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INFLUENCE OF HALL EFFECTS ON PERISTALTIC FLOW OF A CARREAU FLUID IN AN ASYMMETRIC CHANNEL

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Abstract: In this paper, the peristaltic flow of a Carreau fluid in an asymmetric channel under the long wavelength assumption is discussed in the presence of Hall. The flow is examined in a wave frame of reference moving with velocity of the wave. A regular perturbation technique is employed to solve the present problem and solutions are expanded in a power of small Weissenberg number. Expressions for the velocity, axial pressure gradient and pressure rise over a one wavelength are obtained. The effects of various emerging parameters on axial pressure gradient and pumping characteristics are discussed in detail.

Keywords: asymmetric channel; Carreau fluid; Hall effects; peristaltic flow.

2010 AMS Subject Classification: 76Z05, 76D05

1. INTRODUCTION

The mechanics of peristaltic has been examined by a number of investigators. Latham [10] discussed for the first time about peristalsis in his thesis. Later, Shapiro et al. [18] worked on very similar lines. Lew et al. [11] suggested chyme in the small intestine as a non-Newtonian fluid. Shukla et al. [19] investigated the effects of peripheral - layer viscosity on peristaltic transport of a bio-fluid in a uniform tube and used the long wave length approximation as in Shapiro et al [18]. Bohme and Friedrich [4] discussed the peristaltic flow of a viscoelastic liquid

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assuming that the relevant Reynolds number to be small enough to neglect inertia forces and ratio of the wave length and channel hight to be large which implies that the pressure is constant over the cross-section. Pozrikids [13] considered peristaltic flow under the assumption of creeping motion and used boundary integral method for Stokes flow. Srivastava and Srivastava [21, 22] showed the effects of power-law fluid in uniform and non-uniform tubes and in a channel under zero Reynolds number and long wavelength approximations. Siddiqui and Schwarz [20] illustrated the peristaltic flow of a second order fluid in tubes and used a perturbation method to second order in dimensionless wave number. Provost and Schwartz [14] have studied viscous effects in peristaltic pumping and assumed that the flow is free of inertial effects and that non-Newtonian normal stresses are negligible. El Misery et al. [5] have studied peristaltic transport of Carreau fluid through a uniform channel, under zero Reynolds number and long wave length approximations. Elshahawey et al. [6] have investigated peristaltic transport of Carreau fluid through non-uniform channel, under zero Reynolds number and long wave length approximations. Elshehaway et al. [7] have analyzed peristaltic pumping of Carreau fluid through a porous medium in a channel.

The Hall effect is important when the Hall parameter which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency is high; this can occur if the collision frequency is low or when the magnetic field is high. This is a current trend in magnetohydrodynamics because of its important influence of the electromagnetic force. Hayat et al. [8] studied the Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Abo-Eldahab et al. [1] have investigated the effects of Hall and ion-slip currents on magnetohydrodynamic peristaltic transport and couple stress fluid. Subba Narasimhudu and Subba Reddy [24] have studied the Hall effects on the peristaltic flow of a Hyperbolic tangent fluid in a channel. Shalini and Rajasekhar [17] have investigated the effect of Hall on peristaltic flow of a Newtonian fluid through a porous medium in a two-dimensional channel.

Much attention had been confined to symmetric channels or tubes, but there exist also flows which may not be symmetric. Mishra and Ramachandra Rao [12] studied the peristaltic flow of a Newtonian fluid in an asymmetric channel in a recent research. In another attempt, Ramachandra Rao and Mishra [15] discussed the non-linear and curvature effects on peristaltic flow of a Newtonian fluid in an asymmetric channel when the ratio of channel width to the wave length is small. Subba Reddy et al. [25] have studied the peristaltic motion of a power - law fluid in an asymmetric channel under lubrication approach. Peristaltic flow of a Carreau fluid in an asymmetric channel has been studied by Ali and Hayat [2]. Hayat et al. [9] have studied the peristaltic transport of Johnson-Segalman fluid in an asymmetric channel. Effect of variable viscosity on the peristaltic flow of a Newtonian fluid in an asymmetric channel under the effect of a magnetic field has been investigated by Reddappa et al. [16].

However, the study of the Hall effects on peristaltic flow of a Carreau fluid in an asymmetric channel has received little attention. Hence, an attempt is made to model the Hall effects on peristaltic flow of a Carreau fluid in an asymmetric channel under the long wavelength assumption. The flow is examined in a wave frame of reference moving with velocity of the wave. A regular perturbation technique is employed to solve the present problem and solutions are expanded in a power of small Weissenberg number. Expressions for the velocity, axial pressure gradient and pressure rise over a one wavelength are obtained. The effects of various emerging parameters on axial pressure gradient and pumping characteristics are discussed in detail.

2. MATHEMATICAL FORMULATION

A two-dimensional flow of an incompressible electrically conducting Carreau fluid in an asymmetric channel induced by sinusoidal wave trains propagating with constant speed along the channel walls is considered. A uniform magnetic field B_0 applied in the transverse direction to the flow. Fig. 1 represents the physical model of the flow field.

The channel walls are given by

$$Y = H_1(x,t) = d + b_1 \cos\left[\frac{2\pi}{\lambda}(X - ct)\right] \tag{2.1a}$$

$$Y = H_2(X,t) = -d - b_2 \cos\left[\frac{2\pi}{\lambda}(X - ct) + \theta\right] \tag{2.1b}$$

where b_1, b_2 are the amplitudes of the upper and lower waves, λ is the wavelength, θ is the phase difference which varies in the range $0 \le \theta \le \pi$ and t is the time and (X,Y) are the Cartesian co-ordinates in a fixed frame.

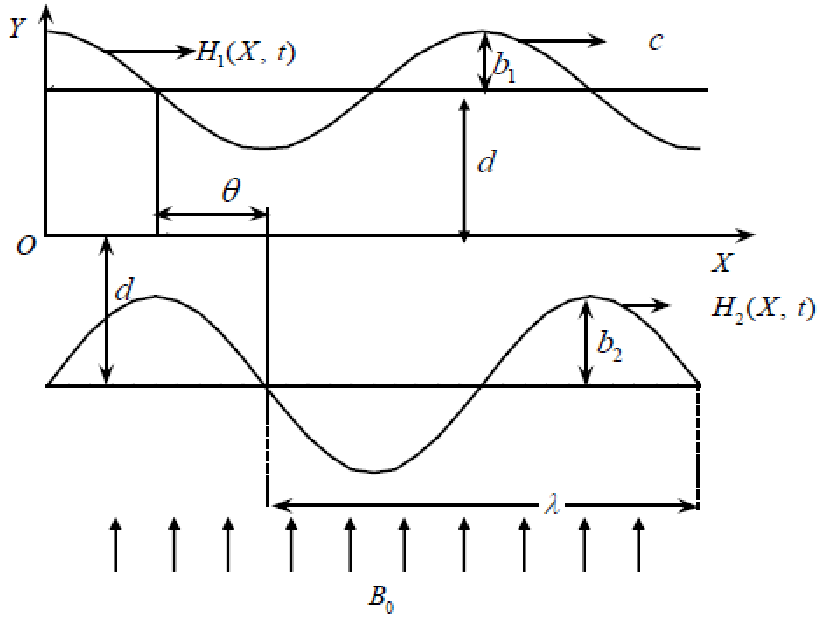

Fig. 1 Schematic diagram of the asymmetric channel

We introduce a wave frame of reference (x, y) moving with the velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant (Shapiro et al., 1969).

The transformation from the fixed frame of reference (X,Y) to the wave frame of reference (x, y) is given by

$$x = X - ct,\ \ y = Y,\ \ u = U - c,\ \ v = V,\ \ p(x) = P(X,t). \tag{2.2}$$

where (u,v) and (U,V) are the velocity components, p and P are the pressures in the wave and fixed frames of reference respectively.

The constitute equation for a Carreau fluid (following Bird et al. [3]) is

$$\tau = -\left[\eta_\infty + (\eta_0 - \eta_\infty)\left(1 + (\Gamma\dot{\gamma})^2\right)^{\frac{n-1}{2}}\right]\dot{\gamma} \tag{2.3}$$

where τ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, n is the dimensionless power-law index and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\pi} \qquad (2.4)$$

where π is the second invariant of strain-rate tensor. We consider in the constitutive Equation (2.3) the case for which $\eta_\infty = 0$ and so we can write

$$\tau = -\eta_0 \left(1 + \left(\Gamma \dot{\gamma}\right)^2\right)^{\frac{n-1}{2}} \dot{\gamma} \qquad (2.5)$$

The above model reduces to a Newtonian model for $n = 1$ (or) $\Gamma = 0$.

The equations governing the flow field, in the wave frame of reference are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.6)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} + \frac{\sigma B_0^2}{1+m^2}\left(mv - \left(u + c\right)\right) \qquad (2.7)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} - \frac{\sigma B_0^2}{1+m^2}\left(m\left(u + c\right) + v\right) \qquad (2.8)$$

where ρ is the density, σ is the electrical conductivity, B_0 is constant transverse magnetic field and m is the Hall parameter.

The boundary conditions for the velocity are

$$u = -c \quad \text{at} \quad y = H_1, H_2 \qquad (2.9)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{d},\ \bar{u} = \frac{u}{c},\ \bar{v} = \frac{v}{c\delta},\ \delta = \frac{d}{\lambda}, \bar{p} = \frac{pa^2}{\eta_0 c\lambda},\ h_1 = \frac{H_1}{d},$$
$$h_2 = \frac{H_2}{d},\ \bar{t} = \frac{ct}{\lambda},\ \bar{\tau}_{xx} = \frac{\lambda}{\eta_0 c}\tau_{xx}, \bar{\tau}_{xy} = \frac{d}{\eta_0 c}\tau_{xy},\ \bar{\tau}_{yy} = \frac{d}{\eta_0 c}\tau_{yy}, \qquad (2.10)$$
$$\mathrm{Re} = \frac{\rho d c}{\eta_0}, We = \frac{\Gamma c}{d},\ \bar{\dot{\gamma}} = \frac{\dot{\gamma} d}{c},\ \bar{q} = \frac{q}{dc}, a = \frac{b_1}{d}, b = \frac{b_2}{d}$$

where Re is the Reynolds number, We - Weissenberg number and δ - the wave number.

In view of Equation (2.10), the Equations (2.6) - (2.8), after dropping bars, reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.11}$$

$$\operatorname{Re}\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}-\delta^2\frac{\partial\tau_{xx}}{\partial x}-\frac{\partial\tau_{yx}}{\partial y}+\frac{M^2}{1+m^2}\left(m\delta v-(u+1)\right) \tag{2.12}$$

$$\operatorname{Re}\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\delta^2\frac{\partial\tau_{xy}}{\partial x}-\delta\frac{\partial\tau_{yy}}{\partial y}-\frac{\delta M^2}{1+m^2}\left(m(u+1)+\delta v\right) \tag{2.13}$$

where $\tau_{xx}=-2\left[1+\left(\frac{n-1}{2}\right)We^2\dot{\gamma}^2\right]\frac{\partial u}{\partial x}$,

$$\tau_{xy}=-\left[1+\left(\frac{n-1}{2}\right)We^2\dot{\gamma}^2\right]\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right),$$

$$\tau_{yy}=-2\delta\left[1+\left(\frac{n-1}{2}\right)We^2\dot{\gamma}^2\right]\frac{\partial u}{\partial y},$$

$$\dot{\gamma}=\left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}-\delta^2\frac{\partial v}{\partial x}\right)+2\delta^2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{1}{2}}.$$

and $M=a\mu_e B_0\sqrt{\frac{\sigma}{\eta_0}}$ is the Hartman number.

Under lubrication approach, neglecting the terms of order δ and Re in the Eqs. (2.12) and (2.13), we get

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left[1+\left(\frac{n-1}{2}\right)We^2\left(\frac{\partial u}{\partial y}\right)^2\right]\frac{\partial u}{\partial y}-\frac{M^2}{1+m^2}(u+1) \tag{2.14}$$

$$\frac{\partial p}{\partial y}=0. \qquad (2.15).$$

The corresponding dimensionless boundary conditions in wave frame of reference are given by

$$u=-1 \quad \text{at} \quad y=h_1,h_2, \tag{2.16}$$

where $h_1=1+a\cos 2\pi x$ and $h_2=-1-b\cos\left(2\pi x+\theta\right)$.

Equation (2.20) implies that $p\neq p(y)$. Therefore Equation (2.19) can be rewritten as

$$\frac{dp}{dx}=\frac{\partial}{\partial y}\left\{\left[1+\left(\frac{n-1}{2}\right)We^2\left(\frac{\partial u}{\partial y}\right)^2\right]\frac{\partial u}{\partial y}\right\}-\frac{M^2}{1+m^2}(u+1)\,, \tag{2.17}$$

The volume flow rate q in a wave frame of reference is given by

$$q=\int_{h_2}^{h_1}udy\,. \tag{2.18}$$

The instantaneous flux $Q(X,t)$ in a fixed frame is

$$Q(X,t)=\int_{h_2}^{h_1}UdY=\int_{h_2}^{h_1}(u+1)dy=q+h_1-h_2\,. \tag{2.19}$$

The time average flux $\overline{Q}$ over one period $T\left(=\lambda/c\right)$ of the peristaltic wave is

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt=\int_0^1(q+h_1-h_2)dx=q+1+d\,. \tag{2.20}$$

3. PERTURBATION SOLUTION

The Equation (2.17) is non-linear and its closed form solution is not possible. Hence, we linearize this equations in terms of We^2, since We is small for the type of flow under consideration. So, we expand u, p and q as

$$u=u_0+We^2u_1+O(We^4)$$
$$\frac{dp}{dx}=\frac{dp_0}{dx}+We^2\frac{dp_1}{dx}+O(We^4) \tag{3.1}$$
$$q=q_0+We^2q_1+O(We^4)$$

Substituting (3.1) in the Equation (2.17) and in the boundary conditions (2.16) and (2.18) and equating the coefficients of like powers of We^2 and neglecting the terms of We^4 and higher order, we get the following equations:

3.1 Equation of order We^0

$$\frac{dp_0}{dx}=\frac{\partial^2u_0}{\partial y^2}-\frac{M^2}{1+m^2}(u_0+1)\,. \tag{3.2}$$

and the respective boundary conditions are

$$u_0 = -1 \quad \text{at} \quad y = h_1, h_2 . \tag{3.3}$$

3.2 Equation of order We^2

$$\frac{dp_1}{dx} = \frac{\partial^2 u_1}{\partial y^2} + \left(\frac{n-1}{2}\right)\frac{\partial}{\partial y}\left[\left(\frac{\partial u_0}{\partial y}\right)^3\right] - \frac{M^2}{1+m^2}u_1 , \tag{3.4}$$

and the respective boundary conditions are

$$u_1 = 0 \qquad \text{at} \qquad y = h_1, h_2 \tag{3.5}$$

3.3 Solution of order We^0

Solving the Equation (3.2) by using the boundary conditions (3.3) , we get

$$u_0 = \frac{1}{\alpha^2}\frac{dp_0}{dx}\left(c_1 \cosh \alpha y + c_2 \sinh \alpha y - 1\right) - 1 \tag{3.6}$$

where $\alpha = M / \sqrt{(1+m^2)}$, $c_1 = \dfrac{\sinh \alpha h_2 - \sinh \alpha h_1}{\sinh \alpha\left(h_2 - h_1\right)}$

and $c_2 = \dfrac{\cosh \alpha h_1 - \cosh \alpha h_2}{\sinh \alpha\left(h_2 - h_1\right)}$.

and the volume flow rate q_0 is given by

$$q_0 = \int_{h_2}^{h_1} u_0 dy = \frac{1}{\alpha^3}\frac{dp_0}{dx} A_1 - h_1 + h_2 \tag{3.7}$$

$$A_1 = c_1\left(\sinh \alpha h_1 - \sinh \alpha h_2\right) + c_2\left(\cosh \alpha h_1 - \cosh \alpha h_2\right) - \alpha\left(h_1 - h_2\right).$$

From Equation (3.9), we get

$$\frac{dp_0}{dx} = \alpha^3 \frac{(q_0 + h_1 - h_2)}{A_1} . \tag{3.8}$$

3.4 Solution of order We^2

Solving the Equation (3.4) by using the Equation (3.6) and the boundary conditions (3.7), we get

$$u_1 = \frac{1}{\alpha^2}\frac{dp_1}{dx}\left(c_1 \cosh \alpha y + c_2 \sinh \alpha y - 1\right) + \frac{3}{16}\left(\frac{n-1}{\alpha^5}\right)\left(\frac{dp_0}{dx}\right)^3 B\left(y\right) \tag{3.9}$$

where

$$B(y)=(c_9-4c_5\alpha y)\cosh\alpha y+(c_{10}-4c_6\alpha y)\sinh\alpha y-c_3\sinh 3\alpha y-c_4\cosh 3\alpha y,$$

$$c_3=\frac{3c_1^2c_2+c_2^3}{4},\ c_4=\frac{3c_1c_2^2+c_1^3}{4},\ c_5=\frac{c_2^3-c_1^2c_2}{4},\ c_6=\frac{c_1c_2^2-c_1^3}{4},$$

$$c_7=c_3\sinh 3\alpha h_1+c_4\cosh 3\alpha h_1+4\alpha h_1c_5\cosh\alpha h_1+4\alpha h_1c_6\sinh\alpha h_1,$$

$$c_8=c_3\sinh 3\alpha h_2+c_4\cosh 3\alpha h_2+4\alpha h_2c_5\cosh\alpha h_2+4\alpha h_2c_6\sinh\alpha h_2,$$

$$c_9=\frac{c_7\sinh\alpha h_2-c_8\sinh\alpha h_1}{\sinh\alpha(h_2-h_1)}\ \text{and}\ c_{10}=\frac{c_8\cosh\alpha h_1-c_7\cosh\alpha h_2}{\sinh\alpha(h_2-h_1)}.$$

and the volume flow rate q_1 is given by

$$q_1=\int_0^h u_1dy=\frac{1}{\alpha^3}\frac{dp_1}{dx}A_1+\frac{3}{16}\left(\frac{n-1}{\alpha^5}\right)\left(\frac{dp_0}{dx}\right)^3A_2 \tag{3.10}$$

where
$$A_2=\begin{bmatrix}\left(\frac{c_9+4c_6}{\alpha}\right)(\sinh\alpha h_1-\sinh\alpha h_2)+\left(\frac{c_{10}+4c_5}{\alpha}\right)(\cosh\alpha h_1-\cosh\alpha h_2)\\ -\frac{c_3}{3\alpha}(\cosh 3\alpha h_1-\cosh 3\alpha h_2)-\frac{c_4}{3\alpha}(\sinh 3\alpha h_1-\sinh 3\alpha h_2)\\ -4c_5(h_1\sinh\alpha h_1-h_2\sinh\alpha h_2)-4c_6(h_1\cosh\alpha h_1-h_2\cosh\alpha h_2)\end{bmatrix}.$$

From Eq. (3.10), we have

$$\frac{dp_1}{dx}=\left(q_1-\frac{3}{16}\left(\frac{n-1}{\alpha^5}\right)\left(\frac{dp_0}{dx}\right)^3A_2\right)\frac{\alpha^3}{A_1} \tag{3.11}$$

Substituting Equations (3.9) and (3.11) into the Equation (3.1) and using the relation $q_0=q-We^2q_1$ and neglecting terms greater than $O(We^2)$, we get

$$\frac{dp}{dx}=\left((q+h_1-h_2)-We^2\alpha^4\frac{3}{16}\left(\frac{n-1}{A_1^3}\right)(q+h_1-h_2)^3A_2\right)\frac{\alpha^3}{A_1}. \tag{3.12}$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \tag{3.13}$$

Note that, as $\theta\to 0$ our results coincide with the results of Subba Narasimhudu [23].

4. MAIN RESULTS

In order to get a the physical insight of the problem, axial pressure gradient and pressure rise per one wavelength are computed numerically for different values of the emerging parameters, viz., Weissenberg number We, power-law index n, Hall parameter m, Hartmann number M, phase shift θ, amplitude ratios a and b and are presented in figures 2-13.

Fig. 2 depicts the variation of the axial pressure gradient $\frac{dp}{dx}$ with We for $n=0.398$, $m=0.2$, $M=1$, $\theta=\frac{\pi}{4}$, $a=0.5$, $b=0.7$ and $\overline{Q}=-1$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing the Weissenberg number We.

The variation of the axial pressure gradient $\frac{dp}{dx}$ with n for $We=0.01$, $m=0.2$, $M=1$, $\theta=\frac{\pi}{4}$, $a=0.5$, $b=0.7$ and $\overline{Q}=-1$ is depicted in Fig. 3. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing the power-law parameter n.

Fig. 4 illustrates the variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $n=0.398$, $We=0.01$, $M=1$, $\theta=\frac{\pi}{4}$, $a=0.5$, $b=0.7$ and $\overline{Q}=-1$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ decreases with an increase in the Hall parameter m.

The variation of the axial pressure gradient $\frac{dp}{dx}$ with M for $n=0.398$, $m=0.2$, $We=0.01$, $\theta=\frac{\pi}{4}$, $a=0.5$, $b=0.7$ and $\overline{Q}=-1$ is illustrated in Fig. 5. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing Hartmann number M.

Fig. 6 shows the variation of the axial pressure gradient $\frac{dp}{dx}$ with θ for $n = 0.398$, $m = 0.2$, $a = 0.5$, $b = 0.7$, $M = 1$, $We = 0.01$ and $\overline{Q} = -1$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing phase shift θ.

The variation of the axial pressure gradient $\frac{dp}{dx}$ with a and b for $n = 0.398$, $m = 0.2$, $\theta = \frac{\pi}{4}$, $M = 1$, $We = 0.01$ and $\overline{Q} = -1$ is shown in Fig. 7. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ is increases with increasing a and b.

Fig. 8 depicts the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Weissenberg number We with $n = 0.398$, $m = 0.2$, $\theta = \frac{\pi}{4}$, $a = 0.5$, $b = 0.7$ and $M = 1$ is shown in Fig. 7. It is observed that, in both the pumping ($\Delta p > 0$) and free pumping regions the time averaged flux $\overline{Q}$ decreases with an increase in We, whereas in the co-pumping ($\Delta p < 0$) region $\overline{Q}$ increases with an increase in We.

The variation of Δp with $\overline{Q}$ for different values of n with $We = 0.1$, $m = 0.2$, $\theta = \frac{\pi}{4}$, $a = 0.5$, $b = 0.7$ and $M = 1$ is depicted in Fig. 9. It is observed that, the time averaged flux $\overline{Q}$ increases with an increase in n in the pumping and free-pumping regions, whereas in the co-pumping region the $\overline{Q}$ decreases with an increase in n. Further, the pumping is more for Newtonian fluid ($n = 1$) than that of a Carreau fluid ($0 < n < 1$).

Fig. 10 illustrates the variation of Δp as a function of $\overline{Q}$ for different values of Hall parameter m with $n = 0.398$, $M = 1$, $We = 0.1$, $\theta = \frac{\pi}{4}$, $a = 0.5$ and $b = 0.7$. It is found that, any of two pumping curves intersect at a point in the first quadrant and to the left of this point time averaged flux $\overline{Q}$ decreases with increasing m and to the right of this point $\overline{Q}$ increases with an increase in m.

The variation of Δp as a function of $\overline{Q}$ for different values of Hartmann number M with $n = 0.398$, $m = 0.2$, $We = 0.1$, $\theta = \frac{\pi}{4}$, $a = 0.5$ and $b = 0.7$ is Fig. 11. It is found that, any of two pumping curves intersect at a point in the first quadrant and to the left of this point time averaged flux $\overline{Q}$ increases with increasing M and to the right of this point $\overline{Q}$ decreases with an increase in M.

Fig. 12 shows the variation of Δp with $\overline{Q}$ for different values of θ with $n = 0.398$, $m = 0.2$, $a = 0.5$, $b = 0.7$, $M = 1$ and $We = 0.1$. It is found that, the $\overline{Q}$ decreases with an increase in θ in both pumping and free pumping regions. But, in the co-pumping region, the $\overline{Q}$ increases with an increase in θ for appropriately chosen Δp (<0).

The variation of Δp as a function of $\overline{Q}$ for different values of amplitude ratios a and b with $n = 0.398$, $m = 0.2$, $We = 0.1$, $\theta = \frac{\pi}{4}$ and $M = 1$ is Fig. 13. It is found that, in both the pumping and free-pumping regions, the time averaged flux $\overline{Q}$ increases with increasing a or b, while $\overline{Q}$ decreases with increasing a or b. Further it is observed that, the pumping is more for unequal amplitudes than that of equal amplitudes.

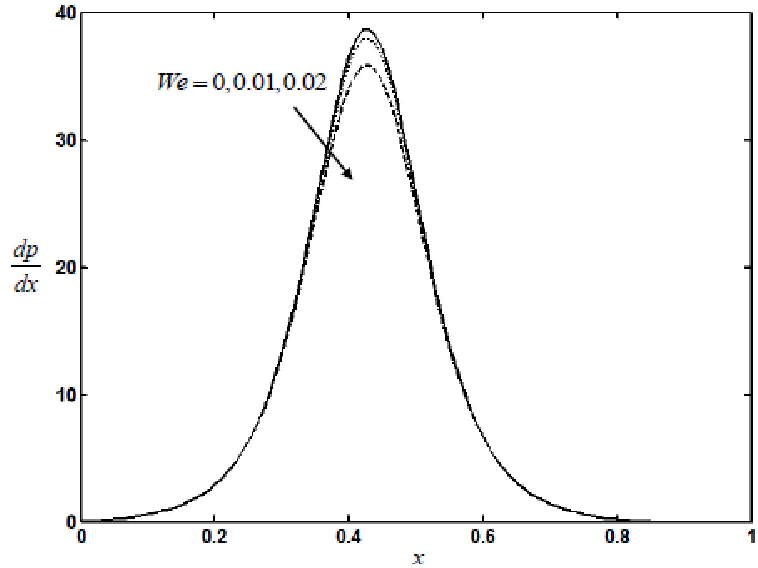

Fig. 2 The variation of the axial pressure gradient $\frac{dp}{dx}$ with We for $n = 0.398$, $m = 0.2$, $M = 1$, $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$ and $\overline{Q} = -1$.

5. CONCLUSIONS

In this paper, the peristaltic flow of a conducting Carreau fluid in an asymmetric channel under the effect of Hall using long wavelength approach is investigated. It is found that the pumping is more for Newtonian fluid $n = 1$ than that of Carreau fluid $0 < n < 1$. The magnitudes of pressure gradient and pressure rise increase with increasing M, a or b whereas, the magnitudes of pressure gradient and pressure rise decrease with increasing We, m or θ.

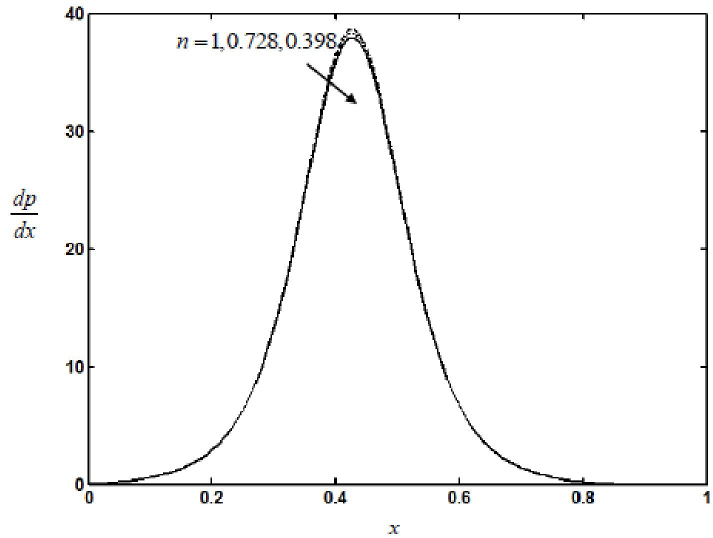

Fig. 3 The variation of the axial pressure gradient $\frac{dp}{dx}$ with n for $We = 0.01$, $m = 0.2$, $M = 1, a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$ and $\overline{Q} = -1$.

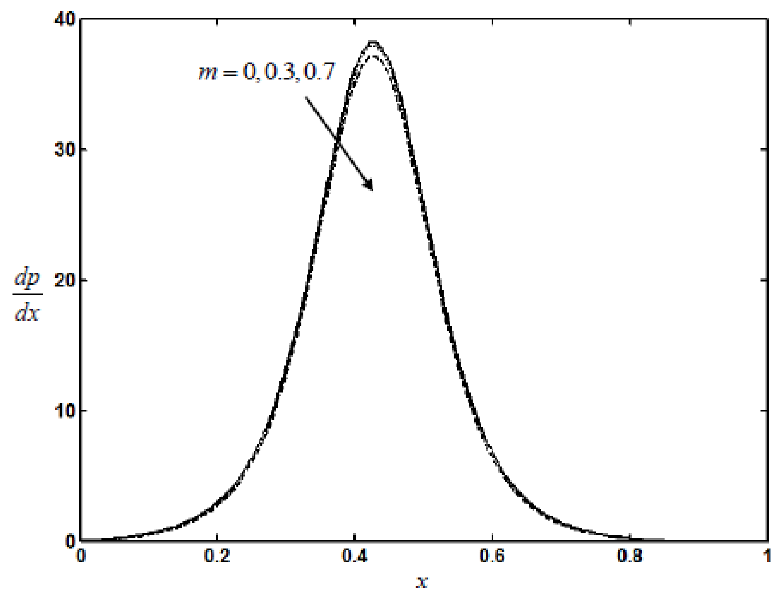

Fig. 4 The variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $n = 0.398$, $We = 0.01$, $M = 1, a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$ and $\overline{Q} = -1$.

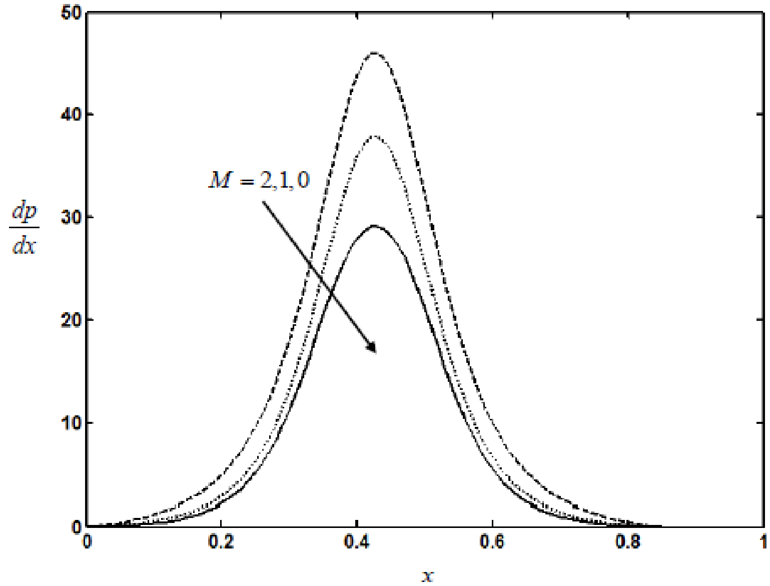

Fig. 5 The variation of the axial pressure gradient $\frac{dp}{dx}$ with M for $n = 0.398$, $m = 0.2$, $We = 0.01$, $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$ and $\overline{Q} = -1$.

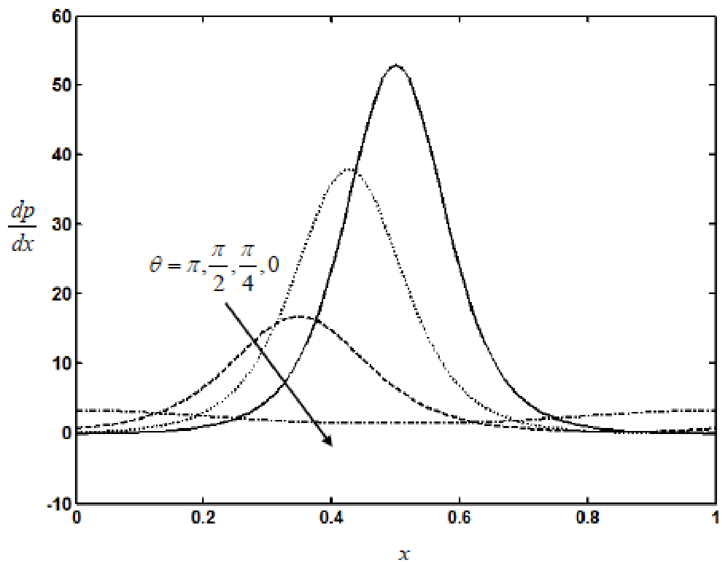

Fig. 6 The variation of the axial pressure gradient $\frac{dp}{dx}$ with θ for $n = 0.398$, $m = 0.2$, $We = 0.01$, $a = 0.5$, $b = 0.7$, $M = 1$ and $\overline{Q} = -1$.

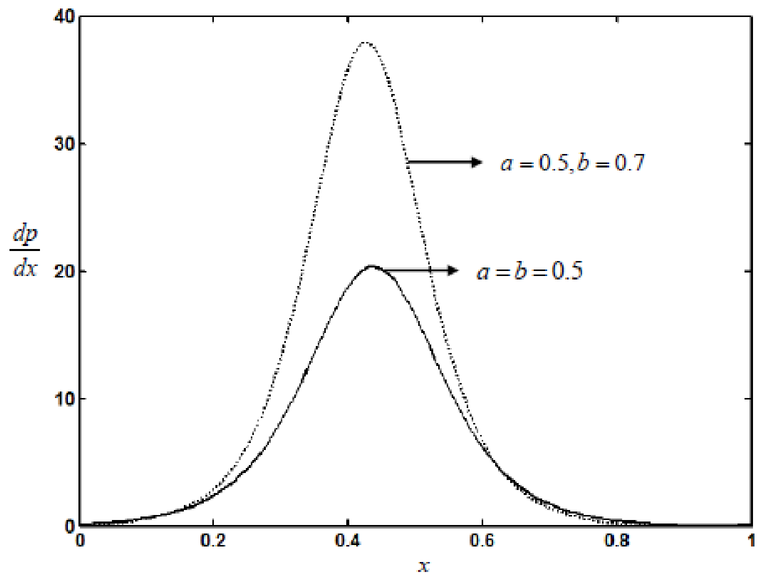

Fig. 7 The variation of the axial pressure gradient $\frac{dp}{dx}$ with a and b for $n = 0.398$, $m = 0.2$, $M = 1$, $\theta = \frac{\pi}{4}$, $We = 0.01$ and $\overline{Q} = -1$.

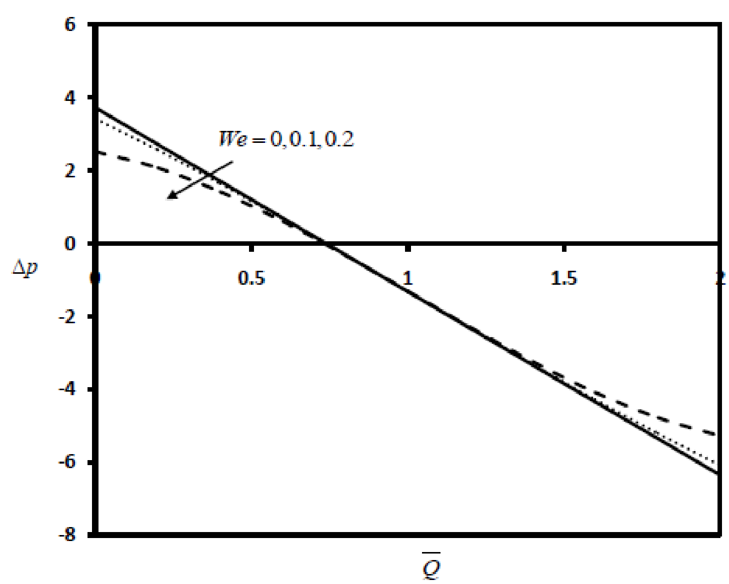

Fig. 7 The variation of the pressure rise Δp with $\overline{Q}$ for different values of We with $n = 0.398$, $m = 0.2$, $M = 1$ $a = 0.5$, $b = 0.7$ and $\theta = \frac{\pi}{4}$.

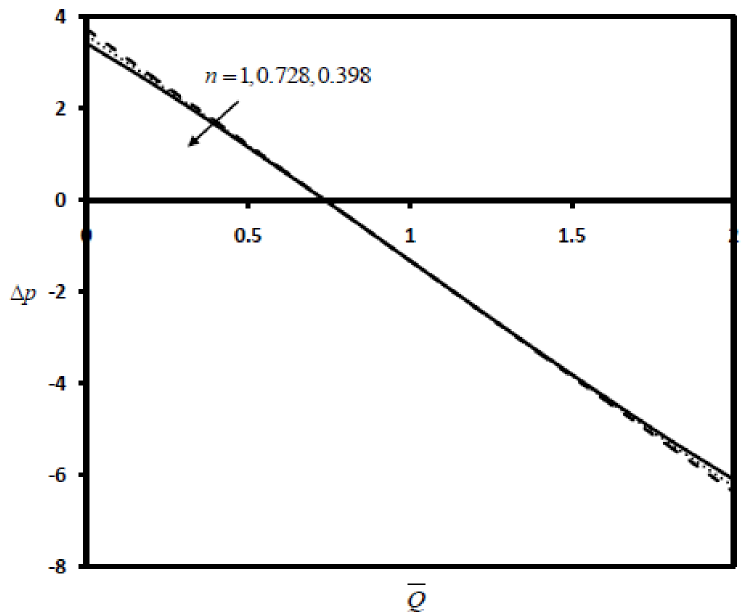

Fig. 8 The variation of the pressure rise Δp with $\overline{Q}$ for different values of n with $We = 0.1$, $m = 0.2$, $M = 1$ $a = 0.5$, $b = 0.7$ and $\theta = \dfrac{\pi}{4}$.

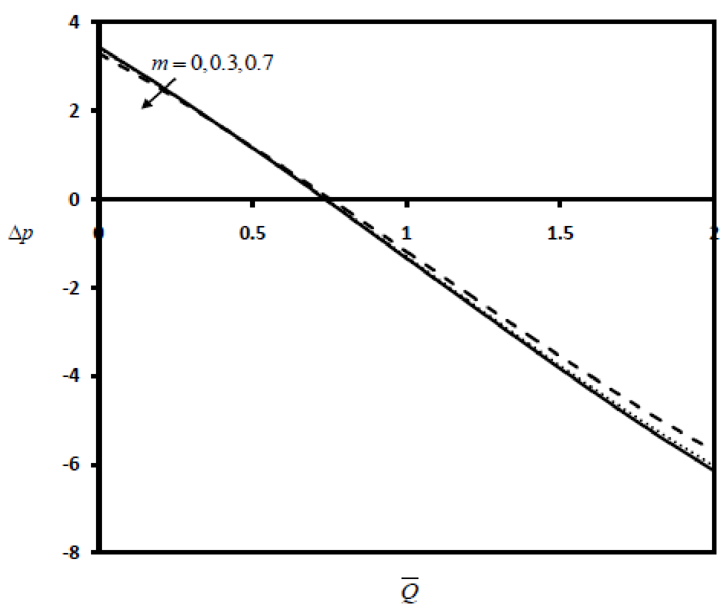

Fig. 9 The variation of the pressure rise Δp with $\overline{Q}$ for different values of m with $n = 0.398$, $We = 0.1$, $M = 1$ $a = 0.5$, $b = 0.7$ and $\theta = \dfrac{\pi}{4}$.

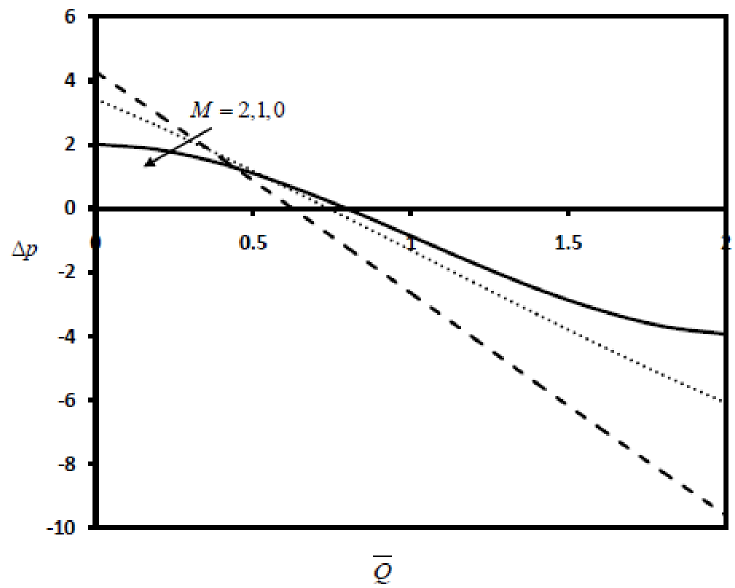

Fig. 10 The variation of the pressure rise Δp with $\overline{Q}$ for different values of M with $n = 0.398$, $m = 0.2$, $We = 0.1$ $a = 0.5$, $b = 0.7$ and $\theta = \dfrac{\pi}{4}$.

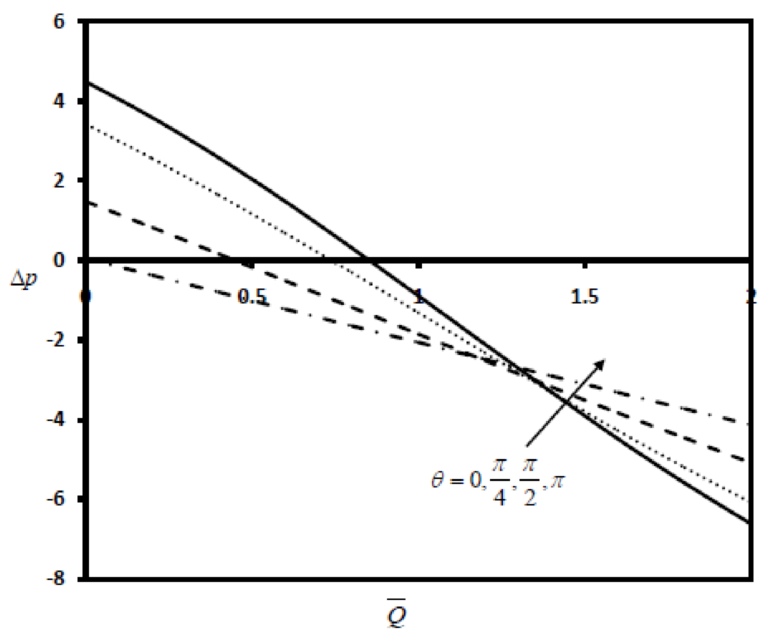

Fig. 11 The variation of the pressure rise Δp with $\overline{Q}$ for different values of θ with $n = 0.398$, $m = 0.2$, $M = 1$ $a = 0.5$, $b = 0.7$ and $We = 0.1$.

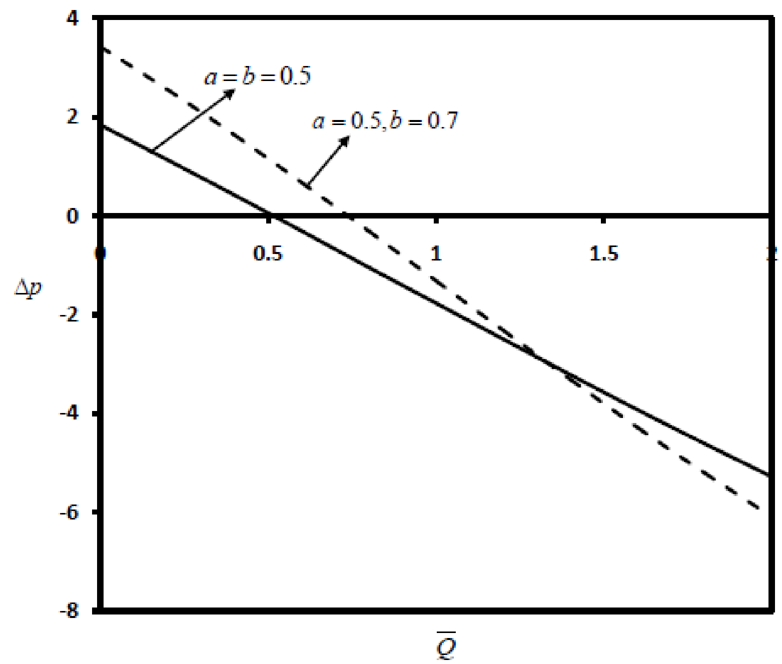

Fig. 11 The variation of the pressure rise Δp with $\bar{Q}$ for different values of ϕ with $n = 0.398$, $m = 0.2$, $M = 1$ and $We = 0.2$.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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