Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 5, 1437-1455
https://doi.org/10.28919/jmcs/4662
ISSN: 1927-5307

APPLICATIONS OF NEUTROSOPHIC $\mathscr{N}$-STRUCTURES IN TERNARY SEMIGROUPS

AMORNRAT RATTANA[1], RONNASON CHINRAM[1,2,*]

[1]Department of Mathematics and Statistics, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla 90110, Thailand

[2]Centre of Excellence in Mathematics, CHE, Si Ayuthaya Road, Bangkok 10400, Thailand

Abstract. In this paper, the notions of neutrosophic $\mathscr{N}$-ternary subsemigroups of ternary semigroups are introduced and several properties are investigated.

Keywords: ternary semigroups; neutrosophic $\mathscr{N}$-structures; neutrosophic $\mathscr{N}$-ternary subsemigroups; (α,β,γ)-level sets; ε-neutrosophic $\mathscr{N}$-ternary subsemigroups.

2010 AMS Subject Classification: 20N10, 20N25.

1. Introduction

The concept of neutrosophic logics was first introduced by Smarandache [11] in 1999. In a neutrosophic set, an element has three associated defining functions characterized by the truth membership function (T), the indeterminate membership function (I) and the false membership function (F) defined on a universe of discourse X. These three functions are completely independent. Jun et al. [3] introduced a new function, called a negative-valued function, and constructed $\mathscr{N}$-structures in 2009. Khan et al. [5] discussed neutrosophic $\mathscr{N}$-structures and their

*Corresponding author
E-mail address: ronnason.c@psu.ac.th
Received April 26, 2020

applications in semigroups in 2017. Jun et al. [3, 4, 12] considered neutrosophic $\mathscr{N}$-structures applied to BCK/BCI-algebras and neutrosophic commutative $\mathscr{N}$-ideals in BCK-algebras in 2017. Rangsuk et al. [8] discussed neutrosophic $\mathscr{N}$-structures and their applications in UP-algebras. Recently, Al-Tahan and Davvaz [1, 2] studied applications of neutrosophic sets and neutrosophic $\mathscr{N}$-structures on some algebraic structures and hyperstructures.

The notion of ternary semigroups was known to Banach (cf. Los [6]) who is credited with an example of a ternary semigroup which does not reduce to a semigroup. Los showed that every ternary semigroup can be imbedded in a semigroup [6]. Many results in semigroups were extended to ternary semigroups. Many applications of fuzzy sets and generalized fuzzy sets were studied in ternary semigroups (for example, [7], [10], [13], [14] and [15]).

The aim of this paper is to extend the results from semigroups [5] to ternary semigroups. We introduce some basic notations and definitions in section 2. The third section contains some results on neutrosophic $\mathscr{N}$-structures in ternary semigroups. The final section is the conclusion.

2. Preliminaries

This section collects some basic notations and definitions needed later.

2.1. Ternary Semigroups

In this subsection, we introduce ternary semigroups, ternary subsemigroups, and homomorphisms (cf. [9]).

Definition 2.1. Let T be a nonempty set. The set T with a ternary operation $(a,b,c) \mapsto [abc]$ is said to be a *ternary semigroup* if it satisfies the associative law:

$$[[abc]uv] = [a[bcu]v] = [ab[cuv]]$$

for all $a,b,c,u,v \in T$.

Any semigroup can be transformed to a ternary semigroup by defining the ternary product to be $[abc] := abc$.

Definition 2.2. Let T be a ternary semigroup. A nonempty subset S of T is said to be a *ternary subsemigroup* of T if $[abc] \in S$ for all $a,b,c \in S$.

Definition 2.3. Let A and B be two ternary semigroups. A mapping $f : A \to B$ is said to be a *homomorphism* if

$$f([xyz]) = [f(x)f(y)f(z)]$$

for all $x, y, z \in A$.

2.2. Neutrosophic $\mathscr{N}$-structures

The purpose of this subsection is to recall the definitions of neutrosophic $\mathscr{N}$-structure, the union and the intersection of two neutrosophic $\mathscr{N}$-structures, and (α, β, γ)-level set (cf. [5]).

The collection of functions from a set X to the interval $[-1,0]$ is denoted by $F(X,[-1,0])$. An element of $F(X,[-1,0])$ is called a negative-valued function from X to the interval $[-1,0]$. An ordered pair (X, f) of X and an $\mathscr{N}$-function f on X is said to be an $\mathscr{N}$-structure. Let X be the nonempty universe of discourse unless otherwise specified.

Definition 2.4. A *neutrosophic $\mathscr{N}$-structure* over X is the structure

$$X_N := \frac{X}{(T_N, I_N, F_N)} = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} \mid x \in X \right\}$$

where T_N, I_N and F_N are $\mathscr{N}$-functions on X which are called the *negative truth membership function*, the *negative indeterminacy membership function*, and the *negative falsity membership function* on X, respectively.

Definition 2.5. Let $X_N := \dfrac{X}{(T_N, I_N, F_N)}$ and $X_M := \dfrac{X}{(T_M, I_M, F_M)}$ be neutrosophic $\mathscr{N}$-structures over X. If X_M satisfies the conditions

$$T_N(x) \geq T_M(x), I_N(x) \leq I_M(x), F_N(x) \geq F_M(x)$$

for all $x \in X$, then X_N is called a *neutrosophic $\mathscr{N}$-substructure* of X_M and denoted by $X_N \subseteq X_M$, If $X_N \subseteq X_M$ and $X_M \subseteq X_N$, we say that $X_N = X_M$.

Definition 2.6. Let $\{a_i \mid i \in \Lambda\}$ be a family of real numbers. We define

$$\bigvee\{a_i \mid i \in \Lambda\} := \begin{cases} \max\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \sup\{a_i \mid i \in \Lambda\} & \text{otherwise} \end{cases}$$

and

$$\bigwedge\{a_i \mid i \in \Lambda\} := \begin{cases} \min\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \inf\{a_i \mid i \in \Lambda\} & \text{otherwise.} \end{cases}$$

For any two real numbers a and b, we use $a \vee b$ and $a \wedge b$ instead of $\vee\{a,b\}$ and $\wedge\{a,b\}$, respectively.

Definition 2.7. Let $X_N := \dfrac{X}{(T_N, I_N, F_N)}$ and $X_M := \dfrac{X}{(T_M, I_M, F_M)}$ be neutrosophic $\mathcal{N}$-structures on X. Then the *union* of X_N and X_M is a neutrosophic $\mathcal{N}$-structure

$$X_{N\cup M} := \frac{X}{(T_{N\cup M}, I_{N\cup M}, F_{N\cup M})}$$

where

$$T_{N\cup M}(x) = T_N(x) \wedge T_M(x), I_{N\cup M}(x) = I_N(x) \vee I_M(x), F_{N\cup M}(x) = F_N(x) \wedge F_M(x)$$

for all $x \in X$.

Definition 2.8. Let $X_N := \dfrac{X}{(T_N, I_N, F_N)}$ and $X_M := \dfrac{X}{(T_M, I_M, F_M)}$ be neutrosophic $\mathcal{N}$-structures on X. Then the *intersection* of X_N and X_M is a neutrosophic $\mathcal{N}$-structure

$$X_{N\cap M} := \frac{X}{(T_{N\cap M}, I_{N\cap M}, F_{N\cap M})}$$

where

$$T_{N\cap M}(x) = T_N(x) \vee T_M(x), I_{N\cap M}(x) = I_N(x) \wedge I_M(x), F_{N\cap M}(x) = F_N(x) \vee F_M(x)$$

for all $x \in X$.

Definition 2.9. Let $X_N := \dfrac{X}{(T_N, I_N, F_N)}$ be a neutrosophic $\mathcal{N}$-structures over X. Then the *complement* of X_N is defined to a neutrosophic $\mathcal{N}$-structure

$$X_{N^c} := \frac{X}{(T_{N^c}, I_{N^c}, F_{N^c})}$$

where

$$T_{N^c}(x) = -1 - T_N(x), I_{N^c}(x) = -1 - I_N(x), F_{N^c}(x) = -1 - F_N(x)$$

for all $x \in X$.

Definition 2.10. Let X_N be a neutrosophic $\mathscr{N}$-structure over X and let α,β,γ be real numbers on the interval $[-1,0]$. Consider the following sets:

$$T_N^{\alpha} := \{x \in X \mid T_N(x) \leq \alpha\},$$

$$I_N^{\beta} := \{x \in X \mid I_N(x) \geq \beta\},$$

$$F_N^{\gamma} := \{x \in X \mid F_N(x) \leq \gamma\}.$$

The set

$$X_N(\alpha,\beta,\gamma) := \{x \in X \mid T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma\}$$

is called an (α,β,γ)-*level set* of X_N. We note that

$$X_N(\alpha,\beta,\gamma) = T_N^{\alpha} \cap I_N^{\beta} \cap F_N^{\gamma}.$$

3. Main Results

In this section, we discuss on neutrosophic $\mathscr{N}$-ternary subsemigroups, the (α,β,γ)-level set, the intersection of neutrosophic $\mathscr{N}$-ternary subsemigroups, neutrosophic $\mathscr{N}$-products, ε-neutrosophic $\mathscr{N}$-ternary subsemigroups, homomorphic preimage of the neutrosophic $\mathscr{N}$-ternary subsemigroup and onto homomorphic image of the neutrosophic $\mathscr{N}$-ternary subsemigroup. Throughout this section, we denote a ternary semigroup X as the universe of discourse unless otherwise specified.

Definition 3.1. Let X_N be a neutrosophic $\mathscr{N}$-structure over X. Then X_N is said to be a neutrosophic $\mathscr{N}$-ternary subsemigroup of X if it satisfies the following conditions:

$$T_N([xyz]) \leq \bigvee\{T_N(x), T_N(y), T_N(z)\},$$

$$I_N([xyz]) \geq \bigwedge\{I_N(x), I_N(y), I_N(z)\},$$

$$F_N([xyz]) \leq \bigvee\{F_N(x), F_N(y), F_N(z)\},$$

for all $x,y,z \in X$.

Theorem 3.1. *Let X_N be a neutrosophic $\mathscr{N}$-structure over X and let α,β,γ be real numbers on the interval $[-1,0]$. If X_N is a neutrosophic $\mathscr{N}$-ternary subsemigroup of X, then the nonempty (α,β,γ)-level set of X_N is a ternary subsemigroup of X.*

Proof. Suppose that $X_N(\alpha,\beta,\gamma) \neq \emptyset$ for $\alpha,\beta,\gamma \in [-1,0]$. Let $x,y,z \in X_N(\alpha,\beta,\gamma)$. Thus

$$T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma,$$

$$T_N(y) \leq \alpha, I_N(y) \geq \beta, F_N(y) \leq \gamma,$$

$$T_N(z) \leq \alpha, I_N(z) \geq \beta, F_N(z) \leq \gamma.$$

It follows that

$$T_N([xyz]) = \bigvee\{T_N(x), T_N(y), T_N(z)\} \leq \alpha,$$

$$I_N([xyz]) = \bigwedge\{I_N(x), I_N(y), I_N(z)\} \geq \beta,$$

$$F_N([xyz]) = \bigvee\{F_N(x), F_N(y), F_N(z)\} \leq \gamma.$$

Hence $[xyz] \in X_N(\alpha,\beta,\gamma)$. It implies that $X_N(\alpha,\beta,\gamma)$ is a ternary subsemigroup of X. □

Theorem 3.2. *Let X_N be a neutrosophic $\mathcal{N}$-structure over X. If $T_N^{\alpha}, I_N^{\beta}$ and F_N^{γ} are ternary subsemigroups of X for all $\alpha,\beta,\gamma \in [-1,0]$, then X_N is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X.*

Proof. We prove this theorem by contradiction. Assume first that there exist $a,b,c \in X$ such that $T_N([abc]) > \bigvee\{T_N(a), T_N(b), T_N(c)\}$. Then $T_N([abc]) > t_\alpha \geq \bigvee\{T_N(a), T_N(b), T_N(c)\}$ for some $t_\alpha \in [-1,0)$. Hence $a,b,c \in T_N^{t_\alpha}$, but $[abc] \notin T_N^{t_\alpha}$, which is a contradiction. Therefore

$$T_N([xyz]) \leq \bigvee\{T_N(x), T_N(y), T_N(z)\}$$

for all $x,y,z \in X$.

Now assume that there are $a,b,c \in X$ such that $I_N([abc]) < \bigwedge\{I_N(a), I_N(b), I_N(c)\}$. Then $a,b,c \in I_N^{t_\beta}$ and $[abc] \notin I_N^{t_\beta}$ for $t_\beta := \bigwedge\{I_N(a), I_N(b), I_N(c)\}$. This is a contradiction. Hence

$$I_N([xyz]) \geq \bigwedge\{I_N(x), I_N(y), I_N(z)\}$$

for all $x,y,z \in X$.

Finally, suppose that there are $a,b,c \in X$ such that $F_N([abc]) > \bigvee\{F_N(a), F_N(b), F_N(c)\}$. Then $F_N([abc]) > t_\gamma \geq \bigvee\{F_N(a), F_N(b), F_N(c)\}$ for some $t_\gamma \in [-1,0)$. This implies that $a,b,c \in T_N^{t_\gamma}$, but $[abc] \notin T_N^{t_\gamma}$, which is a contradiction. Therefore

$$F_N([xyz]) \leq \bigvee\{F_N(x), F_N(y), F_N(z)\}$$

for all $x, y, z \in X$.

Therefore X_N is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X. □

Theorem 3.3. *Let* $X_N := \dfrac{X}{(T_N, I_N, F_N)}$ *and* $X_M := \dfrac{X}{(T_M, I_M, F_M)}$ *be neutrosophic* $\mathcal{N}$*-ternary subsemigroups of* X*. The intersection of* X_N *and* X_M*,* $X_{N\cap M}$*, is also a neutrosophic* $\mathcal{N}$*-ternary subsemigroup of* X*.*

Proof. Let $x, y, z \in X$. Then

$$\begin{aligned}
T_{N\cap M}([xyz]) &= \bigvee\{T_N([xyz]), T_M([xyz])\} \\
&\leq \bigvee\{\bigvee\{T_N(x), T_N(y), T_N(z)\}, \bigvee\{T_M(x), T_M(y), T_M(z)\}\} \\
&= \bigvee\{\bigvee\{T_N(x), T_M(x)\}, \bigvee\{T_N(y), T_M(y)\}, \bigvee\{T_N(z), T_M(z)\}\} \\
&= \bigvee\{T_{N\cap M}(x), T_{N\cap M}(y), T_{N\cap M}(z)\},
\end{aligned}$$

$$\begin{aligned}
I_{N\cap M}([xyz]) &= \bigwedge\{I_N([xyz]), I_M([xyz])\} \\
&\geq \bigwedge\{\bigwedge\{I_N(x), I_N(y), I_N(z)\}, \bigwedge\{I_M(x), I_M(y), I_M(z)\}\} \\
&= \bigwedge\{\bigwedge\{I_N(x), I_M(x)\}, \bigwedge\{I_N(y), I_M(y)\}, \bigwedge\{I_N(z), I_M(z)\}\} \\
&= \bigwedge\{I_{N\cap M}(x), I_{N\cap M}(y), I_{N\cap M}(z)\}
\end{aligned}$$

and

$$\begin{aligned}
F_{N\cap M}([xyz]) &= \bigvee\{F_N([xyz]), F_M([xyz])\} \\
&\leq \bigvee\{\bigvee\{F_N(x), F_N(y), F_N(z)\}, \bigvee\{F_M(x), F_M(y), F_M(z)\}\} \\
&= \bigvee\{\bigvee\{F_N(x), F_M(x)\}, \bigvee\{F_N(y), F_M(y)\}, \bigvee\{F_N(z), F_M(z)\}\} \\
&= \bigvee\{F_{N\cap M}(x), F_{N\cap M}(y), F_{N\cap M}(z)\}
\end{aligned}$$

for all $x, y, z \in X$. Thus $X_{N\cap M}$ is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X. □

Corollary 3.4. *Let $\{X_{N_i} \mid i \in \mathbb{N}\}$ be a family of neutrosophic $\mathscr{N}$-ternary subsemigroup of X. Then the intersection of X_{N_i}, denoted by $X_{\bigcap N_i}$, is also a neutrosophic $\mathscr{N}$-ternary subsemigroup of X.*

Let $X_N := \dfrac{X}{(T_N, I_N, F_N)}, X_M := \dfrac{X}{(T_M, I_M, F_M)}$ and $X_Q := \dfrac{X}{(T_Q, I_Q, F_Q)}$ be neutrosophic $\mathscr{N}$-structures over X. The neutrosophic $\mathscr{N}$-product of X_N, X_M and X_Q is defined to be a neutrosophic $\mathscr{N}$-structure over X

$$X_N \bigodot X_M \bigodot X_Q := \frac{X}{(T_{N\circ M\circ Q}, I_{N\circ M\circ Q}, F_{N\circ M\circ Q})}$$
$$= \left\{ \frac{x}{(T_{N\circ M\circ Q}(x), I_{N\circ M\circ Q}(x), F_{N\circ M\circ Q}(x))} \mid x \in X \right\}$$

where

$$T_{N\circ M\circ Q} = \begin{cases} \bigwedge\limits_{x=[abc]} \{T_N(a) \vee T_M(b) \vee T_Q(c)\} & \text{if } a,b,c \in X \text{ such that } x = [abc], \\ 0 & \text{otherwise,} \end{cases}$$

$$I_{N\circ M\circ Q} = \begin{cases} \bigvee\limits_{x=[abc]} \{I_N(a) \wedge I_M(b) \wedge I_Q(c)\} & \text{if } a,b,c \in X \text{ such that } x = [abc], \\ -1 & \text{otherwise,} \end{cases}$$

$$F_{N\circ M\circ Q} = \begin{cases} \bigwedge\limits_{x=[abc]} \{F_N(a) \vee F_M(b) \vee F_Q(c)\} & \text{if } a,b,c \in X \text{ such that } x = [abc], \\ 0 & \text{otherwise.} \end{cases}$$

For any $x \in X$, the element $\dfrac{x}{(T_{N\circ M\circ Q}(x), I_{N\circ M\circ Q}(x), F_{N\circ M\circ Q}(x))}$ is simply denoted by

$$(X_N \bigodot X_M \bigodot X_Q)(x) := (T_{N\circ M\circ Q}(x), I_{N\circ M\circ Q}(x), F_{N\circ M\circ Q}(x)).$$

Theorem 3.5. *A neutrosophic $\mathscr{N}$-structure X_N over X is a neutrosophic ternary $\mathscr{N}$-ternary subsemigroup of X if and only if $X_N \bigodot X_N \bigodot X_N \subseteq X_N$.*

Proof. To show the necessity condition, we assume that X_N is a neutrosophic $\mathscr{N}$-ternary subsemigroup of X. Let x be an element of X. If $x \neq [abc]$ for all $a,b,c \in X$, then it is clear that

$X_N \odot X_N \odot X_N \subseteq X_N$. Suppose that there are $a,b,c \in X$ such that $x = [abc]$.

$$T_{N\circ N\circ N}(x) = \bigwedge_{x=[abc]} \{T_N(a) \vee T_N(b) \vee T_N(c)\} \geq \bigwedge_{x=[abc]} T_N([abc]) = T_N(x).$$

$$I_{N\circ N\circ N}(x) = \bigvee_{x=[abc]} \{I_N(a) \wedge I_N(b) \wedge I_N(c)\} \leq \bigvee_{x=[abc]} I_N([abc]) = I_N(x),$$

$$F_{N\circ N\circ N}(x) = \bigwedge_{x=[abc]} \{F_N(a) \vee F_N(b) \vee F_N(c)\} \geq \bigwedge_{x=[abc]} F_N([abc]) = F_N(x).$$

Hence $X_N \odot X_N \odot X_N \subseteq X_N$.

Conversely, let X_N be any neutrosophic $\mathcal{N}$-structure over X such that $X_N \odot X_N \odot X_N \subseteq X_N$. Let x,y,z be any elements of X and let $d = [xyz]$. Then

$$T_N([xyz]) = T_N(d) \leq T_{N\circ N\circ N}(d) = \bigwedge_{d=[abc]} \{T_N(a) \vee T_N(b) \vee T_N(c)\} \leq T_N(x) \vee T_N(y) \vee T_N(z),$$

$$I_N([xyz]) = I_N(d) \geq I_{N\circ N\circ N}(d) = \bigvee_{d=[abc]} \{I_N(a) \wedge I_N(b) \wedge I_N(c)\} \geq I_N(x) \wedge I_N(y) \wedge I_N(z),$$

$$F_N([xyz]) = F_N(d) \leq F_{N\circ N\circ N}(d) = \bigwedge_{d=[abc]} \{F_N(a) \vee F_N(b) \vee F_N(c)\} \leq F_N(x) \vee F_N(y) \vee F_N(z).$$

Therefore X_N is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X. □

Theorem 3.6. *Let X be a ternary semigroup with identity e. Let $X_N := \dfrac{X}{(T_N, I_N, F_N)}$ be a neutrosophic $\mathcal{N}$-structure over X such that $X_N(e) \geq X_N(x)$ for all $x \in X$, that is, $T_N(e) \leq T_N(x), I_N(e) \geq I_N(x)$ and $F_N(e) \leq F_N(x)$ for all $x \in X$. If X_N is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X, then $X_N \odot X_N \odot X_N = X_N$.*

Proof. For any $x \in X$, we have

$$T_{N\circ N\circ N}(x) = \bigwedge_{x=[abc]} \{T_N(a) \vee T_N(b) \vee T_N(c)\} \leq T_N(x) \vee T_N(e) = T_N(x),$$

$$I_{N\circ N\circ N}(x) = \bigvee_{x=[abc]} \{I_N(a) \wedge I_N(b) \wedge I_N(c)\} \geq I_N(x) \wedge I_N(e) = I_N(x),$$

$$F_{N\circ N\circ N}(x) = \bigwedge_{x=[abc]} \{F_N(a) \vee F_N(b) \vee F_N(c)\} \leq F_N(x) \vee F_N(e) = F_N(x).$$

It implies that $X_N \subseteq X_N \odot X_N \odot X_N$. By Theorem 3.5, we know that $X_N \odot X_N \odot X_N \subseteq X_N$. Therefore $X_N \odot X_N \odot X_N = X_N$. □

Definition 3.2. Let X_N be a neutrosophic $\mathcal{N}$-structure over X. Then X_N is said to be an ε-*neutrosophic* $\mathcal{N}$*-ternary subsemigroup* of X if it satisfies the conditions

$$T_N([xyz]) \leq \bigvee\{T_N(x), T_N(y), T_N(z), \varepsilon_T\},$$

$$I_N([xyz]) \geq \bigwedge\{I_N(x), I_N(y), I_N(z), \varepsilon_I\},$$

$$F_N([xyz]) \leq \bigvee\{F_N(x), F_N(y), F_N(z), \varepsilon_F\},$$

for all $x, y, z \in X$ where $\varepsilon_T, \varepsilon_I, \varepsilon_F \in [-1, 0]$.

Proposition 3.7. *Let X_N be an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X. Then X_N is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X if it satisfies the condition $X_N(x) \leq (\varepsilon_T, \varepsilon_I, \varepsilon_F)$ for all $x \in X$, that is, $T_N(x) \geq \varepsilon_T, I_N(x) \leq \varepsilon_I$ and $F_N(x) \geq \varepsilon_F$.*

Theorem 3.8. *Let X_N be a neutrosophic $\mathcal{N}$-structure over X and let α, β, γ be real numbers on the interval $[-1, 0]$. If X_N is an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X, then the (α, β, γ)-level set of X_N is a ternary subsemigroup of X whenever $(\alpha, \beta, \gamma) \leq (\varepsilon_T, \varepsilon_I, \varepsilon_F)$, that is, $\alpha \geq \varepsilon_T, \beta \leq \varepsilon_I$ and $\gamma \geq \varepsilon_F$.*

Proof. Assume that $X_N(\alpha, \beta, \gamma) \neq \emptyset$ for $\alpha, \beta, \gamma \in [-1, 0]$. Let $x, y, z \in X_N(\alpha, \beta, \gamma)$. Then

$$T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma,$$

$$T_N(y) \leq \alpha, I_N(y) \geq \beta, F_N(y) \leq \gamma,$$

$$T_N(z) \leq \alpha, I_N(z) \geq \beta, F_N(z) \leq \gamma.$$

It implies that

$$T_N([xyz]) \leq \bigvee\{T_N(x), T_N(y), T_N(z), \varepsilon_T\} \leq \bigvee\{\alpha, \varepsilon_T\} = \alpha,$$

$$I_N([xyz]) \geq \bigwedge\{I_N(x), I_N(y), I_N(z), \varepsilon_I\} \geq \bigwedge\{\beta, \varepsilon_I\} = \beta,$$

$$F_N([xyz]) \leq \bigvee\{F_N(x), F_N(y), F_N(z), \varepsilon_F\} \leq \bigvee\{\gamma, \varepsilon_F\} = \gamma.$$

Hence $[xyz] \in X_N(\alpha, \beta, \gamma)$. It implies that $X_N(\alpha, \beta, \gamma)$ is a ternary subsemigroup of X. □

Theorem 3.9. *Let X_N be a neutrosophic $\mathcal{N}$-structure over X and let α, β, γ be real numbers on the interval $[-1,0]$. If $T_N^{\alpha}, I_N^{\beta}$ and F_N^{γ} are ternary subsemigroups of X for all $\varepsilon_T, \varepsilon_I, \varepsilon_F \in [-1,0]$ and $(\alpha, \beta, \gamma) \leq (\varepsilon_T, \varepsilon_I, \varepsilon_F)$, then X_N is an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X.*

Proof. Assume first that there exist $a,b,c \in X$ such that

$$T_N([abc]) > \bigvee\{T_N(a), T_N(b), T_N(c), \varepsilon_T\}.$$

Then

$$T_N([abc]) > t_\alpha \geq \bigvee\{T_N(a), T_N(b), T_N(c), \varepsilon_T\}$$

for some $t_\alpha \in [-1,0)$. It implies that $a,b,c \in T_N^{t_\alpha}, [abc] \notin T_N^{t_\alpha}$ and $t_\alpha \geq \varepsilon_T$. By the hypothesis, $T_N^{t_\alpha}$ is a ternary subsemigroup of X, this is a contradiction. Thus

$$T_N([xyz]) \leq \bigvee\{T_N(x), T_N(y), T_N(z), \varepsilon_T\}$$

for all $x,y,z \in X$.

Suppose now that there are $a,b,c \in X$ such that

$$I_N([abc]) < \bigwedge\{I_N(a), I_N(b), I_N(c), \varepsilon_I\}.$$

We define $t_\beta := \bigwedge\{I_N(x), I_N(y), I_N(z), \varepsilon_I\}$. Then $a,b,c \in I_N^{t_\beta}, [abc] \notin I_N^{t_\beta}$, and $t_\beta \leq \varepsilon_I$, a contradiction. Hence

$$I_N([xyz]) \geq \bigwedge\{I_N(x), I_N(y), I_N(z), \varepsilon_I\}$$

for all $x,y,z \in X$.

Finally, suppose that there exist $a,b,c \in X$ and $t_\gamma \in [-1,0)$ such that

$$F_N([abc]) > t_\gamma \geq \bigvee\{F_N(a), F_N(b), F_N(c), \varepsilon_F\}.$$

Hence $a,b,c \in F_N^{t_\gamma}, [abc] \notin F_N^{t_\gamma}$ and $t_\gamma \geq \varepsilon_F$, which is a contradiction. Then

$$F_N([xyz]) \leq \bigvee\{F_N(x), F_N(y), F_N(z), \varepsilon_F\}$$

for all $x,y,z \in X$.

Therefore X_N is an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X. □

Theorem 3.10. *Let $\varepsilon_T, \varepsilon_I, \varepsilon_F, \delta_T, \delta_I, \delta_F \in [-1,0]$. If X_N and X_M are ε-neutrosophic $\mathcal{N}$-ternary subsemigroup and a δ-neutrosophic $\mathcal{N}$-ternary subsemigroup of X, respectively, then $X_{N\cap M}$ is a ξ-neutrosophic $\mathcal{N}$-ternary subsemigroup of X for $\xi := \varepsilon \wedge \delta$, that is,*

$$(\xi_T, \xi_I, \xi_F) = (\varepsilon_T \vee \delta_T, \varepsilon_I \wedge \delta_I, \varepsilon_F \vee \delta_F).$$

Proof. Let $x, y, z \in X$. Then

$$\begin{aligned}
T_{N\cap M}([xyz]) &= \bigvee\{T_N([xyz]), T_M([xyz])\} \\
&\leq \bigvee\{\bigvee\{T_N(x), T_N(y), T_N(z), \varepsilon_T\}, \bigvee\{T_M(x), T_M(y), T_M(z), \delta_T\}\} \\
&\leq \bigvee\{\bigvee\{T_N(x), T_N(y), T_N(z), \xi_T\}, \bigvee\{T_M(x), T_M(y), T_M(z), \xi_T\}\} \\
&= \bigvee\{\bigvee\{T_N(x), T_M(x), \xi_T\}, \bigvee\{T_N(y), T_M(y), \xi_T\}, \bigvee\{T_N(z), T_M(z), \xi_T\}\} \\
&= \bigvee\{\bigvee\{T_N(x), T_M(x)\}, \bigvee\{T_N(y), T_M(y)\}, \bigvee\{T_N(z), T_M(z)\}, \xi_T\} \\
&= \bigvee\{T_{N\cap M}(x), T_{N\cap M}(y), T_{N\cap M}(z), \xi_T\},
\end{aligned}$$

$$\begin{aligned}
I_{N\cap M}([xyz]) &= \bigwedge\{I_N([xyz]), I_M([xyz])\} \\
&\geq \bigwedge\{\bigwedge\{I_N(x), I_N(y), I_N(z), \varepsilon_I\}, \bigwedge\{I_M(x), I_M(y), I_M(z), \delta_I\}\} \\
&\geq \bigwedge\{\bigwedge\{I_N(x), I_N(y), I_N(z), \xi_I\}, \bigwedge\{I_M(x), I_M(y), I_M(z), \xi_I\}\} \\
&= \bigwedge\{\bigwedge\{I_N(x), I_M(x), \xi_I\}, \bigwedge\{I_N(y), I_M(y), \xi_I\}, \bigwedge\{I_N(z), I_M(z), \xi_I\}\} \\
&= \bigwedge\{\bigwedge\{I_N(x), I_M(x)\}, \bigwedge\{I_N(y), I_M(y)\}, \bigwedge\{I_N(z), I_M(z)\}, \xi_I\} \\
&= \bigwedge\{I_{N\cap M}(x), I_{N\cap M}(y), I_{N\cap M}(z), \xi_I\}
\end{aligned}$$

and

$$\begin{aligned}
F_{N\cap M}([xyz]) &= \bigvee\{F_N([xyz]), F_M([xyz])\} \\
&\leq \bigvee\{\bigvee\{F_N(x), F_N(y), F_N(z), \varepsilon_F\}, \bigvee\{F_M(x), F_M(y), T_M(z), \delta_F\}\} \\
&\leq \bigvee\{\bigvee\{F_N(x), F_N(y), F_N(z), \xi_F\}, \bigvee\{F_M(x), F_M(y), F_M(z), \xi_F\}\}
\end{aligned}$$

$$= \bigvee\{\bigvee\{F_N(x), F_M(x), \xi_F\}, \bigvee\{F_N(y), F_M(y), \xi_F\}, \bigvee\{F_N(z), F_M(z), \xi_F\}\}$$

$$= \bigvee\{\bigvee\{F_N(x), F_M(x)\}, \bigvee\{F_N(y), F_M(y)\}, \bigvee\{F_N(z), F_M(z)\}, \xi_F\}$$

$$= \bigvee\{F_{N\cap M}(x), F_{N\cap M}(y), F_{N\cap M}(z), \xi_F\}.$$

Therefore $X_{N\cap M}$ is a ξ-neutrosophic $\mathcal{N}$-ternary subsemigroup of X. □

Theorem 3.11. *Let X_N be an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X. If*

$$\kappa := (\kappa_T, \kappa_I, \kappa_F) = (\bigvee_{x\in X}\{T_N(x)\}, \bigwedge_{x\in X}\{I_N(x)\}, \bigvee_{x\in X}\{F_N(x)\}),$$

then the set

$$\Omega := \{x \in X \mid T_N(x) \leq \kappa_T \vee \varepsilon_T, I_N(x) \geq \kappa_I \wedge \varepsilon_I, F_N(x) \leq \kappa_F \vee \varepsilon_F\}$$

is a ternary subsemigroup of X.

Proof. Let $x, y, z \in \Omega$. Then

$$T_N(x) \leq \kappa_T \vee \varepsilon_T = \bigvee_{x\in X}\{T_N(x)\} \vee \varepsilon_T, I_N(x) \geq \kappa_I \wedge \varepsilon_I = \bigwedge_{x\in X}\{I_N(x)\} \wedge \varepsilon_I,$$
$$F_N(x) \leq \kappa_F \vee \varepsilon_F = \bigvee_{x\in X}\{F_N(x)\} \vee \varepsilon_F,$$

$$T_N(y) \leq \kappa_T \vee \varepsilon_T = \bigvee_{y\in X}\{T_N(y)\} \vee \varepsilon_T, I_N(y) \geq \kappa_I \wedge \varepsilon_I = \bigwedge_{y\in X}\{I_N(y)\} \wedge \varepsilon_I,$$
$$F_N(y) \leq \kappa_F \vee \varepsilon_F = \bigvee_{y\in X}\{F_N(y)\} \vee \varepsilon_F,$$

$$T_N(z) \leq \kappa_T \vee \varepsilon_T = \bigvee_{z\in X}\{T_N(z)\} \vee \varepsilon_T, I_N(z) \geq \kappa_I \wedge \varepsilon_I = \bigwedge_{z\in X}\{I_N(z)\} \wedge \varepsilon_I,$$
$$F_N(z) \leq \kappa_F \vee \varepsilon_F = \bigvee_{z\in X}\{F_N(z)\} \vee \varepsilon_F.$$

It follows that

$$T_N([xyz]) \leq \bigvee\{T_N(x), T_N(y), T_N(z), \varepsilon_T\}$$
$$\leq \bigvee\{\kappa_T \vee \varepsilon_T, \kappa_T \vee \varepsilon_T, \kappa_T \vee \varepsilon_T, \varepsilon_T\}$$
$$= \kappa_T \vee \varepsilon_T,$$

$$\begin{aligned} I_N([xyz]) &\geq \bigwedge\{I_N(x), I_N(y), I_N(z), \varepsilon_I\} \\ &\geq \bigwedge\{\kappa_I \wedge \varepsilon_I, \kappa_I \wedge \varepsilon_I, \kappa_I \wedge \varepsilon_I, \varepsilon_I\} \\ &= \kappa_I \wedge \varepsilon_I \end{aligned}$$

and

$$\begin{aligned} F_N([xyz]) &\leq \bigvee\{F_N(x), F_N(y), F_N(z), \varepsilon_F\} \\ &\leq \bigvee\{\kappa_F \vee \varepsilon_F, \kappa_F \vee \varepsilon_F, \kappa_F \vee \varepsilon_F, \varepsilon_F\} \\ &= \kappa_F \vee \varepsilon_F. \end{aligned}$$

Hence $[xyz] \in \Omega$. It implies that Ω is a ternary subsemigroup of X. □

Let $f : X \to Y$ be a mapping of ternary semigroups and $Y_N := \dfrac{Y}{(T_N, I_N, F_N)}$ be a neutrosophic $\mathcal{N}$-structure over Y with $\varepsilon = (\varepsilon_T, \varepsilon_I, \varepsilon_F)$. Then $X_N^{\varepsilon} := \frac{X}{(T_N^{\varepsilon}, I_N^{\varepsilon}, F_N^{\varepsilon})}$ is a neutrosophic $\mathcal{N}$-structure over X where

$$\begin{aligned} T_N^{\varepsilon} &: X \to [-1,0],\ x \mapsto \bigvee\{T_N(f(x)), \varepsilon_T\}, \\ I_N^{\varepsilon} &: X \to [-1,0],\ x \mapsto \bigwedge\{I_N(f(x)), \varepsilon_I\}, \\ F_N^{\varepsilon} &: X \to [-1,0],\ x \mapsto \bigvee\{F_N(f(x)), \varepsilon_F\}. \end{aligned}$$

Theorem 3.12. *Let $f : X \to Y$ be a homomorphism of ternary semigroups. If a neutrosophic $\mathcal{N}$-structure $Y_N := \dfrac{Y}{(T_N, I_N, F_N)}$ over Y is an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of Y, then $X_N^{\varepsilon} := \frac{X}{(T_N^{\varepsilon}, I_N^{\varepsilon}, F_N^{\varepsilon})}$ is an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X.*

Proof. Let $x, y, z \in X$. Then

$$\begin{aligned} T_N^{\varepsilon}([xyz]) &= \bigvee\{T_N(f([xyz])), \varepsilon_T\} \\ &= \bigvee\{T_N([f(x)f(y)f(z)]), \varepsilon_T\} \\ &\leq \bigvee\{\bigvee\{T_N(f(x)), T_N(f(y)), T_N(f(z)), \varepsilon_T\}, \varepsilon_T\} \\ &= \bigvee\{\bigvee\{T_N(f(x)), \varepsilon_T\}, \bigvee\{T_N(f(y)), \varepsilon_T\}, \bigvee\{T_N(f(z)), \varepsilon_T\}, \varepsilon_T\} \\ &= \bigvee\{T_N^{\varepsilon}(x), T_N^{\varepsilon}(y), T_N^{\varepsilon}(z), \varepsilon_T\}, \end{aligned}$$

$$\begin{aligned} I_N^{\varepsilon}([xyz]) &= \bigwedge\{I_N(f([xyz])), \varepsilon_I\} \\ &= \bigwedge\{I_N([f(x)f(y)f(z)]), \varepsilon_I\} \\ &\geq \bigwedge\{\bigwedge\{I_N(f(x)), I_N(f(y)), I_N(f(z)), \varepsilon_I\}, \varepsilon_I\} \\ &= \bigwedge\{\bigwedge\{I_N(f(x)), \varepsilon_I\}, \bigwedge\{I_N(f(y)), \varepsilon_I\}, \bigwedge\{I_N(f(z)), \varepsilon_I\}, \varepsilon_I\} \\ &= \bigwedge\{I_N^{\varepsilon}(x), I_N^{\varepsilon}(y), I_N^{\varepsilon}(z), \varepsilon_I\} \end{aligned}$$

and

$$\begin{aligned} F_N^{\varepsilon}([xyz]) &= \bigvee\{F_N(f([xyz])), \varepsilon_F\} \\ &= \bigvee\{F_N([f(x)f(y)f(z)]), \varepsilon_F\} \\ &\leq \bigvee\{\bigvee\{F_N(f(x)), F_N(f(y)), F_N(f(z)), \varepsilon_F\}, \varepsilon_F\} \\ &= \bigvee\{\bigvee\{F_N(f(x)), \varepsilon_F\}, \bigvee\{F_N(f(y)), \varepsilon_F\}, \bigvee\{F_N(f(z)), \varepsilon_F\}, \varepsilon_F\} \\ &= \bigvee\{F_N^{\varepsilon}(x), F_N^{\varepsilon}(y), F_N^{\varepsilon}(z), \varepsilon_F\} \end{aligned}$$

Hence $X_N^{\varepsilon} := \frac{X}{(T_N^{\varepsilon}, I_N^{\varepsilon}, F_N^{\varepsilon})}$ is an ε-neutrosophic $\mathcal{N}$-ternary subsemigroup of X. □

Let $f : X \to Y$ be a function of sets. If $Y_M := \dfrac{Y}{(T_M, I_M, F_M)}$ is a neutrosophic $\mathcal{N}$-structure over Y, then the preimage of Y_M under f

$$f^{-1}(Y_M) := \frac{X}{(f^{-1}(T_M), f^{-1}(I_M), f^{-1}(F_M))}$$

is defined to be a neutrosophic $\mathcal{N}$-structure over X where

$$f^{-1}(T_M)(x) = T_M(f(x)), f^{-1}(I_M)(x) = I_M(f(x)) \text{ and } f^{-1}(F_M)(x) = F_M(f(x))$$

for all $x \in X$.

Theorem 3.13. *Let X, Y be ternary semigroups and $f : X \to Y$ a homomorphism. If $Y_M := \dfrac{Y}{(T_M, I_M, F_M)}$ is a neutrosophic $\mathcal{N}$-ternary subsemigroup of Y, then the preimage of Y_M under f,*

$$f^{-1}(Y_M) := \frac{X}{(f^{-1}(T_M), f^{-1}(I_M), f^{-1}(F_M))}$$

is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X.

Proof. Let $x,y,z \in X$. Then

$$\begin{aligned} f^{-1}(T_M)([xyz]) &= T_M(f([xyz])) = T_M([f(x)f(y)f(z)]) \\ &\leq \bigvee\{T_M(f(x)),T_M(f(y)),T_M(f(z))\} \\ &= \bigvee\{f^{-1}(T_M)(x),f^{-1}(T_M)(y),f^{-1}(T_M)(z)\}, \end{aligned}$$

$$\begin{aligned} f^{-1}(I_M)([xyz]) &= I_M(f([xyz])) = I_M([f(x)f(y)f(z)]) \\ &\geq \bigwedge\{I_M(f(x)),I_M(f(y)),I_M(f(z))\} \\ &= \bigwedge\{f^{-1}(I_M)(x),f^{-1}(I_M)(y),f^{-1}(I_M)(z)\} \end{aligned}$$

and

$$\begin{aligned} f^{-1}(F_M)([xyz]) &= F_M(f([xyz])) = F_M([f(x)f(y)f(z)]) \\ &\leq \bigvee\{F_M(f(x)),F_M(f(y)),F_M(f(z))\} \\ &= \bigvee\{f^{-1}(F_M)(x),f^{-1}(F_M)(y),f^{-1}(F_M)(z)\}. \end{aligned}$$

Therefore $f^{-1}(Y_M)$ is a neutrosophic $\mathcal{N}$-ternary subsemigroup of X, which completes the proof. □

Let X,Y be sets and $f : X \to Y$ be an onto function. If $X_N := \dfrac{X}{(T_N,I_N,F_N)}$ is a neutrosophic $\mathcal{N}$-structure over X, then the image of X_N under f

$$f(X_N) := \frac{Y}{(f(T_N),f(I_N),f(F_N))}$$

is defined to be a neutrosophic $\mathcal{N}$-structure over Y where

$$\begin{aligned} f(T_N)(y) &= \bigwedge_{x\in f^{-1}(y)} T_N(x), \\ f(I_N)(y) &= \bigvee_{x\in f^{-1}(y)} I_N(x), \\ f(F_N)(y) &= \bigwedge_{x\in f^{-1}(y)} F_N(x). \end{aligned}$$

Theorem 3.14. *Let X,Y be ternary semigroups and $f : X \to Y$ be an onto homomorphism. Let $X_N := \frac{X}{(T_N, I_N, F_N)}$ be a neutrosophic $\mathcal{N}$-structure of X such that*

$$T_N(x_0) = \bigwedge_{z\in A} T_N(z), \quad I_N(x_0) = \bigvee_{z\in A} I_N(z), \quad F_N(x_0) = \bigwedge_{z\in A} F_N(z).$$

for all $A \subseteq X$ and some $x_0 \in A$. If X_N is a neutrosophic $\mathcal{N}$-structure of X, then the image of X_N under f

$$f(X_N) := \frac{Y}{(f(T_N), f(I_N), f(F_N))}$$

is a neutrosophic $\mathcal{N}$-ternary subsemigroup of Y.

Proof. Let $a,b,c \in Y$. Then $f^{-1}(a) \neq \emptyset, f^{-1}(b) \neq \emptyset$ and $f^{-1}(c) \neq \emptyset$ in X. It follows that there exist $x_a \in f^{-1}(a), x_b \in f^{-1}(b)$ and $x_c \in f^{-1}(c)$ such that

$$T_N(x_a) = \bigwedge_{u\in f^{-1}(a)} T_N(u), \quad I_N(x_a) = \bigvee_{u\in f^{-1}(a)} I_N(u), \quad F_N(x_a) = \bigwedge_{u\in f^{-1}(a)} F_N(u),$$

$$T_N(x_b) = \bigwedge_{v\in f^{-1}(b)} T_N(v), \quad I_N(x_b) = \bigvee_{v\in f^{-1}(b)} I_N(v), \quad F_N(x_b) = \bigwedge_{v\in f^{-1}(b)} F_N(v),$$

$$T_N(x_c) = \bigwedge_{w\in f^{-1}(c)} T_N(w), \quad I_N(x_c) = \bigvee_{w\in f^{-1}(c)} I_N(w), \quad F_N(x_c) = \bigwedge_{w\in f^{-1}(c)} F_N(w).$$

Hence

$$\begin{aligned} f(T_N)([abc]) &= \bigwedge_{x\in f^{-1}([abc])} T_N(x) \leq T_N([x_a x_b x_c]) \\ &\leq \bigvee\{T_N(x_a), T_N(x_b), T_N(x_c)\} \\ &= \bigvee\{\bigwedge_{u\in f^{-1}(a)} T_N(u), \bigwedge_{v\in f^{-1}(b)} T_N(v), \bigwedge_{w\in f^{-1}(c)} T_N(w)\} \\ &= \bigvee\{f(T_N)(a), f(T_N)(b), f(T_N)(c)\}, \end{aligned}$$

$$\begin{aligned} f(I_N)([abc]) &= \bigvee_{x\in f^{-1}([abc])} I_N(x) \geq I_N([x_a x_b x_c]) \\ &\geq \bigwedge\{I_N(x_a), I_N(x_b), I_N(x_c)\} \\ &= \bigwedge\{\bigvee_{u\in f^{-1}(a)} I_N(u), \bigvee_{v\in f^{-1}(b)} I_N(v), \bigvee_{w\in f^{-1}(c)} I_N(w)\} \\ &= \bigwedge\{f(I_N)(a), f(I_N)(b), f(I_N)(c)\}, \end{aligned}$$

and

$$\begin{aligned} f(F_N)([abc]) &= \bigwedge_{x\in f^{-1}([abc])} F_N(x) \leq F_N([x_a x_b x_c]) \\ &\leq \bigvee\{F_N(x_a), F_N(x_b), F_N(x_c)\} \\ &= \bigvee\{\bigwedge_{u\in f^{-1}(a)} F_N(u), \bigwedge_{v\in f^{-1}(b)} F_N(v), \bigwedge_{w\in f^{-1}(c)} F_N(w)\} \\ &= \bigvee\{f(F_N)(a), f(F_N)(b), f(F_N)(c)\}. \end{aligned}$$

Then $f(X_N)$ is a neutrosophic $\mathcal{N}$-ternary subsemigroup of Y. □

4. Conclusions

In this paper, we applied neutrosophic $\mathcal{N}$-structure to ternary semigroups. We also investigated the notion of neutrosophic $\mathcal{N}$-ternary subsemigroups and showed some properties. Moreover, the conditions for neutrosophic $\mathcal{N}$-structure to be neutrosophic $\mathcal{N}$-ternary subsemigroup have been investigated. We also defined neutrosophic $\mathcal{N}$-products and discuss about the characterization of neutrosophic $\mathcal{N}$-ternary subsemigroups. In addition, we have introduced ε-neutrosophic $\mathcal{N}$-ternary subsemigroups and shown the relation between neutrosophic ternary subsemigroups and ε-neutrosophic $\mathcal{N}$-ternary subsemigroups. Finally, we showed that the homomorphic preimage of the neutrosophic $\mathcal{N}$-ternary subsemigroup is a neutrosophic $\mathcal{N}$-ternary subsemigroup and the onto homomorphic image of the neutrosophic $\mathcal{N}$-ternary subsemigroup is a neutrosophic $\mathcal{N}$-ternary subsemigroup.

In our future study, we will apply these notion/results to other types of neutrosophic $\mathcal{N}$-structures in ternary semigroups. We will also study the soft set theory/cubic set theory of such neutrosophic $\mathcal{N}$-structures.

Acknowledgments

This work was supported by Algebra and Applications Research, Faculty of Science, Prince of Songkla University.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] M. Al-Taha and B. Davvaz, Neutrosophic $\mathcal{N}$-ideals ($\mathcal{N}$-subalgebras) of subtraction algebra, Int. J. Neutrosophic Sci. 3 (2020), 44-53.

[2] M. Al-Tahan and B. Davvaz, On single valued neutrosophic sets and neutrosophic $\mathcal{N}$-structures: Applications on algebraic structures (hyperstructures), Int. J. Neutrosophic Sci. 3 (2020), 108-117.

[3] Y. B. Jun, K. Lee, and S.-Z. Song. $\mathcal{N}$-ideals of BCK/BCI-algebras. J. Chungcheong Math. Soc. 22 (2009), 417-437.

[4] Y. B. Jun, F. Smarandache, and H. Bordbar. Neutrosophic $\mathcal{N}$-structures applied to BCK/BCI-algebras. Information 8 (2017), 128.

[5] M. Khan, S. Anis, F. Smarandache, and Y. B. Jun. Neutrosophic $\mathcal{N}$-structures and their applications in semigroups. Ann. Fuzzy Math. Inform. 14 (2017), 583-598.

[6] J. Los. On the extending of models I. Fundam. Math. 42 (1955), 38-54.

[7] P. Petchkheaw and R. Chinram, Fuzzy, rough and rough fuzzy ideals in ternary semigroups, Int. J. Pure Appl. Math. 56 (2009), 21-36.

[8] P. Rangsuk, P. Huana and A. Iampan, Neutrosophic $\mathcal{N}$-structures over UP-algebras. Neutrosophic Sets Syst. 28 (2019), 87-127.

[9] M. L. Santiago and S. Sri Bala, Ternary semigroups, Semigroup Forum 81 (2010), 380-388.

[10] S. Saelee and R. Chinram, A study on rough, fuzzy and rough fuzzy bi-ideals of ternary semigroups, IAENG Int. J. Appl. Math. 41 (2011), 172-176.

[11] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, 1999.

[12] S. Z. Song, F. Smarandache, and Y. B. Jun. Neutrosophic commutative $\mathcal{N}$-ideals in BCK-algebras. Information 8 (2017), 130.

[13] S. Suebsung and R. Chinram, Interval valued fuzzy ideal extensions of ternary semigroups, Int. J. Math. Comput. Sci. 13 (2018), 15-27.

[14] A. F. Talee, M. Y. Abbasi and S. A. Khan, Hesitant fuzzy sets approach to ideal theory in ternary semigroups, Int. J. Appl. Math. 31 (2018), 527-539.

[15] P. Yiarayong, Applications of hesitant fuzzy sets to ternary semigroups, Heliyon 6 (2020), e03668.