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AN INTRODUCTION TO THE THEORY OF IMPRECISE SETS: THE MATHEMATICS OF PARTIAL PRESENCE

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Abstract. In this article, we are going to discuss the mathematics of partial presence of an element in a set. A similar theory well known as the theory of fuzzy sets is already in existence since 1965. However, right at the start, the measure theoretic explanations of fuzziness have taken a wrong turn in the sense that from any given law of fuzziness defined on an interval, workers have since been trying to extract a law of probability giving rise thereby to all sorts of misinterpretations of probability theory. Such developments do not have any classical measure theoretic basis. In fact, not one as popularly believed, but two independent laws of randomness are necessary and sufficient to define a law of fuzziness. Secondly, the existing definition of complement of a fuzzy set is logically incorrect, and hence every result in which that definition had been used is incorrect. As long as we would keep on referring to fuzzy sets, the original definitions that include these two unacceptable points would keep coming up creating an unnecessary confusion thereby. We are therefore going to introduce the theory of imprecise sets in which the two fuzzy set theoretic blunders mentioned above would be absent.

Keywords: Uncertainty, randomness, fuzziness.

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1. Introduction

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Preconceived knowledge sometimes creates hindrance towards framing new concepts. If one does not know anything about a particular theory, one may actually end up discovering it all over again. In addition, if in the existing theory there happens to be some flaws, it may be that those flaws may not appear in the rediscovery. If there are flaws in any theory, those who work towards further development of that theory are expected to understand in course of time that there are flaws which should be removed. If however, the workers simply follow the leader blindly without at all trying to realize that there are flaws in the theory, it is no wonder that the theory would finally end up taking an unearthly shape.

This is precisely what has happened to the theory of fuzzy sets. The discovery of fuzzy sets was a paradigm changing event in the history of mathematics. Its discoverer, L. A. Zadeh, would always be hailed for putting forward a new concept of uncertainty way back in 1965. However, soon after this great discovery, he forwarded a *Probability-Possibility Consistency Principle* with a view to linking fuzziness with probability. First saying that a particular type of uncertainty can not be handled by the theory of probability, and that it is indeed a case where the theory of fuzziness would be the only alternative, and thereafter trying to frame a law of probability over the same interval on which a law of fuzziness had been defined sounds illogical anyway. Trying to infer a law of probability from a law of fuzziness is meaningless. As no consistency between probability and fuzziness was reflected by that principle, two more *Probability-Possibility Consistency Principles* were forwarded thereafter by others. Had it been logical to link the two concepts in that manner, there should have been just one such principle and not three different principles anyway.

Didier Dubois and Henry Prade took a positive step towards defining the *membership function* of a *normal fuzzy number*. They defined the membership function in terms of two different functions in two intervals: the *left reference function* and the *right reference function*. However, they did not pursue any further to find out how their definition comes up. They were correct in understanding that fuzzy membership must be expressed in terms of two different functions. Had anyone tried to explain initially why we need two independent functions to define a fuzzy number, the theory of fuzzy sets would have been altogether different from what it currently looks like. That unfortunately did not happen. It was soon observed that not all postulates required to define a measure were followed by the fuzzy sets. This was the unfortunate outcome because the workers did not try to look into the matters using the Dubois-Prade functions independently. Instead of trying to understand that the classical theory of measure should naturally have enveloped the matters of fuzziness, the workers went forward to define a *fuzzy measure* which is something entirely different from a classical measure.

Indeed, yet another blunder entered into the definition of fuzziness right in the beginning. It was declared that the intersection of a fuzzy set and its complement is not the null set. Further, it was declared that the union of a fuzzy set and its complement is not the universal set. The workers should have tried to see that there can not be anything common between a statement and its complement. At least someone should have been able to understand that perhaps the definition of complement of a fuzzy set was defective. That too unfortunately did not happen. Instead of thinking that peculiar sort of a conclusion had been arrived at, the workers went forward to establish more results based on that wrong definition. It is obvious that the results thus found were weird. But with a mindset that fuzzy sets are unearthly already, in the sense that they do not follow the formalisms of both measure theory and field theory, the workers have meanwhile ended up making the mathematics of fuzziness look totally unnatural.

Thousands of articles and hundreds of books on theory and applications of fuzziness have in the meanwhile appeared the world over, and the process of publication has been going on in full swing in a very fast pace. It is now next to impossible to convince those who have published some work in this field to look into the matters from our standpoint. Instead of agreeing that something might have gone wrong nearly half a century ago, they would invariably choose to defend their findings, and anyone who comes forward to challenge their findings would possibly be ridiculed.

Mathematics should be firmly rooted at logic; it must not be based on some popular beliefs. If something is logical, then it can possibly be given a mathematical shape. On the contrary, the process of working out some belief based mathematics first, and then trying to impose some weird logic on it, violates the very philosophy of mathematics. The mathematics of fuzziness is to our knowledge the only branch of mathematics in which a huge amount of results devoid of any logic whatsoever have come up, and now for the workers concerned it is impossible to come out of the symbolic mirage they have created, no physical significance of which should ever be possible. The workers in fuzziness would not agree with us for obvious reasons. They have followed the earlier workers to generalize and to modify results without ever sincerely trying to understand anything of what they are doing. Mathematics must never progress in this way. Instead of trying to see whether the idea of transforming a law fuzziness into a law of probability was logical or not, people went forward to define things like *upper probability* and such other things! Such things are not rooted at considering fuzzy membership in terms of two different Dubois-Prade functions, and that is why they are incorrect. In such things, some sort of symbolic expressions have been found first, and then the workers concerned have started to impose some sort of logic therein. All sorts of such results on *fuzzy measure* have been published. In the process, the theory of probability was misinterpreted in all possible ways.

At the same time, new definitions in topological matters have appeared in the literature based on the wrong definition of the complement of a fuzzy set. Software based on fuzzy logic that in turn includes that definition was built, and conclusions using such software in connection with application of fuzziness have been made in all sorts of fields.

Due to wrong interpretation of the membership function, classical measure theoretic formalisms were found to be insufficient to explain certain matters. Instead of trying to see that something must have gone wrong in the interpretation of the membership function, workers redefined the theory of measure itself. In a similar manner, instead of trying to see that the conclusion that the fuzzy sets do not form a field was a wrong one, the workers have gone forward to redefine the concept of field itself. Unfortunately, the workers of the mathematics of fuzziness have formed a cult already, and it would perhaps be impossible to make them see reason at this point of time at least.

Here then is an appropriate example in which a comment of Henry Bessemer on preconceived knowledge being a hindrance sometimes becomes very meaningful[1]. Indeed, knowledge is needed to develop it further. That is how knowledge has been growing. But if a theory is defective, developing it further hardly makes any sense.

Therefore Bessemer commented that for new thoughts to come up, one should probably be free from any biased ideas existing from earlier times. In the case of the mathematics of fuzziness, the initial definition of a fuzzy set was very correctly forwarded. The arithmetic of fuzziness using the method of α-cuts is absolutely correct. Hence in applications in various fields as long as the users remained within using the arithmetic of fuzziness, things were totally correct. But as soon as people started to try to find a relationship between fuzziness and probability in the sense that one fuzzy space can be transformed into one probability space, things went absolutely wrong. In the findings in which the incorrect definition of complement of a fuzzy set was used, things went further wrong. Trying to establish a law of probability from a law of fuzziness is in our eyes equivalent to going back to the days of alchemy in search of a philosopher's stone. Similarly, first defining the complement of a fuzzy set in a weird manner, and then building up an entire theory based on that, is perhaps comparable to preach for a geocentric solar system all over again.

Our objective is a simple one. Instead of saying (see e.g. [1], [2], [3]) that the theory of fuzzy sets has been incorrectly explained, it is better that we start the whole process anew naming the theory differently. We are now going to introduce *the theory of imprecise sets*, which might initially look similar to the theory of fuzzy sets. But other than just one initial definition, the things are altogether different in the two theories. This is a battle between belief and truth. Those who would like to continue *believing* in the earlier notions on fuzziness will perhaps continue to do so. Those who would like to see reason can come forward to start building up a new theory. A scientific truth does not depend on popular votes. On the contrary, an illogical belief remains illogical even though it may

[1] 'I had an immense advantage over many others dealing with the problem inasmuch as I had no fixed ideas derived from long established practice to control and bias my mind, and did not suffer from the general belief that whatever is, is right.' - Sir Henry Bessemer.

possibly be supported by millions.

To look into the matters from our standpoint, we would need two specific things. First, we would need to define a set operation that we have named *superimposition*. When we overwrite, the overwritten portion looks darker due to superimposition. Similarly, umbra and penumbra are formed due to superimposition when an opaque body is placed in front of a source of light. Secondly, we would need a very classical theorem, known as the Glivenko-Cantelli Theorem, on order statistics to make our conclusions. Unless one looks through the spectacles of set superimposition, and unless we understand how order statistics comes into play to infer a theoretical distribution function from an empirical distribution function based on observed data, we would be unable to explore the exact nature of the membership function of a fuzzy number.

The idea of defining the complement of an imprecise set as we are now going to define is based on a rather simple logic. Not everything can be counted from the zero level. When a glass is partially full of water, depth of the empty portion has to be counted not from the zero level but from the level upto which there is water. If we express this simple logic in symbols, we immediately get the definition of complement of a fuzzy set which we are going to call an imprecise set hencefrom.

Our observations and the explanations are our own. We have not rested on the definition of fuzziness which was already in existence, and we were not influenced by any preconceived knowledge on fuzziness. We now proceed to mention a few definitions based on which our proposed theory of imprecise sets would stand.

2. Definitions and Notations

Defination 2.1. An *imprecise number* $[\alpha, \beta, \gamma]$ is an interval around the real number β with the elements in the interval being *partially present*.

Defination 2.2. Partial presence of an element in an imprecise real number $[\alpha, \beta, \gamma]$ is described by the *presence level indicator function* $p(x)$ which is counted from the *reference function* $r(x)$ such that the presence level for any x, $\alpha \leq x \leq \gamma$, is $(p(x) - r(x))$, where $0 \leq r(x) \leq p(x) \leq 1$.

Defination 2.3. A *normal* imprecise number $N = [\alpha, \beta, \gamma]$ is associated with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \Psi_1(x), & \text{if } \alpha \leq x \leq \beta \\ \Psi_2(x), & \text{if } \beta \leq x \leq \gamma \\ 0, & \text{otherwise,} \end{cases}$$

with a *constant reference function* 0 in the entire real line. Here $\Psi_1(x)$ is continuous and non-decreasing in the interval $[\alpha, \beta]$, and $\Psi_2(x)$ is continuous and non-increasing in the interval $[\beta, \gamma]$, with

$$\Psi_1(\alpha) = \Psi_2(\gamma) = 0,$$

$$\Psi_1(\beta) = \Psi_2(\beta) = 1.$$

Here, the imprecise number would be characterized by $\{x, \mu_N(x), 0 : x \epsilon R\}$, R being the real line.

Definition 2.1 is indeed used to define a fuzzy number. In Definition 2.2, we have deviated from the definition of a fuzzy number. We are using a reference function in our case. We would need this definition later to define the complement of an imprecise number. In fact, if the reference function is zero, we actually get a fuzzy number. In Definition 2.3, we have used the reference function as zero, and therefore this is indeed the definition of the normal fuzzy number. In the Dubois-Prade nomenclature, for a fuzzy number with *fuzzy membership function* $\mu_N(x)$, $\Psi_1(x)$ is called the *Left Reference Function*, and $\Psi_2(x)$ is called the *Right Reference Function* of the normal fuzzy number. We are now coming to two more definitions.

Defination 2.4. For a normal imprecise number $N = [\alpha, \beta, \gamma]$ with presence level indicator function

$$\mu_N(x) = \begin{cases} \Psi_1(x), & \text{if } \alpha \leq x \leq \beta \\ \Psi_2(x), & \text{if } \beta \leq x \leq \gamma \\ 0, & \text{otherwise} \end{cases}$$

such that

$$\Psi_1(\alpha) = \Psi_2(\gamma) = 0,$$

$$\Psi_1(\beta) = \Psi_2(\beta) = 1,$$

with constant reference function equal to 0, $\Psi_1(x)$ is the *distribution function* of a *random variable* defined in the interval $[\alpha, \beta]$, and $\Psi_2(x)$ is the *complementary distribution function* of another random variable defined in the interval $[\beta, \gamma]$.

We are using the term *random variable* here in the broader measure theoretic sense which does not require that the notion of probability need to appear in defining randomness.

Defination 2.5. For a normal imprecise number $N = \{x, \mu_N(x), 0 : x\epsilon R\}$ as defined above, the complement $N^C = \{x, 1, \mu_N(x) : x\epsilon R\}$ will have constant presence level indicator function equal to 1, the reference function being $\mu_N(x)$ for $-\infty < x < \infty$.

In Definition 2.4, we have stated that the Dubois-Prade functions are in fact rooted at two different laws of randomness. Therefore studies to infer one single law of randomness from a given law of fuzziness have consistently failed since the beginning. In Definition 2.5, we have in fact stated that fuzzy membership function and fuzzy membership value are two different things. In the Zadehian definition, these two things were taken to be the same, and that was the root of all troubles. Indeed, we would need the reference function to define an imprecise set only when we talk about the complement of an imprecise set. In all other cases, the reference function assumes the constant value equal to zero.

In what follows, we are going to explain how Definitions 2.4 and 2.5 come up logically and therefore mathematically.

3. The Mathematics of Partial Presence

We now proceed to describe our standpoint. We defined (see e. g. [3]) the operation of superimposition of two real intervals $[a_1, b_1]$ and $[a_2, b_2]$ as

$$[a_1, b_1](S)[a_2, b_2] = [a_{(1)}, a_{(2)}] \cup [a_{(2)}, b_{(1)}]^{(2)} \cup [b_{(1)}, b_{(2)}]$$

where $a_{(1)} = min(a_1, a_2)$, $a_{(2)} = max(a_1, a_2)$, $b_{(1)} = min(b_1, b_2)$, and $b_{(2)} = max(b_1, b_2)$. Here we have assumed without loss of any generality that $[a_1, b_1] \cap [a_2, b_2]$ is not void, or in other words that $max(a_i) \leq min(b_i), i = 1, 2$.

If we increase the number of intervals, with partial presence level of every element for every interval being equal to the inverse of the number of intervals, we shall see that two laws of randomness lead to one law of impreciseness. We can see that formalisms of order statistics would now come into play automatically, and to deal with empirical probability distribution functions we already have the Glivenko-Cantelli theorem, application of which should now lead to the conclusion that superimposition of an infinite number of intervals, with level of partial presence of the elements in every interval tending to zero, would define an imprecise number.

Consider now two probability spaces (Ω_1, A_1, Π_1) and (Ω_2, A_2, Π_2) where Ω_1 and Ω_2 are real intervals $[\alpha, \beta]$ and $[\beta, \gamma]$ respectively. Let $x_1, x_2, \ldots x_n$, and $y_1, y_2, \ldots, y_n$, be realizations in $[\alpha, \beta]$ and $[\beta, \gamma]$ respectively. So for n intervals

$$[x_1, y_1]^{(1/n)}, [x_2, y_2]^{(1/n)}, \ldots, [x_n, y_n]^{(1/n)}$$

all with elements with a constant presence level equal to $1/n$, we shall have

$$[x_1, y_1]^{(1/n)}(S)[x_2, y_2]^{(1/n)}(S)\ldots(S)[x_n, y_n]^{(1/n)} = [x_{(1)}, x_{(2)}]^{(1/n)} \cup [x_{(2)}, x_{(3)}]^{(2/n)} \cup \ldots$$

$$\cup[x_{(n-1)}, x_{(n)}]^{((n-1)/n)} \cup [x_{(n)}, y_{(1)}]^{(1)} \cup [y_{(1)}, y_{(2)}]^{((n-1)/n)} \cup \ldots \cup [y_{(n-2)}, y_{(n-1)}]^{(2/n)} \cup$$

$$[y_{(n-1)}, y_{(n)}]^{(1/n)},$$

where, for example, $[y_{(1)}, y_{(2)}]^{((n-1)/n)}$ represents the interval $[y_{(1)}, y_{(2)}]$ with presence level $((n-1)/n)$ for all elements in the entire interval, $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ being values of $x_1, x_2, \ldots x_n$ arranged in increasing order of magnitude, and $y_{(1)}, y_{(2)}, \ldots, y_{(n)}$ being values of $y_1, y_2, \ldots, y_n$ arranged in increasing order of magnitude again.

Define now

$$\Phi_n(x) = \begin{cases} 0, & \text{if } x < x_{(1)} \\ \frac{r-1}{n}, & \text{if } x_{(r-1)} \leq x \leq x_{(r)}, r = 2, 3, \ldots, n \\ 1, & \text{if } x \geq x_{(n)}. \end{cases}$$

$\Phi_n(x)$ here can be seen to be an empirical distribution function for which the underlying theoretical distribution function is $\Phi(x)$, say. Now the Glivenko-Cantelli theorem states that $\Phi_n(x)$ converges to $\Phi(x)$ uniformly in x. This means,

$$sup|\Phi_n(x) - \Phi(x)| \longrightarrow 0$$

Application of this theorem on the intervals $[\alpha, \beta]$ and $[\beta, \gamma]$ separately gives us Definition-2.4 stated above.

Our standpoint of defining a normal imprecise number does not defy the Dubois-Prade nomenclature of defining a normal fuzzy number. It is known that a distribution function of a random variable is non-decreasing, and that a complementary distribution function of a random variable is non-increasing. The functions are continuous and differentiable. Observe that integration of a distribution function does not make any logical sense, hence trying to infer anything out of integration of such a function is meaningless. In other words, finding the area under the curve $\mu_N(x)$ is of no logical meaning whatsoever. On the other hand, differentiation of $\Psi_1(x)$ and $(1 - \Psi_2(x))$ would give us two density functions. This means, we need two laws of randomness, one in the interval $[\alpha, \beta]$ and the other in $[\beta, \gamma]$, to construct a normal imprecise number $[\alpha, \beta, \gamma]$. For a triangular imprecise number, differentiation of $\Psi_1(x)$ and $(1 - \Psi_2(x))$ would give us two uniform density functions. It is well known that the uniform law of randomness is the simplest of all probability laws. Thus two uniform laws of randomness lead to the simplest imprecise number. Simplicity of the triangular imprecise number is thus rooted at the simplicity of two uniform laws of randomness.

Accordingly, for a normal imprecise number of the type $N = [\alpha, \beta, \gamma]$ with *presence level indicator function*

$$\mu_N(x) = \begin{cases} \Psi_1(x), & \text{if } \alpha \leq x \leq \beta \\ \Psi_2(x), & \text{if } \beta \leq x \leq \gamma \\ 0, & \text{otherwise,} \end{cases}$$

with $\Psi_1(\alpha) = \Psi_2(\gamma) = 0$, $\Psi_1(\beta) = \Psi_2(\beta) = 1$, where the reference function assumes a constant value equal to zero, the partial presence of a value x of the variable X in the interval $[\alpha, \gamma]$ is expressible as

$$\mu_N(x) = \theta\Psi_1(x) + (1-\theta)\Psi_2(x)$$

with

$$\theta = \begin{cases} 1 & \text{if } \alpha \le x \le \beta \\ 0, & \text{if } \beta \le x \le \gamma. \end{cases}$$

In other words, the presence level indicator function is either a distribution function defined with reference to a random variable, or a complementary distribution function defined with reference to another random variable, with randomness defined in the broader measure theoretic sense. This indeed may be named the *randomness-impreciseness consistency principle*. We assure the readers that unlike the existence of a number of probability-possibility consistency principles, there simply can not be a second randomness-impreciseness consistency principle. We are not *proposing* this principle; we have *established* it.

In other words, the presence level indicator function explaining an imprecise variable taking a particular value is either the distribution function of a random event or the complementary distribution function of another random event. Hence, partial presence of an element in an imprecise set can actually be expressed either as a distribution function or as a complementary distribution function.

As an application of this principle, consider an imprecise number $X = [a, b, c]$. Let a function of X, $f(X) = [f(a), f(b), f(c)]$ be another imprecise number. Let the density functions with respect to the distribution functions $\Psi_1(x)$ and $(1 - \Psi_2(x))$ be $\varphi_1(x)$ and $\varphi_2(x)$. If $y = f(x)$ can be written as $x = g(y)$, let $dx/dy = \zeta(y)$. Now replacing x by $g(y)$ in $\varphi_1(x)$ and $\varphi_2(x)$ we can obtain $\varphi_1(x) = \psi_1(y)$ and $\varphi_2(x) = \psi_2(y)$, say. Then the presence level indicator function of $f(X)$ would be given by

$$\mu_{f(X)}(x) = \begin{cases} \int_{f(a)}^{x} \{\psi_1(y)\zeta(y)\}dy, & \text{if } f(a) \le x \le f(b) \\ 1 - \int_{f(b)}^{x} \{\psi_2(y)\zeta(y)\}dy, & \text{if } f(b) \le x \le f(c) \\ 0, & \text{otherwise.} \end{cases}$$

We have verified that this method returns the same presence level indicator function which can be found by using the standard method of α-cuts available in the literature on fuzziness. What we mean is that if we define a normal fuzzy number with reference to two laws of randomness, then our procedure of finding fuzzy membership function returns the same result that would be returned by the method of α-cuts . We have further verified that all sorts of imprecise arithmetic can easily be done using our definitions.

4. The Complement of an Imprecise Set

According to the Zadehian definition, if a normal fuzzy number $N = [\alpha, \beta, \gamma]$ is associated with a membership function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \Psi_1(x), & \text{if } \alpha \le x \le \beta \\ \Psi_2(x), & \text{if } \beta \le x \le \gamma \\ 0, & \text{otherwise,} \end{cases}$$

the complement N^C will have the membership function $\mu_{N^C}(x)$, where

$$\mu_{N^C}(x) = \begin{cases} 1 - \Psi_1(x), & \text{if } \alpha \le x \le \beta \\ 1 - \Psi_2(x), & \text{if } \beta \le x \le \gamma \\ 1, & \text{otherwise.} \end{cases}$$

This definition defies logic, because this leads to a meaningless inference that the intersection of a fuzzy set and its complement is not the null set. In fact, if a normal imprecise number $N = [\alpha, \beta, \gamma]$ is defined with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \Psi_1(x), & \text{if } \alpha \le x \le \beta \\ \Psi_2(x), & \text{if } \beta \le x \le \gamma \\ 0, & \text{otherwise,} \end{cases}$$

with

$$\Psi_1(\alpha) = \Psi_2(\gamma) = 0,$$

$$\Psi_1(\beta) = \Psi_2(\beta) = 1,$$

the complement N^C should have the presence level indicator function $\mu_{N^C}(x)$, with

$$\mu_{N^C}(x) = 1, -\infty < x < \infty,$$

where $\mu_{N^C}(x)$ is to be counted from $\Psi_1(x)$, if $\alpha \leq x \leq \beta$, from $\Psi_2(x)$, if $\beta \leq x \leq \gamma$, and from 0, otherwise. This leads to Definition 2.5 in Section 2.

We are *proposing* this definition. It can also be taken as a postulate. Indeed, we have already explained the logic behind this definition. It can be verified diagrammatically that a normal imprecise number and its complement do complement each other, a necessity that leads to the conclusion that imprecise sets do form a field. Similarly, it can also be verified that the union of a normal imprecise number and its complement is the real line.

5. Discussions

We have seen that the two Dubois-Prade reference functions are to be defined on two probability spaces, the left reference function being a distribution function and the right reference function being a complementary distribution function. In other words, the left reference function is already a function defining an area under a density function. Integrating a function that already defines an area does not make sense. A similar explanation can be forwarded about the right reference function too. Accordingly, the area under a fuzzy membership function $\mu(x)$ from one value of x to another value can hardly be related to a probability law. People have blindly followed what the originator has preached without trying to understand what they are following. In the process, to justify the suggested links between fuzziness and probability, the theory of probability has been misinterpreted by the workers in fuzziness in all possible ways.

Defining a fuzzy measure just for the sake of doing some mathematics was not necessary, particularly because the fuzzy membership function of a normal fuzzy number can be defined in terms of two probability measures already. The workers should have tried to understand that mathematics should follow logic, and that it should never be the other way around. Unfortunately, based on a belief that a law of probability can be defined on the same interval on which a law of fuzziness has already been defined such that these two laws could have some kind of consistency between them led to all sorts of unwanted growth related to the theory of probability in particular and measure theory in general.

Further, the definition of complement of a fuzzy set has led to enough of unearthly mathematics already. Indeed, as the negation of a fuzzy statement was wrongly defined, fuzzy logic itself was baseless right from the beginning. Later on, when software based on fuzzy logic was made available, the applications of such software must have led to all sorts of meaningless results. By now, the things have come to such a pass that we are sure about our voice falling into deaf ears.

Could there really be any physical interpretation of most of the results established so far in the world of the mathematics of fuzziness? Beyond the method of α-cuts, or in other words beyond the arithmetic of fuzziness, results established are hardly correct. Accordingly, application of the arithmetic of fuzziness to analyse data, are correct. However, wherever the definition of the complement of a fuzzy set was used, things went wrong. We would like to cite an example at this point. In the case of the matters related to fuzzy randomness, as soon as the question of statistical testing of rejectability of a null hypothesis regarding a fuzzy parameter came up, the alternative hypothesis invariably used the definition of complement of the fuzzy null hypothesis. In our eyes, immediately the conclusions went wrong because wrong alternative hypotheses were stated.

It is not just very difficult, indeed it is next to impossible to make the workers see reason now. Hence we have decided to restart the whole process by using a different nomenclature to explain partial presence of an element in a set. Perhaps this will lead to correct mathematical propositions regarding imprecise sets in course of time. Continuing to do mathematics just for the sake of doing it, does not really make any sense. In no other branch of mathematics, such preposterous results are to be found. We need to do the amends before it is too late.

Many more such things would come up if we look into the matters from our perspective. It has been continuously said that probability and fuzziness are complementary concepts. This is not true. Probability and fuzziness are not complementary concepts. Every law of fuzziness, and therefore every law of impreciseness, is rooted at two independent laws of randomness, with randomness defined as in measure theory. We have to keep in mind that probability does not have to come into picture while defining randomness in the measure theoretic sense which actually means that anything probabilistic must necessarily be random, but not everything random is probabilistic.

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