ANALYSIS OF FRACTIONAL SUSCEPTIBLE-EXPOSED-INFECTIOUS (SEI) MODEL OF COVID-19 PANDEMIC FOR INDIA

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Abstract. The purpose of this article is to develop and analyse COVID-19 pandemic for India in terms of mathematical equations. We consider the basic Susceptible-Exposed-Infectious (SEI) epidemic model and develop the SEI model of COVID-19 for India. We use Adomian decomposition method to find solution of the group of fractional differential equations. We discuss the stability by using Routh-Hurwitz criterion for disease-free equilibria point and endemic equilibrium point. We obtain approximate solution of the group of fractional differential equations and its solution represented graphically by Mathematica software, that will be helpful to mininize the infection.

Keywords: COVID-19; fractional derivatives; differential equation; Caputo fractional derivative; Adomian decomposition method; Mathematica.

2010 AMS Subject Classification: 34A08, 34A34, 65L20, 92D30, 65H10.

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Received October 04, 2021

1. INTRODUCTION

The Pandemic of a new human corona virus, named as COVID-19 by WHO officially is an ongoing Pandemic. The first case was detected in the month of December 2019 in Wuhan, Hubei, Chiana. As of $03:27$ UTC on 30th April 2020, a total of $3,193,886$ cases are confirmed, in more than 185 countries and 200 territories, including 26 cruise ships and $227,638$ deaths. The first Patient of Corona-Virus was found on 30th January 2020 in India. Ministry of Health and Family Welfare (MHFW) have confirmed 33050 cases upto 30th April 2020.

Now a days, many real world problems in the field of biology, physics, engineering, financial and sociological can be represented in terms of group of non-linear ordinary and fractional order differential equations. Most of the time researchers are not able to find analytical solution of non-linear ordinary and fractional differential equations; due to that we use numerical methods to obtain their approximate solutions. Recently, there are several methods have been studied by many researchers to find solution of ordinary and fractional differential equations. Few methods are exponential Galerkin method introduced by Yuzbasi and Karayir[27], collacation method presented by Mastorakis[15], the exponential collaction method proposed by Yuzbasi[26],Galerkin finite element method given by Al-Omari et.al.[1], the adomian decomposition method improves [3],[9], the multistep method proposed by Hojiati et.al[4]. Many reserachers are investigating numerical and analystical methods to solve differential equation containing fractional derivatives given in [31, 32, 33, 34, 35].

Mathematical system of the transmitted diseases play an important role in understanding spread of disese and taking measures to controll disease. After the start of the emanation in Wuhan, many researchers modelled various mathematical structures for the purpose of estimations and predictions for the Corona virus. In the paper, [21] researcher studied Age structured impact of social distancing on the COVID-19 of India. More literature pertaining to this can be refer in ([28, 29, 30]).

Considering this base data and the SEI model published by WHO on 31st January, 2020 for Chiana, we develop the Fractional Susceptible-Exposed-Infectious(SEI) Epidemic Model of COVID-19 for India, which form the group of non-linear fractional differential equations given in the next section.

1.1. Fractional SEI Model. We know that the WHO used SEI models to characterize and prediction the initial stage of the novel-COVID-19 emanation in Wuhan, China. In this connection, we develop SEI model of COVID-19 for India. The model structure is given below. In case
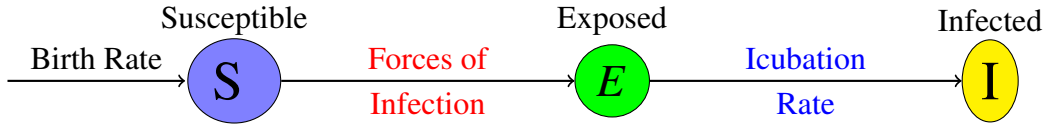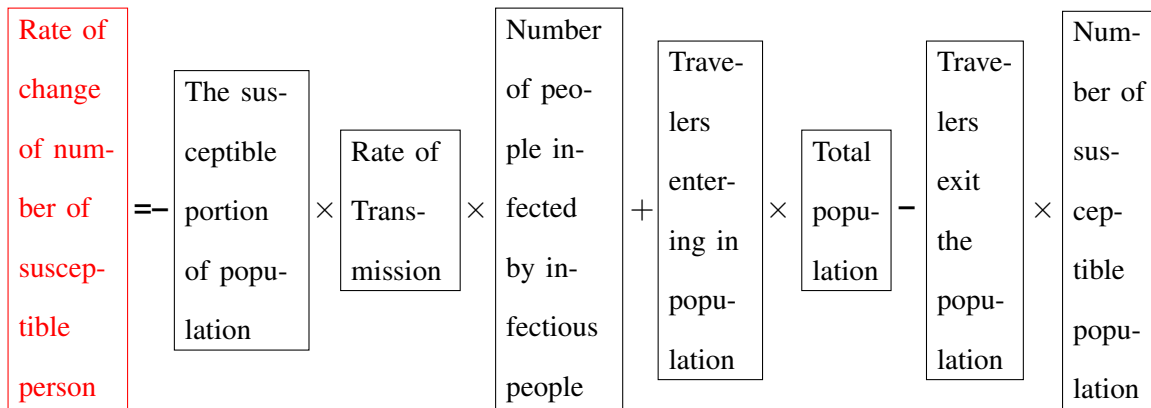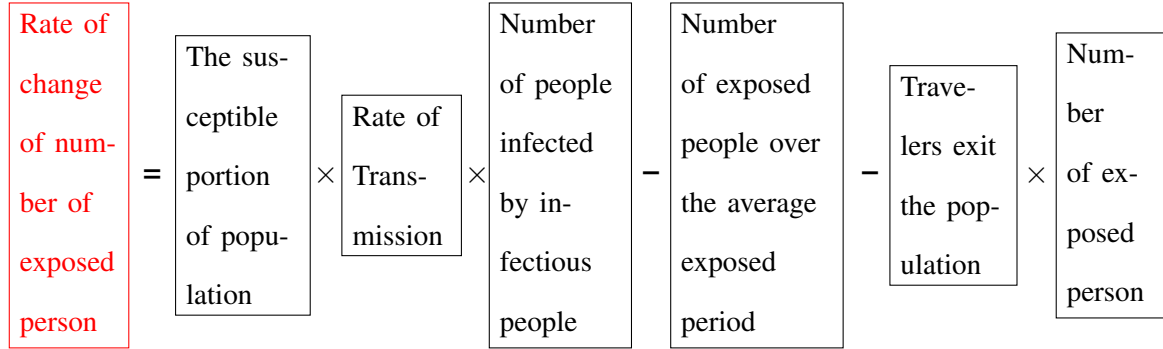

FIGURE 1. SEI Epidemic Model

of India, we considered few modifications like number of people infected by animal source are taken zero, average number of people infected by an infectious person over the average duration of infection is totally considered as rate of transmission, travelers entering Wuhan is replaced by travelers entering in population of India and Population traveling out of Wuhan is replaced by travelers exit from population of India. By observing the model it is clear that the travel data is conclusive, because it directly affects the spread. Note that, COVID 19 enters in India through the persons having foreign travel history. Therefore, our modified SEI model, along with compartment wise explanation and description of defining parameters for COVID 19 for India is presented below
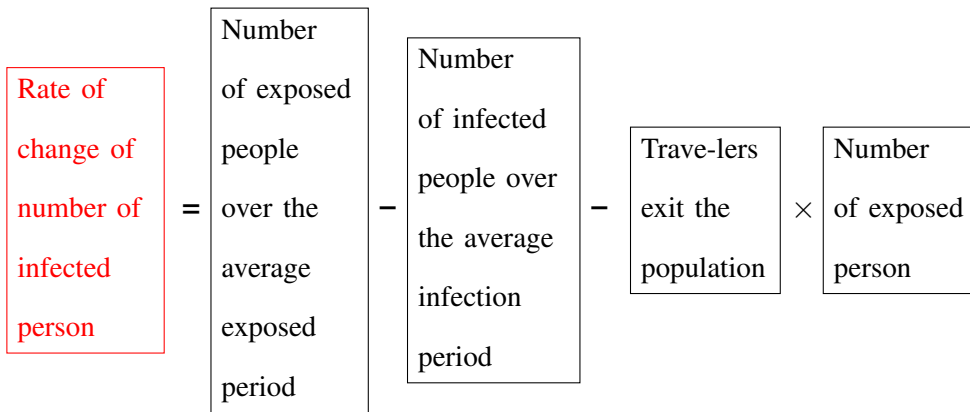
$$\frac{dS}{dt} = -\frac{S}{N} \times [\beta \times I] + \mu_I \times N - \mu_0 \times S$$

$$\frac{dE}{dt} = \frac{S}{N} \times [\beta \times I] - \frac{E}{D_E} - \mu_0 \times E$$

Rate of change of number of exposed person	=	The susceptible portion of population	×	Rate of Transmission	×	Number of people infected by infectious people	−	Number of exposed people over the average exposed period	−	Travelers exit the population	×	Number of exposed person

$$\frac{dI}{dt} = \frac{E}{D_E} - \frac{I}{D_I} - \mu_0 \times I$$

Rate of change of number of infected person	=	Number of exposed people over the average exposed period	−	Number of infected people over the average infection period	−	Trave-lers exit the population	×	Number of exposed person

where S = Number of susceptible persons, N = Total population, E = Number of Exposed persons, I = Number of Infected persons, β = Rate of transmission, μ_I = Number of travelers entering in population, μ_0 = Number of travelers exit the population, D_E = Average exposed period, D_I = Average infected period.

Thus above SEI model can be written as system of ordinary differential equations as follows

(1)
$$\left. \begin{array}{l} \dfrac{dS}{dt} = -\dfrac{\beta}{N}[SI] + \mu N - \mu S \\[2mm] \dfrac{dE}{dt} = \dfrac{\beta}{N}[SI] - (\sigma_E + \mu)E \\[2mm] \dfrac{dI}{dt} = \sigma_E E - (\sigma_I + \mu)I \end{array} \right\}$$

Motivated by the above literature applications of epidemic mathematical models, in this paper we are studying dynamics of novel coronavirus SEI model derived in (1) in the from of system

of the nonlinear differential equations involving Caputo fractional derivative operator of order α such that $\alpha \in (0,1]$ as follows

(2)
$$\left.\begin{array}{l} D^{\alpha} S = -\dfrac{\beta}{N}[SI] + \mu N - \mu S \\[2mm] D^{\alpha} E = \dfrac{\beta}{N}[SI] - (\sigma_E + \mu)E \\[2mm] D^{\alpha} I = \sigma_E E - (\sigma_I + \mu)I \end{array}\right\}$$

with initial conditions

(3)
$$S(0) = S_0, \ E(0) = E_0, \ I(0) = I_0.$$

We arrange the paper as per folowing sequence: In section 2, we discuss few basic definitions of fractional calculus. In Section 3, we discuss about Adomian Decomposition Method to solve fractional SEI model. In Section 4, we comment about equilibrium points and stability and calculate basic reproduction number R_0. In section 5, we find the solution of fractional SEI model and represent their solutions graphically by Mathematica software. Section 6 is devoted to conclusions.

2. PRELIMINARIES

In this section, we study some basic definition of fractional integral, fractional derivative and their properties for further development. We use the Caputo's definition due to its convenience for initial conditions of the differential equations.

Definition 2.1. *A real function $f(t), t > 0$, is said to be in the space $C_\delta, \delta \in R$ if there exist a real number $p > \delta$ such that $f(t) = t^p f_1(t)$, where $f_1(t) \in C[0,\infty)$ and it is said to be in the space C_δ^m if and only if $f^{(m)}(t) \in C_\delta$, $m \in N$.*

Definition 2.2. *Riemann-Liouville Fractional integral:*
If $f(t) \in C[a,b]$ and $a < t < b$ then

(4)
$$_aI_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} f(s)ds,$$

where $\alpha \in (-\infty, \infty)$ is called the Riemann-Liouville fractional integral of order α.

Definition 2.3. *M.Caputo Fractional Derivative:*

If $f(t) \in C[a,b]$ and $a < t < b$ then

$$(5) \qquad {}^C_a D^\alpha_t f(t) = {}_a I^{n-\alpha}_t D^n f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-s)^{n-\alpha-1} f^n(s)ds,$$

where $\alpha \in (n-1,n)$ is called the Caputo fractional derivative of order α.

- **Properties:**

For $f(t) \in C_\delta$, $\delta \geq -1$, $\alpha, \beta > 0$, and $\gamma > -1$, we have

(i)$I^\alpha I^\beta = I^{\alpha+\beta}$,

(ii)$I^\alpha I^\beta = I^\beta I^\alpha$,

(iii)$I^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} t^{\alpha+\gamma}$.

3. ADM FOR THE SYSTEM OF FRACTIONAL ORDINARY DIFFERENTIAL EQUATIONS

Consider the system of fractional ordinary differential equation

$$(6) \qquad D^\alpha u_i = G_i(t, u_1, u_2, \cdots, u_n), \ \ i = 1, 2, \cdots, n.$$

where D^α is the Caputo fractional differential operator.

Applying operator I^α on (6), we have.

$$(7) \qquad u_i = \sum_{i=0}^{[\alpha_i]} \frac{C_i^k u_i}{k!} + I^\alpha G_i(t, u_1, u_2, \cdots, u_n), i = 1, 2, \cdots, n.$$

We consider the series solution of equation (7) is

$$(8) \qquad u_i = \sum_{j=0}^\infty u_{ij}$$

$$(9) \qquad G_i(t, u_1, u_2, \cdots, u_n) = \sum_{j=0}^\infty A_{ij}(u_{i0}, u_{i1}, \cdots, u_{in})$$

where $A_{ij}(u_{i0}, u_{i1}, \cdots, u_{in})$ are called Adomian polynomials.

By substituting (8) and (9) in (7), we get

$$\sum_{j=0}^\infty u_{ij} = \sum_{i=0}^{[\alpha_i]} \frac{C_i^k u_i}{k!} + I^\alpha \left[\sum_{j=0}^\infty A_{ij}(u_{i0}, u_{i1}, \cdots, u_{in}) \right]$$

$$= \sum_{i=0}^{[\alpha_i]} \frac{C_i^k u_i}{k!} + \sum_{j=0}^\infty I^\alpha [A_{ij}(u_{i0}, u_{i1}, \cdots, u_{in})]$$

From this we define,

$$u_{i0} = \sum_{i=0}^{[\alpha_i]} \frac{C_i^k u_i}{k!}$$

(10) $$u_{i(n+1)} = I^\alpha \left[A_{in}(u_{i0}, u_{i1}, \cdots, u_{in}) \right], \ n = 0, 1, 2, 3, \cdots$$

In order to determine the Adomian polynomial, we introduce a parameter λ and (9) becomes,

(11) $$G_i \left(t, \sum_{j=0}^{\infty} u_{1,j} \lambda^j, \cdots, \sum_{j=0}^{\infty} u_{n,j} \lambda^j \right) = \sum_{j=0}^{\infty} A_{ij} \lambda^j$$

Let $G_{i\lambda}(t) = \sum_{j=0}^{\infty} u_{i,j} \lambda^j$, then

(12) $$A_{ij} = \frac{1}{j!} \left[\frac{d^j}{d\lambda^j} G_{i\lambda}(u_1, u_2, \cdots, u_n) \right]_{\lambda=0}$$

where

(13) $$G_{i\lambda}(u_1, u_2, \cdots, u_n) = G_i(t, u_{1\lambda}, u_{2\lambda}, \cdots, u_{n\lambda})$$

In view of (12) and (13), we get

$$
\begin{aligned}
A_{ij} &= \frac{1}{j!} \frac{d^j}{d\lambda^j} \left[G_i(t, u_{1\lambda}, u_{2\lambda}, \cdots, u_{n\lambda}) \right]_{\lambda=0} \\
&= \frac{1}{j!} \frac{d^j}{d\lambda^j} \left[G_i(t, \sum_{j=0}^{\infty} u_{1,j} \lambda^j, \cdots, \sum_{j=0}^{\infty} u_{n,j} \lambda^j) \right]_{\lambda=0} \\
&= \left[\frac{1}{j!} \frac{d^j}{d\lambda^j} G_i(t, \sum_{j=0}^{\infty} u_{1,j} \lambda^j, \cdots, \sum_{j=0}^{\infty} u_{n,j} \lambda^j) \right]_{\lambda=0}
\end{aligned}
$$

(14)

Hence, the equation (10) and (14) leads to following recurrence relation

$$u_{i0} = \sum_{i=0}^{[\alpha_i]} \frac{C_i^k u_i}{k!}$$

(15) $$u_{i(n+1)} = I^\alpha \left[\frac{1}{j!} \frac{d^j}{d\lambda^j} G_i(t, \sum_{j=0}^{\infty} u_{1,j} \lambda^j, \cdots, \sum_{j=0}^{\infty} u_{n,j} \lambda^j) \right]_{\lambda=0}, \ n = 0, 1, 2, 3, \cdots$$

We can approximate the solution u_i by the truncated series

$$G_{ik} = \sum_{j=0}^{k-1} u_{i,j}, \quad \lim_{k \to \infty} G_{ik} = u_i(t).$$

4. EQUILIBRIUM POINTS, STABILITY AND BASIC REPRODUCTIVE NUMBER R_0

Theorem 4.1. *The Diseases free equilibria (DFE) point and the Endemic equilibrium (EE) point for SEI model are* $(s^*, e^*, i^*) = (1, 0, 0)$ *and*

$$(s^*, e^*, i^*) = \left(\frac{(\sigma_E - \mu)(\sigma_I - \mu)}{\beta \sigma_E}, \frac{\mu}{(\sigma_E - \mu)} - \frac{\mu(\sigma_I - \mu)}{\sigma_E \beta}, \frac{\mu \sigma_E}{(\sigma_E - \mu)(\sigma_I - \mu)} - \frac{\mu}{\beta} \right)$$

respectively.

Proof: Now, substituting $s = \frac{S}{N}$, $e = \frac{E}{N}$, $i = \frac{I}{N}$ in to the system of equations (1), we obtain

(16)
$$\left. \begin{array}{l} \dfrac{ds}{dt} = -\beta\, si - \mu + \mu\, s = f(s, e, i) \\[2mm] \dfrac{de}{dt} = \beta\, si - (\sigma_E + \mu)\, e = g(s, e, i) \\[2mm] \dfrac{di}{dt} = \sigma_E e - (\sigma_I + \mu)\, i = h(s, e, i) \end{array} \right\}$$

Now, diseases free equilibria (DFE) can be found by substituting $i^* = 0$ in linearization of system (16) that is

$$f(s, e, i) = 0, \; g(s, e, i) = 0, \; h(s, e, i) = 0$$

Thus

(17)
$$\left. \begin{array}{l} -\beta si + \mu - \mu s = 0 \\[2mm] \beta si - (\sigma_E + \mu)e = 0 \\[2mm] \sigma_E e - (\sigma_I + \mu)i = 0 \end{array} \right\}$$

Putting $i^* = 0$ in third equation of system (17) we get $e^* = 0$.

By adding first two equations of the system (17), we get $\mu(1 - s) - (\sigma_E + \mu)e = 0$, therefore $s^* = 1$.

Thus, DFE point is $(s^*, e^*, i^*) = (1, 0, 0)$.

We obtain the endemic equilibrium point, from third equation of system (17) as follows

$$e^* = \frac{(\sigma_I + \mu)}{\sigma_E} i^*.$$

From the second equation of (17), we get

$$\beta s^* i^* = (\sigma_E + \mu) e^*$$

Thus,

$$s^* = \frac{(\sigma_E + \mu)(\sigma_I + \mu)}{\beta \, \sigma_E}$$

From the first equation of (17), we get

$$\beta \, s^* i^* = \mu \, (1 - s^*)$$

Thus, we obtain

$$i^* = \frac{\mu \, \sigma_E}{(\sigma_E + \mu)(\sigma_I + \mu)} - \frac{\mu}{\beta}$$

Hence,

$$e^* = \frac{\mu}{(\sigma_E + \mu)} - \frac{\mu \, (\sigma_I + \mu)}{\sigma_E \, \beta}$$

Thus, the endemic equilibrium point for SEI model is

$$(s^*, e^*, i^*) = \left(\frac{(\sigma_E + \mu)(\sigma_I + \mu)}{\beta \, \sigma_E}, \frac{\mu}{(\sigma_E + \mu)} - \frac{\mu \, (\sigma_I + \mu)}{\sigma_E \, \beta}, \frac{\mu \, \sigma_E}{(\sigma_E + \mu)(\sigma_I + \mu)} - \frac{\mu}{\beta} \right).$$

Lemma 4.1. *Rourth-Hurwitz Criteria*[20]

For the cubic equation $\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0$ conditions for $Re(\lambda) < 0$ is

$$a_1 > 0, \, a_2 > 0, \, a_3 > 0, \, a_1 \, a_2 - a_3 > 0.$$

Theorem 4.2. *Stability* [18]

If J is the Jacobian matrix of order $(k \times k)$ for a nonlinear system of k first order equations, then trajectory of the system that is equilibrium point will have stable behavior when real part of all eigenvalues is negative.

Proof: The Jacobian matrix of SEI model is evaluated as follows

$$(18) \qquad J(s^*, e^*, i^*) = \begin{bmatrix} f_s & f_e & f_i \\ g_s & g_e & g_i \\ h_s & h_e & h_i \end{bmatrix} = \begin{bmatrix} -\beta \, i - \mu & 0 & -\beta \, s \\ \beta \, i & -\sigma_E - \mu & \beta \, s \\ 0 & \sigma_E & -\sigma_I - \mu \end{bmatrix}$$

The Jacobian Matrix of SEI model at DFE is as below

$$(19) \qquad J(1,0,0) = \begin{bmatrix} -\mu & 0 & -\beta \\ 0 & -\sigma_E - \mu & \beta \\ 0 & \sigma_E & -\sigma_I - \mu \end{bmatrix}$$

To find its eigenvalue, we must have to solve $det[J(1,0,0) - \lambda I_3] = 0$.

Therefore

$$det[J(1,0,0) - \lambda I_3] = (-\mu - \lambda)[\lambda^2 + (\sigma_E + 2\mu + \sigma_I)\lambda + (\sigma_E \sigma_I + \mu \sigma_E + \mu \sigma_I + \mu^2 - \beta \sigma_E)] = 0.$$

Therefore, eigenvalues are

$$\lambda_1 = -\mu, \ \lambda_{2,3} = \frac{-(\sigma_E + 2\mu + \sigma_I) \pm \sqrt{(\sigma_E - \sigma_I)^2 - 4\beta \sigma_E}}{2}$$

According to (4.2), DEF is stable if

$$\sqrt{(\sigma_E - \sigma_I)^2 - 4\beta \sigma_E} < (\sigma_E + 2\mu + \sigma_I)$$

In addition to stability of DFE, we have to discuss stability of EE.

Let us find eigenvalues of EE of SEI model. Consider $det[J(s^*, e^*, i^*) - \lambda I_3] = 0$ that is

$$(20) \qquad det[J(s^*, e^*, i^*) - \lambda I_3] = det \begin{bmatrix} -\beta i - \mu - \lambda & 0 & -\beta s \\ \beta i & -\sigma_E - \mu - \lambda & \beta s \\ 0 & \sigma_E & -\sigma_I - \mu - \lambda \end{bmatrix}$$

To find eigenvalues, characteristic equation will be considered as

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - det[J(s^*, e^*, i^*)] = 0$$

where $S_1 =$ Trace of $J(s^*, e^*, i^*) = -(3\mu + \beta i + \sigma_E + \sigma_I)$, $S_2 =$ sum of minors of diagonal elements of

$$J(s^*, e^*, i^*) = (\sigma_E + \mu)(\sigma_I + \mu) - \beta s \sigma_E + (\beta i + \mu)(\sigma_E + 2\mu + \sigma_I),$$

$$det[J(s^*, e^*, i^*)] = -(\beta i + \mu)(\sigma_E + \mu)(\sigma_I + \mu) + \beta \mu s \sigma_E.$$

Thus, Characteristic equation is

$$\lambda^3 + (3\mu + \beta i + \sigma_E + \sigma_I)\lambda^2 + [(\sigma_E + \mu)(\sigma_I + \mu) - \beta s \sigma_E + (\beta i + \mu)(\sigma_E + 2\mu + \sigma_I)]\lambda$$

$$+ [(\beta i + \mu)(\sigma_E + \mu)(\sigma_I + \mu) - \beta \mu s \sigma_E] = 0.$$

Now, compare with $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, we get

$$a_1 = (3\mu + \beta i + \sigma_E + \sigma_I)$$

$$a_2 = [(\sigma_E + \mu)(\sigma_I + \mu) - \beta s \sigma_E + (\beta i + \mu)(\sigma_E + 2\mu + \sigma_I)]$$

$$a_3 = [(\beta i + \mu)(\sigma_E + \mu)(\sigma_I + \mu) - \beta \mu s \sigma_E].$$

Threfore, from lemma (4.1), EE is stable if $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $a_1 a_2 - a_3 > 0$.

Now, we have to calculate basic reproductive number (R_0) and discuss stability according to that

- **Basic Reproductive Number R_0**

The basic reproductive number (R_0) is define as the number of secondary infectious that one infectious individual create over the duration of the infectious period, provided that every one else is susceptible.

The biological interpretation of R_0 is that if

$$R_0 < 1 \implies \text{Infection dies out.}$$

$$R_0 > 1 \implies \text{Infection persist.}$$

To compute basic reproductive number R_0 of our model, we employ the NGM as applied by Diekmann et. al.[19]. We will refer the second and third equations of (17) as the linearized infection subsystem as it only describe the production of new infected and changes in the states of already existing infected. The matrix

$$(21) \qquad F = \begin{bmatrix} 0 & \beta \\ \sigma_E & 0 \end{bmatrix}$$

corresponding to transmissions and the matrix

$$(22) \qquad V = \begin{bmatrix} -\sigma_E - \mu & 0 \\ 0 & -\sigma_I - \mu \end{bmatrix}$$

to transitions. Thus

$$(23) \qquad V^{-1} = \begin{bmatrix} \frac{-1}{\sigma_I + \mu} & 0 \\ 0 & \frac{-1}{\sigma_E + \mu} \end{bmatrix}$$

(24)
$$FV^{-1} = \begin{bmatrix} 0 & \frac{-\beta}{\sigma_E + \mu} \\ \frac{-\sigma_E}{\sigma_I + \mu} & 0 \end{bmatrix}$$

Therefore, $det[FV^{-1} - \lambda] = 0 \Rightarrow \lambda^2 - \frac{\beta \sigma_E}{(\sigma_E + \mu)(\sigma_I + \mu)} = 0$.

As per the NGM, R_0 is the dominant eigenvalue of matrix FV^{-1}, hence

$$R_0 = \sqrt{\frac{\beta \sigma_E}{(\sigma_E + \mu)(\sigma_I + \mu)}}.$$

By using values of $\beta = 1.7$, $\sigma_E = \frac{1}{14} = 0.07142857$, $\sigma_I = \frac{1}{5} = 0.2$, $\mu = 0.00001956$, we obtain

$$R_0 = 2.92$$

According to theory, infection persist as $R_0 > 1$, which result in Pandemic situation in India. So authors will recommend to the government to increase measures for reducing R_0 (i.e. < 1) which control Pandemic situation. This is possible by reducing contact rate between peoples through effective social distancing, lockdown, by taking safety measures.

5. NUMERICAL SIMULATIONS

In this section, we obtain the solution of fractional SEI model (2) by Adomian Decomposition Method as discussed in previous section.

Consider the series solution of system (2) as

$$u_i = \sum_{j=0}^{\infty} u_{i,j}$$

where $u_1 = S$, $u_2 = E$, $u_3 = I$

Thus

$$u_i = u_i(0) + I^{\alpha}[G_i(t, u_1, u_2, u_3)]$$

Consider

$$G_1(t, u_1, u_2, u_3) = -\frac{\beta}{N} u_1 u_3 + \mu N - \mu u_1$$

$$G_2(t, u_1, u_2, u_3) = \frac{\beta}{N} u_1 u_3 - (\sigma_E + \mu) u_2$$

$$G_3(t, u_1, u_2, u_3) = \sigma_E u_2 - (\sigma_I + \mu) u_3.$$

By using (14), we obtain the Adomian polynomial as follow

$$A_{10} = -\frac{\beta}{N} u_{10} u_{30} + \mu N - \mu u_{10},$$

$$A_{11} = -\frac{\beta}{N} [u_{10} u_{31} + u_{11} u_{30}] - \mu u_{11}$$

$$A_{12} = -\frac{\beta}{N} [u_{10} u_{32} + u_{11} u_{31} + u_{12} u_{30}] - \mu u_{12}$$

$$A_{20} = \frac{\beta}{N} u_{10} u_{30} - (\sigma_E + \mu) u_{20}$$

$$A_{21} = \frac{\beta}{N} [u_{10} u_{31} + u_{11} u_{30}] - (\sigma_E + \mu) u_{21}$$

$$A_{22} = \frac{\beta}{N} [u_{10} u_{32} + u_{11} u_{31} + u_{12} u_{30}] - (\sigma_E + \mu) u_{22}$$

$$A_{30} = \sigma_E u_{20} - (\sigma_I + \mu) u_{30}$$

$$A_{31} = \sigma_E u_{21} - (\sigma_I + \mu) u_{31}$$

$$A_{32} = \sigma_E u_{22} - (\sigma_I + \mu) u_{32}.$$

The Adomian decomposition series given in (10), leads to following result

$$u_{10} = 1300000000, \quad u_{1(n+1)} = I^\alpha [A_{1n}]$$

$$u_{20} = 1300, \quad u_{2(n+1)} = I^\alpha [A_{2n}]$$

$$u_{30} = 44, \quad u_{3(n+1)} = I^\alpha [A_{3n}].$$

We obtain, the first three iterations of the solution of fractional SEI model of COVID-19 for India as follows

$$S = u_1 = 1300000000 - 74.80 \frac{t^\alpha}{\Gamma(\alpha+1)} - 142.89421783 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 15.39024526 \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots$$

$$E = u_2 = 1300 - 18.082569 \frac{t^\alpha}{\Gamma(\alpha+1)} + 144.18763805 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 20.53981631 \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots$$

$$I = u_3 = 44 + 84.05628036 \frac{t^\alpha}{\Gamma(\alpha+1)} + -18.10451226 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 6.96018669 \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots$$

Next, We have analysed the above model by considering the assumptions given below and further estimating or fitting various parameters given by table

(i) $\mu_I = \mu_0 = \mu$ that is the emigration from the population is equal to emigration into population.

(ii) $\frac{1}{D_E} = \sigma_E$ and $\frac{1}{D_I} = \sigma_I$.

(iv) Average exposed(Incubation) period is 14 days and average infection period is 5 days [24].

(vi) Initial case are only from emigrated people (i.e. persons having foreign travel history).

(vii) Initial rate of transmission is 1.7

Parameter	Description	Value	Reference
S	Susceptible population	1300000000	[36]
E	Exposed population	1300	Fitted
I	Infected population	44	Fitted
β	Infection rate	1.7	Fitted
σ_E	Reciprocal of average exposed period	0.07142857	Estimated
σ_I	Reciprocal of average infected period	0.2	Estimated
μ	rate of emigration	0.00001956	Estimated

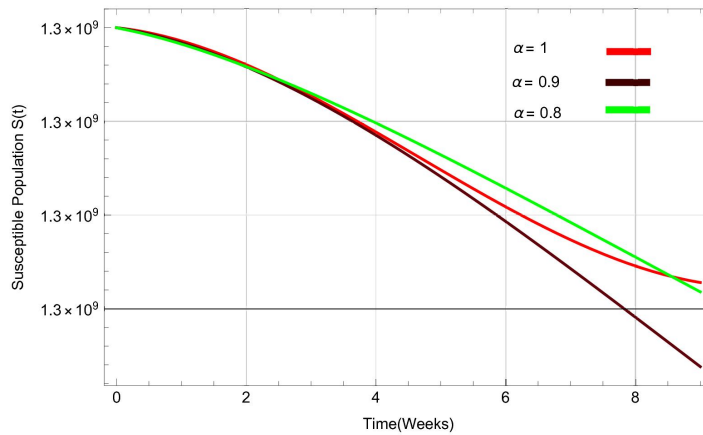TABLE 1. The parametric values for Fractional SEI model of COVID-19 for India.
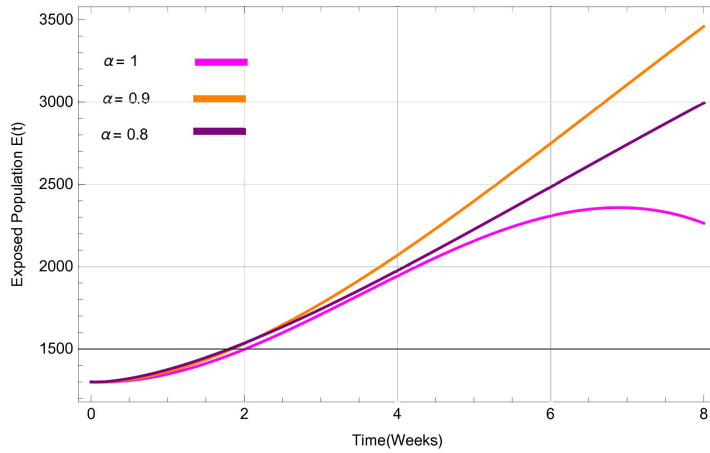

FIGURE 2. $S(t)$ for $\alpha (= 1, 0.9, 0.8)$
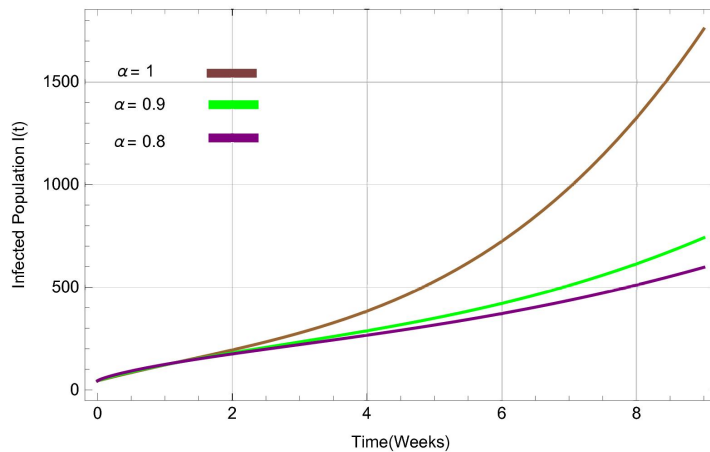
FIGURE 3. $E(t)$ for $\alpha(=1, 0.9, 0.8)$

FIGURE 4. $I(t)$ for $\alpha(=1, 0.9, 0.8)$

Figs. 2, 3 and 4 shows the graphical representation of suspectable population $S(t)$, exposed population $E(t)$ and infected population $I(t)$ for various values of $\alpha(=1, 0.9, 0.8)$ using Adomian Decomposition method which predicts that this method can foresee the conduct of said variables precisely for the considered region. We observe that infection dies out slowly as value of α decreases. Hence, the non-integer order has a significant effect on the dynamics of SEI model of COVID-19 for India.

6. CONCLUSIONS

In this work, we have developed fractional SEI mathematical models and analysed dynamics of COVID-19, which is Pandemic throughout the world. Further, we have obtained series solutions of this model by Adomian Decomposition Method. We found basic reproduction number for this disease is $R_0 = 2.92$, which is unstable because it is greater than one. We also observe that R_0 is depend upon β and μ(note that σ_E and σ_I are constant). Results of above mathematical model are agree with recommendation of WHO that "It is still possible to interrupt virus spread, provided that countries put in place strong, measures to direct disease only". To keep $R_0 < 1$ that is asymptotically stable, we have to control β specially. This can be done by promoting social distancing measures, avoid large social gatherings, applying lock down and travel ban. To control μ, we may increasing effectiveness of passenger screening at airports. To stop spread of virus Indian government has already suspended all commercial passenger flight from 23rd March 2020. Also, declare lock down from 25th March 2020. This steps will help to reduce β and μ and hence R_0. We further Also, we represented solution graphically by Mathematica software. However, the result obtained above is depends on the limited data available in various research paper and note that real situation at initial stage of transmission is may be different.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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