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SOME COMMON FIXED POINT THEOREMS FOR SEQUENCE OF SELF MAPPINGS IN FUZZY METRIC SPACE WITH PROPERTY (CLRg)

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Abstract: In this research article we generalize and improve the results of K. Jha and V. Pant [K. Jha and V. Pant, Some Common fixed point theorem in fuzzy metric space with property (E.A.), Thai Journal of Mathematics, Vol 15, No.1 (2017), 217-226] using weak compatible maps along with property (CLRg).We also demonstrate an example in support of our main result. K.Jha and V.Pant proved their main result that is Theorem-A by applying the concept of property (E-A),to prove this result they take closeness of range of subspaces that is they proved their main result by using stronger contractive conditions, while in this paper we prove the same result as a Theorem 3.1 by removing the condition of 'closeness of range of subspaces' and instead of applying the concept of property (E-A) we do apply the concept of property (CLRg).The importance of property 'CLR'(Common Limit in Range) ensures that one does not require the closeness of range subspaces. That is we prove the same result for a weaker contractive conditions. By proving the result as Theorem-A, Jha and Pant improved and generalized many similar results on fixed points. The purpose of this paper is to further improve and generalize the result of Jha and Pant and some earlier similar results on fixed point.

Keywords: fuzzy metric space; common fixed point; weakly compatible maps; property CLRg.

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1. INTRODUCTION

Zadeh [24] in 1965 introduced the concept of fuzzy sets, and in the next decade in 1975 the concept of fuzzy metric spaces (briefly, FM-spaces) was introduced by Kramosil and Michalek [11], And then, the contraction principle in the setting of the fuzzy metric space was proved by Grabiec [6]. Consequently, George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norm. In 1994, Mishra et al. [16] extended the notion of compatible maps (introduced byJungck [9] in metric space) under the name of asymptotically commuting maps. Singh and Jain [21] studied the notion of weakly compatible maps in fuzzy metric spaces that was introduced by Jungck [10] in the setting of metric space. In 2007, Pant and Pant [18] extended the study of common fixed points of a pair of non-compatible maps (studied by Pant [17] in metric space) and the property (E-A) to FM-spaces. On the other hand, Aamri and Moutawakil [2], in 2002, studied a new property for pair of maps, that is, so called property (E-A) which is the generalization of the concept of non-compatible maps in fuzzy metric spaces. Employing property (E-A), several results have been obtained in fuzzy metric space (see [1], [3], [12], [15]. Imdad et al. [7] introduced the notion of pairwise commuting maps in 2009.Implicit relations are used as a tool for finding out common fixed point of contraction maps. Aalam et al. [1] proved a common fixed point theorem without completeness of space and continuity of involved mappings in FM-space, which generalizes the result of Singh and Jain [21]. Thereafter, Kumar and Chauhan [13] extends the results of Aalam et al. [1] and proved a common fixed point theorem for six self maps in FM–spaces satisfying contractive type implicit relations. As an application they extended their main result to four finite families of self maps in FM–spaces.

Most recently, Sintunavarat and Kumam [23] defined the notion of *common limit in the range* property or CLR property in fuzzy metric spaces. It is observed that the notion of property CLR never requires the condition of the closedness of the subspace while property (E-A) requires this condition for the existence of the fixed point. Now a days, scholars are paying attention to this property for generalizing or improving previous results which were proved by using the concept of property (E-A).

Recently, Jha and Pant [8] proved some common fixed point theorems in fuzzy metric space with property (E-A),in this paper they generalized and improved various results on fixed point in fuzzy metric space under this property (E-A) by removing the continuity of mappings even the completeness.

Purpose of research**:** Our purpose is to further generalize and improve the result of Jha and Pant [8] by using the concept of 'Common Limit in Range' property or CLRg property and by relaxing many conditions involved in the result.

2. PRELIMINARIES

For proving our main result, the following definitions are required:

Definition 2.1 ([24]): A fuzzy set A in X is a function with domain X and values in$[0, 1]$.

Definition 2.2 ([20]): A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $([0, 1],*)$ is a topological abelian monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3 ([11]): The 3-tuple $(X, M,*)$ is called a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$:

(FM-1) $M(x, y, 0) = 0$;

(FM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;

(FM-3) $M(x, y, t) = M(y, x, t)$;

(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

(FM-5) $M(x, y, .): [0, \infty) \to [0, 1]$ is left continuous

Throughout this paper, we consider M to be a fuzzy metric space with condition:

(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$.

In the following example (see [5]), we see that every metric induces a fuzzy metric:

Example 2.4. Let (X, d) be a metric space. Define $a * b = \min\{a, b\}$ for all $x, y \in X$ and

$t > 0$, $M(x,y,t) = \frac{t}{t + d(x,y)}$. Then $(X, M,*)$ is an FM-space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Definition 2.5 ([6]). Let $(X, M,*)$ be an FM-space. Then

(1) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if $lim_{n\to\infty}$ $M(x_n, x, t) = 1$ for all $t > 0$.

(2) a sequence $\{x_n\}$ in X is called a Cauchy sequence if $lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for all $t > 0$ and $p > 0$.

(3) an FM-space in which every Cauchy sequence is convergent is called complete.

Lemma 2.6 ([6]). For all, $x, y \in X$, $M(x, y, .)$ is non-decreasing.

Lemma 2.7 ([14]). Let $M(x, y,*)$ be an FM-space. Then M is a continuous function on $X^2 \times (0,\infty)$.

Definition 2.8 ([16]). Let A and S maps from an FM-space $(X, M,*)$ into itself. The maps A and S are said to be compatible (or asymptotically commuting), if for all t, $lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = z = lim_{n\to\infty} Sx_n$ for some $z \in X$.

Definition 2.9 ([22]). Let A and S be maps from an FM-space $(X, M,*)$ into itself. The maps are said to be weakly compatible if they commute at their coincidence points, that is, $Az = Sz$ implies that $ASz = SAz$.

Remark 2.10. Every pair of compatible maps is weakly compatible but converse is not always true.

Definition 2.11 ([18]). Let A and S be two self-maps of an FM-space $(X, M,*)$.We say that A and S satisfy the property (E-A) if there exists a sequence $\{x_n\}$ such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Notice that weakly compatible and property (E-A) are independent to each other (see [19], Example 2.2).

From Definition 2.11, it is inferred that two self maps A and S on an FM-space $(X, M,*)$ are non-compatible if and only if there exists at least one sequence $\{x_n\}$ in X such that

$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z$ for some $z \in X$, but for some $t > 0$, either $lim_{n\to\infty}$ $M(ASx_n, SAx_n, t) \neq 1$ or the limit does not exist. Therefore, it is easy to see that any two non-compatible self-maps of $(X, M,*)$ satisfy

the property (E-A) from Definition 2.11. But, two maps satisfying the property (E-A) need not be noncompatible (see [4], Example 1).

Definition 2.12 ([23]). A pair of self mappings (f, g) defined on fuzzy metric space $(X, M,*)$ is said to satisfy the property $(CLRg)$ if there exists a sequence $\{x_n\}$ in X such that, $lim_{n\to\infty} f x_n = lim_{n\to\infty} gx_n = gu$ for some $u \in X$.

Lemma 2.13 ([16]). Let $(X, M,*)$ be a fuzzy metric space. If there exists $h \in (0, 1)$ such that $M(x, y, ht) \geq M(x, y, t)$ then $x = y$.

Let Φ a class of implicit relations be the set of all continuous functions $\phi : [0,1]^5 \to [0,1]$ which are increasing in each coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$.

If $\{A_i\}$, $i = 1,2,3 ...$, S and T are self mappings of fuzzy metric space $(X, M,*)$ in the sequel, we shall denote

$M_{1i}(x, y, t) = \{M(A_1x, Sx, t), M(A_1y, T y, t), M(Sx, T y, t), M(A_1x, T y, \alpha t), M(Sx, A_i y, (2 - \alpha)t)\}$ for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $\phi \in \Phi$.

Jha and Pant [8] proved the following result:

Theorem A. Let $(X, M,*)$ be a fuzzy metric space. Let $\{A_i\}, i = 1,2,3, \ldots, S$ and T be mappings of a fuzzy metric space from X into itself such that

(i) $A_1X \subseteq TX$, $A_iX \subseteq SX$, for $i > 1$, and

(ii) There exists a constant $r \in (0,1/2)$ such that

$M(A_1x, A_iy, rt) \geq \emptyset\big(M_{1i}(x, y, t)\big)$, for all $x, y \in X, \propto \in (0,2), t > 0$ and $\phi \in \Phi$.

If one of A_iX, SX *and* TX is a closed subset of X; for some $k > 1$ if the pair (A_1, S) and (A_k, T) are weakly compatible , and the pair (A_1, S) or (A_k, T) satisfies (E-A) property , then all the mappings A_i, S and T have a unique common fixed point in X.

3. MAIN RESULTS

Now we prove the following theorem by generalizing the above Theorem-A.

Theorem 3.1:

Let $(X, M,*)$ be a fuzzy metric space. Let $\{A_i\}, i = 1,2,3, \dots, S$ and T be mappings of a fuzzy metric space from X into itself such that

(i) $A_K X \subseteq SX$, (A_K, T) satisfies property (CLR_T)

or $A_1 X \subseteq TX$, (A_1, S) satisfies property (CLR_S).

(ii) The pairs (A_1, S) and (A_K, T) are weakly compatible.

(iii) There exists a constant $r \in (0,1/2)$ such that

$M(A_1x, A_iy, rt) \geq \emptyset\big(M_{1i}(x, y, t)\big)$, for all $x, y \in X, \propto \in (0,2), t > 0$ and $\phi \in \Phi$.

Then all the mappings A_i, S and T have a unique common fixed point in X.

Proof. Assume that $A_K X \subseteq SX$, and the pair (A_K, T) satisfies property (CLR_T) then there exists a sequence $\{x_n\}$ in X such that $A_K x_n \to Tx$ and $Tx_n \to Tx$, for some $x \in X$ as $n \to \infty$.

Since $\mathrm{A_K X} \subseteq \mathrm{SX}$ so there exists a sequence $\{y_n\}$ in X such that $\mathrm{A_K} x_n = Sy_n$.

Hence, $Sy_n \to Tx$ as $n \to \infty$.

Now we show that $A_1 y_n \to Tx$ as $n \to \infty$. For α = 1, by setting $x = y_n$ and $y = x_n$ in (iii), We have

$M(A_1y_n, A_Kx_n, rt) \geq \emptyset(M(A_1y_n, Sy_n, t), M(A_Kx_n, Tx_n, t), M(Sy_n, Tx_n, t), M(A_1y_n,$
$Tx_n, t), M(Sy_n, A_Kx_n, t))$.

Taking limit as $n \to \infty$, we get

$M(lim_{n\to\infty} A_1y_n, lim_{n\to\infty} A_Kx_n, rt) \geq \emptyset\ (M(lim_{n\to\infty} A_1y_n, lim_{n\to\infty} Sy_n, t),$
$M(lim_{n\to\infty} A_Kx_n, lim_{n\to\infty} Tx_n, t), M(lim_{n\to\infty} Sy_n, lim_{n\to\infty} Tx_n, t),$
$M(lim_{n\to\infty} A_1y_n, lim_{n\to\infty} Tx_n, t), M(lim_{n\to\infty} Sy_n, lim_{n\to\infty} A_Kx_n, t)).$

$M(lim_{n\to\infty} A_1y_n, Tx, rt) \geq \emptyset(M(lim_{n\to\infty} A_1y_n, Tx, t), M(Tx, Tx, t), M(Tx, Tx, t),$
$M(lim_{n\to\infty} A_1y_n, Tx, t), M(Tx, Tx, t)).$

Since $\emptyset$ is increasing in each of its coordinate and $\emptyset(\mathrm{t,t,t,t,t}) > t$ for all $t \in [0,1]$ **,** so we get $M(lim_{n\to\infty}A_1y_n, Tx, rt) > M(lim_{n\to\infty} A_1y_n\ Tx, t)$.

Using lemma 2.13, we get $lim_{n\to\infty}A_1y_n = Tx$ or $z = Tx$

Which implies that A_1y_n, A_Kx_n, Tx_n and Sy_n converges to z.

Again since $A_1X \subseteq TX$ and the pair (A_1, S) satisfies property (CLR_S)

As a pair (A_1, S) satisfies property (CLR_S) then there exists a sequence

$\{x_n\}$ in X such that $A_1x_n \to Sx$ and $Sx_n \to Tx$, for some $x \in X$ as $n \to \infty$.

Since $A_1X \subseteq TX$ so there exists a sequence $\{y_n\}$ in X such that $A_1x_n = Ty_n$.

Hence $Ty_n \to Sx$.

For $\alpha = 1$, by setting $x = x_n$ and $y = y_n$ in (iii), we have

$$M(A_1x_n, A_Ky_n, rt) \geq \emptyset(M(A_1x_n, Sx_n, t), M(A_Ky_n, Ty_n, t), M(Sx_n, Ty_n, t),$$
$$M(A_1x_n, Ty_n, t), M(Sx_n, A_Ky_n, t)).$$

Taking limit as $n \to \infty$, we get

$$M(lim_{n\to\infty}A_1x_n, lim_{n\to\infty} A_Ky_n, rt) \geq \emptyset(M(lim_{n\to\infty} A_1x_n, lim_{n\to\infty} Sx_n, t),$$
$$M(lim_{n\to\infty} A_Ky_n, lim_{n\to\infty} Ty_n, t), M(lim_{n\to\infty} Sx_n, lim_{n\to\infty} Ty_n, t),$$
$$M(lim_{n\to\infty} A_1x_n, lim_{n\to\infty} Ty_n, t), M(lim_{n\to\infty}Sx_n, lim_{n\to\infty} A_Ky_n, t)).$$

Which implies that

$$M(Ty_n, lim_{n\to\infty} A_Ky_n, rt) > M(Ty_n, lim_{n\to\infty}A_Ky_n, t).$$

And hence we get $Ty_n = lim_{n\to\infty}A_Ky_n = Sx$.

Thus we have $\boldsymbol{z} = A_1y_n = A_Kx_n = Tx_n = Sy_n$.

Now since $\boldsymbol{z} = A_1y_n = Sy_n$, so by the weak compatibility of (A_1, S), it follows that

$SA_1\ y_n$=A_1Sy_n.

So we get A_1z =Sz.

Now since $\boldsymbol{z} = A_Kx_n = Tx_n$, so by the weak compatibility of $(\mathrm{A_K}, \mathrm{T})$, it follows that

A_KTx_n= TA_Kx_n.

So we get $A_Kz = Tz$.

Now we claim that $A_Kz = z$. For this, setting $x = x_n$ and $y = z$ in (iii) with $\alpha = 1$, we get

$M(A_1x_n, A_Kz, rt) \geq \emptyset$ (M(A_1x_n, Sx_n, t), M(A_Kz, Tz, t), M(Sx_n, Tz, t), M(A_1x_n,Tz, t), M(Sx_n, $A_K z$, t)), which implies that $M(z, A_K z, rt) > M(z, A_K z, t)$), and hence we get $z = A_K z$.

Similarly by using condition (iii) with $\alpha = 1$, one can show that $z = A_1 z$.

Therefore we have $z = A_1 z = Sz = A_K z = Tz$, for $K > 1$. Hence, the point z is a common fixed point of all mappings A_i, S and T.

Uniqueness: The uniqueness of a common fixed point of the mappings A_1, S and T be easily verified by using (iii). In fact, if u' be another fixed point for mappings A_1, A_K, S and T, for some $k > 1$. Then, for α = 1, we have

$$M(u,u',rt) = M(A_1 u, A_K u', rt)$$
$$\geq \emptyset\,(M(A_1 u, Su, t), M(A_K u', T u', t), M(Su, T u', t), M(A_1 u, T u', t), M(Su, A_K u', t))$$
$$> M\,(u,u',t)$$

This implies that $u = u'$.Hence, u is the unique common fixed point of the mappings.

Example-3.2. Let $X = [2,19)$ with the metric d defined by $d(x,y) = |x - y|$ and for each t define

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, if\ t > 0 \\ 0 \quad\quad , if\ t = 0 \end{cases} \text{ for all } x,y \in X\ .$$

Clearly $(X, M, *)$ be a fuzzy metric space with t-norm defined by $a * b = \min\{a,b\}$ for all $a,b \in [0,1]$ .Let $\emptyset: (R^+)^5 \to R$ be defined as in lemma 2.13.

Define the self mappings A_1,A_K, S and T on X as follows:

$$A_1 x = \begin{cases} 2\ \ if\ x \in \{2\} \cup (3,19); \\ 15\ \ if\ x \in (2,3], \end{cases}, \quad Sx = \begin{cases} 2\ if\ x = 2 \\ 12\ if\ x \in (2,3] \\ \frac{x+1}{2}\ if\ x \in (3,19) \end{cases}$$

$$A_K x = \begin{cases} 2\ if\ x = 2 \\ 7\ if\ x \in (2,3] \\ \frac{x+5}{4}\ if\ x \in (3,19) \end{cases} \quad \text{and } Tx = \begin{cases} 2\ if\ x = 2 \\ 15\ if\ x \in (2,3] \\ \frac{x+7}{5}\ if\ x \in (3,19) \end{cases}$$

Taking $\{x_n\} = \{3 + \frac{1}{n}\}_{n \in N}$ or $\{x_n\} = \{2\}$, it is clear that the pair (A_1, S) satisfies the property (CLR_S) since

$$lim_{n\to\infty} A_1 x_n = lim_{n\to\infty} Sx_n = 2 = S(2) \in X.$$

Here, It may be pointed out that $S(X) = [2,10) \cup \{12\}$ is not a closed subspace of X.

Clearly the pair (A_1, S) is weakly compatible.

Similarly, on taking $\{x_n\} = \{3 + \frac{1}{n}\}_{n\in N}$ or $\{x_n\} = \{2\}$ for the pair (A_K, T), we have

$$lim_{n\to\infty} A_K x_n = lim_{n\to\infty} T x_n = 2 = T(2) \in X.$$

This implies that the pair (A_K, T) satisfies the property (CLR_T).Clearly the pair (A_K, T) is weakly compatible. Here, it may also be pointed out that $T(X) = [2,5.2) \cup \{15\}$ is not a closed subspace of X.

Thus, all the conditions of Theorem-3.1 are satisfied and 2 is the unique common fixed point of the mappings A_1, A_K, S and T.Here, it is noted that none of A_1, A_K, S and T is a closed subspace of X.

4. CONCLUSION AND ITS CONTRIBUTION TO THE FIELD

In [8], Authors have made statement that (E-A) property buys containment of ranges without any continuity requirements, besides minimize the commutativity conditions of the maps to the commutativity at their points of coincidence. Further, they have also stated that property (E-A) allows replacing the completeness requirement of the whole space with a more natural condition of completeness of the range space. And as a result in [8] they improved many results by applying the concept of (E-A) property.

Whereas, as an improvement / generalization of a result (Theorem A which was proved by Jha and Pant in [8] by applying the concept of ‘E-A’ property) we proved our principal result (Theorem-3.1) by applying the concept of property $(CLRg)$ which does not require the condition of closeness of range subspaces, and hence in this article we proved the same result by removing the condition of closeness of subspaces from the result which was proved by Jha and Pant in [8]. As a fixed point theorem in [8], Theorem-A has been established using stronger contractive conditions, while in this paper we proved the same result as a Theorem-3.1 for a weaker contractive conditions. An example-3.2 is furnished which demonstrates the validity of Theorem-3.1.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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