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COMMON FIXED POINT THEOREM FOR MAPPING WITH THE GENERALIZED WEAKLY CONTRACTIVE CONDITIONS IN METRIC SPACES

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Abstract. In this paper, we discuss the existence and uniqueness of fixed point and common fixed point theorems in metric spaces by introducing a generalized weakly contractive mapping, which generalize and unify some well known results in the literature.

Keywords: metric space; fixed point; generalized weak contractive mapping.

2010 AMS Subject Classification: 47H10, 54H25.

1. INTRODUCTION

The Banach contraction principle [1] plays a vital role in the fixed point theory. Alber et al. [2] introduced the concept of weak contraction mappings in Hilbert spaces. The weak contraction principle was further extended by Rhoads [3] in metric spaces. Then many authors developed the generalization and extension of the weak contraction principle. Some remarkable results in fixed point theorems were proved by Khan et al. [4] by using the way of altering distance functions. Choudhury et al. [5], Seonghoon Cho [6] have also obtained the fixed point

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theorems in metric spaces by generalizing the weak contraction principle. In this paper, we extend and develope the study of fixed point theorems in metric spaces by using the generalized weakly contractive mapping.

2. PRELIMINARIES

Definition 2.1. [10] Let W be a non-empty set. A function $d_m : W \times W \to \mathbb{R}^+$ ($\mathbb{R}^+$means non-negative reals) called a metric on W if for all $l, m, n \in W$ the following conditions are satisfied:

(MS1) $d_m(l,m) = 0$ if and only if $l = m$;

(MS2) $d_m(l,m) = d_m(m,l)$;

(MS3) $d_m(l,m) \leq d_m(l,n) + d_m(n,m)$.

Then the pair (W, d_m) is called a metric space.

Definition 2.2. [11] Let (W, d_m) be a metric space and $\{l_n\}$ be a sequence in W and $l \in W$

(i) If for every $c \in \mathbb{R}$, with $0 < c$, there is $N \in \mathbb{N}$ such that for all $n > N$, $d_m(l_n, l) < c$, then $\{l_n\}$ is said to be convergent, $\{l_n\}$ converges to l, and l is the limit point of $\{l_n\}$. It is denoted by $\lim_{n\to\infty} l_n = l$ or $l_n \to l$ as $n \to \infty$.

(ii) If for every $c \in \mathbb{R}$, with $0 < c$, there is $N \in \mathbb{N}$ such that for all $n > N$, $d_m(l_n, l_{n+p}) < c$, where $p \in \mathbb{N}$ then $\{l_n\}$is said to be a Cauchy sequence.

(iii) If every Cauchy sequence in W is convergent, then (W, d_m) is said to be a complete metric space.

Definition 2.3. [6] Let (W, d_m) be a metric space.

(i) A function $V : W \to [0,\infty)$ is called a semicontinuous function if for all $l \in W$ and $\{l_n\} \subset W$ with $\lim_{n\to\infty} l_n = l$, we have $V(l) \leq \lim_{n\to\infty} \inf V(l_n)$.

(ii) The set of all continuous functions $\omega : [0,\infty) \to [0,\infty)$ which satisfy $\omega(h) = 0 \Leftrightarrow h = 0$ is denoted by Ω and the set of all semicontinuous functions $\gamma : [0,\infty) \to [0,\infty)$ which satisfy $\gamma(h) = 0 \Leftrightarrow h = 0$ is denoted by Γ.

Lemma 2.4. [7] *If a sequence $\{l_n\}$ in W is not a Cauchy, then there exists $c>0$ and two subsequences $\{l_{m_k}\}$ and $\{l_{n_k}\}$ of $\{l_n\}$ such that m_k is the smallest index for which $m_k > n_k > k$,*

$$d_m\left(l_{m_k}, l_{n_k}\right) \geq c \tag{2.1}$$

and

$$d_m\left(l_{m_k-1}, l_{n_k}\right) < c. \tag{2.2}$$

Moreover, suppose that $\lim_{n\to\infty} d_m\left(l_n, l_{n+1}\right) = 0$. Then, we have

(i) $\lim_{k\to\infty} d_m\left(l_{m_k}, l_{n_k}\right) = c$;
(ii) $\lim_{k\to\infty} d_m\left(l_{m_k-1}, l_{n_k-1}\right) = c$;
(iii) $\lim_{k\to\infty} d_m\left(l_{m_k}, l_{n_k-1}\right) = c$;
(iv) $\lim_{k\to\infty} d_m\left(l_{m_k-1}, l_{n_k}\right) = c$.

Theorem 2.5. [6] *Let (W, d_m) be a metric space. If $U : W \to W$ satisfies the following condition:*

(2.3)
$$\omega\left(d_m(Uu, Uv)\right) \leq \omega\left(\max\left\{d_m(u,v), d_m(u,Uu), d_m(v,Uv), \frac{1}{2}\left[d_m(u,Uv)+d_m(v,Uu)\right]\right\}\right) - \gamma\left(\max\left\{d_m(u,v), d_m(v,Uv)\right\}\right)$$

for all $u, v \in W$, where $\gamma : [0,\infty) \to [0,\infty)$ is a continuous function, and $\omega : [0,\infty) \to [0,\infty)$ is an altering distance function, that is, ω is a nondecreasing and continuous function, and $\omega(h) = 0$ if and only if $h = 0$. Then U has a unique fixed point.

3. Main Results

Definition 3.1. Let (W, d_m) be a metric space, let $U : W \to W$, and let $V : W \to [0,\infty)$ be a lower semicontinuous function. Then U is called a weakly generalized contractive mapping if it satisfies the following: for each $u, v \in W$

$$\omega\left(d_m(Uu, Uv) + V(Uu) + V(Uv)\right) \leq \omega\left(s(u,v,d_m,U,V)\right) - \gamma\left(t(u,v,d_m,U,V)\right) \tag{3.1}$$

where $\omega \in \Omega$, $\gamma \in \Gamma$ and

(3.2)
$$s(u,v,d_m,U,V) = \max \left\{ \begin{array}{l} d_m(u,v) + V(u) + V(v), d_m(u,Uu) + V(u) + V(Uu), \\ d_m(v,Uv) + V(v) + V(Uv), \dfrac{d_m(u,v) + V(u) + V(v)}{1 + d_m(u,v) + V(u) + V(v)}, \\ \dfrac{1}{2}\left[d_m(u,Uv) + V(u) + V(Uv) + d_m(v,Uu) + V(v) + V(Uu)\right] \end{array} \right\}$$

$$t(u,v,d_m,U,V) = \max \left\{ \begin{array}{l} d_m(u,v) + V(u) + V(v), \\ d_m(v,Uv) + V(v) + V(Uv), \\ \dfrac{d_m(u,v) + V(u) + V(v)}{1 + d_m(u,v) + V(u) + V(v)} \end{array} \right\}. \quad (3.3)$$

Theorem 3.2. *Let (W,d_m) be a complete metric space. Let $U: W \to W$, and let $V: W \to [0,\infty)$ be a lower semicontinuous function. If U is a weakly generalized contractive mapping, then U has a unique fixed point z in W with $V(z) = 0$.*

Proof. For any arbitrary point $l_0 \in W$, define a sequence $\{l_n\}$ in W such that $l_{n+1} = Ul_n$, for all $n = 0,1,2,\ldots$ Suppose for some n, $l_n = l_{n+1} = Ul_n$. Hence l_n is a fixed point of U, the proof is finished.

Assume that $l_n \neq l_{n+1}$ for all $n = 0,1,2,\ldots$ Let $u = l_{n-1}$ and $v = l_n$.

from (3.2)

$$s(l_{n-1},l_n,d_m,U,V) = \max \left\{ \begin{array}{l} d_m(l_{n-1},l_n) + V(l_{n-1}) + V(l_n) \\ d_m(l_{n-1},Ul_{n-1}) + V(l_{n-1}) + V(Ul_{n-1}), \\ d_m(l_n,Ul_n) + V(l_n) + V(Ul_n), \\ \dfrac{d_m(l_{n-1},l_n) + V(l_{n-1}) + V(l_n)}{1 + d_m(l_{n-1},l_n) + V(l_{n-1}) + V(l_n)}, \\ \dfrac{1}{2}\Bigg[d_m(l_{n-1},Ul_n) + V(l_{n-1}) + V(Ul_n) \\ \qquad + d_m(l_n,Ul_{n-1}) + V(l_n) + V(Ul_{n-1})\Bigg] \end{array} \right\}.$$

Now,

$$\begin{aligned}
&\frac{1}{2}\left[d_m(l_{n-1},Ul_n)+V(l_{n-1})+V(Ul_n)+d_m(l_n,Ul_{n-1})+V(l_n)+V(Ul_{n-1})\right]\\
&=\frac{1}{2}\left[d_m(l_{n-1},l_{n+1})+V(l_{n-1})+V(l_{n+1})+d_m(l_n,l_n)+V(l_n)+V(l_n)\right]\\
&\leq\frac{1}{2}\left[d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n)+d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\right]\\
&\leq\max\left\{d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n),\ d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\right\},
\end{aligned}$$

we get,

(3.4)

$$s(l_{n-1},l_n,d_m,U,V)=\max\left\{d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n),\ d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\right\}.$$

Also,

(3.5)

$$t(l_{n-1},l_n,d_m,U,V)=\max\left\{\begin{array}{l} d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n),\\ d_m(l_n,Ul_n)+V(l_n)+V(Ul_n),\\ \dfrac{d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n)}{1+d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n)}\end{array}\right\}$$

$$t(l_{n-1},l_n,d_m,U,V)=\max\left\{d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n),\ d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\right\}$$

by using (3.1), we obtain

$$\omega(d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1}))=\omega(d_m(Ul_{n-1},Ul_n)+V(Ul_{n-1})+V(Ul_n))$$

$$\omega(d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1}))\leq\omega(s(l_{n-1},l_n,d_m,U,V))-\gamma(t(l_{n-1},l_n,d_m,U,V)). \tag{3.6}$$

if

$$\max\left\{d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n),d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\right\}$$
$$=d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})$$

for some $n \in \mathbb{N}$ then from (3.6), we get

$$\omega\left(d_m\left(l_n, l_{n+1}\right)+V\left(l_n\right)+V\left(l_{n+1}\right)\right) \leq \omega\left(d_m\left(l_n, l_{n+1}\right)+V\left(l_n\right)+V\left(l_{n+1}\right)\right)$$
$$-\gamma\left(d_m\left(l_n, l_{n+1}\right)+V\left(l_n\right)+V\left(l_{n+1}\right)\right),$$

which gives

$$\gamma(d_m(l_n, l_{n+1})+V(l_n)+V(l_{n+1}))=0$$

and so

$$d_m(l_n, l_{n+1})+V(l_n)+V(l_{n+1})=0$$

which implies

$$l_n=l_{n+1}, V\left(l_n\right)=V\left(l_{n+1}\right)=0$$

which is contradiction.

Therefore

$$\max\left\{d_m\left(l_{n-1}, l_n\right)+V\left(l_{n-1}\right)+V\left(l_n\right), d_m\left(l_n, l_{n+1}\right)+V\left(l_n\right)+V\left(l_{n+1}\right)\right\}$$
$$=d_m\left(l_{n-1}, l_n\right)+V\left(l_{n-1}\right)+V\left(l_n\right) \tag{3.7}$$

for all $n=1,2,3\ldots$

and so,

$$s\left(l_{n-1}, l_n, d_m, U, V\right)=d_m\left(l_{n-1}, l_n\right)+V\left(l_{n-1}\right)+V\left(l_n\right)$$

and

$$t\left(l_{n-1}, l_n, d_m, U, V\right)=d_m\left(l_{n-1}, l_n\right)+V\left(l_{n-1}\right)+V\left(l_n\right)$$

for all $n=1,2,3,\ldots,$.

then by using (3.6) we have

$$\begin{aligned} \omega\left(d_m\left(l_n, l_{n+1}\right)+V\left(l_n\right)+V\left(l_{n+1}\right)\right) &= \omega\left(d_m\left(Ul_{n-1}, Ul_n\right)+V\left(Ul_{n-1}\right)+V\left(Ul_n\right)\right) \\ &\leq \omega\left(s\left(l_{n-1}, l_n, d_m, U, V\right)\right)-\gamma\left(s\left(l_{n-1}, l_n, d_m, U, V\right)\right) \\ &= \omega\left(d_m\left(l_{n-1}, l_n\right)+V\left(l_{n-1}\right)+V\left(l_n\right)\right) \\ &\quad -\gamma\left(d_m\left(l_{n-1}, l_n\right)+V\left(l_{n-1}\right)+V\left(l_n\right)\right). \end{aligned} \tag{3.8}$$

with the help of (3.7), the sequence $\{d_m\left(l_n, l_{n+1}\right)+V\left(l_n\right)+V\left(l_{n+1}\right)\}$ is a decreasing.

Thus $\{d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\}$ converges to a non negative real l as $n\to\infty$.

Suppose $l>0$,

When $n\to\infty$ in (3.8), by the continuity of ω and the lower semicontinuity of γ it follows that

$$\begin{aligned}\omega(l) &\leq \omega(l)-\lim_{n\to\infty}\gamma(d_m(l_{n-1},l_n)+V(l_{n-1})+V(l_n))\\ &\leq \omega(l)-\gamma(l),\end{aligned}$$

since $l>0$, $\gamma(l)>0$.

Hence $\omega(l)\leq\omega(l)-\gamma(l)<\omega(l)$, a contradiction.

Then $d_m(l_n,l_{n+1})+V(l_n)+V(l_{n+1})\to 0$ as $n\to\infty$,

which implies

$$\lim_{n\to\infty} d_m(l_n,l_{n+1})=0 \text{ and } \lim_{n\to\infty} V(l_n)=0 \tag{3.9}$$

We show that $\{l_n\}$ is a Cauchy sequence.

Suppose$\{l_n\}$ is not a Cauchy, then there exists a $\zeta>0$ and subsequences $\{l_{m_k}\}$ and $\{l_{n_k}\}$ of $\{l_n\}$ such that (2.1) and (2.2) hold.

From (3.2), we can get

$$s(l_{n_k},l_{m_k},d_m,U,V)=\max\left\{\begin{array}{l} d_m(l_{n_k},l_{m_k})+V(l_{n_k})+V(l_{m_k})\\ d_m(l_{n_k},Ul_{n_k})+V(l_{n_k})+V(Ul_{n_k})\\ d_m(l_{m_k},Ul_{m_k})+V(l_{m_k})+V(Ul_{m_k}),\\ \dfrac{d_m(l_{n_k},l_{m_k})+V(l_{n_k})+V(l_{m_k})}{1+d_m(l_{n_k},l_{m_k})+V(l_{n_k})+V(l_{m_k})},\\ \dfrac{1}{2}\Big[d_m(l_{n_k},Ul_{m_k})+V(l_{n_k})+V(Ul_{m_k})\\ \quad +d_m(l_{m_k},Ul_{n_k})+V(l_{m_k})+V(Ul_{n_k})\Big] \end{array}\right\}$$

$$s\left(l_{n_k}, l_{m_k}, d_m, U, V\right)=\max\left\{\begin{array}{l} d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right) \\ d_m\left(l_{n_k}, l_{n_k+1}\right)+V\left(l_{n_k}\right)+V\left(l_{n_k+1}\right) \\ d_m\left(l_{m_k}, l_{m_k+1}\right)+V\left(l_{m_k}\right)+V\left(l_{m_k+1}\right), \\ \dfrac{d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right)}{1+d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right)}, \\ \dfrac{1}{2}\left[d_m\left(l_{n_k}, l_{m_k+1}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k+1}\right)\right. \\ \qquad \left.+d_m\left(l_{m_k}, l_{n_k+1}\right)+V\left(l_{m_k}\right)+V\left(l_{n_k+1}\right)\right] \end{array}\right\}. \tag{3.10}$$

Taking as $k \to \infty$ and applying the Lemma 2.4 and (3.9), we obtain

$$\lim_{k\to\infty} s\left(l_{n_k}, l_{m_k}, d_m, U, V\right)=\zeta. \tag{3.11}$$

And, from (3.3)

$$t\left(l_{n_k}, l_{m_k}, d_m, U, V\right)=\max\left\{\begin{array}{l} d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right), \\ d_m\left(l_{m_k}, U l_{m_k}\right)+V\left(l_{m_k}\right)+V\left(U l_{m_k}\right), \\ \dfrac{d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right)}{1+d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right)} \end{array}\right\}$$

$$t\left(l_{n_k}, l_{m_k}, d_m, U, V\right)=\max\left\{\begin{array}{l} d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right), \\ d_m\left(l_{m_k}, l_{m_k+1}\right)+V\left(l_{m_k}\right)+V\left(l_{m_k+1}\right), \\ \dfrac{d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right)}{1+d_m\left(l_{n_k}, l_{m_k}\right)+V\left(l_{n_k}\right)+V\left(l_{m_k}\right)} \end{array}\right\} \tag{3.12}$$

Hence

$$\lim_{k\to\infty} t\left(l_{n_k}, l_{m_k}, d_m, U, V\right)=\zeta \tag{3.13}$$

From (3.1), we are having

$$\omega(d_m\left(l_{n_k+1}, l_{m_k+1}\right)+V\left(l_{n_k+1}\right)+V\left(l_{m_k+1}\right) \leq \quad \omega\left(s\left(l_{n_k}, l_{m_k}, d_m, U, V\right)\right)-\gamma(t\left(l_{n_k}, l_{m_k}, d_m, U, V\right)).$$

Taking as $k \to \infty$ and applying the Lemma 2.4 , the continuity of ω and the lower semicontinuity of γ, (3.9), (3.11) and (3.13), we obtain

$\omega(\zeta) \leq \omega(\zeta) - \gamma(\zeta)$, which gives a contradiction, since $\gamma(\zeta) > 0$.

Hence $\{l_n\}$ is a Cauchy sequence.

Since W is a complete metric space. We obtain $\lim_{n\to\infty} l_n = z$ where z in W.

Since, V is semicontinuous,

$$V(z) \leq \lim_{n\to\infty} \inf V(l_n) \leq \lim_{n\to\infty} V(l_n) = 0$$

which implies

$$V(z) = 0. \tag{3.14}$$

Now,

$$s(l_n, z, d_m, U, V) = \max \left\{ \begin{array}{l} d_m(l_n, z) + V(l_n) + V(z), \\ d_m(l_n, Ul_n) + V(l_n) + V(Ul_n), \\ d_m(z, Uz) + V(z) + V(Uz), \\ \dfrac{d_m(l_n, z) + V(l_n) + V(z)}{1 + d_m(l_n, z) + V(l_n) + V(z)}, \\ \dfrac{1}{2}\Big[d_m(l_n, Uz) + V(l_n) + V(Uz) \\ \quad + d_m(z, Ul_n) + V(z) + V(Ul_n)\Big] \end{array} \right\}$$

$$s(l_n, z, d_m, U, V) = \max \left\{ \begin{array}{l} d_m(l_n, z) + V(l_n) + V(z), \\ d_m(l_n, l_{n+1}) + V(l_n) + V(l_{n+1}), \\ d_m(z, Uz) + V(z) + V(Uz), \\ \dfrac{d_m(l_n, z) + V(l_n) + V(z)}{1 + d_m(l_n, z) + V(l_n) + V(z)}, \\ \dfrac{1}{2}\Big[d_m(l_n, Uz) + V(l_n) + V(Uz) \\ \quad + d_m(z, Ul_n) + V(z) + V(l_{n+1})\Big] \end{array} \right\}.$$

Then

$$\lim_{n\to\infty} s(l_n, z, d_m, U, V) = d_m(z, Uz) + V(Uz). \tag{3.15}$$

Also, we obtain

$$t(l_n,z,d_m,U,V)=\max\left\{\begin{array}{l} d_m(l_n,z)+V(l_n)+V(z),\\ d_m(z,Uz)+V(z)+V(Uz),\\ \dfrac{d_m(l_n,z)+V(l_n)+V(z)}{1+d_m(l_n,z)+V(l_n)+V(z)}\end{array}\right\}.$$

Then

$$\lim_{n\to\infty} t(l_n,z,d_m,U,V)=d_m(z,Uz)+V(Uz). \tag{3.16}$$

From (3.1),

$$\begin{aligned}\omega(d_m(l_{n+1},Uz)+V(l_{n+1})+V(Uz)) &= \omega(d_m(Ul_n,Uz)+V(Ul_n)+V(Uz))\\ &\leq \omega(s(l_n,z,d_m,U,V))-\gamma(t(l_n,z,d_m,U,V)).\end{aligned} \tag{3.17}$$

Taking as $n\to\infty$ and applying the continuity of ω and the lower semicontinuity of V, (3.15) and (3.16), we obtain

$$\omega(d_m(z,Uz)+V(Uz))\leq\omega(d_m(z,Uz)+V(Uz))-\gamma(d_m(z,Uz)+V(Uz)).$$

Therefore, $d_m(z,Uz)+V(Uz)=0$. Hence $Uz=z$ and $V(Uz)=0$.

Suppose that z' is another fixed point of U.

Then

$$Uz'=z' \text{ such that } V(z')=0.$$

By using with (3.1) we are having

$$\begin{aligned}\omega(d_m(z,z')) &= \omega(d_m(Uz,Uz'))\\ &= \omega(d_m(Uz,Uz')+V(Uz)+V(Uz'))\\ &\leq \omega(s(z,z',d_m,U,V))-\gamma(t(z,z',d_m,U,V))\\ &= \omega(d_m(z,z'))-\gamma(d_m(z,z'))\end{aligned}$$

which gives $z=z'$.

Hence the theorem. □

Corollary 3.3. *Let (W, d_m) be a complete metric space. If U is satisfying the condition for all $u, v \in W$*

$$\omega(d_m(Uu, Uv) + V(Uu) + V(Uv)) \leq \omega(s(u, v, d_m, U, V)) - \gamma(s(u, v, d_m, U, V))$$

where $\omega \in \Omega$, $\gamma \in \Gamma$, then U has a fixed point z in W with $V(z) = 0$.

Proof. With the help of the Theorem 3.2, we can prove this result. □

Corollary 3.4. *Let (W, d_m) be a complete metric space. If U is satisfying the condition for all $u, v \in W$*

$$\omega(d_m(Uu, Uv) + V(Uu) + V(Uv)) \leq \omega(d_m(u, v) + V(u) + V(v)) - \gamma(d_m(u, v) + V(u) + V(v))$$

where $\omega \in \Omega$, $\gamma \in \Gamma$, then U has a fixed point z in W with $V(z) = 0$.

Proof. Which can be proved as earlier. □

Corollary 3.5. *Let (W, d_m) be a complete metric space. If U is satisfying the condition for all $u, v \in W$*

$$\omega\left(d_m\left(U^i u, U^i v\right) + V\left(U^i u\right) + V\left(U^i v\right)\right) \leq \omega\left(s\left(u, v, d_m, U^i, V\right)\right) - \gamma\left(t\left(u, v, d_m, U^i, V\right)\right)$$

where $\omega \in \Omega$, $\gamma \in \Gamma$ and i is any positive integer, then U has a fixed point z in W with $V(z) = 0$.

Proof. With the help of Theorem 3.2 by taking $U^i = Q$ in Theorem 3.2, we can prove this result. □

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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