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COMMON FIXED POINT THEOREM FOR FOUR SELF MAPS SATISFYING COMMON LIMIT RANGE PROPERTY

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Abstract: In this paper, we present a common fixed point theorem for four self-maps using CLR_F-property and also give an example in support of the main result.

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Keywords: fixed point; metric space; weakly compatible mappings; CLR_F-property.

1. INTRODUCTION

In 1986, G Jungck introduced the notion of compatible maps by generalizing the concept of weakly commutativity. Later on, Jungck and Rhoades introduced the concept of weakly compatible mappings and proved some fixed point theorems for such mappings. In 2011, Sintunavarat and Kumam introduced a new property called common limit range property (CLR_G-property).

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The purpose of this paper is to establish a common fixed point theorem for four maps using weakly compatibility and common limit in the range of F (CLR_F) property.

2. PRELIMINARIES

We begin with

Definition 2.1 [1,2]. Two self-mappings E and F of a metric space (X, d) are said to be compatible if $\lim_{n\to\infty} d(EFx_n, FEx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ex_n = \lim_{n\to\infty} Fx_n = t$ for some $t \in X$.

Definition 2.2 [3]. Two self-mappings E and F of a metric space (X, d) are said to be weakly compatible, if they commute at their coincidence points.

Obviously every pair of compatible maps is weakly compatible but the converse may not hold.

Definition 2.3 [5]. Two self-maps E and F of a metric space (X, d) are said to satisfy the common limit in the range of F property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ex_n = \lim_{n\to\infty} Fx_n = Ft$, for some $t \in X$ and it is denoted by CLR_F property.

Example 2.4 Let X= [0, ∞) with the usual metric on X. Define E, F: X→X by

$Ex = \dfrac{x}{3}$ and $Fx = 3x$ for all $x \in X$. Consider the sequence $\{x_n\}$, where $x_n = \dfrac{1}{3n}$.

Since $\lim_{n\to\infty} Ex_n = \lim_{n\to\infty} Fx_n = 0 = F(0)$, E and F satisfy the ($CLR_F$) property.

Kenan Tas et al. proved the following Theorem.

2.5 Theorem [4]: Let E, F, G and H be self-mappings of a complete metric space (X, d) into itself satisfying the following conditions:

(2.5.1) $E(X) \subset H(X)$ and $F(X) \subset G(X)$

(2.5.2) one of E, F, G and H is continuous

(2.5.3) $[d(Ex, Fy)]^2 \le c_1 \max\{[d(Gx, Ex)]^2, [d(Hy, Fy)]^2, [d(Gx, Hy)]^2\}$

$$+c_2 \max\{d(Gx,Ex)d(Gx,Fy), d(Ex,Hy)d(Fy,Hy)\}$$

$$+c_3 d(Gx,Fy)d(Hy,Ex)$$

for all $x, y \in X$ where $c_1, c_2, c_3 \geq 0, c_1 + 2c_2 < 1$ and $c_1 + c_3 < 1$.

(2.5.4) the pairs (E, G) and (F, H) are compatible on X.

Then E, F, G and H have a unique common fixed point in X.

We now generalize the above theorem as follows.

3. MAIN RESULT

3.1 Theorem: Let E, F, G and H be self-mappings of a metric space (X, d) satisfying the following conditions:

3.1.1 $E(X) \subseteq H(X)$ and $F(X) \subseteq G(X)$,

3.1.2 (E, G) and (F, H) are weakly compatible and

3.1.3 $[d(Ey,Fz)]^2 \leq \alpha` \max\{[d(Gy,Ey)]^2, [d(Hz,Fz)]^2, [d(Gy,Hz)]^2\}$

$$+\beta` \max\{d(Gy,Ey)d(Gy,Fz), d(Ey,Hz)d(Fz,Hz)\} + \delta` d(Gy,Fz)d(Hz,Ey)$$

for all $y, z \in X$, where $\alpha`, \beta`, \delta` \geq 0$ , $\alpha` + 2\beta` < 1$ and $\alpha` + \delta` < 1$.

3.1.4 Further, if the pair (E, G) satisfies (CLR_G)-property or the pair (F, H) satisfies (CLR_H)-property, then the self-maps E, F, G and H have a unique common fixed point.

Proof: We prove this theorem in two cases.

Case(i): Let us first suppose that the pair (E, G) satisfies (CLR_G)-property .

So there is a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} Ey_n = \lim_{n\to\infty} Gy_n = Gu$,for some $u \in X$. (1)

Since $E(X) \subseteq H(X)$, for each $\{y_n\} \subset X$ there is a sequence $\{z_n\} \subset X$

such that $Ey_n = Hz_n$. (2)

Therefore $\lim_{n\to\infty} Hz_n = \lim_{n\to\infty} Ey_n = Gu$ where $u \in X$. (3)

Putting $y = y_n$ and $z = z_n$ in (3.1.3), we get

$$[d(Ey_n, Fz_n)]^2 \le \alpha` \max\left\{[d(Gy_n, Ey_n)]^2, [d(Hz_n, Fz_n)]^2, [d(Gy_n, Hz_n)]^2\right\}$$

$$+\beta` \max\{d(Gy_n, Ey_n)d(Gy_n, Fz_n), d(Ey_n, Hz_n)d(Fz_n, Hz_n)\}$$

$$+\delta` d(Gy_n, Fz_n)d(Hz_n, Ey_n)$$

Taking limit as n→∞, we get

$$\lim_{n\to\infty}[d(Ey_n, Fz_n)]^2 \le \lim_{n\to\infty}\left\{\begin{array}{l}\alpha` \max\left\{[d(Gy_n, Ey_n)]^2, [d(Hz_n, Fz_n)]^2, [d(Gy_n, Hz_n)]^2\right\} \\ +\beta` \max\{d(Gy_n, Ey_n)d(Gy_n, Fz_n), d(Ey_n, Hz_n)d(Fz_n, Hz_n)\} \\ +\delta` d(Gy_n, Fz_n)d(Hz_n, Ey_n)\end{array}\right\}$$

$$= \lim_{n\to\infty}\left\{\alpha` \max\left\{0, [d(Ey_n, Fz_n)]^2, 0\right\} + \beta` \max\{0,0\} + \delta` d(Ey_n, Fz_n).0\right\}$$

$$= \lim_{n\to\infty}\alpha`[d(Ey_n, Fz_n)]^2$$

$\lim_{n\to\infty}(1-\alpha`)[d(Ey_n, Fz_n)]^2 \le 0$ and since $1-\alpha` \ge 0$, we have $\lim_{n\to\infty} Ey_n = \lim_{n\to\infty} Fz_n$.

So that $\lim_{n\to\infty} Fz_n = Gu$.

Now since $E(X) \subseteq H(X)$, there exists a point $v \in X$ such that $Ey_n = Hv$.

Putting $y = y_n$ and $z = v$ in (3.1.3), we get

$$[d(Ey_n, Fv)]^2 \le \alpha` \max\left\{[d(Gy_n, Ey_n)]^2, [d(Hv, Fv)]^2, [d(Gy_n, Hv)]^2\right\}$$

$$+\beta` \max\{d(Gy_n, Ey_n)d(Gy_n, Fv), d(Ey_n, Hv)d(Fv, Hv)\}$$

$$+\delta` d(Gy_n, Fv)d(Hv, Ey_n)$$

Taking limit as n→∞ ,we get

$$\lim_{n\to\infty}[d(Ey_n, Fv)]^2 \le \lim_{n\to\infty}\left\{\begin{array}{l}\alpha` \max\left\{[d(Gy_n, Ey_n)]^2, [d(Hv, Fv)]^2, [d(Gy_n, Hv)]^2\right\} \\ +\beta` \max\{d(Gy_n, Ey_n)d(Gy_n, Fv), d(Ey_n, Hv)d(Fv, Hv)\} \\ +\delta` d(Gy_n, Fv)d(Hv, Ey_n)\end{array}\right\}$$

$$=\lim_{n\to\infty}\left\{\alpha`\max\left\{0,\left[d(Ey_n,Fv)\right]^2,0\right\}+\beta`\max\{0,0\}+\delta`d(Gy_n,Fv).0\right\}$$

$$=\lim_{n\to\infty}\alpha`\left[d(Ey_n,Fv)\right]^2$$

$(1-\alpha`)\lim_{n\to\infty}\left[d(Ey_n,Fv)\right]^2\le 0$ and since $1-\alpha`\ge 0$, we have $\lim_{n\to\infty}Ey_n=Fv$.

Therefore $Gu=Fv$ and so that $Fv=Hv=Gu=w$ (say). (4)

Hence v is a coincidence point of F and H.

Since the pair (F, H) is weakly compatible, $FHv=HFv$ and so that $Fw=Hw$.

Now since $F(X)\subseteq G(X)$, there exists a point $p\in X$, such that $Fv=Gp$. (5)

Putting $y=p$ and $z=v$ in (3.1.3), we get

$$\left[d(Ep,Fv)\right]^2\le\alpha`\max\left\{\left[d(Gp,Ep)\right]^2,\left[d(Hv,Fv)\right]^2,\left[d(Gp,Hv)\right]^2\right\}$$

$$+\beta`\max\left\{d(Gp,Ep)d(Gp,Fv),d(Ep,Hv)d(Fv,Hv)\right\}+\delta`d(Gp,Fv)d(Hv,Ep)$$

Using (4) and (5) in the above inequality , we get

$$\left[d(Ep,Gp)\right]^2\le\left\{\begin{array}{l}\alpha`\max\left\{\left[d(Gp,Ep)\right]^2,0,0\right\}\\+\beta`\max\left\{d(Gp,Ep).0,d(Ep,Gp).0\right\}\\+\delta`d(Gp,Gp).0\end{array}\right\}$$

$$=\alpha`\left[d(Ep,Gp)\right]^2 \text{ implies } (1-\alpha`)\left[d(Ep,Gp)\right]^2\le 0 \text{ and since } 1-\alpha`\ge 0$$

we have $Ep=Gp$. Therefore $Ep=Gp=Fv=w$.

Hence p is a coincidence point of E and G.

Since the pair (E, G) is weakly compatible, $EGp=GEp$ and so that $Ew=Gw$.

Putting $y=w$ and $z=v$ in (3.1.3), we get

$$\left[d(Ew,Fv)\right]^2\le\alpha`\max\left\{\left[d(Gw,Ew)\right]^2,\left[d(Hv,Fv)\right]^2,\left[d(Gw,Hv)\right]^2\right\}$$

$$+\beta`\max\left\{d(Gw,Ew)d(Gw,Fv),d(Ew,Hv)d(Fv,Hv)\right\}+\delta`d(Gw,Fv)d(Hv,Ew)$$

Using $Ew=Gw$, $Fw=Hw$, $Fv=Hv$ in the above inequality,

$$\left[d(Ew,Fv)\right]^2 \le \alpha`\max\left\{\left[d(Ew,Ew)\right]^2,\left[d(Fv,Fv)\right]^2,\left[d(Ew,Fv)\right]^2\right\}$$

$$+\beta`\max\left\{d(Ew,Ew)d(Ew,Fv),d(Ew,Fv)d(Fv,Fv)\right\} +\delta`d(Ew,Fv)d(Fv,Ew)$$

$$=\alpha`\left[d(Ew,Fv)\right]^2+\delta`\left[d(Ew,Fv)\right]^2$$ implies $(1-\alpha`-\delta`)\left[d(Ew,Fv)\right]^2 \le 0$ and since $(1-\alpha`-\delta`)>0$ and $Fv=w$, we have $d(Ew,w)=0$ so that $Ew=w$.

Therefore $Ew=Gw=w$.

Putting $y=w$ and $z=w$ in (3.1.3), we get

$$\left[d(Ew,Fw)\right]^2 \le \alpha`\max\left\{\left[d(Gw,Ew)\right]^2,\left[d(Hw,Fw)\right]^2,\left[d(Gw,Hw)\right]^2\right\}$$

$$+\beta`\max\left\{d(Gw,Ew)d(Gw,Fw),d(Ew,Hw)d(Fw,Hw)\right\} +\delta`d(Gw,Fw)d(Hw,Ew)$$

Using $Ew=w$, $Ew=Gw$ and $Fw=Hw$ in the above inequality, we get

$$\left[d(w,Fw)\right]^2 \le \alpha`\max\left\{\left[d(w,w)\right]^2,\left[d(Fw,Fw)\right]^2,\left[d(w,Fw)\right]^2\right\}$$

$$+\beta`\max\left\{d(w,w)d(w,Fw),d(w,Fw)d(Fw,Fw)\right\} \qquad +\delta`d(w,Fw)d(Fw,w)$$

$$=\alpha`\left[d(w,Fw)\right]^2+\delta`\left[d(w,Fw)\right]^2$$ which implies $(1-\alpha`-\delta`)\left[d(w,Fw)\right]^2 \le 0.$

So $\left[d(w,Fw)\right]^2=0$, since $1-\alpha`-\delta`>0$ and hence $Fw=w$ and therefore $Fw=Hw=w$.

Hence $Ew=Fw=Gw=Hw=w$, showing w is a common fixed point of E, F, G and H.

Case (ii): Suppose that the pair (F, H) satisfies (CLR_H) – property. So there is a sequence $\{z_n\}$ in X such that $\lim_{n\to\infty} Fz_n=\lim_{n\to\infty} Hz_n=Hu$, for some $u\in X$. Since $F(X)\subseteq G(X)$, for each $\{z_n\}\subset X$ there is a sequence $\{y_{n+1}\}\subset X$ such that $Fz_n=Gy_{n+1}$.

Therefore $\lim_{n\to\infty} Fz_n=\lim_{n\to\infty} Gy_{n+1}=Hu$, where $u\in X$.

Putting $y=y_{n+1}$ and $z=z_n$ in (3.1.3), we get

$$\left[d(Ey_{n+1},Fz_n)\right]^2 \le \alpha`\max\left\{\left[d(Gy_{n+1},Ey_{n+1})\right]^2,\left[d(Hz_n,Fz_n)\right]^2,\left[d(Gy_{n+1},Hz_n)\right]^2\right\}$$

$$+\beta`\max\left\{d(Gy_{n+1},Ey_{n+1})d(Gy_{n+1},Fz_n),d(Ey_{n+1},Hz_n)d(Fz_n,Hz_n)\right\}$$

$$+\delta` d(Gy_{n+1},Fz_n)d(Hz_n,Ey_{n+1})$$

Taking limit as n→∞, we get

$$\lim_{n\to\infty}\left[d(Ey_{n+1},Fz_n)\right]^2 \le \lim_{n\to\infty}\begin{Bmatrix} \alpha`\max\left\{\left[d(Gy_{n+1},Ey_{n+1})\right]^2,\left[d(Hz_n,Fz_n)\right]^2,\left[d(Gy_{n+1},Hz_n)\right]^2\right\} \\ +\beta`\max\begin{Bmatrix} d(Gy_{n+1},Ey_{n+1})d(Gy_{n+1},Fz_n), \\ d(Ey_{n+1},Hz_n)d(Fz_n,Hz_n)\end{Bmatrix} \\ +\delta` d(Gy_{n+1},Fz_n)d(Hz_n,Ey_{n+1}) \end{Bmatrix}$$

$$\lim_{n\to\infty}\left[d(Ey_{n+1},Hu)\right]^2 \le \lim_{n\to\infty}\left\{\alpha`\max\left\{\left[d(Hu,Ey_{n+1})\right]^2,0,0\right\}+\beta`\max\{0,0\}+\delta`0.d(Hu,Ey_{n+1})\right\}$$

$= \lim_{n\to\infty}\alpha`\left[d(Ey_{n+1},Hu)\right]^2$ which implies $\lim_{n\to\infty}(1-\alpha`)\left[d(Ey_{n+1},Hu)\right]^2 \le 0$ and since $1-\alpha`\ge 0$, we have $\lim_{n\to\infty}Ey_{n+1}=Hu.$

Now since $F(X)\subseteq G(X)$, there exists a point $v\in X$ such that $Fz_n = Gv$.

Putting $y=v$ and $z=z_n$ in (3.1.3), we get

$$\left[d(Ev,Fz_n)\right]^2 \le \alpha`\max\left\{\left[d(Gv,Ev)\right]^2,\left[d(Hz_n,Fz_n)\right]^2,\left[d(Gv,Hz_n)\right]^2\right\}$$

$$+\beta`\max\left\{d(Gv,Ev)d(Gv,Fz_n),d(Ev,Hz_n)d(Fz_n,Hz_n)\right\}$$

$$+\delta` d(Gv,Fz_n)d(Hz_n,Ev)$$

Taking limit as n→∞, we get

$$\lim_{n\to\infty}\left[d(Ev,Fz_n)\right]^2 \le \lim_{n\to\infty}\begin{Bmatrix}\alpha`\max\left\{\left[d(Ev,Fz_n)\right]^2,0,0\right\}+\beta`\max\{0,0\} \\ \delta`0.d(Hz_n,Ev)\end{Bmatrix}$$

$= \lim_{n\to\infty}\alpha`\left[d(Ev,Fz_n)\right]^2$ implies $(1-\alpha`)\lim_{n\to\infty}\left[d(Ev,Fz_n)\right]^2 \le 0$ and since $1-\alpha`\ge 0$, we have $\lim_{n\to\infty}Fz_n = Ev.$ Therefore $Ev=Gv=Hu=w$ (say) which shows that v is a coincidence point of E and G.

Since the pair (E, G) is weakly compatible, we have $EGv=GEv$ and so $Ew=Gw$.

Since $E(X)\subseteq H(X)$, there exists a point $p\in X$ such that $Ev=Hp$.

Putting $y=v$ and $z=p$ in (3.1.3), we get

$$[d(Ev,Fp)]^2 \le \alpha`\max\left\{[d(Gv,Ev)]^2,[d(Hp,Fp)]^2,[d(Gv,Hp)]^2\right\}$$

$$+\beta`\max\{d(Gv,Ev)d(Gv,Fp),d(Ev,Hp)d(Fp,Hp)\}$$

$$+\delta`d(Gv,Fp)d(Hp,Ev)$$

$[d(Hp,Fp)]^2 \le \alpha`[d(Hp,Fp)]^2$ implies $(1-\alpha`)[d(Hp,Fp)]^2 \le 0$ and since $1-\alpha` \ge 0$, we have $Hp=Fp.$ Therefore $Hp=Fp=Ev=w$ showing that p is a coincidence point of F and H.

Since the pair (F, H) is weakly compatible, $FHp=HFp$ and so $Fw=Hw.$

Putting $y=v$ and $z=w$ in (3.1.3), we get

$$[d(Ev,Fw)]^2 \le \alpha`\max\left\{[d(Gv,Ev)]^2,[d(Hw,Fw)]^2,[d(Gv,Hw)]^2\right\}$$

$$+\beta`\max\{d(Gv,Ev)d(Gv,Fw),d(Ev,Hw)d(Fw,Hw)\}$$

$$+\delta`d(Gv,Fw)d(Hw,Ev)$$

Using $Ew=Gw$, $Fw=Hw$ and $Fv=Hv$ in the above inequality, we get

$$[d(Ev,Fw)]^2 \le \alpha`\max\left\{[d(Ev,Ev)]^2,[d(Fw,Fw)]^2,[d(Ev,Fw)]^2\right\}$$

$$+\beta`\max\{d(Ev,Ev)d(Ev,Fw),d(Ev,Fw)d(Fw,Fw)\}$$

$$+\delta`d(Ev,Fw)d(Fw,Ev)$$

$=\alpha`[d(Ev,Fw)]^2+\delta`[d(Ev,Fw)]^2$ implies that $(1-\alpha`-\delta`)[d(Ev,Fw)]^2 \le 0$ and

Since $1-\alpha`-\delta`>0$ and $Ev=w$, we have $d(w,Fw)=0$ which implies $Fw=w.$

Therefore $Fw=Hw=w.$

Putting $y=w$ and $z=w$ in (3.1.3), we get

$$[d(Ew,Fw)]^2 \le \alpha`\max\left\{[d(Gw,Ew)]^2,[d(Hw,Fw)]^2,[d(Gw,Hw)]^2\right\}$$

$$+\beta`\max\{d(Gw,Ew)d(Gw,Fw),d(Ew,Hw)d(Fw,Hw)\}$$

$$+\delta`d(Gw,Fw)d(Hw,Ew)$$

Using Fw =Hw = w, $Ew=Gw$ and $Fw=Hw$ in the above inequality, we get

$$[d(Ew,w)]^2 \le \alpha`\max\left\{[d(Ew,Ew)]^2,[d(w,w)]^2,[d(Ew,w)]^2\right\}$$

$$+\beta`\max\{d(Ew,Ew)d(Ew,w),d(Ew,w)d(w,w)\} +\delta`d(Ew,w)d(w,Ew)$$

$=\alpha`[d(Ew,w)]^2+\delta`[d(Ew,w)]^2$ implies that $(1-\alpha`-\delta`)[d(Ew,w)]^2 \le 0$ and since $1-\alpha`-\delta`>0$, we have Ew = w and hence Ew = Gw = w.

Therefore $Ew=Fw=Gw=Hw=w$, showing that w is a common fixed point of

E, F, G and H.

Uniqueness: Let w* be another common fixed point of E, F, G and H. Then we have

w* = Ew* = Fw* = Gw* = Hw*.

Writing $y=w$ and $z=w^*$ in (3.1.3), we get

$$[d(Ew,Fw^*)]^2 \le \alpha`\max\left\{[d(Gw,Ew)]^2,[d(Hw^*,Fw^*)]^2,[d(Gw,Hw^*)]^2\right\}$$

$$+\beta`\max\{d(Gw,Ew)d(Gw,Fw^*),d(Ew,Hw^*)d(Fw^*,Hw^*)\}$$

$$+\delta`d(Gw,Fw^*)d(Hw^*,Ew)$$

$$[d(w,w^*)]^2 \le \alpha`\max\left\{[d(w,w)]^2,[d(w^*,w^*)]^2,[d(w,w^*)]^2\right\}$$

$$+\beta`\max\{d(w,w)d(w,w^*),d(w,w^*)d(w^*,w^*)\} +\delta`d(w,w^*)d(w^*,w)$$

$$=\alpha`\max\left\{0,0,[d(w,w^*)]^2\right\}+\beta`\max\{0,0\}+\delta`[d(w,w^*)]^2$$

$$(1-\alpha`-\delta`)d(w,w^*) \le 0$$

Since $\alpha`,\beta`,\delta`\ge 0$ , $\alpha`+2\beta`<1$ and $\alpha`+\delta`<1$, we have $d(w,w^*)=0$

which implies that w = w*.

Hence w is a unique common fixed point of E, F, G and H.

The following example illustrates the Theorem 3.1.

3.2 Example: Let X = (0, 1] with the usual metric $d(y,z)=|y-z|$ for all $y,z\in X$. We define the self-maps E, F, G and H of X by

$$E(y)=F(y)=\begin{cases}\frac{1}{2} & if\ 0<y\le\frac{1}{2}\\ \frac{2}{3} & if\ \frac{1}{2}<y\le 1\end{cases} \quad \text{and} \quad G(y)=H(y)=\begin{cases}1-y & if\ 0<y\le\frac{1}{2}\\ y & if\ \frac{1}{2}<y\le 1\end{cases}$$

Then $G(X)=H(X)=\left[\frac{1}{2},1\right]$ and $E(X)=F(X)=\left\{\frac{1}{2},\frac{2}{3}\right\}$.

Clearly $E(X)\subseteq H(X)$ and $\mathrm{F}(X)\subseteq G(X)$.

We now verify the conditions 3.1.2, 3.1.3 and 3.1.4 of the Theorem 3.1.

Consider a sequence $\{y_n\}$, where $y_n=\frac{1}{2}-\frac{1}{5n}$ for $n\ge 1$. Then

$E\left(\frac{1}{2}\right)=G\left(\frac{1}{2}\right)=\frac{1}{2}$ so that $EG\left(\frac{1}{2}\right)=\frac{1}{2}$ and $GE\left(\frac{1}{2}\right)=\frac{1}{2}$

Also $F\left(\frac{1}{2}\right)=H\left(\frac{1}{2}\right)=\frac{1}{2}$ so that $FH\left(\frac{1}{2}\right)=\frac{1}{2}$ and $HF\left(\frac{1}{2}\right)=\frac{1}{2}$, showing that the pairs (E, G) and (F, H) are weakly compatible.

Now $\lim\limits_{n\to\infty}Ey_n=\lim\limits_{n\to\infty}E\left(\frac{1}{2}-\frac{1}{5n}\right)=\lim\limits_{n\to\infty}\left(\frac{1}{2}\right)=\frac{1}{2}$ and also

$\lim\limits_{n\to\infty}Gy_n=\lim\limits_{n\to\infty}G\left(\frac{1}{2}-\frac{1}{5n}\right)=\lim\limits_{n\to\infty}\left(1-\left(\frac{1}{2}-\frac{1}{5n}\right)\right)=\lim\limits_{n\to\infty}\left(\frac{1}{2}+\frac{1}{5n}\right)=\frac{1}{2}.$ Therefore

$\lim\limits_{n\to\infty}Ey_n=\lim\limits_{n\to\infty}Gy_n=G\left(\frac{1}{2}\right)$. This shows that the pair (E, G) satisfies (CLR_G)-property. Moreover the condition 3.1.3 can easily be verified for suitable values of $\alpha`,\beta`,\delta`$. Thus all the conditions of the Theorem 3.1 are satisfied and $\frac{1}{2}$ is the unique common fixed point of E, F, G and H.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] G. Jungck, Compatible mappings and common fixed points, Int. J. Math. Math. Sci. 9 (1986), 771-778.

[2] G. Jungck, Compatible mappings and common fixed points (2), Int. J. Math. Math. Sci. 11(1988), 285- 288.

[3] G.Jungck and B.E. Rhoades, Fixed point for set valued functions without continuity, Indian Pure. Appl. Math. 29(3) (1998), 227-238.

[4] K. Taş, M. Telci, B. Fisher, Common fixed point theorems for compatible mappings, Int. J. Math. Math. Sci. 19 (1996), Article ID 414167.

[5] W. Sintunavarat, P. Kumam, Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces, J. Appl. Math, 2011 (2011), Article ID 637958.