Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 4, 1298-1308
https://doi.org/10.28919/jmcs/4600
ISSN: 1927-5307

COMMON FIXED POINT THEOREM FOR RATIONAL CONTRACTION IN PARTIAL b-METRIC SPACE

VENKATESH BHATT[1,*], GINISWAMY[2], C. JEYANTHI[3]

[1]Department of Mathematics, PET Research Foundation, Mandya, Karnataka, India-560104

[2]Department of Mathematics, PES College of Science, Arts and Commerce, Mandya, Karnataka, India-571401

[3]Department of Mathematics, Teresian College, Mysore, Karnataka, India-560104

Abstract. In this paper, we proved some common fixed point theorem for rational contractions in partial b metric space. As a consequence, the obtained result is extended to an integral type class of mappings. Our result is an extension of some well known results in the literature.

Keywords: fixed point; partial-b-metric spaces.

2010 AMS Subject Classification: 54H25, 47H10.

1. Introduction

Fixed point theory is a major tool in mathematical analysis due to its applications in different areas of engineering and sciences. In the 1920s, Banach[3] proposed and proved the famous Banach Contraction Principle. Many scholars have proposed a series of new concepts of contraction mapping and new fixed point theorems. S.G.Matthews (in 1994) introduced the concept of partial metric space and proved the Banach Contraction Principle in the partial metric space [10].

*Corresponding author
E-mail address: bhatt.venkatesh@gmail.com
Received March 30, 2020

In 1993, Bakthin [2] and Czerwik([7], [8]) introduced the concept of b-metric space which is a generalisation of metric space. He proved the Banach contraction principle in the b-metric space. Recently, many researchers have focused on b-metric, partial metric spaces and obtained many useful fixed point results in these spaces ([1], [5], [6], [9], [12], [14], [15]).

2. Preliminaries

Definition 2.1. [7] *Let X be a non empty set and $s \geq 1$ be a given real number. A function $d : X \times X \to [0,\infty]$ is called a b-metric if $\forall x,y,z \in X$ the following conditions are satisfied*

(1) $d(x,y) = 0$ *iff* $x = y$.

(2) $d(x,y) = d(y,x)$.

(3) $d(x,y) \leq s[d(x,z) + d(z,y)]$.

The pair (X,d) is called b-metric space.The number $s \geq 1$ is called the coefficient of (X,d).

Definition 2.2. [10] *A partial metric on non empty set X is a function $p : X \times X \to [0,\infty]$ is called a b-metric if $\forall x,y,z \in X$ the following conditions are satisfied*

(1) $x = y$ *iff* $p(x,y) = p(x,x) = p(y,y)$,

(2) $p(x,x) \leq p(x,y)$

(3) $p(x,y) = p(y,x)$

(4) $p(x,y) \leq [p(x,z) + p(z,y)] - p(z,z)$.

The pair (X,p) is called partial metric space and p is partial metric on X.

Definition 2.3. [13] *Let X be a non empty set and $s \geq 1$ be a given real number. A function $p_b : X \times X \to [0,\infty]$ is called a partial b-metric if $\forall x,y,z \in X$ the following conditions are satisfied*

(1) $x = y$ *iff* $p_b(x,y) = p_b(x,x) = p_b(y,y)$,

(2) $p_b(x,x) \leq p_b(x,y)$

(3) $p_b(x,y) = p_b(y,x)$

(4) $p_b(x,y) \leq s[p_b(x,z) + p_b(z,y)] - p_b(z,z)$.

The pair (X,p_b) is called partial-b- metric space and the number $s \geq 1$ is called the coefficient of (X,p_b).

Definition 2.4. [11] *A sequence $\{x_n\}$in partial-b-metric space (X, p_b) is said to be*

(1) convergent to a fixed point $x \in X$ if $p_b(x,x) = \lim_{n\to\infty} p_b(x,x_n)$.

(2) Cauchy sequence if $\lim_{n\to\infty} p_b(x_n,x_m)$ exists and finite.

A partial b-metric space$(X, p_b)$is said to be complete if every p_b-Cauchy sequence in X converges to a point $x \in X$, such that $p_b(x,x) = \lim_{n\to\infty} p_b(x,x_n) = \lim_{n,m\to\infty} p_b(x_n,x_m)$.

3. Main Results

Theorem 3.1. *Let(X, p_b) be a complete partial b metric space with $s > 1$.Suppose that the mappings $S,T : X \to X$ satisfy*

$$p_b(Sx,Ty) \le k[\frac{p_b(x,Sx)p_b(x,Ty) + [p_b(x,y)]^2 + p_b(x,Sx)p(x,y)}{p_b(x,Sx) + p_b(x,y) + p_b(x,Ty)}] \tag{1}$$

$\forall x, y \in X$,$0 < k < 1$ with $sk < 1$ and $p_b(x,Sx) + p_b(x,y) + p_b(x,Ty) \neq 0$. Then S and T have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary point,define a sequence $\{x_n\}$ in X by
$Sx_{2n} = x_{2n+1}$ and $Tx_{2n+1} = x_{2n+2}$,n=0,1,2,...
Let $p_b(x,Sx) + p_b(x,y) + p_b(x,Ty) \neq 0$. Then from (1) we have

$$\begin{aligned} p_b(x_{2n+1},x_{2n+2}) &= p_b(Sx_{2n},Tx_{2n+1}) \\ &\le k[\frac{p_b(x_{2n},Sx_{2n})p_b(x_{2n},Tx_{2n+1}) + [p_b(x_{2n},x_{2n+1})]^2 + p_b(x_{2n},Sx_{2n})p_b(x_{2n},x_{2n+1})}{p_b(x_{2n},Sx_{2n}) + p_b(x_{2n},x_{2n+1}) + p_b(x_{2n},Tx_{2n+1})}] \\ &\le k[\frac{p_b(x_{2n},x_{2n+1})p_b(x_{2n},x_{2n+2}) + [p_b(x_{2n},x_{2n+1})]^2 + p_b(x_{2n},x_{2n+1})p_b(x_{2n},x_{2n+1})}{p_b(x_{2n},x_{2n+1}) + p_b(x_{2n},x_{2n+1}) + p_b(x_{2n},x_{2n+2})}] \\ &\le kp_b(x_{2n},x_{2n+1})[\frac{p_b(x_{2n},x_{2n+2}) + 2p_b(x_{2n},x_{2n+1})}{2p_b(x_{2n},x_{2n+1}) + p_b(x_{2n},x_{2n+2})}] \\ &\le kp_b(x_{2n},x_{2n+1}) \end{aligned} \tag{2}$$

Similarly, we have

(3)

$$
\begin{aligned}
p_b(x_{2n},x_{2n+1}) &= p_b(Sx_{2n-1},Tx_{2n}) \\
&\leq k[\frac{p_b(x_{2n-1},Sx_{2n-1})p_b(x_{2n-1},Tx_{2n})+[p_b(x_{2n-1},x_{2n})]^2+p_b(x_{2n-1},Sx_{2n-1})p_b(x_{2n-1},x_{2n})}{p_b(x_{2n-1},Sx_{2n-1})+p_b(x_{2n-1},x_{2n})+p_b(x_{2n-1},Tx_{2n})}] \\
&\leq k[\frac{p_b(x_{2n-1},x_{2n})p_b(x_{2n-1},x_{2n+1})+[p_b(x_{2n-1},x_{2n})]^2+p_b(x_{2n-1},x_{2n})p_b(x_{2n-1},x_{2n})}{p_b(x_{2n-1},x_{2n})+p_b(x_{2n-1},x_{2n})+p_b(x_{2n-1},x_{2n+1})}] \\
&\leq kp_b(x_{2n-1},x_{2n})[\frac{p_b(x_{2n-1},x_{2n+1})+2p_b(x_{2n-1},x_{2n})}{2p_b(x_{2n-1},x_{2n})+p_b(x_{2n-1},x_{2n+1})}] \\
&\leq kp_b(x_{2n-1},x_{2n}).
\end{aligned}
$$

By Continuing this process,we get

(4) $$p_b(x_{2n+1},x_{2n+2}) \leq kp_b(x_{2n},x_{2n+1}) \leq k^2p_b(x_{2n-1},x_{2n}) \leq ... \leq k^{2n+1}p_b(x_1,x_0).$$

Now we show that $\{x_n\}$ is Cauchy sequence.Let $m,n \geq 1$ and $m \geq n$

we have

$$
\begin{aligned}
p_b(x_n,x_m) &\leq s[p_b(x_n,x_{n+1})+p_b(x_{n+1},x_m)]-p_b(x_{n+1},x_{n+1}) \\
&= sp_b(x_n,x_{n+1})+sp_b(x_{n+1},x_m) \\
&\leq sp_b(x_n,x_{n+1})+s[s\{p_b(x_{n+1},x_{n+2})+p_b(x_{n+2},x_m)\}-p_b(x_{n+2},x_{n+2})] \\
&= sp_b(x_n,x_{n+1})+s^2p_b(x_{n+1},x_{n+2})+s^2p_b(x_{n+2},x_m) \\
&\leq sp_b(x_n,x_{n+1})+s^2p_b(x_{n+1},x_{n+2})+...+s^{n+m-1}p_b(x_{m+n-1},x_m) \\
&\leq sk^n[1+sk+(sk)^2+...+(sk)^{m-1}]p_b(x_1,x_0) \\
&\leq [\frac{sk^n}{1-sk}]p_b(x_1,x_0)
\end{aligned}
$$

Since $0 < sk < 1$ and by taking the limit $m,n \to \infty$, we get

$\lim_{m,n\to\infty} p(x_n,x_m) = 0$.

Hence $\{x_n\}$ is Cauchy sequence in partial-b-metric space X.Since X is complete,so there exist $u \in X$ such that $\lim_{n\to\infty} x_n = u$.

Now we show that u is common fixed point of S and T.

Consider

$$p_b(x_{2n+1},Tu) = p_b(Sx_{2n},Tu)$$
$$\leq k[\frac{p_b(x_{2n},Sx_{2n})p_b(x_{2n},Tu)+p_b(x_{2n},u)^2+p_b(x_{2n},Sx_{2n})p_b(x_{2n},u)}{p_b(x_{2n},Sx_{2n})+p_b(x_{2n},u)+p_b(x_{2n},Tu)}]$$
$$\leq k[\frac{p_b(x_{2n},x_{2n+1})p_b(x_{2n},Tu)+p_b(x_{2n},u)^2+p_b(x_{2n},x_{2n+1})p_b(x_{2n},u)}{p_b(x_{2n},x_{2n+1})+p_b(x_{2n},u)+p_b(x_{2n},Tu)}]$$

As $n \to \infty$,we have $p(u,Tu) \leq 0$. This implies Tu=u, i.e u is fixed point of T.

In the same manner we can prove Su=u.

$\Rightarrow$ u is fixed point of S.

Uniqueness:

Now we show that u is unique fixed point of S and T.

Let v is another fixed point of S and T.i.e Sv=Tv=v. Suppose that $u \neq v$ then

we have

$$p_b(u,v) = p_b(Su,Tv)$$
$$\leq k[\frac{p_b(u,Su)p(u,Tv)+[p_b(u,v)]^2+p_b(u,Su)p_b(u,v)}{p_b(u,Su)+p_b(u,v)+p_b(u,Tv)}]$$
$$\leq k[\frac{p_b(u,u)p_b(u,v)+[p_b(u,v)]^2+p_b(u,u)p_b(u,v)}{p_b(u,u)+p_b(u,v)+p_b(u,v)}] \tag{5}$$
$$\leq k\frac{p_b(u,v)}{2}.$$

which is a contradiction,since $o < k < 1$,hence $u = v$.

Therefore u is a unique common fixed point of S and T. □

Corollary 3.2. *Let (X,p) be complete partial b-metric space with $s \geq 1$.*

Suppose $T : X \to X$satisfies

$$p_b(Tx,Ty) \leq k[\frac{p_b(x,Tx)p_b(x,Ty)+p_b(x,y)^2+p_b(x,Tx)p_b(x,y)}{p_b(x,Tx)+p_b(x,y)+p_b(x,Ty)}] \tag{6}$$

$\forall x,y \in X, k \in [0,1)$.*Then T has a unique fixed point in X.*

Proof. If we take $S = T$ in theorem 3.1 then proof is over. □

Theorem 3.3. *Let(X,p) be a complete partial b metric space with* $s>1$*.Suppose that the mappings* $S,T:X\to X$ *satisfy*

$$p_b(Sx,Ty)\le k\max\{p_b(x,y),\frac{p_b(x,Sx)p_b(y,Ty)}{1+p_b(x,y)},\frac{p_b(x,Ty)p_b(y,Sx)}{1+p_b(Sx,Ty)}\} \tag{7}$$

$\forall x,y\in X, 0<k<1$ *and* $sk<1$ *. Then S and T have a unique common fixed point in X.*

Proof. Let $x_0\in X$ be arbitrary point,define sequences $\{x_n\}$ in X by

$Sx_n=x_{n+1}$ and$Tx_{n+1}=x_{n+2}$, n=0,1,2,...

Then from (7) we have

$$\begin{aligned}p_b(x_{n+1},x_{n+2}) &= p_b(Sx_n,Tx_{n+1})\\ &\le k\max\{p_b(x_n,x_{n+1}),\frac{p_b(x_n,Sx_n)p_b(x_{n+1},Tx_{n+1})}{1+p_b(x_n,x_{n+1})},\\ &\qquad \frac{p_b(x_n,Tx_{n+1})p_b(x_{n+1},Sx_n)}{1+p_b(Sx_n,Tx_{n+1})}\}\\ &\le k\max\{p_b(x_n,x_{n+1}),\frac{p_b(x_n,x_{n+1})p_b(x_{n+1},x_{n+2})}{1+p_b(x_n,x_{n+1})},\\ &\qquad \frac{p_b(x_n,x_{n+2})p_b(x_{n+1},x_{n+1})}{1+p_b(x_{n+1},x_{n+2})}\}\\ &= k\max\{p_b(x_n,x_{n+1}),p_b(x_{n+1},x_{n+2})\}\end{aligned} \tag{8}$$

If $\max\{p_b(x_n,x_{n+1}),p_b(x_{n+1},x_{n+2})\}=p_b(x_{n+1},x_{n+2})$ then

$p_b(x_{n+1},x_{n+2})\le kp_b(x_{n+1},x_{n+2})$. Since $sk<1$.

This implies $p_b(x_{n+1},x_{n+2})\le p_b(x_{n+1},x_{n+2})$.

Which is a contradiction.

Hence, if $\max\{p(x_n,x_{n+1}),p_b(x_{n+1},x_{n+2})\}=p_b(x_n,x_{n+1})$ then

$p_b(x_{n+1},x_{n+2})\le kp_b(x_n,x_{n+1})$.

$p_b(x_{n+1},x_{n+2})\le kp_b(x_n,x_{n+1})\le k^2p_b(x_{n-1},x_n)\le ...$

$p_b(x_{n+1},x_{n+2})\le k^{n+1}p_b(x_0,x_1)$.

Now we show that $\{x_n\}$is Cauchy sequence in X. Let $m,n\in N$ and $m>n$

we have

$$
\begin{aligned}
p_b(x_n,x_m) &\le s[p_b(x_n,x_{n+1})+p_b(x_{n+1},x_m)]-p_b(x_{n+1},x_{n+1})\\
&\le s[p_b(x_n,x_{n+1})+p_b(x_{n+1},x_m)]\\
&\le sp_b(x_n,x_{n+1})+s[s\{p_b(x_{n+1},x_{n+2})+p_b(x_{n+2},x_m)\}-p_b(x_{n+2},x_{n+2})]\\
&\le sp_b(x_n,x_{n+1})+s[s\{p_b(x_{n+1},x_{n+2})+p_b(x_{n+2},x_m)\}]\\
&\le sk^n p_b(x_0,x_1)+s^2k^{n+1}p_b(x_0,x_1)+....\\
&\le sk^n p_b(x_0,x_1)[1+sk+(sk)^2+....]\\
&\le \frac{sk^n}{1-sk}p_b(x_0,x_1).
\end{aligned} \tag{9}
$$

Since $0<sk<1$ and by taking the limit $m,n\to\infty$, we get $\lim_{m,n\to\infty} p(x_n,x_m)=0$.

Hence $\{x_n\}$ is Cauchy sequence. Since X is complete , so it converges to x^*. Now we show x^* is fixed point of T.

$$
\begin{aligned}
p_b(x^*,Tx^*) &\le s[p_b(x^*,x_{n+1})+p_b(x_{n+1},Tx^*)]-p_b(x_{n+1},x_{n+1})\\
&\le sp_b(x^*,x_{n+1})+sp_b(Sx_n,Tx^*)\\
&\le sp_b(x^*,x_{n+1})+s[k\max\{p_b(x_n,x^*),\frac{p_b(x_n,Sx_n)p_b(x^*,Tx^*)}{1+p_b(x_n,x^*)},\\
&\qquad \frac{p_b(x_n,Tx^*)p_b(x^*,Sx_n)}{1+p_b(Sx_n,Tx^*)}\}]\\
&\le sp_b(x^*,x_{n+1})+sk\max\{p_b(x_n,x^*),\frac{p_b(x_n,x_{n+1})p_b(x^*,Tx^*)}{1+p_b(x_n,x^*)},\\
&\qquad \frac{p_b(x_n,x^*)p_b(x^*,x_{n+1})}{1+p_b(x_{n+1},x^*)}\}
\end{aligned} \tag{10}
$$

Taking the limit $n\to\infty$, we get $p_b(x^*,Tx^*)\le 0$, therefore $x^*=Tx^*$.

$\Rightarrow x^*$ is fixed point of T.

Now we show x^* is fixed point of S.

$$
\begin{aligned}
p_b(x^*,Sx^*) &\leq s[p_b(x^*,x_{n+2})+p_b(x_{n+2},Sx^*)]-p_b(x_{n+2},x_{n+2})\\
&\leq sp_b(x^*,x_{n+2})+sp_b(Sx^*,Tx_{n+1})\\
&\leq sp_b(x^*,x_{n+1})+s[k\max\{p_b(x^*,x_{n+1}),\frac{p_b(x^*,Sx^*)p_b(x_{n+1},Tx_{n+1})}{1+p_b(x^*,x_{n+1})},\\
&\qquad \frac{p_b(x^*,Tx_{n+1})p_b(x_{n+1},Sx^*)}{1+p(Sx^*,Tx_{n+1})}\}]\\
&\leq sp_b(x^*,x_{n+1})+s[k\max\{p_b(x^*,x_{n+1}),\frac{p_b(x^*,Sx^*)p_b(x_{n+1},x_{n+2})}{1+p_b(x^*,x_{n+1})},\\
&\qquad \frac{p_b(x^*,x_{n+2})p_b(x_{n+1},Sx^*)}{1+p_b(Sx^*,x_{n+2})}\}]
\end{aligned} \tag{11}
$$

Taking the limit $n\to\infty$, we get $p_b(x^*,Sx^*)\leq 0$, therefore $x^*=Sx^*$.

$\Rightarrow x^*$ is fixed point of S.

Uniqueness:

If u and v are two fixed point of S and T ,then we have $Su=u$,$Sv=v$,$Tu=u$,$Tv=v$.

Consider

$$
\begin{aligned}
p_b(u,v) &= p_b(Su,Tv)\\
&\leq k\max\{p_b(u,v),\frac{p_b(u,Su)p_b(v,Tv)}{1+p_b(u,v)},\frac{p_b(u,Tv)p_b(v,Su)}{1+p_b(Su,Tv)}\}\\
&\leq k\max\{p_b(u,v),\frac{p_b(u,u)p_b(v,v)}{1+p_b(u,v)},\frac{p_b(u,v)p_b(v,u)}{1+p_b(u,v)}\}\\
&\leq k\max\{0,0,\frac{p_b(u,v)p_b(v,u)}{1+p_b(u,v)}\}
\end{aligned}
$$

The above inequality is true only if $p_b(u,v)=0$.Therefore $u=v\Rightarrow$ S and T have a unique common fixed point in X.

□

If $\max\{p_b(x,y),\frac{p_b(x,Sx)p_b(y,Ty)}{1+p_b(x,y)},\frac{p_b(x,Ty)p_b(y,Sx)}{1+p_b(Sx,Ty)}\}=P_b(x,y)$
then from theorem 3.3 we have the following result as

Corollary 3.4. *Let X be a complete partial b-metric space with $s>1$.Suppose $S,T:X\to X$ satisfies $p_b(Sx,Ty)\leq kp_b(x,y)$,$\forall x,y\in X$,$0<k<1$ and $sk<1$.*

Then S and T have a unique common fixed point in X.

Corollary 3.5. *Let X be a complete partial b-metric space with* $s > 1$.*Suppose* $T : X \to X$ *satisfies*

$p_b(Tx,Ty) \leq kp_b(x,y)$, $\forall x,y \in X, 0 < k < 1$ *and* $sk < 1$.

Then T has a unique fixed point in X.

Proof. Put S=T in the above theorem 3.3 we get the result. □

We can extend our result for the mappings involving contraction of integral type as follows. Let Ψ the set of functions $\rho : [0,\infty) \to [0,\infty)$ satisfies

(1) ρ is Lebesgue integrable mapping on compact subset $[0,\infty)$.

(2) for any $\varepsilon > 0$, we have $\int_0^{\varepsilon} \rho(t)dt \geq 0$.

Theorem 3.6. *Let*(X, p_b) *be a complete partial b metric space with* $s > 1$.*Suppose that the mappings* $S,T : X \to X$ *satisfy*

$$\int_0^{d(Sx,Ty)} \rho(t)dt \leq a \int_0^{[\frac{p_b(x,Sx)p_b(x,Ty)+[p_b(x,y)]^2+p_b(x,Sx)p(x,y)}{p_b(x,Sx)+p_b(x,y)+p_b(x,Ty)}]} \rho(t)dt$$

$\forall x,y \in X, 0 < k < 1$ *with* $sk < 1$ *and* $p_b(x,Sx) + p_b(x,y) + p_b(x,Ty) \neq 0$. *Then S and T have a unique common fixed point in X.*

If we put $S = T$ in theorem 3.6 , then we get the following theorem

Theorem 3.7. *Let*(X, p_b) *be a complete partial b metric space with* $s > 1$.*Suppose that the mappings* $T : X \to X$ *satisfy*

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq k \int_0^{[\frac{p_b(x,Tx)p_b(x,Ty)+[p_b(x,y)]^2+p_b(x,Tx)p(x,y)}{p_b(x,Tx)+p_b(x,y)+p_b(x,Ty)}]} \rho(t)dt$$

$\forall x,y \in X, 0 < k < 1$ *with* $sk < 1$ *and* $p_b(x,Tx) + p_b(x,y) + p_b(x,Ty) \neq 0$. *Then T has a unique common fixed point in X.*

Theorem 3.8. *Let* (X, p_b) *be a complete partial b metric space with* $s > 1$*.Suppose that the mappings* $S, T : X \to X$ *satisfy*

$$\int_0^{p_b(Sx,Ty)} \rho(t)dt \leq a \int_0^{[\max\{p_b(x,y), \frac{p_b(x,Sx)p_b(y,Ty)}{1+p_b(x,y)}, \frac{p_b(x,Ty)p_b(y,Sx)}{1+p_b(Sx,Ty)}\}]} \rho(t)dt$$

$\forall x, y \in X$*,* $0 < k < 1$ *and* $sk < 1$ *. Then S and T have a unique common fixed point in X.*

Example:Let$X = [0, \infty)$ be equipped with partial order relation defined by $x > y$ and $p_b : X \times X \to X$defined by $p_b(x, y) = |x - y|^2 + 3$ for all$x, y \in X$, where $s \geq 2$.It is obvious that (X, p_b) is a complete partial b-metric space .

Let the mapping $S, T : X \to X$ defined by $S(x) = \frac{x}{2}$ and $T(y) = \frac{y}{3}$.This satisfies all the conditions of the theorem 3.1 and theorem 3.3, thus 0 is the only fixed point of S and T.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] I. Altun, A. Erduran, Fixed Point Theorems for Monotone Mappings on Partial Metric Spaces, Fixed Point Theory and Applications. 2011 (2011), 508730.

[2] I. A. Bakhtin, The contraction mapping principle in almost metric spaces, Funct.Anal. Gos. Ped. Inst. Unianowsk, 30 (1989), 26-37.

[3] S. Banach, Surles operation dans les ensembles abstraits et leur application auxequation integrals, Fund. Math. 3 (1922), 133-181.

[4] A. Branciari, A fixed point theorem for mapping satisfying a general contractive condition of integral type, Int. J. Math. Math. Sci. 29 (2002), 531-536.

[5] J. Harjania and K. Sadarangyani, Fixed point theorems for weakly contractive mappings in partial ordered set, Nonlinear Anal., Theory Methods Appl. 71 (2009), 3403-3410.

[6] E. Karapinar, Generalizations of Caristi Kirk's theorem on partial metric spaces, Fixed Point Theory Appl. 2011 (2011), 4.

[7] S. Czerwik, Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostrav. 1 (1993), 5-11.

[8] S. Czerwik, Nonlinear set-valued contraction mappings in b-metric spaces, Atti.Semin. Mat. Fis. Univ. Modena, 46 (1998), 263-276.

[9] M. Kir, H. Kizitune, On some well known fixed point theorems in b-metric spaces, Turk. J. Anal. Number Theory, 1 (2013), 13-16.

[10] S.G. Matthews, Partial metric topology, Ann. N. Y. Acad. Sci. 728 (1994), 183-197.

[11] Z. Mustafa, J. Roshan, V. Parvaneh, Z. Kadelburg, Some common fixed point results in ordered partial b-metric spaces. J. Inequal. Appl. 2013 (2013), 562.

[12] S. Romaguera, Fixed point theorems for generalized contractions on partial metric spaces. Topol. Appl. 159 (2012), 194–199.

[13] S. Shukla, Partial b-metric space and fixed point theorems, Mediterr. J. Math. 2 (2) (2014), 703-711.

[14] S. Agrawal, K. Qureshi, J. Nema, A fixed point theorem for b-metric space, Int. J. Pure Appl. Math. Sci. 9 (2016), 45-50.

[15] O. Valero, On banach fixed point theorems for partial metric spaces, Appl. Gen. Topol. 6 (2005), 229-240.