Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 3, 633-638
https://doi.org/10.28919/jmcs/4471
ISSN: 1927-5307

STRONGLY spZc-CONNECTED SPACES IN TOPOLOGY

RM. SIVAGAMA SUNDARI*, AP. DHANA BALAN

Department of Mathematics, Alagappa University, Karaikudi, Tamil Nadu, India

Abstract: In this paper, we introduce the concept of strongly spZc-connectedness, spZc-continuum using the concept of spZc-open sets. Some properties and theorems using locally spZc-connected space are also discussed.

Keywords: spZc -open; spZc –closed; strongly spZc-connected space; spZc-continuous; spZc-continuum.

2010 AMS Subject Classification: 54A05, 54C10.

1. INTRODUCTION

The notion of connectedness [1] is useful not only in General topology but also in other advanced branches of Mathematics. In 2011, EL-Magharabi, A.I. and Mubarki, A.M [2] introduced the concept of Z-open sets. Throughout this paper, (X,τ) represents a topological space on which no separation axiom is assumed unless otherwise stated.

2. PRELIMINARIES

Definition 2.1 [3]: A subset A of a space X is Zc-open if for each $x \in A \in ZO(X)$, there exists a closed set F such that $x \in F \subset A$. A subset A of a space X is Zc-closed if $X-A$ is Zc-open. The

*Corresponding author
E-mail address: kanish9621@hotmail.com
Received January 18, 2020

family of all Zc-open (resp. *Zc*-closed) subsets of a topological space (X,τ) is denoted by $ZcO\ (X,\tau)$ or $ZcO(X)$ (resp. $ZcC\ (X,\tau)$ or $ZcC(X)$).

Definition 2.2[4]: A subset A of (X,τ) is called

(i) *spZc* open if $A \subseteq Zccl(Zc\ int(Zccl(A)))$ and is denoted by $spZcO(X)$;

(ii) *spZc* closed if $X-A$ is *spZc* open and is denoted by $spZcC(X)$.

Definition 2.2 [4]:

(i) The semi pre *Zc* interior of a subset A of X is the union of all semi pre *Zc* open sets contained in A and is denoted by $spZcInt\ (A)$.

(ii) The semi pre *Zc closure* of a subset A of X is the intersection of all semi pre *Zc* closed sets containing A and is denoted by $spZcCl(A)$.

Example 2.3: `Let $X=\{a,b,c,d\}$ with $\tau=\{\emptyset,X,\{a\},\{c,d\},\{a,c,d\}\}$ then the family of *Zc*-open sets are $ZcO(X)=\{X,\emptyset,\{a,b\}\{b,c,d\}\}$ and

$spZcO(X)=\{X,\emptyset,\{b\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\},\{a,c,d\},\{c\},\{d\}\}$.

Definition 2.4 [4]: Let f: $X\rightarrow Y$ is called

(i) *spZc* continuous if $f^{-1}(V)$ is *spZc* open in X for every open set V in Y.

(ii) *spZc* irresolute if $f^{-1}(V)$ is *spZc* open in X for each open set V in Y.

(iii) contra *spZc*-continuous if $f^{-1}(V)$ is *spZc*-closed in X, for every open set in Y.

Definition 2.5 [4]: Let (X,τ) be a topological space. X is *spZc*-connected if X cannot be written as the disjoint union of two non-empty *spZc* open sets in X.

Definition 2.6[4]: $X=A\cup B$ is said to be a *spZc* separation of X if A and B are non-empty, disjoint, *spZc* open sets in X.

3. STRONGLY *SPZC*-CONNECTED SPACES

Definition 3.1: A Mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be *spZc* open (resp. *spZc* closed) if $f(V)\in ZcO(Y)$ (resp. $ZcC(Y)$), for each $V\in ZcO(X)$ (resp. $ZcC(X)$).

Theorem 3.2: A Mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is *spZc* open if and only if , $f(Int(U))\subseteq spZcInt(f(U))$ for each $U\subseteq X$.

Proof: Let *f* be a *spZc* open mapping and $U \subseteq X$, then

$spZcInt(f(Int(U))) = f(Int(U)) \in spZcO(Y)$.

Therefore $spZcInt(f(Int(U))) = f(Int(U)) \subseteq spZcInt(f(U))$.

Conversely, Let $U \in \tau$ and $f(U) = f(Int(U)) \subseteq spZcInt(f(U))$. Then $f(U) = spZcInt(f(U))$. Thus, $f(U)$ is *spZc* open in *Y* and hence *f* is *spZc* open.

Theorem 3.3: Let $f: X \to Y$ be a *spZc*-continuous function of *X* into a discrete space *Y* with at least two points which is a constant map then empty set and *X* are the only subsets of *X* that are both *spZc*-open and *spZc*-closed.

Proof: Let *A* be both *spZc*-open and *spZc*-closed in *X* and $A \neq \emptyset$.Let $f: X \to Y$ be a *spZc*-continuous function defined by $f(A) = \{y\}$ and $f(X - A) = \{w\}$ for some distinct points *y* and *w* in *Y*. Since *f* is a constant function, we get $A = X$ and hence the proof follows.

Definition 3.4: A space (X,τ) is said to be strongly *spZc*-connected if and only if it is not a disjoint union of countably many but more than one *spZc*-closed set. In other words, if E_i are non-empty disjoint closed sets of *X*, then $X \neq E_1 \cup E_2 \cup E_3 \cup \ldots\ldots$, or else *X* is said to be strongly *spZc*-connected.

Lemma 3.5: For any surjective *spZc*-irresolute function $f: X \to Y$ .The image $f(X)$ is strongly *spZc*-connected if *X* is strongly *spZc*-connected.

Proof: Assume, $f(X)$ is strongly *spZc*-disconnected. Then by definition 3.4 it is a disjoint union of countably many but more than one *spZc*-closed sets. Since *f* is *spZc*-irresolute, then the inverse image of *spZc*-closed sets are still *spZc*-closed, *X* is also a disjoint union of *spZc*-closed sets and hence $f(X)$ is strongly *spZc*-connected.

Theorem 3.6: A space *X* is strongly *spZc*-connected if there exists a constant surjective *spZc*-irresolute function $f: X \to D$,where *D* denote the discrete space of *X*.

Proof: Let *X* be strongly *spZc*-connected and function $f: X \to D$ be a surjective *spZc*-irresolute function, then by previous lemma, $f(X)$ is strongly *spZc*-connected. The only strongly *spZc*-connected subset of *D* are the one-point spaces. Hence *f* is constant. Conversely, suppose *X* is a disjoint union of countably many but more than one *spZc*-closed sets, $X = \bigcup_i E_i$. Define $f: X \to D$

by taking $f(x) = i$ whenever $x \in E_i$ .This f is a surjective *spZc*-irresolute and not constant. So X is strongly *spZc*-connected.

Definition 3.7: A compact *spZc*-connected set is called a *spZc*-continuum.

Definition 3.8: A space X is called:

(1) *spZc* T_1 if for each $x, y \in X$, $x \neq y$, there exist two disjoint *spZc*-open sets U, V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

(2) *spZc* T_2 if for each $x, y \in X$, $x \neq y$,there exists two disjoint *spZc*-open sets U, V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

(3) *spZc*-normal for any pair of disjoint *spZc*-closed sets F_1 and F_2 ,there exist disjoint *spZc*-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$ such that $U \cap V = \emptyset$.

Lemma 3.9: If A is any *spZc*-continuum in a *spZc* T_2 space X and B is any *spZc*-open set such that $A \cap B \neq \emptyset \neq A \cap (X - B)$, then every component of $(A \cap spZc\text{-}cl(B)) \cap spZc\text{-}bd(B) \neq \emptyset$.

Theorem 3.10: Let X be a compact *spZc* T_2-space. Then X is *spZc* - connected if and only if X is strongly *spZc*-connected.

Proof: Assume, X is strongly *spZc*-connected, then X is *spZc*-connected. Now let us consider that X is a compact *spZc* T_2- *spZc*-connected space and it is strongly *spZc*-disconnected, then X is a union of a countably many but more than one disjoint *spZc*-closed sets. Then $X = \bigcup K_i$,where K_i are *spZc*-closed disjoint sets. Since a compact *spZc* T_2-space is *spZc*-normal, then X is a *spZc*-normal space. So by definition there exists a *spZc*-open sets U such that $K_2 \subset U$ and $spZccl(U) \cap K_1 = \emptyset$. Let X_1 be a component of $spZccl(U)$ which intersects K_2. Then X_1 is compact and *spZc*-connected. Then by previous lemma $X_1 \cap spZc\text{-}bd(U) \neq \emptyset$. So X_1 contains a point $p \in spZc\text{-}bd(U)$ so that $p \in spZc\text{-}bd(U)$ such that $p \notin U$ and $p \notin K_1$.Thus $X_1 \cap K_i \neq \emptyset$ for some $i > 2$. Let K_{n_2} be the first K_i for $i > 2$ which intersects X_1 and let V be a *spZc*-open set satisfying $K_{n_2} \subset V$, and $spZccl(V) \cap K_2 = \emptyset$. Then X_2 be an element of $X_1 \cap spZccl(V)$ which contains a point of K_{n_2}.Again we have $X_2 \cap spZc\text{-}bd(V) \neq \emptyset$, and X_2 contains some point $p \in spZc\text{-}bd(V)$ such that $p \notin V$, $p \notin K_1 \cup K_2$. Thus $X_2 \cap K_i \neq \emptyset$ for some $i > n_2$ and $X_2 \cap K_i = \emptyset$

for $i < n_2$. Let K_{n_3} be the first K_i for $i > n_2$, which intersects X_2, then by methods stated above we can find a compact *spZc*-connected X_3 so that $X_3 \subset X_2 \subset X_1$ and X_3 intersects some K_i with $i > n_3$ but $X_3 \cap K_i = \emptyset$ for $i < n_3$.In this way, we obtain a sequence of sub continum of $X : X_1\, X_2\, X_3 \ldots$..,so that for every j, $X_j \cap K_i = \emptyset$ for $i < n_j$ and $n_j \to \infty$ as $j \to \infty$. We know that $\bigcap_i X_i \neq \emptyset$. Also ,($\bigcap_i X_i$) $\cap K_j = \emptyset$ for all j , so that $(\bigcap_i X_i) \cap (\bigcup_i K_j) = \emptyset$ or $(\bigcap_i X_i) \cap X = \emptyset$. But $(\bigcap_i X_i) \subset X$, which contradicts the fact that $\bigcap_i X_i \neq \emptyset$ .Thus X is strongly *spZc*-connected.

Lemma 3.11: For a space X the following holds: (i) X is a *spZc* T_1-space. (ii) For any point $x \in X$, the singleton set $\{x\}$ is *spZc*-closed.

Corollary 3.12: A strongly *spZc*-connected *spZc* T_1-space having more than one point is uncountable.

Proof: By previous lemma, a one-point set in a *spZc* T_1-space is *spZc*-closed. Then by definition the proof follows that a *spZc* T_1 space cannot have countably many but more than one point.

Theorem 3.13: Let X be a locally compact *spZc* T_2-space. If X is locally *spZc*-connected, then X is locally strongly *spZc*-connected.

Proof: Let O be a *spZc*-open *spZc*-nbd of a point $x \in X$. Then there exists a compact *spZc*-nbd U of x lying inside O. Let M be a *spZc*-connected component of U containing x. Since U is a *spZc*-nbd of x and X is locally *spZc*-connected, M is a *spZc*-nbd of x. Since M is *spZc*-closed in U and U is compact, then M is compact. So M is a compact *spZc*-connected *spZc*-nbd of x lying inside O. By theorem 3.10, M is strongly *spZc*-connected.

Theorem 3.14: Let X be a locally compact *spZc* T_2-space. If X is locally *spZc*-connected and *spZc*-connected, then X is strongly *spZc*-connected.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] A.V. Arhangelskii, R. Wiegandt, Connectedness and disconnectedness in topology, General Topology Appl. 5 (1) (1975), 9-33. https://doi.org/10.1016/0016-660X(75)90009-4.

[2] A. I. EL-Magharabi, A. M. Mubarki, Z-open sets and Z-continuity in topological spaces. Int. J. Math. Arch, 2 (10) (2011), 1819-1827.

[3] RM. Sivagamasundari, AP. Dhanabalan, Zc-open sets and Zs-open sets in topological spaces, Int. J. Sci. Res. 5 (3) (2016), 153-160.

[4] RM. Sivagamasundari, AP. Dhanabalan, Semi pre Zc-connected spaces in general topology, Int. J. Math. Trends Technol. 52 (6) (2017), 407-410.