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APPLICATION OF ADOMIAN DECOMPOSITION METHOD TO SOLVE THE FRACTIONAL MATHEMATICAL MODEL OF CORONA VIRUS

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Abstract. The aim of this paper is to solve the fractional mathematical model of Corona Virus by Adomian Decomposition Method. The researchers developed mathematical model which is based on the network Bats-Hosts-Reservoir-People. We used Adomian Decomposition Method to solve the system of time fractional ordinary differential equation. The fractional model is solved numerically by using Caputo fractional derivative and solutions are presented graphically. These solutions may be useful for further research to minimize the infection.

Keywords: corona virus; time fractional differential equations; caputo fractional derivative; adomian decomposition method; plots.

2010 AMS Subject Classification: 35A20, 35A22, 34A08, 35R11.

1. Introduction

The COVID-19 pandemic is an ongoing pandemic of corona virus disease 2019. The outbreak was first identified in Wuhan, China in December 2019. It was recognized as a pandemic by the World Health Organization (WHO) on 11 March 2020. According to WHO vector-borne

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diseases account for more than 17 percent of all infectious diseases causing more than 700000 deaths annually. They can be caused by either parasites, bacteria or viruses that can transmit infections between humans or from animals to humans. Many times human beings interact with animals. These animals carry harmful germs that can spread to people and causes illness, which are known as Zoonotic diseases. Corona viruses are Zoonotic. There were many Zoonotic diseases like HIV, Ebola, SARS, MERS and Zika etc. Novel corona virus has expanded to every corner of the world and affected lakhs of people. It has been emerging as global health emergency.

The objective of developing mathematical models using fractional differential system of equations is to improve and generalize several differential systems of integer order. Hence, demonstrating some real world problems with the help of fractional derivative operator has attracted many researchers in the field of applied mathematics and related fields [11, 12, 13, 14, 15, 16, 17, 18]. In paper [1], the researchers developed a mathematical model for calculating the transmissibility of corona virus on 28 February 2020. They developed Bats-Hosts-Reservoir-People transmission network model for simulating the potential transmission from the infection source to the human infection. The mathematical model was made by considering transmission as a geometric random walk process and used sequential Monte Carlo simulation to infer the transmission rate over time in [2]. We will be using the mathematical model from [3], which describes the details of interaction among the bats and unknown hosts, then among the people and the infectious reservoir (seafood market). In fractional model [3], the researchers used Atangana-Baleanu derivative to solve the equations. It is our one of the attempt to solve the fractional model by Adomian Decomposition Method with Caputo fractional derivative.

We consider the base study model of following system of ordinary nonlinear differential equations from [3] . The total population of people is classified into z_1, z_2, z_3, z_4, z_5, which represent respectively susceptible, exposed, symptomatic infected, asymptomatic infected, recovered or removal people, z_6 denotes the seafood market (reservoir) place and z_7 denotes the total population. The network of Bats-Hosts-Reservoir-People is described through the following system of nonlinear ordinary differential equations.

(1) $$\frac{dz_1}{dt} = \Pi_p - \mu_p z_1 - \frac{\eta_p z_1 z_3}{N_p} - \frac{\eta_p \psi z_1 z_4}{N_p} - \eta_w z_1 z_6$$

(2) $$\frac{dz_2}{dt} = \frac{\eta_p z_1 z_3}{N_p} + \frac{\eta_p \psi z_1 z_4}{N_p} + \eta_w z_1 z_6 - (\omega_p - \omega_p \theta_p + \theta_p \rho_p + \mu_p) z_2$$

(3) $$\frac{dz_3}{dt} = (1 - \theta_p)\omega_p z_2 - (\tau_p + \mu_p) z_3$$

(4) $$\frac{dz_4}{dt} = \theta_p \rho_p z_2 - (\tau_{ap} + \mu_p) z_4$$

(5) $$\frac{dz_5}{dt} = \tau_p z_3 + \tau_{ap} z_4 - \mu_p z_5$$

(6) $$\frac{dz_6}{dt} = \rho_p z_3 + \varpi_p z_4 - \xi z_6$$

(7) $$\frac{dz_7}{dt} = \Pi_p - \mu_p z_7$$

where $\Pi_p =$ Birth rate of the people, $\mu_p =$ Natural death rate of people, $\eta_p =$ Disease transmission coefficient, $N_p =$ Total population of people, $\eta_w =$ Disease transmission coefficient from seafood market to susceptible people, $\psi =$ Transmissibility multiple of asymptotically infected people, $\theta_p =$ Proportion of asymptomatic infection, $\omega_p =$ Transmission rate after completing the incubation period and becomes infected, joining the class of infected (symptomatic) people, $\rho_p =$ Transmission rate after completing the incubation period and becomes infected, joining the class of asymptotically infected people, $\tau_p =$ Removal or recovery rate of symptomatic class of people joining to removal or recovered class of people, $\tau_{ap} =$ Removal or recovery rate of asymptomatic class of people joining to removal or recovered class of people, $\rho_p =$ Parameter of infected symptomatic people contributing the virus into seafood market, $\varpi_p =$ Parameter of asymptomatically infected people contributing the virus into seafood market, and $\xi =$ Removing rate of virus from the seafood market.

Over the last twenty years, the Adomain decomposition approach has been applied to obtain formal solutions to a wide class of deterministic and stochastic differential equations. Recently, ADM is extended to obtain the solution of fractional differential equations [4]. This method yields rapidly convergent series solutions by using a few iterations for both linear and nonlinear deterministic and stochastic equations. In papers [5] and [6] researchers have applied Adomian

Decomposition Method to solve system of nonlinear differential equations. The advantage of this method is to provide a direct scheme for solving the fractional differential equations without linearization, perturbation, massive computation and transformations. Therefore, we develop the fractional Adomain decomposition method for system of fractional nonlinear ordinary differential equations.

We organize the paper as follows: We define some basic definitions and properties of fractional calculus in section 2. The section 3 is devoted for detailed description of the fractional ADM for system of nonlinear ordinary differential equation. The numerical solution of system of Time Fractional Ordinary Differential Equation (TFODE) for Corona Virus model is obtained and it is represented graphically by Mathematica software in the section 4. Section 5 is conclusions.

2. PRELIMINARIES

In this section, we study some definitions and properties of fractional calculus.

Definition 2.1. *A real function $f(x)$, $x > 0$, is said to be in the space C_μ, $\mu \in R$ if there exists a real number $p > \mu$, such that $f(x) = x^p f_1(x)$, where $f_1(x) \in C[0,\infty)$ and it is said to be in the space C_μ^m if and only if $f^{(m)}(x) \in C_\mu$, $m \in N$.*

Definition 2.2. *The Caputo fractional derivative of the function $f(x)$ is defined as follows* [9]

$$D^\beta f(x) = J^{(m-\beta)} D^m f(x) = \frac{1}{\Gamma(m-\beta)} \int_0^x \frac{1}{(x-t)^{(1-m+\beta)}} f^{(m)}(t)dt, \tag{8}$$

$$\textit{for } m-1 < \beta \leq m,\ m \in N, x > 0,\ f \in C_{-1}^m$$

Properties:

For $f(x) \in C_\mu$, $\mu \geq -1$, $\alpha, \beta \geq 0$ and $\gamma > -1$, we have

(i) $J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x)$,

(ii) $J^\alpha J^\beta f(x) = J^\beta J^\alpha f(x)$,

(iii) $J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{(\alpha+\gamma)}$.

In the next section, we develop the time fractional Adomian Decomposition Method for system of TFODE for Corona Virus model.

3. Time Fractional Adomian Decomposition Method (TFADM) for Model

We consider the following system of TFODE for Corona Virus model

$$D^{\alpha_i} z_i(t) = L_i(t, z_1, z_2, z_3, z_4, z_5, z_6, z_7),\ z_i^k(0) = C_k^i,\ 0 < k \leq [\alpha_i],\ 1 \leq i \leq 7 \tag{9}$$

Applying J^α on both sides, we have

$$z_i(t) = \sum_{k=0}^{[\alpha_i]} \frac{C_k^i z^k}{k!} + J^\alpha L_i(t, z_1, z_2, z_3, z_4, z_5, z_6, z_7)\ 1 \leq i \leq 7 \tag{10}$$

We apply Adomian Decomposition Method to solve the system of (9), for this we consider the series

$$z_i(t) = \sum_{m=0}^{\infty} z_{im} \tag{11}$$

$$L_i(t, z_1, z_2, z_3, z_4, z_5, z_6, z_7) = \sum_{m=0}^{\infty} A_{im}. \tag{12}$$

Now, substituting values from (11) and (12) in equation (10), we have

$$\sum_{m=0}^{\infty} z_{im} = \sum_{k=0}^{[\alpha_i]} \frac{C_k^i z^k}{k!} + J^\alpha \left[\sum_{m=0}^{\infty} A_{im}(z_{10}, z_{11}, \cdots, z_{17}, \cdots, z_{70}, \cdots, z_{71}, \cdots, z_{77}) \right]. \tag{13}$$

We set

$$z_{i0}(t) = \sum_{k=0}^{[\alpha_i]} \frac{C_k^i z^k}{k!} \tag{14}$$

$$z_{i(m+1)}(t) = J^\alpha \left[A_{im}(z_{10}, z_{11}, \cdots, z_{17}, \cdots, z_{70}, z_{71}, \cdots, z_{77}) \right]. \tag{15}$$

In order to determine the Adomian Polynomials, we suppose a parameter λ and equation (12) becomes,

$$L_i\left(t, \sum_{m=0}^{\infty} z_{1m}\lambda^m, \cdots, \sum_{m=0}^{\infty} z_{6m}\lambda^m\right) = \sum_{m=0}^{\infty} A_{im}\lambda^m.$$

Let

$$z_{i\lambda}(t) = \sum_{m=0}^{\infty} z_{im}(t)\lambda^m \tag{16}$$

$$A_{im} = \frac{1}{m!} \left[\frac{d^m}{d\lambda^m} (L_{i\lambda}(z_1, z_2, z_3, z_4, z_5, z_6, z_7)) \right]_{\lambda=0} \tag{17}$$

where

$$L_{i\lambda}(z_1,z_2,\cdots,z_6,z_7) = L_i(t,z_1\lambda,z_2\lambda,\cdots,z_7\lambda) \tag{18}$$

From equation (17) and equation (18), we get

$$A_{im} = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m}(L_{i\lambda}(t,z_1\lambda,z_2\lambda,\cdots,z_7\lambda))\right]_{\lambda=0}$$

From equation (16), we have

$$A_{im} = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m}\left(L_{i\lambda}\left(t,\sum_{m=0}^{\infty} z_{1m}\lambda^m,\cdots,\sum_{m=0}^{\infty} z_{7m}\lambda^m\right)\right)\right]_{\lambda=0} \tag{19}$$

From equation (14) and equation (19), we have recurrence relations

$$z_{i0}(t) = \sum_{k=0}^{[\alpha_i]} \frac{C_k^i z^k}{k!} \tag{20}$$

$$z_{i(m+1)}(t) = J^\alpha \frac{1}{m!}\left[\frac{d^m}{d\lambda^m}\left(L_i\left(t,\sum_{m=0}^{\infty} z_{1m}\lambda^m,\cdots,\sum_{m=0}^{\infty} z_{7m}\lambda^m\right)\right)\right]_{\lambda=0} \tag{21}$$

We can approximate the solution z_i by truncated series

$$U_{ik} = \sum_{m=0}^{k-1} z_{im} \tag{22}$$

$$\lim_{k\to\infty} U_{ik} = z_i(t). \tag{23}$$

In the next section, we illustrate example and its solution is represented graphically by Mathematica software.

4. Illustration

We apply the ADM for the system of time fractional ordinary differential equations. Consider the following system of time fractional ordinary differential equations with order α, where $0 < \alpha \leq 1$.

$$\frac{d^\alpha z_1}{dt^\alpha} = \Pi_p - \mu_p z_1 - \frac{\eta_p z_1 z_3}{N_p} - \frac{\eta_p \psi z_1 z_4}{N_p} - \eta_w z_1 z_6$$

$$\frac{d^\alpha z_2}{dt^\alpha} = \frac{\eta_p z_1 z_3}{N_p} + \frac{\eta_p \psi z_1 z_4}{N_p} + \eta_w z_1 z_6 - (\omega_p - \omega_p\theta_p + \theta_p\rho_p + \mu_p) z_2$$

$$\frac{d^\alpha z_3}{dt^\alpha} = (1-\theta_p)\omega_p z_2 - (\tau_p + \mu_p) z_3$$

$$\frac{d^{\alpha}z_4}{dt^{\alpha}} = \theta_p\rho_p z_2 - (\tau_{ap}+\mu_p)z_4$$
$$\frac{d^{\alpha}z_5}{dt^{\alpha}} = \tau_p z_3 + \tau_{ap}z_4 - \mu_p z_5$$
$$\frac{d^{\alpha}z_6}{dt^{\alpha}} = \rho_p z_3 + \varpi_p z_4 - \xi z_6$$
$$\frac{d^{\alpha}z_7}{dt^{\alpha}} = \Pi_p - \mu_p z_7$$

with initial conditions $z_1(t,0) = 8065518, z_2(t,0) = 200000, z_3(t,0) = 282, z_4(t,0) = 200,$

$$z_5(t,0) = 0, z_6(t,0) = 50000, z_7(t,0) = 8266000.$$

Suppose that the series solution of the above system is

$$z_i(t) = \sum_{m=0}^{\infty} z_{im}$$

From (19) and references [4, 5], we evaluate Adomian Polynomials as follows.

Therefore, the first iteration of the system is

$$A_{10} = \Pi_p - \mu_p z_{10} - \frac{\eta_p z_{10}z_{30}}{N_p} - \frac{\eta_p\psi z_{10}z_{40}}{N_p} - \eta_w z_{10}z_{60}$$
$$A_{11} = \Pi_p - \mu_p z_{11} - \frac{\eta_p z_{10}z_{31}}{N_p} - \frac{\eta_p z_{11}z_{30}}{N_p} - \frac{\eta_p\psi z_{10}z_{41}}{N_p} - \frac{\eta_p\psi z_{11}z_{40}}{N_p} - \eta_w z_{10}z_{61} - \eta_w z_{11}z_{60}$$
$$A_{12} = \Pi_p - \mu_p z_{12} - \frac{\eta_p z_{12}z_{30}}{N_p} - \frac{\eta_p z_{11}z_{31}}{N_p} - \frac{\eta_p z_{10}z_{32}}{N_p} - \frac{\eta_p\psi z_{10}z_{42}}{N_p} - \frac{\eta_p\psi z_{11}z_{41}}{N_p} - \frac{\eta_p\psi z_{12}z_{40}}{N_p}$$
$$- \eta_w z_{10}z_{62} - \eta_w z_{11}z_{61} - \eta_w z_{12}z_{60}$$
$$\vdots$$

Now, the second iteration of the system is

$$A_{20} = \frac{\eta_p z_{10}z_{30}}{N_p} + \frac{\eta_p\psi z_{10}z_{40}}{N_p} + \eta_w z_{10}z_{60} - (\omega_p - \omega_p\theta_p + \theta_p\rho_p + \mu_p)z_{20}$$
$$A_{21} = \frac{\eta_p z_{10}z_{31}}{N_p} + \frac{\eta_p z_{11}z_{30}}{N_p} + \frac{\eta_p\psi z_{10}z_{41}}{N_p} + \frac{\eta_p\psi z_{11}z_{40}}{N_p} + \eta_w z_{11}z_{61} + \eta_w z_{11}z_{61}$$
$$- (\omega_p - \omega_p\theta_p + \theta_p\rho_p + \mu_p)z_{21}$$

$$A_{22}=\frac{\eta_p z_{10}z_{32}}{N_p}+\frac{\eta_p z_{11}z_{31}}{N_p}+\frac{\eta_p z_{12}z_{30}}{N_p}+\frac{\eta_p\psi z_{10}z_{42}}{N_p}+\frac{\eta_p\psi z_{11}z_{41}}{N_p}+\frac{\eta_p\psi z_{12}z_{40}}{N_p}$$
$$+\eta_w z_{10}z_{62}+\eta_w z_{11}z_{61}+\eta_w z_{12}z_{60}-(\omega_p-\omega_p\theta_p+\theta_p\rho_p+\mu_p)z_{22}$$
$$\vdots$$

The third and fourth iterations of the system are obtained as follows

$$A_{30}=(1-\theta_p)\omega_p z_{20}-(\tau_p+\mu_p)z_{30},\ A_{40}=\theta_p\rho_p z_{20}-(\tau_{ap}+\mu_p)z_{40}$$
$$A_{31}=(1-\theta_p)\omega_p z_{21}-(\tau_p+\mu_p)z_{31},\ A_{41}=\theta_p\rho_p z_{21}-(\tau_{ap}+\mu_p)z_{41}$$
$$A_{32}=(1-\theta_p)\omega_p z_{22}-(\tau_p+\mu_p)z_{32},\ A_{42}=\theta_p\rho_p z_{22}-(\tau_{ap}+\mu_p)z_{42}$$
$$A_{33}=(1-\theta_p)\omega_p z_{23}-(\tau_p+\mu_p)z_{33},\ A_{43}=\theta_p\rho_p z_{23}-(\tau_{ap}+\mu_p)z_{43}$$
$$\vdots$$

The fifth, sixth and seventh iterations of the system are obtained as follows

$$A_{50}=\tau_p z_{30}+\tau_{ap}z_{40}-\mu_p z_{50},\ A_{60}=\rho_p z_{30}+\varpi_p z_{40}-\xi z_{60},\ A_{70}=\Pi_p-\mu_p z_{70}$$
$$A_{51}=\tau_p z_{31}+\tau_{ap}z_{41}-\mu_p z_{51},\ A_{61}=\rho_p z_{31}+\varpi_p z_{41}-\xi z_{61},\ A_{71}=\Pi_p-\mu_p z_{71}$$
$$A_{52}=\tau_p z_{32}+\tau_{ap}z_{42}-\mu_p z_{52},\ A_{62}=\rho_p z_{32}+\varpi_p z_{42}-\xi z_{62},\ A_{72}=\Pi_p-\mu_p z_{72}$$
$$A_{53}=\tau_p z_{33}+\tau_{ap}z_{43}-\mu_p z_{53},\ A_{63}=\rho_p z_{33}+\varpi_p z_{43}-\xi z_{63},\ A_{73}=\Pi_p-\mu_p z_{73}$$
$$\vdots$$

From (20) the recurrence relations for system of equations are as follows

$$z_{10}=8065518,\ z_{20}=200000,\ z_{30}=282,\ z_{1(m+1)}=J^{\alpha}A_{1m},\ z_{2(m+1)}=J^{\alpha}A_{2m},$$
$$z_{3(m+1)}=J^{\alpha}A_{3m}\ z_{40}=200,\ z_{50}=0,\ z_{60}=50000,\ z_{70}=8266000$$
$$z_{4(m+1)}=J^{\alpha}A_{4m},\ z_{5(m+1)}=J^{\alpha}A_{5m},\ z_{6(m+1)}=J^{\alpha}A_{6m},\ z_{7(m+1)}=J^{\alpha}A_{7m}.$$

After solving above iterations with the values
$\Pi_p=107644.22451,\ \mu_p=.0000356782,\ \eta_p=.05,\ \psi=.02,\ \eta_w=.000001231,\ \theta_p=.1243,$
$\omega_p=.00047876,\ \rho_p=.005,\ \tau_p=.09871,\ \tau_{ap}=.854302,\ \rho_p=.0001,\ \varpi_p=.0001,\ \xi=.01$

we have

$$z_1(t) = 8065518 - 281445.75549\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 28923.36950\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots$$

$$z_2(t) = 200000 + 49637.6713\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 23978.66\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots$$

$$z_3(t) = 282 + 56\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 15.28087\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots$$

$$z_4(t) = 200 - 46.56753564\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 39.7844002782\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots$$

$$z_5(t) = 0 + 198.69662\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 34.2620679701\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots$$

$$z_6(t) = 50000 - 499.867\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 11.95481\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$z_7(t) = 8266000 + 214993.5330188\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 3.8300300988\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots$$

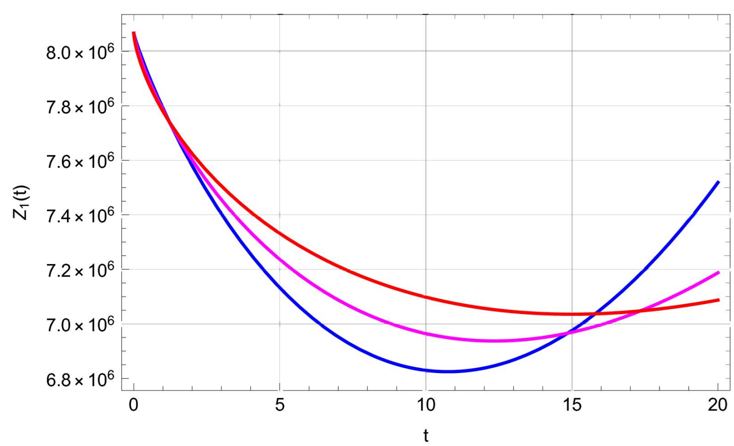

FIGURE 1. The plots for the susceptible population $z_1(t)$ vs. time t in days for distinct values of α.

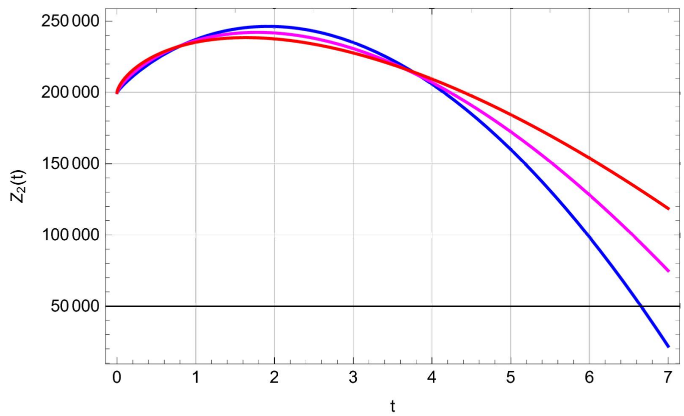

FIGURE 2. The plots for the exposed people $z_2(t)$ vs. time t in days for distinct values of α.

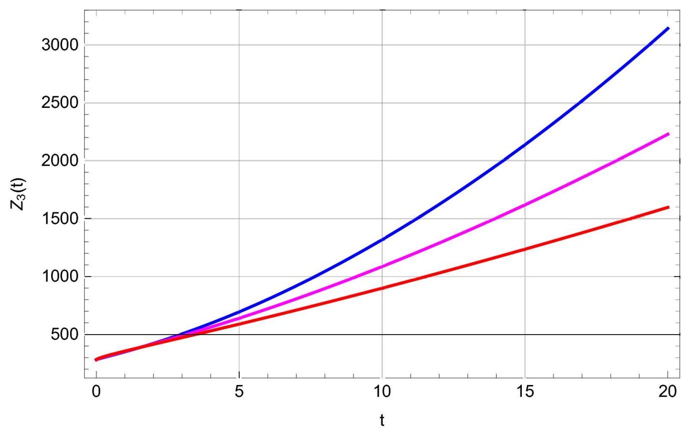

FIGURE 3. The plots for the symptomatic infected $z_3(t)$ vs. time t in days for distinct values of α.

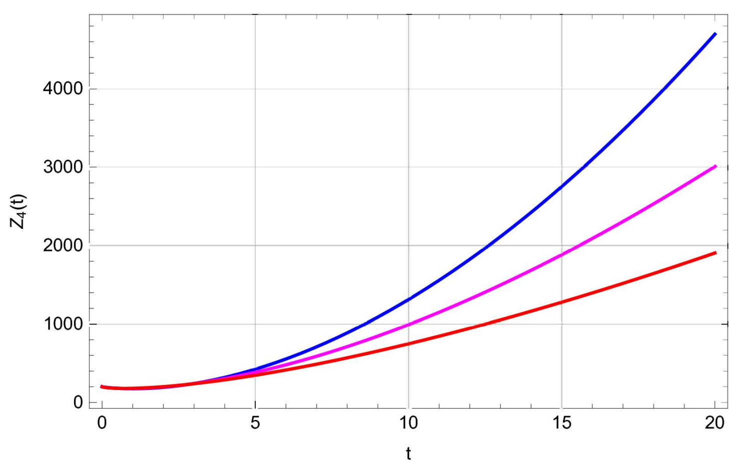

FIGURE 4. The plots for the aymptotically infected $z_4(t)$ vs. time t in days for distinct values of α.

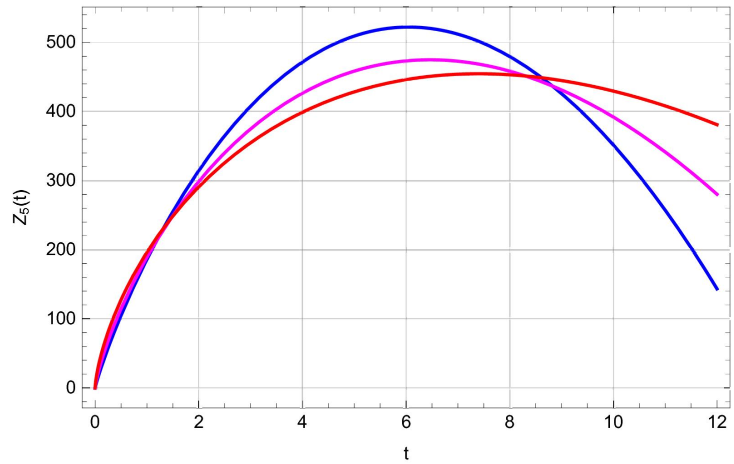

FIGURE 5. The plots for the recovered population $z_5(t)$ vs. time t in days for distinct values of α.

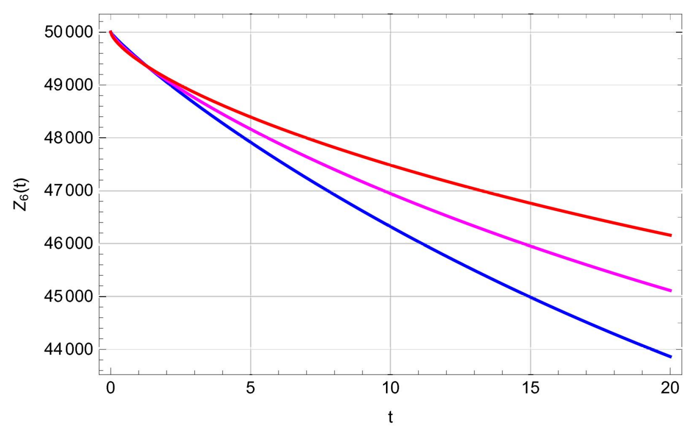

FIGURE 6. The plots for the population in reservoir $z_6(t)$ vs. time t in days for distinct values of α.

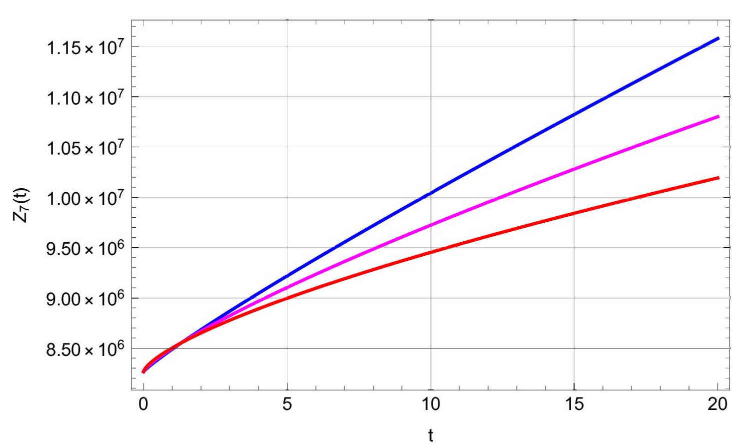

FIGURE 7. The plots for the total population $z_7(t)$ vs. time t in days for distinct values of α.

Figs. 1 to 7 shows the graphical representations of susceptible people $Z_1(t)$, exposed people $z_2(t)$, symptomatic infected $z_3(t)$, asymptotically infected $z_4(t)$, recovered or removed population $z_5(t)$, population in reservoir $z_6(t)$ and total population $z_7(t)$ acquired for various values of $(\alpha = 0.9, 0.8, 0.7)$ using Adomian decomposition method which predicts that this method can foresee the conduct of said variables precisely for the considered region. The simulations uncover that a difference in the esteem influences the dynamics of the pandemic. The non-integer order has a significant effect on the dynamics of the corona virus model.

5. CONCLUSIONS

In this study, we have examined fractional corona virus transmission model of disease, which considers the routes from reservoir to person and from person to person of SARS-CoV-2 respectively. The series solutions obtained by this powerful approach demonstrate a decent consent to control the effect of various types of populations for the several time period. The effectiveness of this technique can be drastically enhanced by reducing steps and computing more components. It is also noted that there is a remarkable difference at various estimations of α and this model depend continuously on the time-fractional derivative. However, this result was based on the limited data from a published literature, and it might not show the real situation at the early stage of the transmission.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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