Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 4, 1167-1175
https://doi.org/10.28919/jmcs/4578
ISSN: 1927-5307

ON p*gp-LOCALLY CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In this article, we consider a new class of sets which are called p*gp-locally closed sets and obtain some of their properties and also their relationships with some other classes of topological spaces. In addition, we found P*GPLC continuous function and P*GPLC irresolute function. Moreover, several examples are providing to illustrate the behavior of these new classes of sets.

Keywords: p*gp-locally closed; P*GPLC continuous; P*GPLC irresolute.

2010 AMS Subject Classification: 54A05, 54D05.

1. INTRODUCTION

Kuratowski and Sierpinski [7] have been studied the notion of a locally closed sets in a topological space. Bourbaki [1] defined by locally closed sets in topological spaces. Ganster and Reilly [4] used locally closed sets to define LC-continuity and LC-irresoluteness. The concept of generalized closed sets was considered by Levine [8] plays a significant role in general topology.

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Received March 14, 2020

Noiri, Maki, and Umehara [10] provided the class of pre generalized closed sets and used them to obtain properties of pre-$T_{1/2}$ spaces. Selvi [11] further investigated pre*closed sets using the g-closure operator due to Dunham [2, 3]. The notion of pre open set was discovered by Mashhour [9]. This characterization paved a new direction.

The authors [5, 6] brings out the p*gp-closed sets and p*gp-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The purpose of this paper is to discuss about the concept of p*gp-locally closed sets in topological spaces and study their basic properties. Also, we provide P*GPLC continuous function, P*GPLC* continuous function and P*GPLC** continuous function and discuss P*GPLC irresolute function. We obtain many interesting results, to substantiate these result, suitable examples are given at the respective places.

This paper is organized as follows. In the second section, a brief survey of basic concepts and results in topological spaces which are essentially needed are given Section 3, we consider the properties of p*gp-locally closed sets and some basic results, while section 4, introduces the classes of P*GPLC continuous function, P*GPLC* continuous function, P*GPLC** continuous function and P*GPLC irresolute function and some of the properties of these functions. Last section, we provide a brief summary of work done in this paper.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. (X, τ) will be replaced by X if there are no changes of confusion. For a subset A of a topological space X, cl(A), int(A) and X\A denote the closure of A, the interior of A and the complement of A respectively. Further, we denote the collection of all locally closed subsets of (X, τ) by LC(X, τ). We recall the following definitions and results which are prerequisites for our present work.

Definition 2.1. [8] Let (X, τ) be a topological space. Then the subset A of X is said to be

(i) generalized closed (briefly g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an open in (X, τ).

(ii) generalized open (briefly g-open) if its complement, X\A is g-closed.

Definition 2.2. Let (X, τ) be a topological space and A $\subseteq$ X. The generalized closure of A [2], denoted by cl*(A) and is defined by the intersection of all g-closed sets containing A and generalized interior of A [3], denoted by int*(A) and is defined by union of all g-open sets contained in A.

Definition 2.3. Let (X, τ) be a topological space and A $\subseteq$ X. Then

(i). A is pre open if A $\subseteq$ int(cl(A)) and pre closed if cl(int(A)) $\subseteq$ A [9].

(ii). A is pre*open if A $\subseteq$ int$*$(cl(A)) and pre*closed if cl*(int(A)) $\subseteq$ A [11].

Definition 2.4. [9] Let (X, τ) be a topological space and A $\subseteq$ X. The pre closure of A denoted by pcl(A) and is defined by the intersection of all pre closed sets containing A.

Definition 2.5. [5] A subset A of a topological space (X, τ) is said to be pre*generalized pre closed set (briefly p*gp-closed) if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is pre*open in (X, τ). The collection of all p*gp-closed sets of X is denoted by p*gp-C(X).

Lemma 2.6. [5] Let (X, τ) be a topological space. Then

(i). Every closed set is p*gp-closed.

(ii). Intersection of any two p*gp-closed sets is p*gp-closed.

Definition 2.7. [6] A subset A of a topological space (X, τ) is said to be p*gp-open if X\A is p*gp-closed. The collection of all p*gp-open sets of X is denoted by p*gp-O(X).

Lemma 2.8. [6] Let (X, τ) be a topological space. Then

(i). Every open set is p*gp-open.

(ii). Union of any two p*gp-open sets is p*gp-open.

Definition 2.9. A subset A of a topological space (X, τ) is called a locally closed (briefly lc) set [4] if A = U∩V where U is open and V is closed in (X, τ).

Definition 2.10. A function f : (X, τ) $\rightarrow$ (Y, σ) is called LC-continuous [4] if f^{-1}(F) is locally closed set in (X, τ) for each closed set F of (Y, σ).

Definition 2.11. A function f : (X, τ) $\rightarrow$ (Y, σ) is called LC-irresolute [4] if f^{-1}(F) is locally closed set in (X, τ) for locally closed set F of (Y, σ).

3. PRE*GENERALIZED PRE LOCALLY CLOSED SETS

In this section, p*gp-locally closed sets are introduced to obtain some of their properties and their relationships with other existing sets.

Definition 3.1. A subset A of a topological space (X, τ) is said to be a p*gp-locally closed (briefly p*gplc) set if A=V∩F where V is p*gp-open and F is p*gp-closed.

The class of all p*gp-locally closed sets in (X, τ) is denoted by $P^*GPLC(X, \tau)$.

Definition 3.2. A subset A of a topological space (X, τ) is said to be p*gplc* if there exist a p*gp-open set V and a closed set F of (X, τ) such that $A = V \cap F$.

The class of all p*gplc* sets in (X, τ) is denoted by $P^*GPLC^*(X, \tau)$.

Definition 3.3. A subset A of a topological space (X, τ) is said to be p*gplc** if there exist an open set V and a p*gp-closed set F of (X, τ) such that $A=V \cap F$.

The class of all p*gplc** sets in (X, τ) is denoted by $P^*GPLC^{**}(X, \tau)$.

Theorem 3.4. If a subset A of (X, τ) is locally closed then it is a p*gplc set, p*gplc* set and p*gplc** set.

Proof. Let A be a locally closed subset of X. Then $A=V \cap F$, where V is open and F is closed in (X, τ). By Lemma 2.8 and Lemma 2.6, A is a p*gplc set, p*gplc* set and p*gplc** set.

Remark 3.5. The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{c\}, X\}$. Then the locally closed sets are $\{\phi, \{c\}, \{a, b\}, X\}$, $P^*GPLC(X, \tau) = P^*GPLC^*(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $P^*GPLC^{**}(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\},\{a, b\}, X\}$. Here, $\{a\}$ is p*gplc, p*gplc* and p*gplc** but not locally closed.

Theorem 3.7. If a subset A of (X, τ) is p*gplc** then it is a p*gplc set.

Proof. Let A be a p*gplc** set. Then by Definition 3.3, $A=V \cap F$, where V is an open set in (X, τ) and F is a p*gp-closed set in (X, τ). By Lemma 2.8, A is p*gplc set.

Remark 3.8. The converse of the above theorem need not be true as shown in the following example.

Example 3.9. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Let $A = \{a, c\}$. Then $\{a, c\}$ is p*gplc set but not p*gplc** set.

Theorem 3.10. If $A \in P^*GPLC(X, \tau)$ and B is p*gp-closed in (X, τ), then $A \cap B \in P^*GPLC(X, \tau)$.

Proof. Since $A \in P^*GPLC(X, \tau)$, there exist a p*gp-open set V and a p*gp-closed set F such that $A = V \cap F$. Now $A \cap B = (V \cap F) \cap B = V \cap (F \cap B)$. Since V is p*gp-open and $F \cap B$ is p*gp-closed, $A \cap B \in P^*GPLC(X, \tau)$.

Theorem 3.11. If $A \in P^*GPLC^*(X, \tau)$ and B is closed in (X, τ), then $A \cap B \in P^*GPLC^*(X, \tau)$.

Proof. Since $A \in P^*GPLC^*(X, \tau)$, there exist a p*gp-open set V and a closed set F such that $A = V \cap F$. Since B is a closed set, we have $A \cap B = (V \cap F) \cap B = V \cap (F \cap B)$. Since V is p*gp-open and $F \cap B$ is closed, $A \cap B \in P^*GPLC^*(X, \tau)$.

Theorem 3.12. If $A \in P^*GPLC^{**}(X, \tau)$ and B is p*gp-closed (resp. open) in (X, τ), then $A \cap B \in P^*GPLC^{**}(X, \tau)$.

Proof. Since $A \in P^*GPLC^{**}(X, \tau)$, there exist an open set V and a p*gp-closed set F such that $A = V \cap F$. Now $A \cap B = (V \cap F) \cap B = V \cap (F \cap B)$. Since V is open and $F \cap B$ is p*gp-closed, $A \cap B \in P^*GPLC^{**}(X, \tau)$.

In this case B being an open set, we have $A \cap B = (V \cap F) \cap B = (V \cap B) \cap F$. Since $V \cap B$ is open and F is p*gp-closed, $A \cap B \in P^*GPLC^{**}(X, \tau)$.

Theorem 3.13. Let (X, τ) and (Y, σ) be topological spaces. Then

(i) If $A \in P^*GPLC(X, \tau)$ and $B \in P^*GPLC(Y, \sigma)$, then $A \times B \in P^*GPLC(X \times Y, \tau \times \sigma)$.

(ii) If $A \in P^*GPLC^*(X, \tau)$ and $B \in P^*GPLC^*(Y, \sigma)$, then $A \times B \in P^*GPLC^*(X \times Y, \tau \times \sigma)$.

(iii) If $A \in P^*GPLC^{**}(X, \tau)$ and $B \in P^*GPLC^{**}(Y, \sigma)$, then $A \times B \in P^*GPLC^{**}(X \times Y, \tau \times \sigma)$.

Proof. Let $A \in P^*GPLC(X, \tau)$ and $B \in P^*GPLC(Y, \sigma)$. Then there exist p*gp-open sets V and V_1 of (X, τ) and (Y, σ) and p*gp-closed sets F and F_1 of X and Y respectively such that $A = V \cap F$ and $B = V_1 \cap F_1$.Then $A \times B = (V \times V_1) \cap (F \times F_1)$ holds. Hence $A \times B \in P^*GPLC(X \times Y, \tau \times \sigma)$. This proves (i).

Let $A \in P^*GPLC^*(X, \tau)$ and $B \in P^*GPLC^*(Y, \sigma)$. Then there exist p*gp-open sets V and V_1 of (X, τ) and (Y, σ) and closed sets F and F_1 of (X, τ) and (Y, σ) respectively such that $A = V \cap F$ and $B = V_1 \cap F_1$. Then $A \times B = (V \times V_1) \cap (F \times F_1)$ holds. Hence $A \times B \in P^*GPLC^*(X \times Y, \tau \times \sigma)$. This proves (ii).

Let $A \in P^*GPLC^{**}(X, \tau)$ and $B \in P^*GPLC^{**}(Y, \sigma)$. Then there exist open sets V and V_1 of (X, τ) and (Y, σ) and p*gp-closed sets F and F_1 of $(X, \tau)$and (Y, σ) respectively such that $A = V \cap F$ and $B = V_1 \cap F_1$. Then $A \times B = (V \times V_1) \cap (F \times F_1)$ holds. Hence $A \times B \in P^*GPLC^{**}(X \times Y, \tau \times \sigma)$. This proves (iii).

4. FUNCTIONS VIA PRE*GENERALIZED PRE LOCALLY CLOSED SETS

In this section, we introduce the concept of P*GPLC continuous function, P*GPLC* continuous function and P*GPLC** continuous function in topological spaces and study some of their properties. Also, we describe P*GPLC irresolute function, P*GPLC* irresolute function and P*GPLC** irresolute function in topological spaces and study some of their properties.

Definition 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be P*GLC continuous (resp. P*GPLC* continuous, P*GPLC** continuous) if $f^{-1}(V) \in$ P*GPLC(X, τ) (resp. $f^{-1}(V) \in$ P*GPLC* (X, τ), $f^{-1}(V) \in$ P*GPLC**(X, τ)) for each closed set V of (Y, σ).

Example 4.2. Let X = Y = {a, b, c}, $\tau = \{\phi, \{c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by the identity function. Then, f is P*GPLC continuous, P*GPLC* continuous and P*GPLC** continuous.

Theorem 4.3. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then we have the following:

(i) If f is LC continuous, then f is P*GPLC continuous, P*GPLC* continuous and P*GPLC** continuous.

(ii) If f is P*GPLC** continuous function, then f is P*GPLC continuous.

Proof. Suppose that $f : (X, \tau) \to (Y, \sigma)$ is LC continuous. Let V be a closed set of (X, τ). Then $f^{-1}(V)$ is a locally closed set in (X, τ). By Theorem 3.4, it follows that f is P*GPLC continuous (resp. P*GPLC* continuous and P*GPLC** continuous). This proves (i).

Let $f : (X, \tau) \to (Y, \sigma)$ be a P*GPLC** continuous function. Let V be a closed set of (X, τ). Then $f^{-1}(V)$ is p*gplc** set in (X, τ). By Theorem 3.7, it follows that f is P*GPLC** continuous is P*GPLC continuous. This proves (ii).

Remark 4.4. The converse of the above theorem need not be true as seen from the following example.

Example 4.5. Let X = Y = {a, b, c}, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$.Define $f : (X, \tau) \to (Y, \sigma)$ by f (a) = b, f (b) = c and f (c) = a. Then, f is P*GPLC continuous, P*GPLC* continuous and P*GPLC** continuous. It can be proved that, $f^{-1}(\{a, b\}) = \{a, c\}$ is not a locally closed set in X. Hence f is not LC continuous.

Example 4.6. Let X = Y = {a, b, c}, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{c\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c) = b. Then, f is P*GPLC continuous. It can be found that, $f^{-1}(\{a, b\}) = \{a, c\}$ is not p*gplc** set in X. Hence f is not P*GPLC** continuous.

Theorem 4.7. If f: (X, τ)→(Y, σ) and g: (Y, σ)→ (Z, η) are any two functions. Then

(i) g∘f is P*GPLC continuous if f is P*GPLC continuous and g is continuous.

(ii) g∘f is P*GPLC* continuous if f is P*GPLC* continuous and g is continuous.

(iii) g∘f is P*GPLC** continuous if f is P*GPLC** continuous and g iscontinuous.

Proof. Let F be a closed set in (Z, η). Since g is continuous, $g^{-1}(F)$ is closed set in (Y, σ). Again, since f is P*GPLC continuous, $f^{-1}(g^{-1}(F))$ is p*gplc in (X, τ). Thus g∘f is P*GPLC continuous function. This proves (i).

Let F be a closed set in (Z, η). Since g is continuous, $g^{-1}(F)$ is closed in (Y, σ). Since f is P*GPLC* continuous, $f^{-1}(g^{-1}(F))$ is p*gplc* in (X, τ). Thus g∘f is P*GPLC* continuous function. This proves (ii).

Let F be a closed set in (Z, η). Since g is continuous, $g^{-1}(F)$ is closed in (Y, σ). Since f is P*GPLC** continuous, $f^{-1}(g^{-1}(F))$ is p*gplc** in (X, τ). Thus g∘f is P*GPLC** continuous function. This proves (iii).

Definition 4.8. A function f : (X, τ)→(Y, σ) is said to be P*GPLC irresolute (resp. P*GPLC* irresolute, P*GPLC** irresolute) if $f^{-1}(V)$ ∈ P*GPLC(X, τ)
(resp. $f^{-1}(V)$ ∈ P*GPLC*(X, τ), $f^{-1}(V)$ ∈ P*GPLC**(X, τ)) for each V ∈ P*GPLC (Y, σ**)**
(resp. V ∈ P*GPLC*(Y, σ**),** V ∈ P*GPLC** (Y, σ)).

Example 4.9. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {c}, {a, b}, Y}. Define f : (X, τ)→ (Y, σ) by f (a) = b, f (b) = a and f (c) = c. Then, f is P*GPLC irresolute, P*GPLC* irresolute and P*GPLC** irresolute.

Theorem 4.10. If a function f :(X, τ)→(Y, σ) is LC irresolute, then f is P*GPLC irresolute (resp. P*GPLC* irresolute and P*GPLC** irresolute).

Proof. Suppose that f is LC irresolute. Let V be a locally closed set of (X, τ). Then $f^{-1}(V)$ is a locally closed set in (X, τ). By Theorem 3.4, it follows that f is P*GPLC irresolute (resp. P*GPLC* irresolute and P*GPLC** irresolute).

Remark 4.11. The converse of the above theorem need not be true as seen from the following example.

Example 4.12. Let X = Y = {a, b, c}, τ = {ϕ, {c}, X} and σ = {ϕ, {c}, {a, b}, Y}. Define f : (X, τ) → (Y, σ) by f (a) = c, f (b) = b and f (c) = a. Then, f is P*GPLC irresolute, P*GPLC* irresolute and P*GPLC** irresolute. It can be verified that, $f^{-1}(\{a, c\}) = \{a, c\}$ is not locally closed in X. Hence f is not LC irresolute.

Theorem 4.13. Let $f : (X, \tau)\to (Y, \sigma)$ and $g:(Y, \sigma)\to (Z, \eta)$ be any two functions. Then

(i) $g\circ f : (X, \tau)\to(Z, \eta)$ is P*GPLC irresolute if g is P*GPLC irresolute and f is P*GPLC irresolute.

(ii) $g\circ f : (X, \tau)\to(Z, \eta)$ is P*GPLC continuous if g is P*GPLC continuous and f is P*GPLC irresolute.

Proof. Let F ∈ P*GPLC(Z, η). Since g is P*GPLC irresolute, $g^{-1}(F)$ is p*gplc in (Y, σ). As f is P*GPLC irresolute, $f^{-1}(g^{-1}(F))$ is p*gplc in (X, τ). That is $(g\circ f)^{-1}(F)$ ∈ P* GPLC(X, τ). Thus $g\circ f$ is P*GPLC irresolute. This proves (i).

Let F be a closed set in (Z, η). Since g is P*GPLC continuous, $g^{-1}(F)$ is p*gplc in (Y, σ). Again, since f is P*GPLC irresolute, $f^{-1}(g^{-1}(F))$ is p*gplc in (X, τ). Thus $g\circ f$ is P*GPLC continuous. This proves (ii).

Theorem 4.14. Let $f : (X, \tau)\to (Y, \sigma)$ and $g:(Y, \sigma)\to (Z, \eta)$ be any two functions. Then

(i) $g\circ f$ is P*GPLC* irresolute if f and g are P*GPLC* irresolute.

(ii) $g\circ f$ is P*GPLC** irresolute if f and g are P*GPLC** irresolute.

(iii) $g\circ f$ is P*GPLC* continuous if f is P*GPLC* irresolute and g is P*GPLC* continuous.

(iv) $g\circ f$ is P*GPLC** continuous if f is P*GPLC** irresolute and g is P*GPLC** continuous.

Proof. Let F∈P*GPLC*(Z, η). Since g is P*GPLC* irresolute, $g^{-1}(F)$ is p*gplc* in (Y, σ). As f is P*GPLC* irresolute, $f^{-1}(g^{-1}(F))$ is p*gplc* in (X, τ). That is $(g\circ f)^{-1}(F)$ ∈ P*GPLC*(X, τ). Thus $g\circ f$ is P*GPLC* irresolute. This proves (i).

Let F ∈ P*GPLC**(Z, η). Since g is P*GPLC** irresolute, $g^{-1}(F)$ is p*gplc** in (Y, σ). As f is P*GPLC** irresolute, $f^{-1}(g^{-1}(F))$ is p*gplc** in (X, τ). That is $(g\circ f)^{-1}(F)$ ∈ P*GPLC**(X, τ). Thus $g\circ f$ is P*GPLC** irresolute. This proves (ii).

Let F be a closed set in (Z, η). Since g is P*GPLC* continuous, $g^{-1}(F)$ is p*gplc* in (Y, σ). Again, since f is P*GPLC* irresolute, $f^{-1}(g^{-1}(F))$ is p*gplc* in (X, τ). Thus $g\circ f$ is P*GPLC* continuous. This proves (iii).

Let F be a closed set in (Z, η). Since g is P*GPLC** continuous, $g^{-1}(F)$ is p*gplc** in (Y, σ). Since f is P*GPLC** irresolute, $f^{-1}(g^{-1}(F))$ is p*gplc** in (X, τ). Thus $g∘f$ is P*GPLC** continuous. This proves (iv).

5. CONCLUSION

In this paper, p*gp-locally closed sets in topological spaces are projected. Also P*GPLC continuous function and P*GPLC irresolute function are found.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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