,3} = \{e_3\}, v_{2,1} = \{e_1,e_2\}, v_{2,2} = \{e_1,e_3\}, v_{2,3} = \{e_2,e_3\}, v_{3,1} = \{e_1,e_2,e_3\}$ are depicted below.

Figure 1 depicts the edge-set graph $\mathscr{G}_{P_4}$.

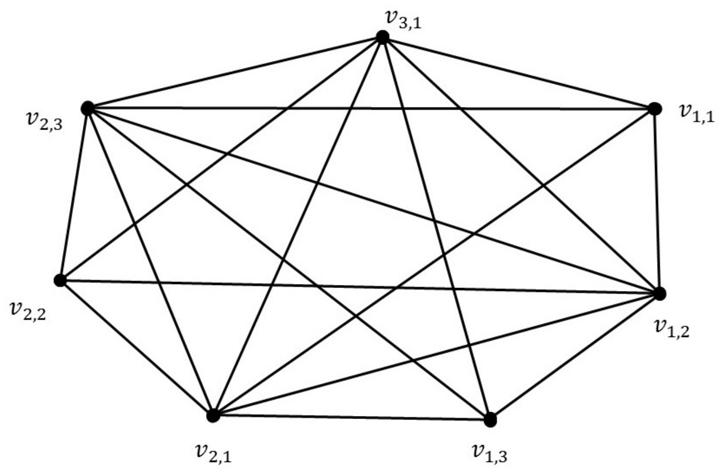

FIGURE 1. Edge-set graph $\mathscr{G}_{P_4}$.

Figure 2 depicts the edge-set graph $\mathscr{G}_{C_3} = \mathscr{G}_{K_{1,3}} = K_7$.

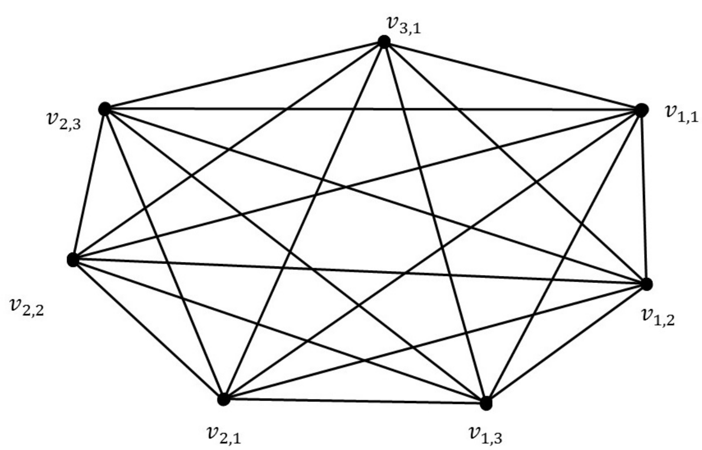

FIGURE 2. Edge-set graph $\mathscr{G}_{C_3} = \mathscr{G}_{K_{1,3}} = K_7$.

Notice that both $\mathscr{G}_{C_3}$ and $\mathscr{G}_{K_{1,3}}$ are complete graphs.

3. Proper Colouring of the Edge-set Graphs of Paths

It is known that for a given size $\varepsilon \geq 1$ a graph of maximum order ν, is a tree. Hence, for a given size the graphs with maximum structor index $si(G)$ are the corresponding trees,

T. It easily follows that for $\varepsilon(T) \geq 3$ only the star graphs have $\mathscr{G}_{S_{\varepsilon+1}}$, complete. Put another way, a tree T has $\mathscr{G}_T$ complete if and only if $diam(T) \leq 2$. From the family of trees, a path corresponding to a given ε, denoted by P_ε, has largest diameter. These observations motivate a detailed study of the proper colouring and associated colour parameters of edge-set graphs of paths to lay the foundation for studying more complex graph classes.

For this section paths of the form $P_{n+1} = v_1e_1v_2e_2v_3\cdots e_nv_{n+1}$, will be considered. Such graph will be abbreviated to $P_{n+1} = v_1e_iv_i \succ$, $1 \leq i \leq n$. To easily relate the results with Definition 2.1, note that $\varepsilon(P_{n+1}) = n$. It can be easily verified that $\mathscr{G}_{P_2} = K_1$. Hence, $\chi(G_{P_2}) = 1$. Also, $\mathscr{G}_{P_3} = K_3$ and hence, $\chi(\mathscr{G}_{P_3}) = 3$. These observations bring the main results. First, we state an important lemma.

Lemma 3.1. *Let $G(V,E)$ be a non-empty finite graph with $|E| = \varepsilon \geq 1$ and $\mathcal{E} = \mathcal{P}(E) - \{\emptyset\}$, where $\mathcal{P}(E)$ is the power set of the edge set $E(G)$. Then each edge e_i is in exactly $2^{\varepsilon-1}$ subsets of $\mathcal{E}$.*

Proof. The result follows directly from the well-definedness and well-ordering of the power set, $\mathcal{P}(E)$. □

It is observed that if the number of subsets which has say, e_i as element is t, then within the corresponding t subsets the edge e_j, $j \neq i$ will be in $\frac{t}{2} = 2^{\varepsilon-2}$ of those subsets.

Theorem 3.2. *The edge-set graph $\mathscr{G}_{P_{n+1}}$, $n \geq 1$ has*

$$\chi(\mathscr{G}_{P_{n+1}}) = \begin{cases} 1 \text{ or } 3, & \text{if } P_2 \text{ or } P_3 \text{ respectively},\\ 5, & \text{if } P_4,\\ 2^{n-1}+2^{n-2}-2, & \text{for } P_{n+1},\ n \geq 4. \end{cases}$$

Proof. Part 1: Trivial is the observation that $\mathscr{G}_{P_2} = K_1$ and that result in equality. It has been observed that $\mathscr{G}_{P_3} = K_3$ and hence $\chi(\mathscr{G}_{P_3}) = 3$.

Part 2: In constructing $\mathscr{G}_{P_4}$ begin with $\mathscr{G}_{P_3}$ which has vertices $\{e_1\}$, $\{e_2\}$, $\{e_1,e_2\}$. Add a disjoint copy of $\mathscr{G}_{P_3}$ and relabel the vertices of this copy to be $\{e_1,e_3\}$, $\{e_2,e_3\}$, $\{e_1,e_2,e_3\}$ to obtain, $\mathscr{G}'_{P_3}$. Clearly, $\mathscr{G}'_{P_3}$ complies with Definition 2.1.

Consider $H = \mathscr{G}_{P_3} \cup \mathscr{G}'_{P_3}$ and add the cut edges, $\{e_2\}\{e_1,e_3\}$, $\{e_2\}\{e_2,e_3\}$, $\{e_2\}\{e_1,e_2,e_3\}$, $\{e_1,e_2\}\{e_1,e_3\}$, $\{e_1,e_2\}\{e_2,e_3\}$, $\{e_1,e_2\}\{e_1,e_2,e_3\}$. Clearly, the induced subgraph, $\langle\{e_2\}, \{e_1,e_2\},\{e_1,e_3\},\{e_2,e_3\},\{e_1,e_2,e_3\}\rangle = K_5$. Now add all additional bridges in accordance with Definition 2.1 to obtain graph H'. Due to symmetry considerations between edges e_1 and e_2 in P_3, exactly two maximum cliques K_5 come into existence hence, $\omega(H') = 5$. Finally, by adding vertex $\{e_3\}$ and the corresponding edges in accordance with Definition 2.1 and by symmetry considerations between edges e_1 and e_3 in P_4, the edge-set graph $\mathscr{G}_{P_4}$ has exactly four maximum cliques K_5. Therefore, $\chi(\mathscr{G}_{P_4}) \geq 5$.

Invoking Definition 2.1, consider the following colouring of $\mathscr{G}_{P_4}$. Let $c(v_{1,1}) = c_1$, $c(v_{1,3}) = c_1$, $c(v_{2,2}) = c_1$, $c(v_{1,2}) = c_2$, $c(v_{2,1}) = c_3$, $c(v_{2,3}) = c_4$, $c(v_{3,1}) = c_5$. Clearly, the colouring is proper and hence $\chi(\mathscr{G}_{P_4}) \leq 5$. Hence we have $\chi(\mathscr{G}_{P_4}) = 5$.

Part 3: For $n \geq 4$, and the path path P_{n+1} the edge-set graph $\mathscr{G}_{P_{(n-1)+1}}$ of the preceding path hence, the $(n-1)$-edge path $P_{(n-1)+1}$, is incomplete. In accordance with the procedure described in Part 2, consider $\mathscr{G}_{P_{(n-1)+1}}$ and $\mathscr{G}'_{P_{(n-1)+1}}$. Since in $\mathscr{G}'_{P_{(n-1)+1}}$ the edge e_n has been added to each vertex corresponding to the vertices $v_{i,j} \in V(\mathscr{G}_{P_{(n-1)+1}})$, the new edges in accordance with Definition 2.1 are those between all pairs of vertices for which at least one vertex has $e_{n-1} \in v'_{i,j}$. From Lemma 3.1, it follows that at least one complete induced subgraph, $K_{2^{n-2}}$ exists in $\mathscr{G}'_{P_{(n-1)+1}}$. All pairs of vertices which has both $e_{n-2}, e_{n-1} \in v'_{i,j}$ is an edge in $\mathscr{G}'_{P_{(n-1)+1}}$ so least one complete induced subgraph, $K_{2^{n-2}+1}$ exists in $\mathscr{G}'_{P_{(n-1)+1}}$. Proceeding to vertices for which edge $e_{n-3} \in v'_{i,j}$ and so on until the edge e_1 has been accounted for results in $\mathscr{G}'_{P_{(n-1)+1}}$ being complete. Hence, $\chi(\mathscr{G}'_{P_{(n-1)+1}}) = 2^{n-1} - 1$.

Finally, by adding the bridges between $\mathscr{G}_{P_{(n-1)+1}}$ and $\mathscr{G}'_{P_{(n-1)+1}}$ and through similar arguments in respect of edges $e_{n-2}, e_{n-1} \in v_{i,j} \in V(\mathscr{G}_{P_{(n-1)+1}})$ and so on, it follows that at least one maximum induced clique, of order $2^{n-2} - 1 + \chi(\mathscr{G}_{P_{(n-1)+1}})$, exists in $\mathscr{G}_{P_{n+1}}$. Therefore, $\chi(\mathscr{G}_{P_{n+1}}) \geq 2^{n-1} + 2^{n-2} - 2$. By allocating colours similar to the procedure described in Part-2, it follows that $2^{n-1} + 2^{n-2} - 2 \leq \chi(\mathscr{G}_{P_{n+1}}) \leq 2^{n-1} + 2^{n-2} - 2 \Leftrightarrow \chi(\mathscr{G}_{P_{n+1}}) = 2^{n-1} + 2^{n-2} - 2$. Therefore, by immediate induction, the result follows for all $n \geq 4$. □

Corollary 3.3. *(a) Each vertex in an edge-set graph $\mathscr{G}_{P_{n+1}}$, $n \geq 2$ belongs to some maximum clique in $\mathscr{G}_{P_{n+1}}$.*

(b) The edge-set graphs $\mathscr{G}_{P_{n+1}}$, $n \geq 1$ has clique number, $\omega(\mathscr{G}_{P_{n+1}}) = 2^{n-1} + 2^{n-2} - 2$.

(c) The edge-set graphs $\mathscr{G}_{P_{n+1}}$, $n \geq 1$ are perfect graphs.

(d) The edge-set graph $\mathscr{G}_{P_{n+1}}$ has, $r_c^- hi(\mathscr{G}_{P_{n+1}}) = r_c^+ hi(\mathscr{G}_{P_{n+1}}) = 2^n - 1$.

Proof. The results are a direct consequence from the proof of Theorem 3.2. □

Theorem 3.4. *An edge-set graph $\mathscr{G}_{P_{n+1}}$, $n \geq 1$ is a perfect graph.*

Proof. For P_1, P_2 the result is trivial. From Theorem 3.2 and Corollary 3.3(b) we have, $n \geq 2$ and hence it follows that $\omega(\mathscr{G}_{P_{n+1}}) = 2^{n-1} + 2^{n-2} - 2 = \chi(\mathscr{G}_{P_{n+1}})$. Hence, an edge-set graph is weakly perfect. From Definition 2.1, it follows that an edge-set graph has a unique maximum independent set X. Furthermore, $\langle X \rangle$ is a null graph hence, any subgraph thereof is perfect.

Also, from Corollary 3.3(a), each vertex in $V(\mathscr{G}_{P_{n+1}})$ is in some induced maximum clique. It then follows that $\omega(H) = \chi(H)$, $\forall H \subseteq \mathscr{G}_{P_{n+1}}$, $n \geq 1$. Hence the result. □

Conjecture 1. *The edge-set graphs of acyclic graphs are perfect graphs.*

4. Conclusion

Research problem: The notion of a chromatic core subgraph of a graph G was introduced in [9]. We recall that, for a graph G its *structural size* is measured by its *structor index* denoted and defined as, $si(G) = \nu(G) + \varepsilon(G)$. We say that the smaller of graphs G and H is the graph satisfying the condition, $min\{si(G), si(H)\}$. If $si(G) = si(H)$ the graphs are of equal structural size but not necessarily isomorphic. A straight forward example is the path, P_4 and the star graph, S_3.

Definition 4.1. For a finite, undirected simple graph G of order $\nu(G) = n \geq 1$ a chromatic core subgraph H is a smallest induced subgraph H (smallest in respect of $si(H)$) such that, $\chi(H) = \chi(G)$.

From the construction used in the proof of Theorem 3.2 it follows that a finite number of distinct maximum cliques can be associated with a given edge-set graph $\mathscr{G}_{P_{n+1}}$. As an application, the largest number of vertices common to the maximum number of chromatic core subgraphs can be considered the most strategic vertices for protection from a disaster management and recovery plan in the event of graph destruction. The aforesaid observation motivates us to introduce a new graph parameter called the *chromatic cluster number* of a graph G. It is denoted by $\mathtt{C}(G)$. From Theorem 3.2 it follows that $\mathtt{C}(\mathscr{G}_{P_2}) = \mathtt{C}(\mathscr{G}_{P_3}) = 1$ and $\mathtt{C}(\mathscr{G}_{P_4}) = 4$. Note that the vertices $v_{1,1} = \{e_1\}$, $v_{1,3} = \{e_3\}$, $v_{2,2} = \{e_1, e_3\}$ and $v_{1,3} = \{e_1, e_2, e_3\}$ corresponds to $\mathtt{C}(\mathscr{G}_{P_4})$.

Problem 1. For the edge-set graph $\mathscr{G}_{P_{n+1}}$, $n \geq 4$, determine $\mathtt{C}(\mathscr{G}_{P_{n+1}})$.

The research on set-graphs (see [3]) and edge-set graphs naturally leads to new concepts such as vertex degree sequence set-graphs and colour set-graphs and colour-string set-graphs. Preliminary definitions are provided below.

(1) If the degree sequence of a graph G of order $n \geq 1$ is $(d_1 \leq d_2 \leq d_3 \leq \cdots, d_n)$, then for a subsequence $(d_{t+1} = d_{t+2} = \cdots = d_{t+\ell} = m_i)$, $t \geq 0$, $1 \leq \ell \leq n$, label the corresponding vertices to be $m_{i,1}, m_{i,2}, m_{i,3}, \ldots, m_{i,\ell}$. Consider the set $\mathscr{V}(G) = \mathcal{P}(V) - \emptyset$ where, $\mathcal{P}(V)$ is the power set of $V(G)$.

Definition 4.2. *The degree sequence set-graph corresponding to G, denoted by $\mathscr{G}_{\mathscr{V}(G)}$, is the graph with the following properties.*

(i) $|\mathscr{G}_{\mathscr{V}(G)}| = 2^\nu - 1$ so that there exists a one to one correspondence between $V(\mathscr{G}_{\mathscr{V}(G)})$ and $\mathscr{V}(G)$.

(ii) Two vertices, say $v_{s,i}$ and $v_{t,j}$, in $\mathscr{G}_{\mathscr{V}(G)}$ are adjacent if some element(s) (specific vertex degree(s) of G) in $v_{s,i}$ is adjacent to some element(s) of $v_{t,j}$ in G.

It follows easily that for a complete graph K_n, $n \geq 1$ has its corresponding degree sequence set-graph, a complete graph.

Problem 2. Discuss the properties of the degree sequence set-graph corresponding to graph G.

(2) Let the minimum colour set $\mathscr{C} = \{c_1, c_2, c_3, \ldots, c_\chi\}$ permit a chromatic colouring of G in accordance with the rainbow neighbourhood convention. Let $\mathscr{C}^{\{\}}(G) = \mathcal{P}(\mathscr{C}) - \emptyset$ where, $\mathcal{P}(\mathscr{C})$ is the power set of $\mathscr{C}$.

Definition 4.3. *The colour set-graph corresponding to G, denoted by $\mathscr{G}_{\mathscr{C}^{\{\}}(G)}$, is the graph with the following properties.*

(i) $|\mathscr{G}_{\mathscr{C}\{\}(G)}| = 2^{\chi} - 1$ *so that there exists a one to one correspondence between* $V(\mathscr{G}_{\mathscr{C}\{\}(G)})$ *and* $\mathscr{C}^{\{\}}(G)$.

(ii) Two vertices, say $v_{s,i}$ *and* $v_{t,j}$*, in* $\mathscr{G}_{\mathscr{C}\{\}(G)}$ *are adjacent if some element(s) (specific vertex degree(s) of G) in* $v_{s,i}$ *is adjacent to some element(s) of* $v_{t,j}$ *in G.*

Clearly, for all graphs G with $\chi(G) = 2$ the colour set-graph is K_3.

Problem 3. *Discuss the properties of the colour set-graph corresponding to a chromatic colouring of a graph G.*

This problem is similar to (1). For a minimum colour set $\mathscr{C} = \{c_1, c_2, c_3, \ldots, c_\chi\}$ the corresponding colour weight sequence is $(\underbrace{c_1, c_1, c_1, \ldots, c_1}_{\theta(c_1)\ entries}, \cdots, \underbrace{c_\chi, c_\chi, c_\chi, \ldots, c_\chi}_{\theta(c_\chi\ entries})$.

Let $\mathscr{C}^{\circ}(G) = \{c_{1,1}, c_{1,2}, c_{1,3} \ldots, c_{1,\theta(c_1)}, \cdots, c_{\chi,1}, c_{\chi,2}, c_{\chi,3}, \ldots, c_{\chi,\theta(c_\chi)}\}$. We can define the colour-string set-graph, $\mathscr{G}_{\mathscr{C}^{\circ}(G)}$ similar to Definition 4.2.

Problem 4. *Research the properties of the colour-string set-graph corresponding to a chromatic colouring of a graph G.*

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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