



Available online at <http://scik.org>

J. Math. Comput. Sci. 10 (2020), No. 4, 881-890

<https://doi.org/10.28919/jmcs/4540>

ISSN: 1927-5307

4-TOTAL DIFFERENCE CORDIAL LABELING OF CORONA OF SNAKE GRAPHS WITH K_1

R. PONRAJ^{1,*}, S. YESU DOSS PHILIP^{2,†}, R. KALA²

¹Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India

²Department of Mathematics, Manonmaniam Sundarnar University, Abishekapatti, Tirunelveli, 627012, Tamilnadu, India

Copyright © 2020 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \dots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv , assign the label $|f(u) - f(v)|$. f is called k -total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \dots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with x . A graph with admits a k -total difference cordial labeling is called k -total difference cordial graphs. In this paper we investigate the 4-total difference cordial labeling behaviour of corona of snake graphs with K_1 .

Keywords: $T_n \odot K_1$; $Q_n \odot K_1$; $A(T_n \odot K_1)$.

2010 AMS Subject Classification: 05C78.

1. INTRODUCTION

All graphs in this paper are finite, simple and undirecte. The k -total difference cordial graph was introduced in [3]. In [3, 4], 3-total difference cordial labeling behaviour of path, complete graph, comb, armed crown, crown, wheel, star etc have been investigated . Also 4-total difference cordial labeling of path, star , bistar, comb, crown, $P_n \cup K_{1,n}$, $S(P_n \cup K_{1,n})$, $P_n \cup B_{n,n}$

*Corresponding author

†Research scholar, Register number 182240120910010

E-mail address: ponrajmaths@gmail.com

Received February 24, 2020

etc., have been investigated [5]. In this paper we investigate 4-total difference of cordial labeling of Corona of triangular snake and quadrilateral snake graphs with K_1 .

2. PRELIMINARIES

Definition 2.1. Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \dots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv , assign the label $|f(u) - f(v)|$. f is called k -total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \dots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labelled with x . A graph with a k -total difference cordial labeling is called k -total difference cordial graph.

Definition 2.2. The Triangular snake T_n is obtained from the path $P_n : u_1u_2 \dots u_n$ with $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$ and $E(T_n) = E(P_n) \cup \{u_iv_i, u_iv_{i+1} : 1 \leq i \leq n-1\}$.

Definition 2.3. The Quadrilateral snake Q_n is obtained from the path $P_n : u_1u_2 \dots u_n$ with $V(Q_n) = V(P_n) \cup \{v_i, w_i : 1 \leq i \leq n-1\}$ and $E(Q_n) = E(P_n) \cup \{u_iv_i, u_{i+1}w_i : 1 \leq i \leq n-1\}$.

Definition 2.4. The Alternate triangular snake of $A(T_n)$ is obtained from the path $P_n : u_1u_2 \dots u_n$ by joining u_i and u_{i+1} (alternatively) to the vertex v_i . That is every alternate edge of a path is replaced by C_3 .

Definition 2.5. Let G_1, G_2 respectively be p_1, q_1, p_2, q_2 graphs. The corona of G_1 with $G_2, G_1 \odot G_2$ is the graph is obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2 .

3. MAIN RESULTS

Theorem 3.1. The corona of triangular snake T_n with $K_1, T_n \odot K_1$ is 4-total difference cordial.

Proof. Take the vertex set and edge set of T_n as in definition 2.1. Let $x_i (1 \leq i \leq n-1)$ be the pendent vertices adjacent to $u_i (1 \leq i \leq n-1)$ and $y_i (1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i (1 \leq i \leq n-1)$. Clearly $|V(T_n)| + |E(T_n)| = 9n - 6$.

Case 1. $n > 3$. Fix the labels 1, 1, 3, 3, 3, 3, 3, 1, 1 and 1 to the vertices $x_1, x_2, v_1, v_2, u_1, u_2, u_3, y_1, y_2$ and y_3 . Next assign the label 3 to the all path vertices $u_1u_2 \dots u_n$. Next assign the labels 1, 2, 1 and 3 to the vertices v_3, v_4, v_5 and v_6 . Similarly assign the labels 1, 2, 1 and 3 to

the next four vertices v_7, v_8, v_9 and v_{10} . Continue in this pattern until we reach the vertex v_{n-1} . Clearly the vertex v_{n-1} receive the label 1 when $n \equiv 1, 3 \pmod{4}$ and 2 or 3 according as $n \equiv 0 \pmod{4}$ or $n \equiv 2 \pmod{3}$.

Next assign the labels 2, 3, 2 and 1 to the vertices x_3, x_4, x_5 and x_6 . Assign the labels 2, 3, 2 and 1 to the next four vertices x_7, x_8, x_9 and x_{10} . Proceeding in this way until we reach the vertex x_{n-1} . Clearly the vertex x_{n-1} receive the label 2 when $n \equiv 1, 3 \pmod{4}$ and 3 or 1 according as $n \equiv 0 \pmod{4}$ or $n \equiv 2 \pmod{3}$.

Next assign the labels 1, 3, 3 and 3 to the vertices y_3, y_4, y_5 and y_6 . Assign the labels 1, 3, 3 and 3 to the next four vertices y_7, y_8, y_9 and y_{10} . Proceeding like this until we reach the vertex y_n . Clearly the vertex y_n receive the label 3 or 1 when $n \equiv 0, 1, 2 \pmod{4}$ or $n \equiv 3 \pmod{4}$.

Case 2. $n \leq 3$.

Table 1 gives a 4-total difference cordial labeling for this case.

| | | | | | | | | | | |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| n | u_1 | u_2 | u_3 | v_1 | v_2 | x_1 | x_2 | y_1 | y_2 | y_3 |
| 2 | 3 | 3 | | 3 | | 1 | | 1 | 1 | |
| 2 | 3 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 |

TABLE 1

The table 2 shows that this vertex labeling is a 4-total difference cordial labeling.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|------------------|------------------|------------------|------------------|
| $n \equiv 0 \pmod{4}$ | $\frac{9n-4}{4}$ | $\frac{9n-8}{4}$ | $\frac{9n-4}{4}$ | $\frac{9n-8}{4}$ |
| $n \equiv 1 \pmod{4}$ | $\frac{9n-5}{4}$ | $\frac{9n-5}{4}$ | $\frac{9n-9}{4}$ | $\frac{9n-5}{4}$ |
| $n \equiv 2 \pmod{4}$ | $\frac{9n-6}{4}$ | $\frac{9n-6}{4}$ | $\frac{9n-6}{4}$ | $\frac{9n-6}{4}$ |
| $n \equiv 3 \pmod{4}$ | $\frac{9n-7}{3}$ | $\frac{9n-7}{4}$ | $\frac{9n-7}{4}$ | $\frac{9n-7}{4}$ |

TABLE 2

□

Example 3.1. A 4-total difference cordial labeling of $T_6 \odot K_1$ is shown in Figure 1

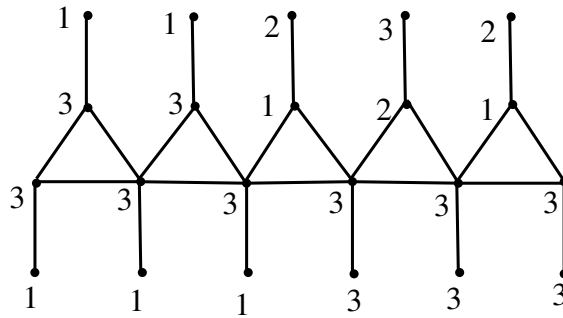


FIGURE 1

Theorem 3.2. The corona of quadrilateral snake Q_n with K_1 , $Q_n \odot K_1$ is 4-total difference cordial.

Proof. Take the vertex set and edge set of Q_n as in definition 2.2. Let x_i be the pendent vertices adjacent to v_i and z_i be the pendent vertices adjacent to $w_i (1 \leq i \leq n - 1)$. Let $y_i (1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i (1 \leq i \leq n)$. It is easy to verify that $|V(Q_n)| + |E(Q_n)| = 13n - 10$.

Assign the label 3 to the all the path vertices $u_1 u_2 \dots u_n$. Next assign the labels 3, 3, 1 and 1 to the vertices v_1, v_2, v_3 and v_4 . Assign the labels 3, 3, 1 and 1 to the vertices v_5, v_6, v_7 and v_8 . Continue in this pattern until we reach the vertex v_{n-1} . Clearly the vertex v_{n-1} receive the label 3 or 1 according as $n \equiv 1, 2 \pmod{4}$ or $n \equiv 0, 3 \pmod{4}$.

We now consider the vertices w_i . Assign the labels 3, 3, 1 and 2 to the vertices w_1, w_2, w_3 and w_4 . Next assign the labels 3, 3, 1 and 2 to the vertices w_5, w_6, w_7 and w_8 . Proceeding like this until we reach the vertex w_{n-1} . Clearly the vertex w_{n-1} receive the label 3 when $n \equiv 1, 2 \pmod{4}$ and 1 or 2 when $n \equiv 0, 3 \pmod{4}$.

Now we consider the vertices x_i . Assign the labels 1, 1, 2 and 3 to the vertices x_1, x_2, x_3 and x_4 . Next assign the labels 1, 1, 2 and 3 to the vertices x_5, x_6, x_7 and x_8 . Proceeding like this until we reach the vertex x_{n-1} . Clearly the vertex x_{n-1} receive the label 1 when $n \equiv 1, 2 \pmod{4}$ and 2 or 3 when $n \equiv 3, 0 \pmod{4}$.

We now move to the vertices z_i . Assign the labels 1, 1, 3 and 3 to the vertices z_1, z_2, z_3 and z_4 . Next assign the labels 1, 1, 3 and 3 to the vertices z_5, z_6, z_7 and z_8 . Proceeding like this until

we reach the vertex z_{n-1} . Clearly the vertex z_{n-1} receive the labels 1 or 3 according as $n \equiv 1, 2 \pmod{4}$ or $n \equiv 3, 0 \pmod{4}$.

Next we move to the pendent vertices of path. Fix the label 1 to the vertex y_i . Assign the labels 1, 1, 3 and 3 to the vertices y_2, y_3, y_4 and y_5 . Next assign the labels 1, 1, 3 and 3 to the vertices y_6, y_7, y_8 and y_9 . Proceeding like this until we reach the vertex y_n . Clearly the vertex y_n receive the label 1 or 3 according as $n \equiv 2, 3 \pmod{4}$ or $n \equiv 0, 1 \pmod{4}$.

The table 3 shows that this vertex labeling is a 4-total difference cordial labeling.

□

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|--------------------|--------------------|--------------------|--------------------|
| $n \equiv 0 \pmod{4}$ | $\frac{13n-8}{4}$ | $\frac{13n-12}{4}$ | $\frac{13n-8}{4}$ | $\frac{13n-12}{4}$ |
| $n \equiv 1 \pmod{4}$ | $\frac{13n-13}{4}$ | $\frac{13n-9}{4}$ | $\frac{13n-9}{4}$ | $\frac{13n-9}{4}$ |
| $n \equiv 2 \pmod{4}$ | $\frac{13n-10}{4}$ | $\frac{13n-10}{4}$ | $\frac{13n-10}{4}$ | $\frac{13n-10}{4}$ |
| $n \equiv 3 \pmod{4}$ | $\frac{13n-7}{3}$ | $\frac{13n-11}{4}$ | $\frac{13n-11}{4}$ | $\frac{13n-11}{4}$ |

TABLE 3

Example 3.2. A 4-total difference cordial labeling of $Q_5 \odot K_1$ is shown in Figure 2

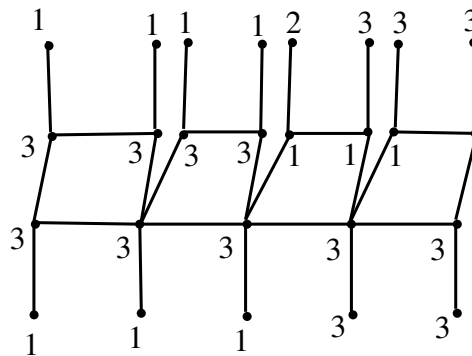


FIGURE 2

Theorem 3.3. The corona of alternate triangular snake $A(T_n)$ with K_1 , $A(T_n) \odot K_1$ is 4-total difference cordial.

Proof. Take the vertex set and edge set of $A(T_n)$ as in definition 2.3.

Case 1. The edge u_1u_2 lies in a triangle and the edge $u_{n-1}u_n$ lies in a triangle.

Let $x_i(1 \leq i \leq n - 1)$ be the pendent vertices adjacent to $v_i(1 \leq i \leq n - 1)$ and $y_i(1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i 1 \leq i \leq n - 1$. Clearly n is even. In this case $|V(A(T_n)) \odot K_1| + |E(A(T_n))| = \frac{13n-2}{2}$.

Assign the label 3 to the path vertices $u_1u_2 \dots u_n$. Next fix the label 3 and 3 to the vertices v_1 and v_2 . Fix the label 1 to the vertices x_1, x_2, y_1 and y_2 . Next assign the labels 2, 3, 2 and 1 to the vertices x_3, x_4, x_5 and x_6 . Assign the labels 2, 3, 2 and 1 to the next four vertices x_7, x_8, x_9 and x_{10} . Continue in this pattern until we reach the vertex $x_{\frac{n}{2}}$. Clearly the vertex $x_{\frac{n}{2}}$ receive the label 2 when $n \equiv 1, 3 \pmod{4}$ and 3 or 1 according as $n \equiv 0 \pmod{4}$ or $n \equiv 2 \pmod{3}$.

We now consider the vertices v_i . Assign the labels 1, 2, 1 and 3 to the vertices v_3, v_4, v_5 and v_6 . Similarly assign the labels 1, 2, 1 and 3 to the next four vertices v_7, v_8, v_9 and v_{10} . Continue in this pattern until we reach the vertex $v_{\frac{n}{2}}$. Clearly the vertex $v_{\frac{n}{2}}$ receive the label 1 when $n \equiv 1, 3 \pmod{4}$ and 2 or 3 according as $n \equiv 2 \pmod{4}$ or $n \equiv 0 \pmod{3}$.

Consider the vertices y_i . Assign the labels 1, 1, 1, 3, 1, 3, 1 and 3 to the vertices $y_3, y_4, y_5, y_6, y_7, y_8, y_9$ and y_{10} . Next assign the labels 1, 1, 1, 3, 1, 3, 1 and 3 to the vertices $y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, y_{16}, y_{17}$ and y_{18} . Continue in this pattern until we reach the vertex y_n . Clearly the vertex y_n receive the label 1 or 3 according as $n \equiv 1, 3, 4, 5, 7 \pmod{8}$ or $n \equiv 0, 2, 6 \pmod{4}$.

The table 4 shows that this vertex labeling is a 4-total difference cordial labeling.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|-------------------|-------------------|-------------------|-------------------|
| $n \equiv 0 \pmod{8}$ | $\frac{13n}{8}$ | $\frac{13n}{8}$ | $\frac{13n-8}{8}$ | $\frac{13n}{8}$ |
| $n \equiv 2 \pmod{8}$ | $\frac{13n-2}{8}$ | $\frac{13n-2}{8}$ | $\frac{13n-2}{8}$ | $\frac{13n-2}{8}$ |
| $n \equiv 4 \pmod{8}$ | $\frac{13n+4}{8}$ | $\frac{13n-4}{8}$ | $\frac{13n-4}{8}$ | $\frac{13n-4}{8}$ |
| $n \equiv 6 \pmod{8}$ | $\frac{13n+2}{8}$ | $\frac{13n-6}{8}$ | $\frac{13n+2}{8}$ | $\frac{13n-6}{8}$ |

TABLE 4

Case 2. The edge u_1u_2 lies in a triangle and the edge $u_{n-2}u_{n-1}$ lies in a triangle. In this case n is odd.

Clearly removal of the edge $u_{n-1}u_n$ is the graph as in case(i). Assign the label to the vertices $u_i(1 \leq i \leq n-1)$ and $v_i(1 \leq i \leq \frac{n-1}{2})$ as in case (i). Finally assign the labels 3 and 1 respect to the vertices u_n and v_n .

The table 5 shows that this vertex labeling is a 4-total difference cordial labeling.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|--------------------|--------------------|--------------------|--------------------|
| $n \equiv 1 \pmod{8}$ | $\frac{13n-5}{8}$ | $\frac{13n-5}{8}$ | $\frac{13n-13}{8}$ | $\frac{13n-5}{8}$ |
| $n \equiv 3 \pmod{8}$ | $\frac{13n-7}{8}$ | $\frac{13n-7}{8}$ | $\frac{13n-7}{8}$ | $\frac{13n-7}{8}$ |
| $n \equiv 5 \pmod{8}$ | $\frac{13n-9}{8}$ | $\frac{13n-17}{8}$ | $\frac{13n-17}{8}$ | $\frac{13n-17}{8}$ |
| $n \equiv 7 \pmod{8}$ | $\frac{13n-11}{8}$ | $\frac{13n-19}{8}$ | $\frac{13n-11}{8}$ | $\frac{13n-19}{8}$ |

TABLE 5

Case 3. The edge u_2u_3 lies in a triangle and the edge $u_{n-2}u_{n-1}$ lies in a triangle.

Obviously removal of the edge u_1u_2 as in case(ii). Assign the label to the vertices $u_i(2 \leq i \leq n)$ and $v_i(2 \leq i \leq \frac{n-2}{2})$ as in case (i). Next assign the labels 3 and 1 respect to the vertices u_1 and v_1 .

The table 6 shows that this vertex labeling is a 4-total difference cordial labeling.

□

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|--------------------|--------------------|--------------------|--------------------|
| $n \equiv 0 \pmod{8}$ | $\frac{13n-8}{8}$ | $\frac{13n-16}{8}$ | $\frac{13n-8}{8}$ | $\frac{13n-16}{8}$ |
| $n \equiv 2 \pmod{8}$ | $\frac{13n-10}{8}$ | $\frac{13n-10}{8}$ | $\frac{13n-18}{8}$ | $\frac{13n-10}{8}$ |
| $n \equiv 4 \pmod{8}$ | $\frac{13n-12}{8}$ | $\frac{13n-12}{8}$ | $\frac{13n-12}{8}$ | $\frac{13n-12}{8}$ |
| $n \equiv 6 \pmod{8}$ | $\frac{13n-6}{8}$ | $\frac{13n-14}{8}$ | $\frac{13n-14}{8}$ | $\frac{13n-14}{8}$ |

TABLE 6

Theorem 3.4. The corona of alternate quadrilateral snake $A(Q_n)$ with K_1 , $A(Q_n) \odot K_1$ is 4-total difference cordial.

Proof. Take the vertex set and edge set of $A(Q_n)$ as in definition 2.3.

Case 1. The edge u_1u_2 lies in a Quadrilateral and the edge $u_{n-1}u_n$ lies in a Quadrilateral.

Let $x_i(1 \leq i \leq n)$ be the pendent vertices adjacent to $v_i(1 \leq i \leq n)$ and $z_i(1 \leq i \leq \frac{n}{2})$ be the pendent vertices adjacent to $w_i(1 \leq i \leq n)$ and $y_i(1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i(1 \leq i \leq n)$. Clearly n is even. In this case $|V(A(Q_n)) \odot K_1| + |E(A(Q_n))| = \frac{17n-11}{2}$.

Assign the label 3 to the path vertices $u_1u_2 \dots u_n$. Next fix the label 3 to the vertices v_1 and w_1 . Fix the label 1 to the vertices x_1, z_1 and $y_i(1 \leq i \leq n)$. Next assign the labels 1, 3, 3 and 3 to the vertices x_2, x_3, x_4 and x_5 . Assign the labels 1, 3, 3 and 3 to the next four vertices x_6, x_7, x_8 and x_9 . Continue in this pattern until we reach the vertex $x_{\frac{n}{2}}$. Clearly the vertex $x_{\frac{n}{2}}$ receive the label 1 when $n \equiv 2 \pmod{4}$ and 3 when $n \equiv 0, 1, 3 \pmod{4}$.

We now consider the vertices $z_i(1 \leq i \leq \frac{n}{2})$. Assign the labels 1, 2, 3 and 2 to the vertices z_2, z_3, z_4 and z_5 . Similarly assign the labels 1, 2, 3 and 2 to the next four vertices z_6, z_7, z_8 and z_9 . Continue in this pattern until we reach the vertex $z_{\frac{n}{2}}$. Clearly the vertex $z_{\frac{n}{2}}$ receive the label 2 when $n \equiv 1, 3 \pmod{4}$ and 1 or 3 according as $n \equiv 0, 2 \pmod{4}$.

Consider the vertices $v_i(1 \leq i \leq \frac{n}{2})$. Assign the label 3 to the vertices $v_1, v_2, v_{\frac{n}{2}}$. Next assign the labels 3, 1, 2 and 1 to the vertices w_2, w_3, w_4 and w_5 . Assign the labels 3, 1, 2 and 1 to the next four vertices w_6, w_7, w_8 and w_9 . Continue in this way until we reach the vertex $w_{\frac{n}{2}}$. Clearly the vertex $w_{\frac{n}{2}}$ receive the label 1 when $n \equiv 1, 3 \pmod{4}$ and 3 or 2 when $n \equiv 0, 2 \pmod{4}$.

The table 7 shows that this vertex labeling is a 4-total difference cordial labeling.

Case 2. The edge u_1u_2 lies in a quadrilateral and the edge $u_{n-2}u_{n-1}$ lies in a quadrilateral. In this case n is odd.

Clearly removal of the edge $u_{n-1}u_n$ is the graph as in case(i). Assign the label to the vertices

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|-------------------|-------------------|-------------------|-------------------|
| $n \equiv 0 \pmod{8}$ | $\frac{17n}{8}$ | $\frac{17n}{8}$ | $\frac{17n-8}{8}$ | $\frac{17n}{8}$ |
| $n \equiv 2 \pmod{8}$ | $\frac{17n-2}{8}$ | $\frac{17n-2}{8}$ | $\frac{17n-2}{8}$ | $\frac{17n-2}{8}$ |
| $n \equiv 4 \pmod{8}$ | $\frac{17n+4}{8}$ | $\frac{17n-4}{8}$ | $\frac{17n-4}{8}$ | $\frac{17n-4}{8}$ |
| $n \equiv 6 \pmod{8}$ | $\frac{17n+2}{8}$ | $\frac{17n-6}{8}$ | $\frac{17n+2}{8}$ | $\frac{17n-6}{8}$ |

TABLE 7

$u_i(1 \leq i \leq n-1)$ and $v_i(1 \leq i \leq \frac{n}{2})$ and $w_i(1 \leq i \leq \frac{n}{2})$ as in case (i). Finally assign the labels 3 and 1 respect to the vertices u_n and $v_{\frac{n}{2}}$.

The table 8 shows that this vertex labeling is a 4-total difference cordial labeling.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|--------------------|--------------------|--------------------|--------------------|
| $n \equiv 1 \pmod{8}$ | $\frac{17n-9}{8}$ | $\frac{17n-9}{8}$ | $\frac{17n-17}{8}$ | $\frac{17n-9}{8}$ |
| $n \equiv 3 \pmod{8}$ | $\frac{17n-11}{8}$ | $\frac{17n-11}{8}$ | $\frac{17n-11}{8}$ | $\frac{17n-11}{8}$ |
| $n \equiv 5 \pmod{8}$ | $\frac{17n-5}{8}$ | $\frac{17n-13}{8}$ | $\frac{17n-13}{8}$ | $\frac{17n-13}{8}$ |
| $n \equiv 7 \pmod{8}$ | $\frac{17n-7}{8}$ | $\frac{17n-15}{8}$ | $\frac{17n-7}{8}$ | $\frac{17n-15}{8}$ |

TABLE 8

Case 3. The edge u_2u_3 lies in a Quadrilateral and the edge $u_{n-2}u_{n-1}$ lies in a Quadrilateral. Obviously removal of the edge u_1u_2 as in case(ii). Assign the label to the vertices $u_i(2 \leq i \leq n)$ and $v_i(2 \leq i \leq n-1)$ and $w_i(1 \leq i \leq \frac{n}{2})$ as in case (i). Next assign the labels 3 and 1 respect to the vertices u_1 and v_1 .

The table 9 shows that this vertex labeling is a 4-total difference cordial labeling.

□

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ | $t_{df}(3)$ |
|-----------------------|--------------------|--------------------|--------------------|--------------------|
| $n \equiv 0 \pmod{8}$ | $\frac{17n-32}{8}$ | $\frac{17n-40}{8}$ | $\frac{17n-32}{8}$ | $\frac{17n-40}{8}$ |
| $n \equiv 2 \pmod{8}$ | $\frac{17n-34}{8}$ | $\frac{17n-34}{8}$ | $\frac{17n-42}{8}$ | $\frac{17n-34}{8}$ |
| $n \equiv 4 \pmod{8}$ | $\frac{17n-20}{8}$ | $\frac{17n-20}{8}$ | $\frac{17n-20}{8}$ | $\frac{17n-20}{8}$ |
| $n \equiv 6 \pmod{8}$ | $\frac{17n-30}{8}$ | $\frac{17n-38}{8}$ | $\frac{17n-38}{8}$ | $\frac{17n-38}{8}$ |

TABLE 9

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

- [1] J.A. Gallian, A Dynamic survey of graph labeling, Electron. J. Comb. 19 (2017), #Ds6.
- [2] F. Harary, Graph theory, Addison wesley, New Delhi, 1969.
- [3] R. Ponraj, S. Yesu Doss Philip and R. Kala, k -total difference cordial graphs, J. Algorithms Comput. 51(2019), 121-128.
- [4] R. Ponraj, S. Yesu Doss Philip and R. Kala, 3-total difference cordial graphs, Glob. Eng. Sci. Res. 6 (2019), 46-51.
- [5] R. Ponraj, S. Yesu Doss Philip and R. Kala, Some families of 4-total difference cordial graphs J. Math. Comput. Sci. 10 (2020), 150-156.