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TWO WAREHOUSE INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK DEPENDENT DEMAND UNDER PERMISSIBLE DELAY IN PAYMENT

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Abstract: In this paper, a two warehouse inventory model for deteriorating items is studied with stock dependent demand rate. Holding cost of rented warehouse has higher than the owned warehouse due to better preservation facilities in rented warehouse. Due to the improved services offer in rented warehouse, the deterioration rate in rented warehouse is less than deterioration rate in owned warehouse. To reduce inventory cost, items of rented warehouse are consumed first and then the items of owned warehouse are consumed because it will be profitable for the organization. Permissible delay in payments allowed to the inventory manager is also taken into account. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal fruits and vegetables, newly launched fashion items, etc. The optimum replenishment policies are determined by minimizing the total cost in a replenishment interval. Sensitivity analysis has also been performed by changing (increasing or decreasing) one parameter at a time keeping the remaining parameters unchanged.

Keywords: two warehouses; stock dependent demand; deteriorating item; delay in payment.

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1. INTRODUCTION

In classical inventory models it is assumed that organization have a single warehouse with the facility of unlimited storage capacity. But in reality, when suppliers provide attractive price discount for bulk purchase at a time, the inventory manager may purchase more goods. These large amount of goods can not to be stored in its own warehouse (OW) due to its limited capacity. For these excess quantities, additional warehouse is required and items are stored in rented warehouse (RW). Due to different preservation facilities the inventory costs in RW are assumed to be higher than those in OW. So, it will be economical for the inventory manager to store items in OW before RW, but the items of RW are consumed first and the items of OW are the next to reduce the inventory cost. Two warehouse inventory model was first discussed by Hartley. Sarma (1987) developed two warehouse inventory model for deteriorating items with an infinite replenishment rate and shortage. Pakkala and Achary (1992) extended the two warehouse inventory model for deteriorating items with finite rate of replenishment and shortages, taking time as discrete and continuous variable, respectively. Bhunia and Maiti (1998) considered a two warehouse inventory model for deteriorating items with linearly increasing demand and shortages. Zhou (2003) studied two warehouse inventory models with time varying demand. Wee et al. (2005) presented a two warehouse model with constant demand and Weibull distribution deterioration under inflation. Shaikh et al. (2019) developed a two-warehouse inventory model with advanced payment, partial backlogged shortages. Subsequently, the ideas of two warehouse modelling were considered by some other authors, such as Pasandideh et al. (2015), Tiwari et al. (2016), Jaggi et al. (2017), Shaikh et al. (2019), Panda et al. (2019) and others.

It is generally assumed that the demand rate is independent of factors like price of items, stock availability, etc. However, in real life, it is observed that for certain type of inventory, particularly consumer goods in supermarkets; customers are highly influenced by the stock level. The sale at a retail level is directly proportional to the amount of inventory displayed. Levin et al. (1972) pointed out that large piles of consumer goods displayed in a supermarket attract the customer to buy more. Kar et al. (2001) studied an inventory model for deteriorating items sold from two shops, under single management dealing with limitations on investment and the total floor space area. They considered that demand of the fresh units varies with the amount in stock and its selling price. Alfares (2007) determined the optimum inventory policy for an inventory system with inventory level dependent demand rate and a time dependent holding cost. Yadav et al. (2012)

developed a multi item inventory model for deteriorating items with stock dependent demand rate under inflation and time value of money. Pal and Chandra (2014) studied a periodic review inventory model for non-instantaneous deteriorating items with stock dependent and time decreasing demand. Aggarwal and Tyagi (2017) determined optimal inventory and credit decisions in an inventory system when demand is dependent on day-terms credit period as well as on instantaneous inventory-level. Shaikh et al. (2017) developed an inventory model where the demand function is dependent on price and stock, and in shortage time the demand is depend only price of the product. Bhunia et al. (2018) considered the impact of marketing decisions and the displaced stock level on the demand in their inventory model. Tripathi (2018) proposed a model of deteriorating items with inventory induced demand and inflation. Masud et al. (2018) studied inventory model with consideration of price, stock dependent demand, partially backlogged shortages, and two constant deterioration rates. Shaikh et al. (2019) developed an inventory model for deteriorating item with variable demand dependent on the selling price and the deterioration rate follows a three parameters Weibull distribution. Khan et al. (2019) studied two supply chain models by assuming the demand to be dependent on price. They also considered that deterioration rate is dependent on expiration and shortages with partial backlogging. Shaikh developed two different inventory models, namely (a) inventory model for zero-ending case and (b) inventory model for shortages case, considering demand as price and stock dependent for both models, and shortages are partially backlogged.

In the traditional inventory model, it is assumed that the customer must pay for the items as soon as the items are received. But, in practice, the supplier allows the inventory manager a certain period of time to settle his accounts. No interest is charged during this period, but beyond it the manager has to pay an interest to the supplier. Goyal (1985) is the pioneer researcher who formed inventory models taking the condition of permissible delay in payments. Pal and Ghosh (2007) studied an inventory model for deteriorating items with stock-dependent demand under permissible delay in payments. Misra et al. (2011) derived an optimal inventory replenishment policy for two parameters weibull deteriorating items with a permissible delay in payment under inflation over the finite planning horizon. Pal and Chandra (2014) studied a periodic review inventory model with stock dependent demand rate, allowing shortages and permissible delay in payments. Shaikh et al. (2018) developed an inventory model for deteriorating item with permissible delay in payments.

Major contribution of the proposed model

Literature	Warehouse facility	Type of payment	Demand rate
Pal and Chandra (2014)	Single	Permissible delay in payment	Stock dependent
Aggarwal and Tyagi (2017)	Single	Credit period	Stock dependent
Md Mashud et al. (2018)	Single	Advance	Stock and price dependent
Bhunia et al. (2018)	Single	Advance	Displayed stock dependent
Shaikh et al. (2019)	Single	Trade credit	Price dependent
Panda et al. (2019)	Two	Advance	Price and stock dependent
Khan et al. (2019)	Single	Advance	Price dependent
This paper	Two	Permissible delay in payment	Stock dependent

In this paper, a two warehouse inventory model for deteriorating items is considered with stock dependent demand. It is assumed that the items of rented warehouse are consumed first and then the items of owned warehouse are consumed because rented warehouse has higher unit holding cost than the owned warehouse. The supplier allows the inventory manager a fixed time interval to settle his dues. The objective of this model is to find the best replenishment policies for minimizing the total appropriate inventory cost. The paper is organized as follows. Assumptions and notations are presented in Section 2. In Section 3, the model is formulated and the optimal value of decision variables are determined. In Section 4, numerical examples are cited to illustrate the policy, and analyze the sensitivity of the model with respect to the cost parameters. Concluding remarks are given in Section 5.

2. Notations and Assumptions

To develop the model, the following notations and assumptions have been used.

Notations

$I_0(t)$ = inventory level in owned warehouse (OW) at time point t

$I_r(t)$ = inventory level in rented warehouse (RW) at time point t

K = ordering cost per order

P = purchase cost per unit

h_r = inventory holding cost per unit per unit time in RW

h_0 = inventory holding cost per unit per unit time in OW

θ_1 = deterioration rate in RW, $0 < \theta_1 < 1$

θ_2 = deterioration rate in OW, $0 < \theta_2 < 1$, $\theta_2 > \theta_1$

I_e = interest that can be earned per unit time

I_r = interest payable per unit time beyond the permissible delay period ($I_r > I_e$)

M = permissible delay in settling the accounts, $0 < M < T$

T = length of a replenishment cycle

T_1 = time taken for stock on hand to be exhausted at RW, $0 < T_1 < T$

S = maximum stock height in a replenishment cycle at OW

Assumptions

1. The model considers only one item in inventory.
2. Replenishment of inventory occurs instantaneously on ordering i.e., lead time is zero.
3. The OW has the limited capacity of storage (S) and RW has unlimited capacity.
4. Items of RW are consumed first and then the items of OW are consumed due to the more holding cost in RW than in OW ($h_r > h_0$).
5. Due to the improved services offer in RW, the deterioration rate in RW is less than deterioration rate in OW ($\theta_2 > \theta_1$).
6. Demand rate $R(t)$ at time t is
$$R(t) = \alpha + \beta I(t) \quad for \ 0 < t < T$$
where α = fixed demand per unit time, $\alpha > 0$, β = fraction of total inventory demanded per unit time under the influence of stock on hand, $0 < \beta < 1$.
7. No payment to the supplier is outstanding at the time of placing an order, i.e., T is assumed to be greater than M.

3. MODEL FORMULATION

The planning period is divided into reorder intervals, each of length T units. Orders are placed at time points 0, T, 2T, 3T, …. At the beginning of the reorder interval order quantity being just sufficient to bring the stock height at OW to a certain maximum level S and the remaining order quantity in RW. Due to different preservation facilities the inventory costs (including holding cost and deterioration cost) in RW are assumed to be higher than those in OW. So, it will be economical for the inventory manager to store items in OW before RW, but the items of RW are consumed first and the items of OW are the next to reduce the inventory cost. Stocks on hand of both warehouses are exhausted at time point T.

Depletion of inventory at RW occurs due to demand and deterioration during the period $(0, T_1)$, $T_1 < T$. Hence, the variation in inventory level at RW with respect to time is given by

$$\frac{d}{dt}I_r(t) + \theta_1 I_r(t) = -\alpha - \beta I_r(t), \quad \text{if } 0 \le t \le T_1$$

Since $I_r(T_1) = 0$, we get

$$I_r(t) = \frac{\alpha}{\theta_1 + \beta}\left(e^{(\theta_1+\beta)(T_1-t)} - 1\right), \quad \text{if } 0 \le t \le T_1 \tag{3.1}$$

Depletion of inventory at OW occurs due to deterioration during the period $(0, T_1)$, and due to demand and deterioration both during the period (T_1, T), $T_1 < T$. Hence, the variation in inventory level at OW with respect to time is given by

$$\begin{aligned}\frac{d}{dt}I_0(t) + \theta_2 I_0(t) &= 0, && \text{if } 0 \le t \le T_1 \\ &= -\alpha - \beta I_0(t), && \text{if } T_1 \le t \le T\end{aligned}$$

Since $I_0(0) = S$ and $I_0(T) = 0$, we get

$$\begin{aligned}I_0(t) &= Se^{-\theta_2 t}, && \text{if } 0 \le t \le T_1 \\ &= \frac{\alpha}{\theta_2 + \beta}\left(e^{(\theta_2+\beta)(T-t)} - 1\right), && \text{if } T_1 \le t \le T\end{aligned} \tag{3.2}$$

Considering the continuity of $I_0(t)$ at $t = T_1$, it follows that

$$I_0(T_1) = Se^{-\theta_2 T_1} = \frac{\alpha}{\theta_2 + \beta}\left(e^{(\theta_2+\beta)(T-T_1)} - 1\right)$$

Hence, $$T = \frac{1}{\theta_2 + \beta}\ln\left(\frac{(\theta_2+\beta)Se^{-\theta_2 T_1}}{\alpha} + 1\right) + T_1 \tag{3.3}$$

Then,

Ordering cost during a cycle (OC) = K

Holding cost of inventories at RW during a cycle (HC_r)

$$= h_r \int_0^{T_1} I_r(t)dt$$

$$= \frac{h_r \alpha}{\theta_1 + \beta} \left(\frac{1}{\theta_1 + \beta} \left(e^{(\theta_1+\beta)T_1} - 1 \right) - T_1 \right)$$

Holding cost of inventories at OW during a cycle (HC_0)

$$= h_0 \int_0^{T} I_0(t)dt = h_0 \left(\int_0^{T_1} I_0(t)dt + \int_{T_1}^{T} I_0(t)dt \right)$$

$$= h_0 \left(\frac{S}{\theta_2} \left(1 - e^{-\theta_2 T_1} \right) + \frac{\alpha}{\theta_2 + \beta} \left(\frac{1}{\theta_2 + \beta} \left(e^{(\theta_2+\beta)(T-T_1)} - 1 \right) - (T - T_1) \right) \right)$$

Deterioration cost of inventories at RW during a cycle (DC_r)

$$= \theta_1 \int_0^{T_1} I_r(t)dt$$

$$= \frac{\theta_1 \alpha}{\theta_1 + \beta} \left(\frac{1}{\theta_1 + \beta} \left(e^{(\theta_1+\beta)T_1} - 1 \right) - T_1 \right)$$

Deterioration cost of inventories at OW during a cycle (DC_0)

$$= \theta_2 \int_0^{T} I_0(t)dt = \theta_2 \left(\int_0^{T_1} I_0(t)dt + \int_{T_1}^{T} I_0(t)dt \right)$$

$$= S \left(1 - e^{-\theta_2 T_1} \right) + \frac{\theta_2 \alpha}{\theta_2 + \beta} \left(\frac{1}{\theta_2 + \beta} \left(e^{(\theta_2+\beta)(T-T_1)} - 1 \right) - (T - T_1) \right)$$

As regards the permissible delay in payment, there can be two possibilities: $M \le T_1$ and $M > T_1$.

Case 1: $M \le T_1$

For $M \le T_1$, the inventory manager has stock on hand beyond M, and so he can use the sale revenue to earn interest at a rate I_e during $(0, T_1)$. The interest earn by the inventory manager is, therefore,

$$IE_1 = PI_e \left(\int_0^{M} I_r(t)dt + \int_0^{M} I_0(t)dt \right)$$

$$= PI_e \left(\frac{\alpha}{\theta_1 + \beta} \left(\frac{1}{\theta_1 + \beta} \left(e^{(\theta_1+\beta)T_1} - e^{(\theta_1+\beta)(T_1-M)} \right) - M \right) + \frac{S}{\theta_2} \left(1 - e^{-\theta_2 M} \right) \right)$$

Beyond the fixed settlement period, the unsold stock is financed with an interest rate I_r, so that the interest payable by the inventory manager is

$$IP_1 = PI_r\left(\int_M^{T_1} I_r(t)dt + \int_M^{T_1} I_0(t)dt + \int_{T_1}^{T} I_0(t)dt\right)$$

$$= PI_r\left(\begin{aligned}&\frac{\alpha}{\theta_1+\beta}\left(\frac{1}{\theta_1+\beta}\left(e^{(\theta_1+\beta)(T_1-M)}-1\right)-\left(T_1-M\right)\right)+\frac{S}{\theta_2}\left(e^{-\theta_2 M}-e^{-\theta_2 T_1}\right)\\&+\frac{\alpha}{\theta_2+\beta}\left(\frac{1}{\theta_2+\beta}\left(e^{(\theta_2+\beta)(T-T_1)}-1\right)-\left(T-T_1\right)\right)\end{aligned}\right)$$

Hence, the cost per unit length of a replenishment cycle is given by

$$C_1(S,T_1) = \frac{1}{T}[\mathrm{OC}+\mathrm{HC_r}+\mathrm{HC_0}+\mathrm{DC_r}+\mathrm{DC_0}+\mathrm{IP_1}\text{-}\mathrm{IE_1}]$$

$$= \frac{1}{T}\left(\begin{aligned}&K+\frac{\alpha\left(h_r+\theta_1\right)}{\theta_1+\beta}\left(\frac{1}{\theta_1+\beta}\left(e^{(\theta_1+\beta)T_1}-1\right)-T_1\right)\\&+\left(\begin{aligned}&\frac{\alpha\left(h_0+\theta_2+PI_r\right)}{\theta_2+\beta}\left(\frac{1}{\theta_2+\beta}\left(e^{(\theta_2+\beta)(T-T_1)}-1\right)-(T-T_1)\right)\\&+S\left(1-e^{-\theta_2 T_1}\right)\left(\frac{h_0}{\theta_2}+1\right)\end{aligned}\right)\\&+\frac{\alpha P\left(I_r+I_e\right)}{\left(\theta_1+\beta\right)^2}e^{(\theta_1+\beta)(T_1-M)}-\frac{\alpha P\left(I_r+I_e e^{(\theta_1+\beta)T_1}\right)}{\left(\theta_1+\beta\right)^2}-\frac{\alpha P}{\theta_1+\beta}\left(I_r T_1-M\left(I_r+I_e\right)\right)\\&+\frac{SP}{\theta_2}\left(e^{-\theta_2 M}\left(I_r+I_e\right)-I_r e^{-\theta_2 T_1}-I_e\right)\end{aligned}\right)$$

$$= \frac{N_1(S,T_1)}{T}$$

The optimal values of S and T_1, which minimize $C_1(S, T_1)$, must satisfy the following equations:

$$\frac{\alpha\left(h_0+\theta_2+PI_r\right)}{\theta_2+\beta}e^{-\theta_2 T_1}\left(\frac{1}{\alpha}-\frac{1}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}\right)+\left(1-e^{-\theta_2 T_1}\right)\left(\frac{h_0}{\theta_2}+1\right)$$

$$+\frac{P}{\theta_2}\left(e^{-\theta_2 M}\left(I_r+I_e\right)-I_r e^{-\theta_2 T_1}-I_e\right)=\frac{e^{-\theta_2 T_1}}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}C_1(S,T_1) \tag{3.4}$$

$$\frac{\alpha\left(h_r+\theta_1\right)\left(e^{(\theta_1+\beta)T_1}-1\right)}{\left(\theta_1+\beta\right)}+\frac{\alpha S\theta_2\left(h_0+\theta_2+PI_r\right)e^{-\theta_2T_1}}{\left(\theta_2+\beta\right)}\left(\frac{1}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2T_1}}-\frac{1}{\alpha}\right)+S\left(h_0+\theta_2\right)e^{-\theta_2T_1}$$

$$+\frac{\alpha P}{\left(\theta_1+\beta\right)}\left((I_e+I_r)e^{(\theta_1+\beta)(T_1-M)}-(I_e e^{(\theta_1+\beta)T_1}+I_r)\right)+SPI_r e^{-\theta_2T_1}=\left(1-\frac{S\theta_2}{\alpha e^{\theta_2T_1}+\left(\theta_2+\beta\right)S}\right)C_1(S,T_1) \tag{3.5}$$

<u>Case 2</u>: $M > T_1$

The inventory manager has stock on hand beyond M, and so he can use the sale revenue to earn interest at a rate I_e.

The interest earned by the inventory manager is given by

$$IE_2=PI_e\left(\int_0^{T_1}I_r(t)dt+\int_0^{T_1}I_0(t)dt+\int_{T_1}^{M}I_0(t)dt\right)$$

$$=PI_e\left(\begin{array}{l}\frac{\alpha}{\theta_1+\beta}\left(\frac{1}{\theta_1+\beta}\left(e^{(\theta_1+\beta)T_1}-1\right)-T_1\right)+\frac{S}{\theta_2}\left(1-e^{-\theta_2T_1}\right)\\+\frac{\alpha}{\theta_2+\beta}\left(\frac{1}{\theta_2+\beta}\left(e^{(\theta_2+\beta)(T-T_1)}-e^{(\theta_2+\beta)(T_1-M)}\right)-\left(M-T_1\right)\right)\end{array}\right)$$

Beyond the fixed settlement period, the unsold stock is financed with an interest rate I_r, so that the interest payable by the inventory manager is

$$IP_2=PI_r\int_M^T I_0(t)dt$$

$$=\frac{\alpha PI_r}{\theta_2+\beta}\left(\frac{1}{\theta_2+\beta}\left(e^{(\theta_2+\beta)(T-M)}-1\right)-\left(T-M\right)\right)$$

Hence, the cost per unit length of a replenishment cycle is given by

$$C_2(S,T_1)=\frac{1}{T}[\mathrm{OC}+\mathrm{HC_r}+\mathrm{HC_0}+\mathrm{DC_r}+\mathrm{DC_0}+\mathrm{IP_2}\text{-}\mathrm{IE_2}]$$

$$=\frac{1}{T}\left(\begin{array}{l} K+\dfrac{\alpha\left(h_r+\theta_1-PI_e\right)}{\theta_1+\beta}\left(\dfrac{1}{\theta_1+\beta}\left(e^{(\theta_1+\beta)T_1}-1\right)-T_1\right) \\ +\left(\begin{array}{l}\dfrac{\alpha\left(h_0+\theta_2\right)}{\theta_2+\beta}\left(\dfrac{1}{\theta_2+\beta}\left(e^{(\theta_2+\beta)(T-T_1)}-1\right)-(T-T_1)\right) \\ +S\left(1-e^{-\theta_2 T_1}\right)\left(\dfrac{h_0-PI_e}{\theta_2}+1\right)\end{array}\right)+\dfrac{\alpha P\left(I_r+I_e\right)}{\left(\theta_1+\beta\right)^2}e^{(\theta_2+\beta)(T-M)} \\ -\dfrac{\alpha PI_r}{\theta_2+\beta}\left(\dfrac{1}{\theta_2+\beta}+\left(T-M\right)\right)-\dfrac{\alpha PI_e}{\theta_2+\beta}\left(\dfrac{e^{(\theta_2+\beta)(T-T_1)}}{\theta_2+\beta}+\left(M-T_1\right)\right)\end{array}\right)$$

$$=\frac{N_2(S,T_1)}{T}$$

The optimal values of S and T_1, which minimize $C_2(S,T_1)$, must satisfy the following equations:

$$\frac{\alpha e^{-\theta_2 T_1}}{\theta_2+\beta}\left(\left(h_0+\theta_2\right)\left(1-\frac{\alpha}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}\right)+P\left(I_r+I_e\right)e^{(\theta_2+\beta)(T-M_1)}-\frac{\alpha PI_r}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}-PI_e\right)$$
$$+\left(1-e^{-\theta_2 T_1}\right)\left(\frac{h_0-PI_e}{\theta_2}+1\right)=\frac{e^{-\theta_2 T_1}}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}C_2(S,T_1) \tag{3.6}$$

$$\frac{\alpha\left(h_r+\theta_1-PI_e\right)\left(e^{(\theta_1+\beta)T_1}-1\right)}{\left(\theta_1+\beta\right)}+S\theta_2 e^{-\theta_2 T_1}\left\{\begin{array}{l}\dfrac{\alpha\left(h_0+\theta_2\right)}{\left(\theta_2+\beta\right)}\left(\dfrac{1}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}-\dfrac{1}{\alpha}\right)+\left(\dfrac{h_0-PI_e}{\theta_2}+1\right) \\ +\dfrac{P}{\left(\theta_2+\beta\right)}\left(\dfrac{\alpha I_r}{\alpha+\left(\theta_2+\beta\right)Se^{-\theta_2 T_1}}+I_e\right)\end{array}\right\}$$
$$+\frac{\alpha P(I_e+I_r)}{\left(\theta_2+\beta\right)}\left\{e^{(\theta_2+\beta)(T_1-M)}\left(\frac{S\beta e^{-\theta_2 T_1}}{\alpha}+1\right)-1\right\}=\left(1-\frac{S\theta_2}{\alpha e^{\theta_2 T_1}+\left(\theta_2+\beta\right)S}\right)C_2(S,T_1) \tag{3.7}$$

The total expected cost per unit length of a replenishment cycle is, therefore, given by

$$\begin{aligned}C(S,T_1)&=C_1(S,T_1)\quad \text{if}\ \ T_1\geq M\\ &=C_2(S,T_1)\quad \text{if}\ \ T_1<M\end{aligned}$$

The optimal values of the decision variables (S,T_1) minimizing $C(S,T_1)$ will be the set of values minimizing $C_1(S,T_1)$ if min $C_1(S,T_1) \leq$ min $C_2(S,T_1)$, or the set of values minimizing $C_2(S,T_1)$ if min $C_2(S,T_1) \leq$ min $C_1(S,T_1)$.

4. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

Since it is difficult to find closed form solutions to the sets of equations (3.4) - (3.5) and (3.6) - (3.7), we numerically find solutions to the equations for given sets of model parameters using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed.

Table 1: Showing the optimal inventory policy for different values of h_r, when K = 500, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, P = 5, M = 0.6, I_r = 0.04 and I_e = 0.03.

h_r	S	T_1	T	$C(S,T_1)$
0.2	0.75	2.46	2.47	298.75
0.25	29.72	2.32	2.43	305.64
0.3	54.07	2.20	2.40	311.52
0.35	74.95	2.08	2.37	316.60
0.4	93.16	1.98	2.35	321.02
0.45	109.27	1.88	2.32	324.90
0.5	123.68	1.79	2.30	328.32
0.55	136.74	1.71	2.28	331.35
0.6	148.67	1.63	2.26	334.04

Table 2: Showing the optimal inventory policy for different values of h_0, when K = 500, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_r = 0.25, P = 5, M = 0.6, I_r = 0.04 and I_e = 0.03.

h_0	S	T_1	T	$C(S,T_1)$
0.1	53.45	2.25	2.45	304.65
0.13	39.13	2.29	2.44	305.31
0.15	29.72	2.32	2.43	305.64
0.17	20.40	2.35	2.43	305.87
0.2	6.55	2.40	2.42	306.07

Table 3: Showing the optimal inventory policy for different values of P when K = 500, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, M = 0.6, I_r = 0.04 and I_e = 0.03.

P	S	T_1	T	$C(S,T_1)$
1	23.29	2.40	2.48	300.04
3	26.46	2.36	2.46	302.89
5	29.72	2.32	2.43	305.64
7	33.06	2.28	2.41	308.28
8	34.77	2.26	2.40	309.56
10	38.26	2.22	2.37	312.06

Table 4: Showing the optimal inventory policy for different values of M when K = 500, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5, I_r = 0.04 and I_e = 0.03.

M	S	T_1	T	$C(S,T_1)$
0.3	36.34	2.24	2.38	318.86
0.6	29.72	2.32	2.43	305.64
1	23.39	2.40	2.48	292.42
1.5	19.32	2.47	2.54	281.21
2	18.61	2.51	2.58	274.10
2.5	20.03	2.54	2.60	269.72

Table 5: Showing the optimal inventory policy for different values of θ_1 when K = 500, M = 0.6, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5, I_r = 0.04 and I_e = 0.03.

θ_1	S	T_1	T	$C(S,T_1)$
0.4	29.72	2.32	2.43	305.64
0.45	72.71	2.09	2.37	314.57
0.5	104.76	1.90	2.32	321.58
0.55	130.13	1.73	2.28	327.21
0.6	151.06	1.60	2.25	331.83
0.65	168.86	1.48	2.22	335.66

Table 6: Showing the optimal inventory policy for different values of θ_2 when K = 500, M = 0.6, θ_1 = 0.4, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5, I_r = 0.04 and I_e = 0.03.

θ_2	S	T_1	T	$C(S,T_1)$
0.45	169.42	1.78	2.72	278.02
0.5	149.69	1.87	2.65	285.73
0.55	127.90	1.96	2.58	292.57
0.6	102.55	2.06	2.52	298.39
0.65	71.26	2.18	2.47	302.90
0.7	29.72	2.32	2.43	305.64

Table 7: Showing the optimal inventory policy for different values of I_r when K = 500, M = 0.6, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5 and I_e = 0.03.

I_r	S	T_1	T	$C(S,T_1)$
0.04	29.72	2.32	2.43	305.64
0.1	47.07	2.09	2.28	332.04
0.15	58.56	1.92	2.18	351.51
0.2	69.24	1.76	2.09	369.08
0.25	80.75	1.58	2.00	384.92
0.3	96.97	1.37	1.90	399.03

Table 8: Showing the optimal inventory policy for different values of I_e when K = 500, M = 0.6, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5 and I_r = 0.04.

I_e	S	T_1	T	$C(S,T_1)$
0.01	34.96	2.27	2.40	314.43
0.015	33.75	2.28	2.41	312.25
0.02	32.47	2.29	2.42	310.06
0.025	31.13	2.31	2.42	307.85
0.03	29.72	2.32	2.43	305.64
0.035	28.24	2.34	2.44	303.41

The above tables show that, for other parameters remaining constant,

(a) both T_1 and T are decreasing in h_r, P, θ_1 and I_r but increase as I_e and M increase;

(b) S decreases with increase in h_0, I_e and θ_2, but increases with h_r, P, θ_1 and I_r;

(c) the minimum cost per unit length of a reorder interval increases as h_r, P, θ_1, θ_2 and I_r increase, but decreases with increase in I_e and M.

The above observations indicate that, with a view to minimizing total cost, the policy should be to maintain high inventory level in OW for low holding cost and deterioration rate in OW but high holding cost and deterioration rate in RW. Also, higher the permissible delay period lower should be the inventory level in OW.

The following table gives the percentage change in the total cost over an inventory cycle with change in the model parameters.

Let us consider the following model parameters: K = 500, M = 0.6, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5, I_e = 0.03 and I_r = 0.04. Table 9 gives the percentage change in the total cost over an inventory cycle with change in the model parameters.

Table 9: Percentage change in total cost with change in the model parameters

Parameter		% change in total cost	Parameter		% change in total cost
Name	Value		Name	Value	
h_r	0.2	-2.25	h_0	0.10	-0.32
	0.3	1.92		0.13	-0.11
	0.4	5.03		0.16	0.04
	0.5	7.42		0.18	0.11
	0.6	9.30		0.20	0.14
θ_1	0.45	2.92	θ_2	0.45	-9.03
	0.50	5.22		0.50	-6.51
	0.55	7.06		0.55	-4.27
	0.60	8.57		0.60	-2.37
	0.65	9.82		0.65	-0.90
M	0.3	4.33	P	1	-1.83
	1	-4.32		3	-0.90
	1.5	-7.99		7	0.87
	2	-10.32		8	1.29
	2.5	-11.75		10	2.10
I_r	0.05	1.53	I_e	0.01	2.88
	0.10	8.64		0.015	2.16
	0.15	15.01		0.02	1.45
	0.20	20.76		0.025	0.73
	0.25	25.94		0.035	-0.73

From the above table it is quite evident that the model is highly sensitive to changes in the holding cost in RW (h_r), deterioration rate in RW (θ_1), permissible delay period (M), interest earned per unit time (I_r) compare to other model parameters.

Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 5% and 10% and taking one parameter at a time, keeping the remaining parameters at their original values. Let us consider the following model parameters: K = 500, M = 0.6, θ_1 = 0.4, θ_2 = 0.7, α = 50, β = 0.5, h_0 = 0.15, h_r = 0.25, P = 5, I_e = 0.03 and I_r = 0.04. The following table gives the percentage change in the decision variables and total cost over an inventory cycle with change in the model parameters.

Table 10: The results of sensitivity analysis

Parameter	**% change**	**% change in *S***	**% change in T_1**	**% change in *T***	**% change in $C(S,T_1)$**
M	-10	4.04	-0.62	-0.37	0.78
	-5	1.99	-0.30	-0.19	0.39
	5	-1.94	0.30	0.18	-0.38
	10	-3.82	0.59	0.36	-0.74
θ_1	-10	-100.00	7.71	2.86	-2.97
	-5	-74.04	4.80	1.22	-1.41
	5	63.67	-4.34	-1.09	1.26
	10	119.31	-8.29	-2.08	2.40
θ_2	-10	185.00	-8.30	2.33	-1.43
	-5	102.41	-4.46	1.05	-0.56
	5	-100.00	4.07	-0.61	0.15
	10	-100.00	4.07	-0.61	0.15
I_r	-10	-4.72	0.73	0.45	-0.62
	-5	-2.34	0.36	0.22	-0.31
	5	2.31	-0.36	-0.22	0.31
	10	4.58	-0.72	-0.44	0.62
I_e	-10	2.87	-0.37	-0.20	0.44
	-5	1.45	-0.18	-0.10	0.22
	5	-1.47	0.19	0.10	-0.22
	10	-2.96	0.37	0.20	-0.44
h_r	-10	-46.49	2.95	0.69	-1.08
	-5	-22.73	1.45	0.34	-0.53
	5	21.77	-1.41	-0.33	0.51
	10	42.64	-2.79	-0.65	1.00
h_0	-10	23.72	-0.95	0.15	-0.08
	-5	11.84	-0.48	0.08	-0.04
	5	-11.79	0.48	-0.08	0.03
	10	-23.55	0.95	-0.15	0.06
P	-10	-2.76	0.42	0.25	-0.22
	-5	-1.38	0.21	0.13	-0.11
	5	1.39	-0.21	-0.13	0.11
	10	2.78	-0.42	-0.25	0.22

From the above table it is quite evident that the optimum value of S is highly sensitive to changes in the holding cost in RW (h_r), holding cost in OW (h_0), deterioration rate in RW (θ_1), deterioration rate in OW (θ_2) compare to other model parameters.

5. CONCLUSION

This paper studies two warehouse inventory model for deteriorating items under stock dependent demand environment. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal fruits and vegetables, newly launched fashion items, etc. The replenishment source allows the inventory manager a certain time period to settle his accounts. No interest is charged during this period, but beyond it the manager has to pay an interest. The optimum ordering policies are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that the policy should be to maintain high inventory level in OW for low holding cost and deterioration rate in OW but high holding cost and deterioration rate in RW. Also, higher the permissible delay period lower should be the inventory level in OW. From the result of sensitivity analysis, it is also observed that the optimum value of stock height in OW is highly sensitive to changes in the holding cost in RW, holding cost in OW, deterioration rate in RW, deterioration rate in OW compare to other model parameters. The model is highly sensitive to changes in the holding cost in RW, deterioration rate in RW, permissible delay period, interest earned per unit time compare to other model parameters.

The proposed inventory model can be extended by considering ordered quantity dependent permissible delay period. Since larger the quantity ordered, larger is the money involved and hence the supplier is expected to allow a longer time interval to pay the dues. Also, one may extend this model by taking nonlinear holding cost.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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