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MODIFIED FORMS OF SOFT NANO CONTRA CONTINUOUS FUNCTIONS

P. G. PATIL*, SPOORTI S. BENAKANAWARI

Department of Mathematics, Karnatak University Dharwad-580003, Karnataka, India

Abstract: The objective of this paper is to present two such contexts where the results obtained for soft nano contra continuous functions widely differ from those known for soft nano continuous functions. Significant facts concerned with soft nano contra continuous, soft nano contra gω-continuous, soft nano contra gω-irresolute functions, R_{sn}-maps, soft nano almost contra gω - continuous functions are developed. Also, the stronger forms of soft nano contra continuous and soft nano contra gω-continuous functions called the soft nano-bi-contra continuous functions, soft nano-bi-contra gω - continuous functions. Soft nano- strongly-bi-contra gω - continuous functions are studied with their notable properties.

Keywords: soft nano gω-open maps; soft nano gω-closed maps; soft nano almost contra gω-continuous; soft nano-bi-contra-continuous functions; soft nano-bi-contra gω - continuous maps; soft nano-strongly-bi-contra gω-continuous maps.

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1. INTRODUCTION

Generalized closed maps in topological spaces were proposed by Malghan[12]. After the introduction of these functions, extensive work was continued and it includes some of the

*Corresponding author

E-mail address: pgpatil@kud.ac.in

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concepts of generalized open, β-open, almost open, α-open, gs-closed, gα-closed, sg-closed, wg-closed maps. Dontchev[7] introduced the concept of contra continuity. Later, strongly contra g#p-continuous functions, almost-contra g#p-continuous functions and related maps were studied by Salih[18]. Several researchers followed the theory of nano topology to introduce and study the properties of nano open maps and nano closed maps and related aspects in [2], [3], [4]. [9], [10], [13],[14], [17], [20], [21].

Patil et.al. [16] introduced soft nano gω-continuous functions in soft nano topological spaces and analyzed the properties of weaker forms of soft nano continuous functions with soft nano generalized continuous functions and in specific soft nano gω-continuous functions and its compositions. They also introduced soft nano gω-closed, soft nano gω-open maps and soft nano gω-irresolute functions. The characterizations of such functions in comparison with the generalized soft nano functions are derived. In this paper, the contrasting character of soft nano continuous functions gave the idea of soft nano contra continuous functions. The modified forms of such functions are soft nano strongly contra continuous, soft nano contra gω-continuous, soft nano contra strongly gω-continuous and soft nano contra gω-irresolute functions in soft nano topological spaces.Various properties of soft nano contra g-closedness and soft nano g-closedness, soft nano contra wg-closedness, soft nano wg-closedness, soft nano contra gω-closedness and soft nano gω-closedness are independent notions. Important results involving soft nano almost contra gω-continuous, soft nano perfectly continuous, soft nano-bi-contra continuous maps, R_{sn}-maps and several others.

2. Preliminaries

Definition 2.1: [1] Let set of objects be denoted by U, R^1 is a soft equivalence relation and $\tau_{R^1}(X_1) = \{U, \emptyset, (L_{R^1}(X_1), O_1), (U_{R^1}(X_1), O_1), (B_{R^1}(X_1), O_1)\}$ satisfies the following axioms:

i) U and $\emptyset \in \tau_{R^1}(X_1)$

ii) the union of the elements of any finite subcollection $\tau_{R^1}(X_1)$ is in $\tau_{R^1}(X_1)$.

iii) the intersection of the elements of any finite subcollection $\tau_{R^1}(X_1)$ is in $\tau_{R^1}(X_1)$.

Then, $\tau_{R^1}(X_1)$ is soft nano topology on U with respect to X_1, elements of the soft nano topology are known as the soft nano open sets and $(\tau_{R^1}(X_1), U, O_1)$ is called a soft nano topological space.

Definition 2.2:[16] In a soft nano topological space $(\tau_{\mathbb{R}'}(X_1), U_1, O_1)$, $\mathcal{B}_{sn} = \{U_1, L_{\mathbb{R}'}(X_1), B_{\mathbb{R}'}(X_1)\}$ is soft nano basis for $\tau_{\mathbb{R}'}(X_1)$.

Definition 2.3: **[16]** A function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is sn-gω- continuous if the inverse image of every sn-open in U_2 is sn-gω open in U_1.

3. Types of Soft Nano Contra-Continuous Functions (Sn-C-Continuous Functions)

Definition 3.1: A mapping $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is a

i) soft nano contra continuous **(**briefly, sn-c-continuous) if$\mathcal{F}^{-1}(E_1^*, O_1)$ is $sn\text{-}C(X_1, O_1)$ for every $sn\text{-}O(X_2, O_2)$.

ii) soft nano contra–gω-continuous (briefly, sn-c-gω-continuous) if $\mathcal{F}^{-1}(E_1^*, O_1)$ is $sn\text{-}C(X_1, O_1)$ for every sn-gω-$O(X_2, O_2)$.

iii) soft nano contra strongly-gω-continuous (briefly, sn-c$\mathcal{S}_{g\omega}$-continuous), if $\mathcal{F}^{-1}(E_1^*, O_1)$ is soft nano clopen inU_1 for every sn-gω-$O(X_2, O_2)$.

Remark 3.2: Soft nano continuity is independent of soft nano contra continuity.

Example3.3: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$, $X_1 = \{\varepsilon_1, \varepsilon_4\} \subseteq U_1$ with $\tau_{\mathbb{R}'}(X_1) = \{U_1, \emptyset, (\kappa_1, \{\varepsilon_1, \varepsilon_4\}), (\kappa_2, \{\varepsilon_1, \varepsilon_4\}), (\kappa_3, \{\varepsilon_1, \varepsilon_4\})\}$.,$U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4'\}$, $X_2 = \{\varepsilon_1', \varepsilon_3'\}$ with $\tau_{\mathbb{R}''}(X_2) = \{U_2, \emptyset, (\kappa_1', \{\varepsilon_1', \varepsilon_3'\}), (\kappa_2', \{\varepsilon_1', \varepsilon_3'\}), (\kappa_3', \{\varepsilon_1', \varepsilon_3'\})\}$. Here,sn-closed sets are $\{\varepsilon_2, \varepsilon_3\}$ and sn-closed sets are $\{\varepsilon_2', \varepsilon_4'\}$ respectively. We define the function as $\mathcal{F}(\varepsilon_1) = \varepsilon_2'$, $\mathcal{F}(\varepsilon_2) = \varepsilon_1'$, $\mathcal{F}(\varepsilon_3) = \varepsilon_3'$ and $\mathcal{F}(\varepsilon_4) = \varepsilon_4'$. Then $\mathcal{F}^{-1}(\varepsilon_1') =\{\varepsilon_2\}$, $\mathcal{F}^{-1}(\varepsilon_2') =\{\varepsilon_1\}$, $\mathcal{F}^{-1}(\varepsilon_3') =\{\varepsilon_3\}$ and $\mathcal{F}^{-1}(\varepsilon_4') =\{\varepsilon_4\}$. Here $\mathcal{F}^{-1}(\varepsilon_2', \varepsilon_4') = \{\varepsilon_1, \varepsilon_4\}$, that is inverse image of every $sn\text{-}C(X_2, O_2)$ is $sn\text{-}O(X_1, O_1)$.

Theorem 3.4: Every soft nano perfectly continuous is soft nano contra continuous function.

Proof: As $\mathcal{F}$ is soft nano perfectly continuous,$\mathcal{F}^{-1}(V^*, O_1)$ is soft nano clopen inU_1. Let (V^*, O_1) be $sn\text{-}O(X_2, O_2)$, then$\mathcal{F}^{-1}(V^*, O_1)$ is $sn\text{-}cl(X_1, O_1)$. Therefore $\mathcal{F}$ is soft nano contra continuous.

Remark 3.5: Converse of the theorem need not be true in general.

Example 3.6: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$, $U_1/_{\mathbb{R}'} = \{\{\varepsilon_1\}, \{\varepsilon_2, \varepsilon_3\}, \{\varepsilon_4\}\}$ and let $X_1 = \{\varepsilon_1, \varepsilon_4\} \subseteq U_1$, then $\tau_{\mathbb{R}'}(X_1) = \{U_1, \emptyset, (\kappa_1, \{\varepsilon_1, \varepsilon_4\}), (\kappa_2, \{\varepsilon_1, \varepsilon_4\}), (\kappa_3, \{\varepsilon_1, \varepsilon_4\})\}$. Let $U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4'\}$, with $U_2/_{\mathbb{R}''} = \{\{\varepsilon_1', \varepsilon_3'\}, \{\varepsilon_2', \varepsilon_4'\}\}$. Let $X_2 = \{\varepsilon_1', \varepsilon_3'\} \subseteq U_2$, then

$\tau_{\mathbb{R}''}(X_2) = \{U_2, \emptyset, (\kappa_1', \{\varepsilon_1', \varepsilon_3'\}), (\kappa_2', \{\varepsilon_1', \varepsilon_3'\}), (\kappa_3', \{\varepsilon_1', \varepsilon_3'\})\}$. Then a function $\mathcal{F}$ is defined as $\mathcal{F}(\varepsilon_1) = \varepsilon_2'$, $\mathcal{F}(\varepsilon_2) = \varepsilon_1'$, $\mathcal{F}(\varepsilon_3) = \varepsilon_3'$, $\mathcal{F}(\varepsilon_4) = \varepsilon_4'$. Here $\mathcal{F}^{-1}(\{\varepsilon_1', \varepsilon_3'\}) = \{\varepsilon_2, \varepsilon_3\}$ is $sn\text{-}c(X_1, O_1)$ but not $sn\text{-}O(X_1, O_1)$. Therefore, $\mathcal{F}$ is sn contra continuous and not soft nano perfectly continuous.

Theorem 3.7: Every soft nano contra continuous is soft nano contra gω-continuous function.

Proof: Let (V^*, O_1) be sn-O(X_2, O_2), then It is sn-$g\omega$-O(X_2, O_2) [15]. As $\mathcal{F}$ is soft nano contra continuous, then $\mathcal{F}^{-1}(V^*, O_1)$ is $sn\text{-}cl(X_1, O_1)$.So it is sn-$g\omega$-C(X_1, O_1) [15]. Hence, $\mathcal{F}$ is soft nano contra gω-continuous.

Remark 3.8: The converse of the theorem 3.7 need not be true in general.

Theorem 3.9: For a function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$, we have

i) every soft nano strongly $g\omega$-continuous is soft nano continuous.

ii) every soft nano contra strongly $g\omega$-continuous is soft nano contra continuous.

iii) every soft nano contra strongly $g\omega$-continuous is soft nano contra $g\omega$-continuous.

iv) every soft nano contra strongly $g\omega$-continuous is soft nano contra $g\omega$-irresolute.

Proof: i) Let (P^*, O_1) be sn-O(X_2, O_2). Then it is sn-$g\omega$-O(X_2, O_2) [17]. As $\mathcal{F}$ is sn-c$\mathcal{S}_{g\omega}$-continuous, $\mathcal{F}^{-1}(P^*, O_1)$ is sn-O(X_1, O_1). Hence $\mathcal{F}$ is sn-continuous.

ii) Let (P^*, O_1) as a sn-O(X_2, O_2) and it is sn-$g\omega$-O(X_2, O_2)[17]. Here $\mathcal{F}^{-1}(P^*, O_1)$ is sn-O(X_1, O_1) is sn-C(X_1, O_1) as $\mathcal{F}$ is sn-c$\mathcal{S}_{g\omega}$-continuous.

iii) Follows from the fact that every soft nano contra continuous function is soft nano contra $g\omega$-continuous, the proof is obvious

iv) Obvious.

Theorem 3.10: A bijective function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is sn-contra–gω-continuous function if and only if the inverse image of every $sn\text{-}C(X_2, O_2)$ is $sn\text{-}g\omega\text{-}O(X_1, O_1)$.

Proof: Let $\mathcal{F}$ be a sn-$g\omega$-continuous and (V_1^*, O_1) is sn-$C(X_2, O_2)$. Now $U_2 - (V_1^*, O_1)$ is sn-$O(X_2, O_2)$ and $\mathcal{F}^{-1}(U_2 - (V_1^*, O_1))$ is sn-$g\omega$- $cl(X_1, O_1)$ as $\mathcal{F}$ is sn-c–gω-continuous. That implies $\mathcal{F}^{-1}(U_2) - \mathcal{F}^{-1}(V_1^*, O_1) = U_1 - \mathcal{F}^{-1}(V_1^*, O_1)$ is sn-$g\omega$-$cl(X_1, O_1)$. Thus $\mathcal{F}^{-1}(V_1^*, O_1)$ is sn-$g\omega$-$O(X_1, O_1)$, hence $\mathcal{F}$ is sn-c–gω-continuous on U_2.

Conversely, let (S_1^*, O_1) be sn-$O(X_2, O_2)$. So $U_2 - (S_1^*, O_1)$ is sn-$C(X_2, O_2)$. By the hypothesis $\mathcal{F}^{-1}(U_2 - (S_1^*, O_1))$ is sn-$g\omega$-$O(X_1, O_1)$, which implies $\mathcal{F}^{-1}(U_2) - \mathcal{F}^{-1}(S_1^*, O_1)$ = $U_1 - \mathcal{F}^{-1}(S_1^*, O_1)$ is sn-$g\omega$- $O(X_1, O_1)$. Thus $\mathcal{F}^{-1}(S_1^*, O_1)$ is sn-$g\omega$- $cl(X_1, O_1)$. Therefore the inverse image of every sn-$O(X_2, O_2)$ is sn-$g\omega$- $cl(X_1, O_1)$. Hence $\mathcal{F}$ is sn-c–gω-continuous onU_1.

Remark 3.11: Composition of two soft nano contra gω - continuous functions is a soft nano contra gω - continuous function

Example 3.12: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$, $X_1 = \{\varepsilon_4, \varepsilon_5\} \subseteq U_1$, O$_1$ = {κ$_1$, κ$_2$, κ$_3$}.$U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, O$_2$ = {κ'$_1$, κ'$_2$, κ'$_3$},$X_2 = \{\varepsilon_3', \varepsilon_4', \varepsilon_5'\} \subseteq U_2$, $U_3 = \{\varepsilon_1'', \varepsilon_2'', \varepsilon_3'', \varepsilon_4'', \varepsilon_5''\}$, O$_3$ = {κ''$_1$, κ''$_2$, κ''$_3$},$X_3 = \{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\} \subseteq U_3$. ${}^{U_1}/_{\mathbb{R}'} = \{\{\varepsilon_1\}, \{\varepsilon_2\}, \{\varepsilon_3\}, \{\varepsilon_4\}, \{\varepsilon_5\}\}$,

${}^{U_2}/_{\mathbb{R}''} = \{\{\varepsilon_1', \varepsilon_2'\}, \{\varepsilon_3', \varepsilon_5'\}, \{\varepsilon_4'\}\}$,${}^{U_3}/_{\mathbb{R}'''} = \{\{\varepsilon_1''\}, \{\varepsilon_2''\}, \{\varepsilon_3''\}, \{\varepsilon_4''\}, \{\varepsilon_5''\}\}$.

$\tau_{\mathbb{R}'}(X_1) = \{U_1, \emptyset, (\kappa_1, \{\varepsilon_4, \varepsilon_5\}), (\kappa_2, \{\varepsilon_4, \varepsilon_5\}), (\kappa_3, \{\varepsilon_4, \varepsilon_5\})\}$.

$\tau_{\mathbb{R}''}(X_2) = \{U_2, \emptyset, (\kappa_1', \{\varepsilon_3', \varepsilon_4', \varepsilon_5'\}), (\kappa_2', \{\varepsilon_3', \varepsilon_4', \varepsilon_5'\}), (\kappa_3', \{\varepsilon_3', \varepsilon_4', \varepsilon_5'\})\}$.

$\tau_{\mathbb{R}'''}(X_3) = \{U_3, \emptyset, (\kappa''_1, \{\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\}\}), (\kappa_2'', \{\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\}\}), (\kappa_3'', \{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\})\}$.

Define the function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ as $\mathcal{F}(\varepsilon_1) = \varepsilon_3'$, $\mathcal{F}(\varepsilon_2) = \varepsilon_4'$, $\mathcal{F}(\varepsilon_3) = \varepsilon_5'$, $\mathcal{F}(\varepsilon_4) = \varepsilon_1'$ and $\mathcal{F}(\varepsilon_5) = \varepsilon_2'$. And a function $\mathcal{F}_1: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ defined as $\mathcal{F}_1(\varepsilon_1') = \varepsilon_5''$, $\mathcal{F}_1(\varepsilon_2') = \varepsilon_4''$, $\mathcal{F}_1(\varepsilon_3') = \varepsilon_3''$, $\mathcal{F}_1(\varepsilon_4') = \varepsilon_2''$ and $\mathcal{F}_1(\varepsilon_5') = \varepsilon_1''$. Here $\mathcal{F}$ and $\mathcal{F}_1$ are contra sn-gω-continuous. sn-gω closed sets in U_1 are $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$, $\{\varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$ and t_n U_2 $\{\varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, $\{\varepsilon_1', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, $\{\varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$. $\mathcal{F}_1^{-1}(\varepsilon_1'', \varepsilon_2'', \varepsilon_3'')$ = $\{\varepsilon_5', \varepsilon_4', \varepsilon_3'\}$ is sn-gω closed in U_2. Here $\mathcal{F}_1$ is contra sn-gω continuous. Let (B_1^*, O_1) =$\{(\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\}), (\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\}), (\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\})\}$ then $(\mathcal{F}_1 \circ \mathcal{F})$ = $\mathcal{F}^{-1}(\mathcal{F}_1^{-1}(\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\}) = \mathcal{F}^{-1}(\varepsilon_3', \varepsilon_4', \varepsilon_5') = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ is again sn-gω continuous.

Theorem 3.13: Let $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ be a soft nano contra $g\omega$-continuous function, then soft nano restriction map induced by soft nano contra $g\omega$-continuous function:

i) $(H^*, O_1) \subseteq sn\text{-}O(X_1, O_1)$ and $\mathcal{F}$ is sn-c-$g\omega$-open, then the restriction $\mathcal{F}_{(H^*,O_1)}: ((H^*, O_1), U_{(H^*,O_1)}, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is sn-c-$g\omega$-open.

ii) $(H^*, O_1) \subseteq sn\text{-}C(X_2, O_2)$ and $\mathcal{F}$ is sn-c-$g\omega$-closed, then the restriction $\mathcal{F}_{(H^*,O_1)}: ((H^*, O_1), U_{(H^*,O_1)}, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is sn-c-$g\omega$-closed.

iii) $(H^*, O_1) = \mathcal{F}^{-1}(G^*, O_1)$ for $sn\text{-}O(X_2, O_2)$ and $\mathcal{F}$ is sn-c-$g\omega$-closed bijective function, then the restriction $\mathcal{F}_{(H^*,O_1)}: ((H^*, O_1), U_{(H^*,O_1)}, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is sn-c-$g\omega$-closed.

Proof: i) Let $(G^*, O_1) \subseteq sn\text{-}O(H^*, O_1)$. Then for some sn-open set (P^*, O_1) of $(\tau_{\mathbb{R}'}(X_1), U_1, O_1)$, $(G^*, O_1) = (H^*, O_1) \cap (P^*, O_1)$ and (G^*, O_1) is $sn\text{-}O(X_1, O_1)$. Given, $\mathcal{F}(G^*, O_1)$ is $sn\text{-}g\omega$ closed. But $\mathcal{F}(G^*, O_1) = \mathcal{F}_{(H^*,O_1)}(G^*, O_1)$ and hence $\mathcal{F}_{(H^*,O_1)}$ is sn-c-$g\omega$-open.

ii) Let $(G^*, O_1) \subseteq sn\text{-}C(H^*, O_1)$. Then for some sn-closed set (Q^*, O_1) of $(\tau_{\mathbb{R}'}(X_1), U_1, O_1)$, $(G^*, O_1) = (H^*, O_1) \cap (Q^*, O_1)$ and so (G^*, O_1) is $sn\text{-}C(X_1, O_1)$. But given, $\mathcal{F}(G^*, O_1) = \mathcal{F}_{(H^*,O_1)}(G^*, O_1)$ and hence $\mathcal{F}_{(H^*,O_1)}$ is sn-c-$g\omega$-closed.

(iii) Let $(R^*, O_1) \subseteq sn\text{-}C(H^*, O_1)$. Then for some sn-closed set (S^*, O_1) of $(\tau_{\mathbb{R}'}(X_1), U_1, O_1)$, $(R^*, O_1) = (H^*, O_1) \cap (S^*, O_1)$. But $\mathcal{F}(R^*, O_1) = \mathcal{F}_{(H^*,O_1)}(R^*, O_1) = \mathcal{F}\{(R^*, H_1) \cap (R^*, O_1)\} = \mathcal{F}\{\mathcal{F}^{-1}(R^*, H_1) \cap (R^*, O_1)\} = (G^*, O_1) \cap \mathcal{F}(R^*, O_1)$. Hence $\mathcal{F}(R^*, O_1)$ is $sn\text{-}g\omega$-open as $\mathcal{F}$ is sn-c-$g\omega$-closed and so $(G^*, O_1) \cap \mathcal{F}(R^*, O_1)$ is $sn\text{-}g\omega\text{-}O(X_2, O_2)$. Therefore $\mathcal{F}$ is sn-c-$g\omega$-closed.

4. Nature of Independency of Few Soft Nano Generalized Continuous with Their Corresponding Contra Continuous Functions

Definition 4.1: The mapping $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is a

i) soft nano contra g-continuous (briefly, sn-c-g-continuous), if $\mathcal{F}^{-1}(A^*, O_1)$ is soft nano g-closed in U_1 for every soft nano open set (A^*, O_1) in U_2 or $sn\text{-}O(X_2, O_2)$.

ii) soft nano contra wg-continuous (briefly, sn-c-wg-continuous), if $\mathcal{F}^{-1}(A^*, O_1)$ is soft nano contra wg-closed in U_1 for every soft nano open set (A^*, O_1) in U_2.

iii) soft nano contra gw-continuous (briefly, sn-c-$g\omega$-continuous), if $\mathcal{F}^{-1}(A^*, O_1)$ is soft nano gω-closed in U_1 for every soft nano open set (A^*, O_1) in U_2.

Remark 4.2: Consider the following statements:

i) The concepts of sn-c-g-continuous and sn-g- continuous functions are independent.

ii) sn-c-wg-continuity and sn-wg-continuity are independent concepts.

iii) sn-c-gω closedness and sn-$g\omega$-closedness are independent notions.

The following examples are proposed to show that the above statements are valid.

Example 4.3: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ with $U_1/_{\mathbb{R}'} = \{\{\varepsilon_1\}, \{\varepsilon_2, \varepsilon_3\}\}$, O1 = {κ1, κ2, κ3}.$X_1 = \{\varepsilon_2, \varepsilon_3\} \subseteq U_1$ then $\tau_{\mathbb{R}'}(X_1) = \{U_1, \emptyset, (\kappa_1, \{\varepsilon_2, \varepsilon_3\}), (\kappa_2, \{\varepsilon_2, \varepsilon_3\}), (\kappa_3, \{\varepsilon_2, \varepsilon_3\})\}$. Let $U_2 = \{\varepsilon'_1, \varepsilon'_2, \varepsilon'_3\}$ with $U_2/_{\mathbb{R}''} = \{\{\varepsilon'_1\}, \{\varepsilon'_2, \varepsilon'_3\}\}$, O2 = {κ'1, κ'2, κ'3}, $X_2 = \{\varepsilon'_1\} \subseteq U_2$ then $\tau_{\mathbb{R}''}(X_2) = \{U_2, \emptyset, (\kappa'_1, \{\varepsilon'_1\}), (\kappa'_2, \{\varepsilon'_1\}), (\kappa'_3, \{\varepsilon'_1\})\}$. Define a function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ as $\mathcal{F}(\varepsilon_1) = \varepsilon'_1$, $\mathcal{F}(\varepsilon_2) = \varepsilon'_3$, $\mathcal{F}(\varepsilon_3) = \varepsilon'_2$. Here $\mathcal{F}$ is sn-c-g-continuous and sn-c-$g\omega$ continuous but not sn-g-continuous and sn-$g\omega$ continuous. Here we define $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ by $\mathcal{F}_1(\varepsilon_1) = \varepsilon'_2$, $\mathcal{F}_1(\varepsilon_2) = \varepsilon'_3$, $\mathcal{F}_1(\varepsilon_3) = \varepsilon'_1$. Thus $\mathcal{F}_1$ is sn-g-continuous but not sn-c-g continuous. Also $\mathcal{F}_1$ is sn-$g\omega$ continuous but not sn-c-$g\omega$-continuous.

Example 4.4: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$, O1 = {κ1, κ2, κ3},$X_1 = \{\varepsilon_1, \varepsilon_3\}$, $U_1/_{\mathbb{R}'} = \{\{\varepsilon_1, \varepsilon_3\}, \{\varepsilon_2\}, \{\varepsilon_4\}, \{\varepsilon_5\}\}$ then$\tau_{\mathbb{R}'}(X_1) = \{U_1, \emptyset, (\kappa_1, \{\varepsilon_1, \varepsilon_3\}), (\kappa_2, \{\varepsilon_1, \varepsilon_3\}), (\kappa_3, \{\varepsilon_1, \varepsilon_3\})\}$. Let $U_2 = \{\varepsilon'_1, \varepsilon'_2, \varepsilon'_3, \varepsilon'_4, \varepsilon'_5\}$, O2 ={κ'1,κ'2,κ'3}$X_2 = \{\varepsilon'_3, \varepsilon'_5\}$,$U_2/_{\mathbb{R}''} = \{\{\varepsilon'_1\}, \{\varepsilon'_2, \varepsilon'_4\}, \{\varepsilon'_3\ \varepsilon'_5\}\}$ then $\tau_{\mathbb{R}''}(X_2) = \{U_2, \emptyset, (\kappa'_1, \{\varepsilon'_3, \varepsilon'_5\}), (\kappa'_2, \{\varepsilon'_3, \varepsilon'_5\}), (\kappa'_3, \{\varepsilon'_3, \varepsilon'_5\})\}$. Then $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is defined as $\mathcal{F}(\varepsilon_1) = \varepsilon'_3$, $\mathcal{F}(\varepsilon_2) = \varepsilon'_1$, $\mathcal{F}(\varepsilon_3) = \varepsilon'_5$, $\mathcal{F}(\varepsilon_4) = \varepsilon'_4$ and $\mathcal{F}(\varepsilon_5) = \varepsilon'_2$. Here $\mathcal{F}$ is sn-wg-continuous but not sn-c-wg-continuous.

Example 4.5: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$, $X_1 = \{\varepsilon_1, \varepsilon_2\}$, O1 = {κ1, κ2, κ3}, $U_1/_{\mathbb{R}'} = \{\{\varepsilon_1\}, \{\varepsilon_2\}, \{\varepsilon_3\}, \{\varepsilon_4\}, \{\varepsilon_5\}\}$ then $\tau_{\mathbb{R}'}(X_1)$ = {U1,ϕ, $(\kappa_1, \{\varepsilon_1, \varepsilon_2\})$, $(\kappa_2, \{\varepsilon_1, \varepsilon_2\})$, $(\kappa_3, \{\varepsilon_1, \varepsilon_2\})\}$. Let $U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, O2 = {κ′1,κ′2,κ′3},$X_2 = \{\varepsilon_2', \varepsilon_3'\}$, $U_2/_{\mathbb{R}''} = \{\{\varepsilon_1', \varepsilon_3'\}, \{\varepsilon_2'\}, \{\varepsilon_4' \ \varepsilon_5'\}\}$ then $\tau_{\mathbb{R}''}(X_2)$ =$\{U_2, \emptyset, (\kappa_1', \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}), (\kappa_2', \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}), (\kappa_3', \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\})\}$. Then $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is defined as $\mathcal{F}(\varepsilon_1) = \varepsilon_4'$, $\mathcal{F}(\varepsilon_2) = \varepsilon_5'$, $\mathcal{F}(\varepsilon_3) = \varepsilon_1'$, $\mathcal{F}(\varepsilon_4) = \varepsilon_2'$ and $\mathcal{F}(\varepsilon_5) = \varepsilon_3'$. Here $\mathcal{F}$ is sn-c-wg-continuous but not sn wg-continuous.

5. SOFT NANO CONTRA gω-IRRESOLUTE FUNCTIONS

Definition 5.1: A function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is said to be soft nano contra gω-irresolute (briefly, sn-c- $g\omega$-irresolute), if $\mathcal{F}^{-1}(M_1^*, O_1)$ is sn-$g\omega$-C(X_1, O_1) for each sn-$g\omega$-C(X_2, O_2) of U_2.

Theorem 5.2: Let a soft nano-$g\omega$-continuous function $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ and soft nano $\mathcal{S}_{g\omega}$-continuous function $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-gω-irresolute.

Proof: Let(P^*, O_1) be sn-$g\omega$-C(X_3, O_3). Here $\mathcal{F}_2^{-1}(P^*, O_1)$ is sn-C(X_2, O_2) as$\mathcal{F}_2$ is $\mathcal{S}_{sn-g\omega}$-continuous and $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(P^*, O_1))$ is sn-$g\omega$-C(X_1, O_1). But $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(P^*, O_1))$ = $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Thus $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-$g\omega$-irresolute.

Theorem 5.3: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ be soft nano contra strongly-$g\omega$-continuous and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ is sn-$g\omega$-continuous, $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-continuous.

Proof: Let (M^*, O_1) be sn-O(X_3, O_3). As $\mathcal{F}_2$ is sn-$g\omega$-continuous, $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-$g\omega$-O(X_2, O_2). Hence $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1))$ is sn-$g\omega$-O(X_1, O_1). Also, $\mathcal{F}_1$ is sn-contra-strongly-$g\omega$-continuous, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1))$ = $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(M^*, O_1)$ is sn-C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-contra continuous.

Theorem 5.4: Consider a soft nano contra $g\omega$-continuous function $\mathcal{F}_1 : (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ and sn-c$\mathcal{S}_{g\omega}$-continuous function $\mathcal{F}_2 : (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-gω-irresolute.

Proof: Let (M^*, O_1) be sn-$g\omega$-O(X_3, O_3). As $\mathcal{F}_2$ is sn-$\mathcal{S}_{g\omega}$-continuous, $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-O(X_2, O_2). Hence $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1))$ is sn-$g\omega$-C(X_1, O_1) as $\mathcal{F}_1$ is sn-c-$g\omega$-continuous. Therefore $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(M^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-$g\omega$-irresolute.

Theorem 5.5: Let $\mathcal{F}_1 : (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ and $\mathcal{F}2$: $(\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ be functions, if

i) $\mathcal{F}_1$ is sn-c-$g\omega$-irresolute and $\mathcal{F}_2$ is sn-c-$g\omega$-continuous, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-gω-continuous.

ii) $\mathcal{F}_1$ is sn-c-$g\omega$-irresolute and $\mathcal{F}_2$ is sn-$g\omega$-irresolute, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-gω-irresolute.

iii) $\mathcal{F}_1$ is sn-$g\omega$-irresolute and $\mathcal{F}_2$ is sn-c-continuous, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-gω-continuous.

iv) $\mathcal{F}_1$ is sn-$g\omega$-irresolute and $\mathcal{F}_2$ is sn-c-$g\omega$-irresolute, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-gω-irresolute.

v) $\mathcal{F}_1$ is sn-c-$g\omega$-continuous and $\mathcal{F}_2$ is sn-continuous, then $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-gω-continuous.

Proof: i) Consider a sn-O(X_3, O_3), (V^*, O_1) and a sn-$g\omega$-continuous function $\mathcal{F}_2$. So $\mathcal{F}_2^{-1}(V^*, O_1)$ is sn-gω-O(X_2, O_2). As $\mathcal{F}_1$ is sn-c-$g\omega$-irresolute, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1))$ is sn-$g\omega$-C(X_1, O_1). Hence $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is sn- -C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-$g\omega$-continuous.

ii) Let (V^*, O_1) be a sn-$g\omega$-O(X_3, O_3). As $\mathcal{F}_2$ is sn-$g\omega$-irresolute, $\mathcal{F}_2^{-1}(V^*, O_1)$ is sn-gω-O(X_2, O_2). As $\mathcal{F}_1$ is sn-c-$g\omega$-irresolute, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-$g\omega$-irresolute.

iii) Let (V^*, O_1) be a sn-O(X_3, O_3), it is sn-$g\omega$-O(X_3, O_3). As $\mathcal{F}_2$ is sn-$g\omega$-irresolute, $\mathcal{F}_2^{-1}(V^*, O_1)$ is sn-gω-O(X_2, O_2). Here $\mathcal{F}_1$ is sn-c-$g\omega$-continuous. Hence $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-$g\omega$-continuous.

iv) Let (V^*, O_1) be a sn-$g\omega$-O(X_3, O_3). As $\mathcal{F}_2$ is sn-c-$g\omega$-irresolute, $\mathcal{F}_2^{-1}(V^*, O_1)$ is sn-gω-C(X_2, O_2). As $\mathcal{F}_1$ is sn-$g\omega$-irresolute, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-$g\omega$-irresolute.

v) Let (V^*, O_1) be a sn-O(X_3, O_3). As $\mathcal{F}_2$ is sn-continuous, $\mathcal{F}_2^{-1}(V^*, O_1)$ is sn-O(X_2, O_2). As $\mathcal{F}_1$ is sn-c-$g\omega$-continuous, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is sn-c-$g\omega$-continuous.

6. SOFT NANO ALMOST CONTRA gω-CONTINUOUS FUNCTIONS

In this section, soft nano almost contra gω-continuous functions are introduced and their properties are studied.

Definition 6.1: A function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano almost contra gω-continuous, if $\mathcal{F}^{-1}(V^*, O_1)$ is sn-$g\omega$-closed in U_1 for each sn-regular open (V^*, O_1) in U_2.

Theorem 6.2: If the function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano almost-contra gω-continuous, then $\mathcal{F}^{-1}(M^*, O_1)$ is sn-$g\omega$-open in U_1 for each soft nano regular closed set (M^*, O_1) is U_2.

Proof: Let (M^*, O_1) be sn-r-C(X_2, O_2). Then $U_2 - (M^*, O_1)$ is sn-regular open. As $\mathcal{F}$ is sn-almost contra-gω-continuous, then $\mathcal{F}^{-1}(U_2 - (M^*, O_1)) = U_1 - \mathcal{F}^{-1}(M^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Hence $\mathcal{F}^{-1}(M^*, O_1)$ is sn-$g\omega$-O(X_1, O_1).

Theorem 6.3: Let (P^*, O_1) be soft nano-subset of U_1 and if the function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano-almost contra gω-continuous function, then the restriction $\mathcal{F}_{(P^*, O_1)}: \left((P^*, O_1), U_{(P^*, O_1)}, O_1\right) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is also soft nano-almost contra gω-continuous.

Proof: Let (M^*, O_1) be sn-r-C(X_2, O_2). As $\mathcal{F}$ is sn-almost-contra-gω-continuous, $\mathcal{F}^{-1}(M^*, O_1)$ is sn-$g\omega$-O(X_1, O_1). Now $(\mathcal{F}_{(P^*, O_1)})^{-1}(M^*, O_1) = (P^*, O_1) \cap \mathcal{F}^{-1}(M^*, O_1)$ where $\mathcal{F}^{-1}(M^*, O_1)$ is sn-$g\omega$-open set in (P^*, O_1). Hence $\mathcal{F}_{(P^*, O_1)}$ is sn-almost-contra-gω-continuous.

Theorem 6.4: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano-almost-contra-gω-continuous and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ is soft nano-almost-contra-gω-continuous, then $\mathcal{F}_2 \circ \mathcal{F}_1$ soft nano-almost-contra-gω-continuous.

Proof: Let (P^*, O_1) be sn-r-O(X_3, O_3). As $\mathcal{F}_2$ is sn-almost-continuous, $\mathcal{F}_2^{-1}(P^*, O_1)$ is sn-O(X_2, O_2). Since $\mathcal{F}_1$ is sn-almost-contra-gω-continuous, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(P^*, O_1))$ = $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Hence $\mathcal{F}_2 \circ \mathcal{F}_1$ soft nano-almost-contra-gω-continuous.

Theorem 6.5: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano-almost contra gω-continuous and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ is soft nano perfectly continuous, then $\mathcal{F}_2 \circ \mathcal{F}_1$ soft nano-contra-gω-continuous.

Proof: Let (P^*, O_1) be sn-O(X_3, O_3). As $\mathcal{F}_2$ is sn-perfectly continuous function, $\mathcal{F}_2^{-1}(P^*, O_1)$ is sn-clopen in U_2. As $\mathcal{F}_1$ is sn-almost contra gω-continuous, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(P^*, O_1))$ = $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Hence $\mathcal{F}_2 \circ \mathcal{F}_1$ sn-contra-gω-continuous.

Theorem 6.6: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano-almost contra gω-continuous and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \to (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ is R_{sn}-map, then $\mathcal{F}_2 \circ \mathcal{F}_1$ soft nano-almost-contra-gω-continuous.

Proof: Let (P^*, O_1) be sn-r-O(X_3, O_3). As $\mathcal{F}_2$ is R_{sn}-map, $\mathcal{F}_2^{-1}(P^*, O_1)$ is sn-r-O(X_2, O_2). As $\mathcal{F}_1$ is sn-almostc contra gω-continuous. Thus $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(P^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1)$ is sn-$g\omega$-C(X_1, O_1). Hence $\mathcal{F}_2 \circ \mathcal{F}_1$ soft nano almost contra gω-continuous.

7. Soft Nano- Bi-Contra-Continuous Functions

Definition 7.1: A surjective function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is called a soft nano bi-contra continuous if $\mathcal{F}$ is soft nano contra $g\omega$-continuous and $\mathcal{F}^{-1}(P^*, O_1)$ is soft nano open in U_1 implied (P^*, O_1) is soft nano $g\omega$-closed in U_2.

Example 7.2: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$ with $U_1/_{\mathbb{R}'} = \{\{\varepsilon_1, \varepsilon_2\}, \{\varepsilon_3, \varepsilon_4\}, \{\varepsilon_5\}\}$, $X_1 = \{\varepsilon_3, \varepsilon_4\} \subseteq U_1$ then $\tau_{\mathbb{R}'}(X_1) = \{U_1, \emptyset, \{\varepsilon_3, \varepsilon_4\}\}$. $U_2 = \{\varepsilon'_1, \varepsilon'_2, \varepsilon'_3, \varepsilon'_4, \varepsilon'_5\}$

with ${}^{U_2}/_{\mathbb{R}''} = \{\{\varepsilon_1', \varepsilon_2'\}, \{\varepsilon_3', \ \varepsilon_4', \ \varepsilon_5'\}\} X_2 = \{\varepsilon_3', \varepsilon_4', \varepsilon_5'\} \subseteq U_2$ then $\tau_{\mathbb{R}''}(X_2) = \{U_2, \emptyset, \{\varepsilon_3', \varepsilon_4', \varepsilon_5'\}\}$.

Then $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is defined as $\mathcal{F}(\varepsilon_1) = \varepsilon_4'$, $\mathcal{F}(\varepsilon_2) = \varepsilon_5'$, $\mathcal{F}(\varepsilon_3) = \varepsilon_1'$, $\mathcal{F}(\varepsilon_4) = \varepsilon_2'$ and $\mathcal{F}(\varepsilon_5) = \varepsilon_3'$. Here $\mathcal{F}$ is soft nano contra continuous as $\mathcal{F}^{-1}(\{\varepsilon_3', \varepsilon_4', \ \varepsilon_5'\}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}$ that is inverse image of is sn-$O(X_2,\ O_2)$ is is sn-$c(X_1,\ O_1)$. Also, $\mathcal{F}^{-1}(\{\varepsilon_1', \varepsilon_2'\}) = \{\varepsilon_3, \varepsilon_4\}$ is sn-$O(X_1,\ O_1)$. Therefore $\mathcal{F}$ is soft nano bi-contra continuous function.

Theorem 7.3: Composition of two soft nano bi-contra continuous map is again soft nano bi-contra continuous map.

Proof: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \rightarrow (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ be soft nano bi-contra continuous. Let (V^*, O_1) be a sn-O(X_3, O_3). As $\mathcal{F}_2$ is soft nano continuous, ${\mathcal{F}_2}^{-1}$ (V^*, O_1) is sn-$O(X_2,\ O_2)$ and as $\mathcal{F}_1$ is soft nano-bi-contra continuous ${\mathcal{F}_1}^{-1}$ $({\mathcal{F}_2}^{-1}(V^*, O_1))$ is sn-C(X_1, O_1). $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ = ${\mathcal{F}_1}^{-1}$ $({\mathcal{F}_2}^{-1}(V^*, O_1))$ and thus $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is sn-$C(X_1,\ O_1)$. Here ${\mathcal{F}_1}^{-1}$ $({\mathcal{F}_2}^{-1}(V^*, O_1))$ is sn-O(X_1, O_1) as $\mathcal{F}_1$ is sn-open, $\mathcal{F}_1({\mathcal{F}_1}^{-1}$ $({\mathcal{F}_2}^{-1}(V^*, O_1)))$ = ${\mathcal{F}_2}^{-1}(V^*, O_1)$ is sn-O(X_2, O_2). Again $\mathcal{F}_2$ is sn-c-continuous. Here (V^*, O_1) is sn-$C(X_3,\ O_3)$. Here $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1)$ is soft nano-bi-contra continuous.

Theorem 7.4: The function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is a soft nano bi-contra continuous map and onto-mapping. Let (M^*, O_1) be sn-clopen subset of U_1, then the restriction $\mathcal{F}_{(M^*, O_1)}$: $[(M^*, O_1),\ (\tau_{\mathbb{R}'}(M^*, O_1), U_1, O_1)] \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is sn-bi-contra continuous map.

Proof: By hypothesis $\mathcal{F}$ is sn-bi-contra continuous. Let (P^*, O_1) be sn-O(X_2, O_2), then $\mathcal{F}^{-1}(P^*, O_1)$ is sn-O(X_1, O_1). Since $\mathcal{F}$ is sn-contra continuous. As (M^*, O_1) is clopen, $\mathcal{F}^{-1}(M^*, O_1) \cap (P^*, O_1)$ is sn-C(M^*, O_1). Here ${\mathcal{F}^{-1}}_{(M^*, O_1)}(P^*, O_1)$ = $\mathcal{F}^{-1}(P^*, O_1) \cap (M^*, O_1)$. Since (M^*, O_1) is sn-clopen, $\mathcal{F}^{-1}(P^*, O_1)$ is sn-O(X_1, O_1) and as $\mathcal{F}$ is sn-bi-contra continuous (P^*, O_1) is sn-C(X_2, O_2).

Remark 7.5: The condition of assuming (M^*, O_1) as both sn-closed and sn-open cannot be given up in the above theorem as taken in the example 7.2.

8. Soft Nano-Bi-Contra gω - Continuous Functions

Definition 8.1: An onto map $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is called a soft nano-bi-contra-gω-continuous, if $\mathcal{F}$ is soft nano contra-gω-continuous and $\mathcal{F}^{-1}(M^*, O_1)$ is soft nano open in U_1 implies (M^*, O_1) is soft nano-gω-closed.

Theorem 8.2: Every soft nano-bi contra continuous is soft nano bi-contra gω- continuous.

Proof: Let $\mathcal{F}^{-1}(M^*, O_1)$ be sn-O(X_1, O_1). We know that soft nano-contra-continuous implies soft-nano-contra-gω-continuous. As $\mathcal{F}$ is soft nano-bi-contra-continuous, (M^*, O_1) is soft nano-gω-closed in U_2.Therefore, (M^*, O_1) is soft nano- gω-C(X_2, O_2).

Remark 8.3: The converse of above theorem is not true always.

Example 8.4: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$, $O_1 = \{k_1, k_2, k_3\}$, $X_1 = \{\varepsilon_3, \varepsilon_4\}$, $\tau_{\mathbb{R}'}(X_1) = \{(k_1, \{\varepsilon_3, \varepsilon_4\}), (k_2, \{\varepsilon_3, \varepsilon_4\}), (k_3, \{\varepsilon_3, \varepsilon_4\})\}$. $U_2 = \{\varepsilon'_1, \varepsilon'_2, \varepsilon'_3, \varepsilon'_4\}$, $O_2 = \{k'_1, k'_2, k'_3\}$, $X_2 = \{\varepsilon'_1, \varepsilon'_3\}$, $\tau_{\mathbb{R}''}(X_2) = \{(k'_1, \{\varepsilon'_1, \varepsilon'_3\}), (k'_2, \{\varepsilon'_1, \varepsilon'_3\}), (k'_3, \{\varepsilon'_1, \varepsilon'_3\})\}$. Define a function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ as $\mathcal{F}(\varepsilon_1) = \mathcal{F}(\varepsilon_2) = \varepsilon'_1$, $\mathcal{F}(\varepsilon_3) = \varepsilon'_2 \mathcal{F}(\varepsilon_4) = \varepsilon'_4$ and $\mathcal{F}(\varepsilon_5) = \varepsilon'_3$. Here sn-open sets in U_1 are $(A^*_1, O_1) = \{(k_1, \{\varepsilon_3, \varepsilon_4\}), (k_2, \{\varepsilon_3, \varepsilon_4\}), (k_3, \{\varepsilon_3, \varepsilon_4\})\}$, sn-closed sets in U_1 are $(A^*_1, O_1)^c = \{(k_1, \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}), (k_2, \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}), (k_3, \{\varepsilon_1, \varepsilon_2, \varepsilon_5\})\}$, sn-g$\omega$-closed sets in U_1 are $(B^*_1, O_1) = \{(k_1, \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}), (k_2, \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}), (k_3, \{\varepsilon_1, \varepsilon_2, \varepsilon_5\})\}$ $(B^*_2, O_1) = \{(k_1, \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_5\}), (k_2, \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_5\}), (k_3, \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_5\})\}$ $(B^*_4, O_1) = \{(k_1, \{\varepsilon_1, \varepsilon_2, \varepsilon_4, \varepsilon_5\}),\}), (k_2, \{\varepsilon_1, \varepsilon_2, \varepsilon_4, \varepsilon_5\}),\}), (k_3, \{\varepsilon_1, \varepsilon_2, \varepsilon_4, \varepsilon_5\}),\})\}$ and sn-open sets in U_2 is $(C^*_1, O_2) = \{(k'_1,\{\varepsilon'_1, \varepsilon'_3\}), (k'_2,\{\varepsilon'_1, \varepsilon'_3\}), (k'_3,\{\varepsilon'_1, \varepsilon'_3\})\}$, sn-closed set in U_2 is $(C^*_1, O_2)^c = \{(k'_1,\{\varepsilon'_2, \varepsilon'_4\}), (k'_2,\{\varepsilon'_2, \varepsilon'_4\}), (k'_3,\{\varepsilon'_2, \varepsilon'_4\})\}$, sn-g$\omega$-closed sets in U_2 are $(D^*_1, O_2) = \{(k'_1,\{\varepsilon'_1, \varepsilon'_3\}), (k'_2,\{\varepsilon'_1, \varepsilon'_3\}), (k'_3,\{\varepsilon'_1, \varepsilon'_3\})\}$, $(D^*_2, O_2) = \{(k'_1,\{\varepsilon'_2, \varepsilon'_4\}), (k'_2,\{\varepsilon'_2, \varepsilon'_4\}), (k'_3,\{\varepsilon'_2, \varepsilon'_4\})\}$, $(D^*_3, O_2) = \{(k'_1,\{\varepsilon'_1, \varepsilon'_2, \varepsilon'_4\}), (k'_2,\{\varepsilon'_1, \varepsilon'_2, \varepsilon'_4\}), (k'_3,\{\varepsilon'_1, \varepsilon'_2, \varepsilon'_4\})\}$, $(D^*_4, O_2) = \{(k'_1, \{\varepsilon'_2, \varepsilon'_3, \varepsilon'_4\}), (k'_2, \{\varepsilon'_2, \varepsilon'_3, \varepsilon'_4\}), (k'_3, \{\varepsilon'_2, \varepsilon'_3, \varepsilon'_4\})\}$.Here $\mathcal{F}^{-1}(\varepsilon'_1, \varepsilon'_3) = \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}$ that is inverse image of sn-open set in U_2 is sn-gω-closed in U_1. Hence $\mathcal{F}$ is sn-contra gω-continuous and $\mathcal{F}^{-1}(\varepsilon'_2, \varepsilon'_4)$ is sn-open in U_1 implies $\{\varepsilon'_2, \varepsilon'_4\}$ is sn-gω-closed in U_2. Therefore, $\mathcal{F}$ is sn-bi-contra-gω-continuous map.If we define $\mathcal{F}(\varepsilon_1) = \varepsilon'_1, \mathcal{F}(\varepsilon_2) = \varepsilon'_2$, $\mathcal{F}(\varepsilon_3) = \varepsilon'_3$, $\mathcal{F}(\varepsilon_4) = \varepsilon'_4$ and $\mathcal{F}(\varepsilon_5) = \varepsilon'_5$, then

$\mathcal{F}^{-1}(\varepsilon_1', \varepsilon_3')$ $=\{\varepsilon_1, \varepsilon_2, \varepsilon_5\}$ that is inverse image of sn-open set in U_2 is not sn-open set in U_1. Hence $\mathcal{F}$ is not sn-bi-contra continuous map.

Theorem 8.5: If $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is onto, soft-nano-contra gω-continuous and soft nano-contra gω-open, then $\mathcal{F}$ is soft nano-bi-contra gω-continuous.

Proof: Let (M^*, O_1) be sn-cl(X_2, O_2), $\mathcal{F}^{-1}(M^*, O_1)$ is sn-O(X_1, O_1). As $\mathcal{F}$ is soft nano contra-gω-open, $\mathcal{F}(\mathcal{F}^{-1}(M^*, O_1))$ = (M^*, O_1) is sn-gω-closed. Therefore $\mathcal{F}$ is soft nao-contra gω-continuous.

Theorem 8.6: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ be an open surjective, sn-gω-irresolute and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \longrightarrow (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ be a soft nano-bi-contra gω-continuous map. Then $\mathcal{F}_2 \circ \mathcal{F}_1$ is soft nano-bi-contra gω-continuous map.

Proof: Let (M^*, O_1) be a sn-O(X_3, O_3). As $\mathcal{F}_2$ is soft nano-bi-contra gω-continuous, $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-gω-cl(X_2, O_2). Since $\mathcal{F}_2$ is soft nano gω-irresolute, $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1)$) is sn-gω-cl(X_1, O_1). Thus $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}$ (M^*, O_1) = $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1)$) is sn-gω-cl(X_1, O_1). Therefore $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}$ is sn-contra gω-continuous.

Let $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}$ (M^*, O_1) = $\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(M^*, O_1)$) be sn-O(X_1, O_1). As $\mathcal{F}_1$ is sn-open and surjective $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-O(X_2, O_2). As $\mathcal{F}_2$ is soft nano-bi-contra-gω-continuous, (M^*, O_1) is sn-gω-cl(X_3, O_3). Therefore $(\mathcal{F}_2 \circ \mathcal{F}_1)$ is sn-bi-contra-gω-continuous.

Theorem 8.7: If $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is soft nano-bi-contra gω-continuous map and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \longrightarrow (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ is a continuous map, where $(\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ is a space that is constant on each set $\mathcal{F}_1^{-1}(\{(P^*, O_1)\})$, for ($P^*$, O_1) $\in (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$, then $\mathcal{F}_2$ induces a soft nano-bi-contra gω-continuous map $\mathcal{F}_3: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \longrightarrow (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ such that $\mathcal{F}_3 \circ \mathcal{F}_1 = \mathcal{F}_2$.

9. Soft Nano Strongly-Bi-Contra gω-Continuous Maps

Definition 9.1: Let $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ be an onto map. Then $\mathcal{F}$ is called a soft nano strongly-bi-contra gω-continuous provided $\mathcal{F}$ is soft nano contra gω-continuous and $\mathcal{F}^{-1}(M^*, O_1)$ is soft nano open in U_1, if and only if (M^*, O_1) is soft nano gω-closed in U_2.

Definition 9.2: The function $\mathcal{F}: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ is called soft nano bi-contra (gω)*-continuous, if $\mathcal{F}$ is soft nano contra gω-irresolute and $\mathcal{F}^{-1}(M^*, O_1)$ is sn-gω open in U_1, if and only if (M^*, O_1) is soft nano closed in U_2.

Theorem 9.3: Each soft nano-bi contra-(gω)*-continuous map is soft nano-bi-contra gω-continuous map.

Proof: Proof is obvious.

Theorem 9.4: Let $\mathcal{F}_1: (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \longrightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2)$ be a onto, soft nano strongly gω-open and soft nano gω-irresolute map and $\mathcal{F}_2: (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \longrightarrow (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)$ be a soft nano-bi-contra (gω)*-continuous map. Then $\mathcal{F}_2 \circ \mathcal{F}_1$ is an soft nano-bi-contra (gω)*-continuous map.

Proof: Let (M^*, O_1) soft nano gω-open in U_3. As $\mathcal{F}_2$ is soft nano-bi-contra (gω)*-continuous, $\mathcal{F}_2^{-1}(M^*, O_1)$ is soft nano gω-closed in U_2. Since $\mathcal{F}_1$ is soft nano strongly-gω-open and onto, $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-gω-O(X_2, O_2). Also, $\mathcal{F}_2$ is soft nano-bi-contra (gω)*-continuous and so $\mathcal{F}_2^{-1}(M^*, O_1)$ is soft nano gω-open if and only if (M^*, O_1) is soft nano closed in U_3. Therefore $\mathcal{F}_2 \circ \mathcal{F}_1$ is an soft nano-bi-contra-(gω)*-continuous.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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