Available online at http://scik.org

J. Math. Comput. Sci. 10 (2020), No. 5, 1687-1696

https://doi.org/10.28919/jmcs/4724

ISSN: 1927-5307

COMMON FIXED POINTS OF FUZZY MAPS UNDER NONEXPANSIVE TYPE CONDITION

ANIL BAKHRU[1,*], MANOJ UGHADE[2], RICHA GUPTA[3]

[1]Department of Mathematics, Sardar Vallabhbhai Polytechnic College, College of University of Technology of Madhya Pradesh, Bhopal 462002, India

[2]Department of Post Graduate Studies and Research in Mathematics, Jaywanti Haksar Government Post Graduate College, College of Chhindawara University, Betul, 460001 India

[3]Department of Engineering Mathematics and Research Center, Sarvepalli Radhakrishanan University, Bhopal 462026, India

Abstract: In this paper, we obtain a common fixed-point theorem for a sequence of fuzzy mappings satisfying a rational contractive condition involving nonexpansive mapping.

Keywords: fuzzy sets; common fixed point; fuzzy mapping; nonexpansive mapping.

1. INTRODUCTION

Fixed point theory plays a basic role in applications of many branches of mathematics. The term metric fixed point theory refers to those fixed point theoretic results in which geometric conditions on the underlying spaces and/or mappings play a crucial role. For the past twenty five years metric fixed point theory has been a flourishing area of research for many mathematicians. Although a substantial number of definitive results now has been discussed, a few questions

---

[*]Corresponding author

E-mail address: anilbakhru@gmail.com

Received May 24, 2020

lying at the heart of the theory remain open and there are many unanswered questions regarding the limit to which theory may extended. The first important result on fixed points for contractive type mappings was the well-known Banach contraction principle [1] appeared in explicit form in Banach's thesis in 1922, where it was used and established the existence of a solution for an integral equation. In 1965, Zadeh [2] introduced the concept of a fuzzy set as a new way to represent vagueness in everyday life. The study of fixed point theorems in fuzzy mathematics was investigated by Weiss [3], Butnariu [4], Singh and Talwar [5], Mihet [6], Qiu et al. [7], and Beg and Abbas [8] and many others. Heilpern [9] first used the concept of fuzzy mappings to prove the Banach contraction principle for fuzzy mappings on a complete metric linear space. The result obtained by Heilpern [9] is a fuzzy analogue of the fixed point theorem for multivalued mappings of Nadler et al. [10]. Bose and Sahani [11], Vijayaraju and Marudai [12], improved the result of Heilpern. In some earlier work, Watson and Rhoades [13], [14] proved several fixed-point theorems involving a very general contractive definition. In this paper, we prove a common fixed point theorem for sequence of fuzzy mappings satisfying a rational contractive condition involving nonexpansive mapping. Our results extend and generalized the corresponding results of Bose and Sahani [11], Vijayaraju and Mohanraj [12] and Rhoades [15], [16], Saluja et al. [18] and Das and Gupta [19].

2. PRELIMINARIES

In this paper, we shall generally follow the notations of Heilpern [9]. We recall some mathematical basics and definitions to make this paper self-sufficient.

Definition 2.1 Let (X, d) be a complete linear metric space and $\mathcal{F}(X)$, the collection of all fuzzy sets in X. A fuzzy set in X is a function with domain X and values in $[0,1]$. If A is a fuzzy set and $x \in X$, then the function value $A(x)$ is called the grade of membership of x in A. The α-level set of A is denoted by

$$A_\alpha = \{x: A(x) \geq \alpha\} \text{ if } \alpha \in (0,1]$$
$$A_0 = \overline{\{x: A(x) > 0\}},$$

where $\bar{B}$ stands for the (non-fuzzy) closure of a set B.

Definition 2.2 A fuzzy set A is said to be an approximate quantity if and only if A_α is compact and convex for each $\alpha \in (0,1]$ and $\sup_{x \in X} A(x) = 1$, when A is an approximate quantity and

$A(x_0) = 1$ for some $x_0 \in X$, A is identified with an approximation of x_0. From the collection $\mathcal{F}(X)$, a sub-collection of all appropriate quantities is denoted as $\mathcal{W}(X)$.

Definition 2.3 The distance between two appropriate quantities is defined by the following scheme. Let $A, B \in \mathcal{W}(X)$ and $\alpha \in [0,1]$,

$$D_\alpha(A,B) = \inf_{x \in A_\alpha, y \in B_\alpha} d(x,y);$$

$$H_\alpha(A,B) = \operatorname{dist} d(A_\alpha, B_\alpha);$$

$$H(A,B) = \sup_\alpha D_\alpha(A,B);$$

wherein the "dist" is in the sense of Hausdorff distance. The function D_α is called an α-distance (induced by d), H_α a α- distance (induced by dist) and H a distance between A and B. Note that D_α is a non-decreasing function of α.

Definition 2.4 Let $A, B \in \mathcal{W}(X)$. Then A is said to be more accurate than B, denoted by $A \subset B$, iff $A(x) \leq B(x)$ for each $x \in X$. The relation $\subset$ induces a partial ordering on the family $\mathcal{W}(X)$.

Definition 2.5 Let Y be an arbitrary set and X be any metric space. F is called a fuzzy mapping if and only if F is a mapping from the set Y into $\mathcal{W}(X)$. A fuzzy mapping F is a fuzzy subset of $Y \times X$ with membership function $F(y,x)$. The function value $F(y,x)$ is the grade of membership of x in $F(y)$. Note that each fuzzy mapping is a set valued mapping. Let $A \in F(X), B \in F(Y)$. Then the fuzzy set $F(A)$ in $F(X)$ is defined by

$$F(A)(x) = \sup_{y \in X} \Big(F(y,x) \bigwedge A(y)\Big), \mathrm{x} \in \mathrm{X}$$

and the fuzzy set $F^{-1}(B)$ in $F(Y)$ is defined by

$$F^{-1}(B)(y) = \sup_{x \in X} F(y,x) \bigwedge B(x), \mathrm{y} \in \mathrm{Y}$$

Lee [17] proved the following.

Lemma 2.6 Let (X,d) be a complete linear metric space, F is a fuzzy mapping from X into $\mathcal{W}(X)$ and $x_0 \in X$, then there exists an $x_1 \in X$ such that $\{x_1\} \subset F(x_0)$.

The following two lemmas are due to Heilpern [9].

Lemma 2.7 Let $x \in X$, $A \in \mathcal{W}(X)$ and $\{x\}$ a fuzzy set with membership function equal to a characteristic function of $\{x\}$. If $\{x\} \subset A$, then $D_\alpha(x,A) = 0$ for each $\alpha \in [0,1]$.

Lemma 2.8 Let $A, B \in \mathcal{W}(X)$, $\alpha \in [0,1]$ and $D_\alpha(A,B) = \inf_{x \in A_\alpha, y \in B_\alpha} d(x,y)$, where $A_\alpha = \{x : A(x) \geq \alpha\}$, then

$$D_\alpha(x, A) \le d(x, y) + D_\alpha(y, A)$$

for each $x, y \in X$.

Lemma 2.9 Let $H_\alpha(A, B) = \text{dist}(A_\alpha, B_\alpha)$, where 'dist' is the Hausdorff distance. If $\{x_0 \subset A\}$, then $D_\alpha(x_0, B) \le H_\alpha(A, B)$ for each $B \in \mathcal{W}(X)$.

Rhoades [15] proved the following common fixed point theorem involving a very general contractive condition, for fuzzy mappings on complete linear metric space. He proved the following theorem.

Theorem 2.10 Let (X, d) be a complete linear metric space and let F, G be fuzzy mappings from X into $\mathcal{W}(X)$ satisfying

$$H(Fx, Gy) \le Q(m(x, y)) \tag{2.1}$$

where

$$m(x, y) = \max\left\{d(x, y), D_\alpha(x, Fx), D_\alpha(y, Gy), \frac{D_\alpha(x, Gy) + D_\alpha(y, Fx)}{2}\right\}$$

and Q is a real-valued function defined on D, the closure of the range of d, satisfying the following three conditions:

a) $0 < Q(s) < s$ for each $s \in D \backslash \{0\}$ and $Q(0) = 0$,
b) Q is non-decreasing on D, and
c) $g(s) = s/s - Q(s)$ is non-increasing on $D \backslash \{0\}$.

Then there exists a point z in X such that $\{z\} \subset Fz \cap Gz$.

In [16] Rhoades, generalized the result of Theorem 2.10 for sequence of fuzzy mappings on complete linear metric space. He proved the following theorem.

Theorem 2.11 Let g be a non-expansive self-mapping of a complete linear metric space (X, d) and $\{F_i\}$ be a sequence of fuzzy mappings from X into $W(X)$. For each pair of fuzzy mappings F_i, F_j and for any $x \in X, \{u_x\} \subset F_i(x)$, there exists a $\{v_y\} \subset F_j(y)$ for all $y \in X$ such that

$$D(\{u_x\}, \{v_y\}) \le Q(m(x, y)) \tag{2.2}$$

Where

$$m(x, y) = \max\left\{(g(x), g(y)), d(g(x), g(u_x)), d\left(g(y), g(v_y)\right), \frac{d\left(g(x), g(v_y)\right) + d(g(y), g(u_x))}{2}\right\}$$

and Q satisfying the conditions (a)-(c) of Theorem 2.10. Then there exists $\{z\} \subset \bigcap_{i=1}^{\infty} F_i(z)$.

3. MAIN RESULTS

Now, we give our first main result.

Theorem 3.1 Let g be a non-expansive self-mapping of a complete linear metric space (X,d) and $\{F_i\}$ be a sequence of fuzzy mappings from X into $W(X)$. For each pair of fuzzy mappings F_i, F_j and for any $x \in X, \{u_x\} \subset F_i(x)$, there exists a $\{v_y\} \subset F_j(y)$ for all $y \in X$ such that

$$D(\{u_x\},\{v_y\}) \leq Q\left(\max\left\{d\big(g(x),g(y)\big), \frac{d\big(g(y),g(v_y)\big)[1+ d(g(x),g(u_x))]}{1+d\big(g(x),g(y)\big)}\right\}\right) \tag{3.1}$$

and Q satisfying the conditions (a)-(c) of Theorem 2.10. Then there exists $\{z\} \subset \bigcap_{i=1}^{\infty} F_i(z)$

PROOF Let $x_0 \in X$. Then we can choose $x_1 \in X$ such that $\{x_1\} \subset Fx_0$ by Lemma 2.6. From the hypothesis, there exists an $x_1 \in X$ such that $\{x_2\} \subset Fx_1$ and since g is a nonexpansive self mapping, from (3.1), we have

$$\begin{aligned} D(\{x_1\},\{x_2\}) &\leq Q\left(\max\left\{d\big(g(x_0),g(x_1)\big), \frac{d\big(g(x_1),g(x_2)\big)\big[1+\ d\big(g(x_0),g(x_1)\big)\big]}{1+d\big(g(x_0),g(x_1)\big)}\right\}\right) \\ &< \max\big\{d\big(g(x_0),g(x_1)\big), d\big(g(x_1),g(x_2)\big)\big\} \\ &\leq \max\{d(x_0,x_1), d(x_1,x_2)\} \end{aligned}$$

Inductively, we obtain a sequence $\{x_n\}$ such that $\{x_{n+1}\} \subset F_{n+1}(x_n)$ and

$$\begin{aligned} &D(\{x_n\},\{x_{n+1}\}) \\ &\leq Q\left(\max\left\{d\big(g(x_{n-1}),g(x_n)\big), \frac{d\big(g(x_n),g(x_{n+1})\big)\big[1+\ d\big(g(x_{n-1}),g(x_n)\big)\big]}{1+d\big(g(x_{n-1}),g(x_n)\big)}\right\}\right) \\ &< \max\big\{d\big(g(x_{n-1}),g(x_n)\big), d\big(g(x_n),g(x_{n+1})\big)\big\} \\ &\leq \max\{d(x_{n-1},x_n), d(x_n,x_{n+1})\} \end{aligned} \tag{3.2}$$

Since $D(\{x_n\},\{x_{n+1}\}) = d(x_n,x_{n+1})$ it follows from (3.2) that

$$d(x_n,x_{n+1}) < d(x_{n-1},x_n). \tag{3.3}$$

Using this fact back in (3.1), we obtain that $d(x_n,x_{n+1}) \leq d(x_{n-1},x_n)$. Substituting into (3.2) we obtain

$$d(x_n,x_{n+1}) < Q\big(d(x_{n-1},x_n)\big) < Q^2\big(d(x_{n-2},x_{n-1})\big) < \cdots < Q^n\big(d(x_0,x_1)\big) \tag{3.4}$$

From Lemma 2 of [17], $\lim_{n\to\infty} Q^n\big(d(x_0,x_1)\big) = 0$. To show that $\{x_n\}$ is Cauchy, choose N so large that $Q^n\big(d(x_0,x_1)\big) \leq \left(\frac{1}{2}\right)^n$ for all $n > N$. Then, for $m > n > N$,

$$d(x_n,x_m) \leq d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + \cdots + d(x_{m-1},x_m)$$

$$= \sum_{j=n}^{m-1} d\big(x_j, x_{j+1}\big) \le \sum_{j=n}^{m-1} Q^j\big(d(x_0, x_1)\big) \le \sum_{j=n}^{m-1} \left(\frac{1}{2}\right)^j$$
$$< \left(\frac{1}{2}\right)^{n-1} \tag{3.5}$$

and $\{x_n\}$ is Cauchy, hence convergent. Call the limit p. Let F_m be an arbitrary member of the sequence $\{F_i\}$. Since $\{x_n\} \subset F_m(x_{n-1})$, there exists a $v_n \in X$ such that $\{v_n\} \subset F_m(p)$ for all n and applying (3.1), we have

$$D\big(\{x_n\}, \{\{v_n\}\}\big) \le Q\left(\max\left\{d\big(g(x_{n-1}), g(p)\big), \frac{d\big(g(p), g(v_n)\big)\big[1 + d\big(g(x_{n-1}), g(x_n)\big)\big]}{1 + d\big(g(x_{n-1}), g(p)\big)}\right\}\right)$$
$$< Q\left(\max\left\{d\big(g(x_{n-1}), g(p)\big), \frac{d\big(g(p), g(v_n)\big)\big[1 + d\big(g(x_{n-1}), g(x_n)\big)\big]}{1 + d\big(g(x_{n-1}), g(p)\big)}\right\}\right)$$
$$\le Q\left(\max\left\{d(x_{n-1}, p), \frac{d(p, v_n)[1 + d(x_{n-1}, x_n)]}{1 + d(x_{n-1}, p)}\right\}\right) \tag{3.6}$$

Suppose that $\lim_{n\to\infty} v_n \ne p$. Taking the limit as $n \to \infty$ yields, since Q is continuous (Lemma 1 of [13]

$$\limsup_{n\to\infty} d(p, v_n) \le Q\left(\limsup_{n\to\infty} d(p, v_n)\right) < \limsup_{n\to\infty} d(p, v_n)$$

This is a contradiction. Therefore, $\lim_{n\to\infty} v_n = p$. Since $F_m(p) \in W(X)$, $F_m(p)$ is upper semi continuous and therefore, $\limsup_{n\to\infty}[F_m(p)](v_n) \le [F_m(p)](p)$. Since $\{v_n\} \subset F_m(p)$ for all n, $[F_m(p)](p) = 1$. Hence $\{p\} \subset F_m(p)$. Since F_m is arbitrary, $\{p\} \subset \bigcap_{i=1}^{\infty} F_i(p)$.

Theorem 3.2 Let g be a nonexpansive self mapping of acomplete linear metric space (X, d) and $\{F_i\}$ be a sequence of fuzzy mappings from X into $W(X)$. For each pair of fuzzy mappings F_i, F_j and for any $x \in X, \{u_x\} \subset F_i(x)$, there exists a $\{v_y\} \subset F_j(y)$ for all $y \in X$ such that

$$D\big(\{u_x\}, \{v_y\}\big) \le \max\left\{d\big(g(x), g(y)\big), \frac{d\Big(g(y), g\big(v_y\big)\Big)\big[1 + d\big(g(x), g(u_x)\big)\big]}{1 + d\big(g(x), g(y)\big)}\right\}$$
$$-\mathrm{w}\left(\max\left\{d\big(g(x), g(y)\big), \frac{d\big(g(y), g(v_y)\big)\big[1 + d\big(g(x), g(u_x)\big)\big]}{1 + d\big(g(x), g(y)\big)}\right\}\right) \tag{3.7}$$

for all $x, y \in X, w: R^+ \to R^+$ be a continuous function such that $0 < w(r) < r$ for all $r > 0$. Then there exists $\{p\} \subset \bigcap_{i=1}^{\infty} F_i(p)$., i.e. p is a common fixed point of the sequence of fuzzy mappings.

Proof. Let $x_0 \in X$. Then we can choose $x_1 \in X$ such that $\{x_1\} \subset Fx_0$ by Lemma 2.6. From the hypothesis, there exists an $x_1 \in X$ such that $\{x_2\} \subset Fx_1$ and since g is a nonexpansive self mapping, from (3.7), we have

$$D(\{x_1\},\{x_2\}) \leq \max\left\{d\big(g(x_0),g(x_1)\big), \frac{d\big(g(x_1),g(x_2)\big)\big[1+\ d\big(g(x_0),g(x_1)\big)\big]}{1+d\big(g(x_0),g(x_1)\big)}\right\}$$

$$-\mathrm{w}\left(\max\left\{d\big(g(x_0),g(x_1)\big), \frac{d\big(g(x_1),g(x_2)\big)\big[1+\ d\big(g(x_0),g(x_1)\big)\big]}{1+d\big(g(x_0),g(x_1)\big)}\right\}\right)$$

$$= \max\big\{d\big(g(x_0),g(x_1)\big), d\big(g(x_1),g(x_2)\big)\big\}$$

$$-\mathrm{w}\big(\max\big\{d\big(g(x_0),g(x_1)\big), d\big(g(x_1),g(x_2)\big)\big\}\big)$$

$$\leq \max\{d(x_0,x_1), d(x_1,x_2)\} - \mathrm{w}(\max\{d(x_0,x_1), d(x_1,x_2)\})$$

The last inequality gives

$$d(x_1,x_2) =\ D(\{x_1\},\{x_2\}) \leq \max\{d(x_0,x_1), d(x_1,x_2)\} - \mathrm{w}(\max\{d(x_0,x_1), d(x_1,x_2)\})$$

which implies that

$$d(x_1,x_2) \leq d(x_0,x_1) - \mathrm{w}\big(d(x_0,x_1)\big) \tag{3.8}$$

Similarly

$$d(x_2,x_3) \leq d(x_1,x_2) - \mathrm{w}\big(d(x_1,x_2)\big) \tag{3.9}$$

Inductively, we obtain a sequence $\{x_n\}$ such that $\{x_{n+1}\} \subset F_{n+1}(x_n)$ and

$$d(x_n,x_{n+1}) \leq d(x_{n-1},x_n) - \mathrm{w}\big(d(x_{n-1},x_n)\big) \tag{3.10}$$

Adding (3.8)-(3.10), we obtain

$$\sum_{i=0}^{n} \mathrm{w}\big(d(x_i,x_{i+1})\big) \leq d(x_0,x_1) - d(x_n,x_{n+1}) < d(x_0,x_1)$$

Therefore

$$\sum_{i=0}^{n} \mathrm{w}\big(d(x_i,x_{i+1})\big) < \infty, \lim_{n\to\infty} \mathrm{w}\big(d(x_n,x_{n+1})\big) = 0$$

Now suppose that $\{x_n\}$ is not a Cauchy sequence, then there is an $\varepsilon > 0$ such that for each positive even integer $2k$, there exists positive even integer $2m > 2n > 2k$ such that

$$d(x_{2n},x_{2m}) \geq \varepsilon \tag{3.11}$$

Also, for each $2k$, we may find the least $2m$ exceeding $2n$ such that

$$d(x_{2n},x_{2m-2}) < \varepsilon \tag{3.12}$$

Since $\{d(x_n, x_{n+1})\}$ is a decreasing sequence of non-negative terms, it converges, call the limit z. Suppose that $z > 0$. Then, since w is continuous, $\lim_{n\to\infty} \mathrm{w}\big(d(x_n, x_{n+1})\big) = w(z)$. But $\lim_{n\to\infty} \mathrm{w}\big(d(x_n, x_{n+1})\big) = 0$. Hence $w(z) = 0$, which is a contradiction to the fact that $0 < w(p) < p$. Hence $z = 0$ and then

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0 \tag{3.13}$$

Now

$$\varepsilon \le d(x_{2n}, x_{2m}) \le d(x_{2n}, x_{2m-2}) + d(x_{2m-2}, x_{2m-1}) + d(x_{2m-1}, x_{2m}) \tag{3.14}$$

Using (3.11)-(3.14), we obtain

$$d(x_{2n}, x_{2m}) \to \varepsilon \text{ as } k \to \infty \tag{3.15}$$

Note that

$$|d(x_{2m}, x_{2n+1}) - d(x_{2m}, x_{2n})| \le d(x_{2n}, x_{2n+1})$$
$$|d(x_{2m+1}, x_{2n+1}) - d(x_{2m}, x_{2n+1})| \le d(x_{2m}, x_{2m+1})$$
$$|d(x_{2m}, x_{2n+2}) - d(x_{2m}, x_{2n+1})| \le d(x_{2n+1}, x_{2n+2})$$
$$|d(x_{2m+1}, x_{2n+2}) - d(x_{2m+1}, x_{2n+1})| \le d(x_{2n+1}, x_{2n+2})$$

which implies that as $k \to \infty$,

$$\begin{gathered} d(x_{2m}, x_{2n+1}) \to \varepsilon, \qquad d(x_{2m+1}, x_{2n+1}) \to \varepsilon, \\ d(x_{2m}, x_{2n+2}) \to \varepsilon, d(x_{2m+1}, x_{2n+2}) \to \varepsilon \end{gathered} \tag{3.16}$$

Again applying (3.7), we get

$$\begin{aligned} & d(x_{2m+1}, x_{2n+2}) = D(\{x_{2m+1}\}, \{x_{2n+2}\}) \\ & \le \max\left\{d\big(g(x_{2m}), g(x_{2n+1})\big), \frac{d\big(g(x_{2n+1}), g(x_{2n+2})\big)\big[1 + \ d\big(g(x_{2m}), g(x_{2m+1})\big)\big]}{1 + d\big(g(x_{2m}), g(x_{2n+1})\big)}\right\} \\ & -\mathrm{w}\left(\max\left\{d\big(g(x_{2m}), g(x_{2n+1})\big), \frac{d\big(g(x_{2n+1}), g(x_{2n+2})\big)\big[1 + \ d\big(g(x_{2m}), g(x_{2m+1})\big)\big]}{1 + d\big(g(x_{2m}), g(x_{2n+1})\big)}\right\}\right) \\ & \le \max\left\{d(x_{2m}, x_{2n+1}), \frac{d(x_{2n+1}, x_{2n+2})[1 + \ d(x_{2m}, x_{2m+1})]}{1 + d(x_{2m}, x_{2n+1})}\right\} \\ & -\mathrm{w}\left(\max\left\{d(x_{2m}, x_{2n+1}), \frac{d(x_{2n+1}, x_{2n+2})[1 + \ d(x_{2m}, x_{2m+1})]}{1 + d(x_{2m}, x_{2n+1})}\right\}\right) \end{aligned}$$

Using (3.13), (3.16) and taking the limit as $k \to \infty$, we get

$$\varepsilon \le \max\{\varepsilon, 0\} - \mathrm{w}(\max\{\varepsilon, 0\})$$

which gives a contradiction. Thus $\{x_n\}$ is a Cauchy sequence and since X is complete, it converges to some $p \in X$.

Let F_m be an arbitrary member of the sequence $\{F_i\}$. Since $\{x_n\} \subset F_m(x_{n-1})$, by Lemma 2.6, there exists a $v_n \in X$ such that $\{v_n\} \subset F_m(p)$ for all n and applying (3.7) again, we have

$$
\begin{aligned}
& d(x_n, v_n) = D(\{x_n\}, \{v_n\}) \\
& \leq \max\left\{d\big(g(x_{n-1}), g(p)\big), \frac{d\big(g(p), g(v_n)\big)\big[1 + d\big(g(x_{n-1}), g(x_n)\big)\big]}{1 + d\big(g(x_{n-1}), g(p)\big)}\right\} \\
& -\mathrm{w}\left(\max\left\{d\big(g(x_{n-1}), g(p)\big), \frac{d\big(g(p), g(v_n)\big)\big[1 + d\big(g(x_{n-1}), g(x_n)\big)\big]}{1 + d\big(g(x_{n-1}), g(p)\big)}\right\}\right) \\
& \leq \max\left\{d(x_{n-1}, p), \frac{d(p, v_n)[1 + d(x_{n-1}, x_n)]}{1 + d(x_{n-1}, p)}\right\} \\
& -\mathrm{w}\left(\max\left\{d(x_{n-1}, p), \frac{d(p, v_n)[1 + d(x_{n-1}, x_n)]}{1 + d(x_{n-1}, p)}\right\}\right)
\end{aligned}
$$

Suppose that $\lim_{n\to\infty} v_n \neq p$. Taking the limit as $n \to \infty$ yields

$$d(p, v_n) \leq d(p, v_n) - \mathrm{w}\big(d(p, v_n)\big)$$

Since w is continuous, we get a contradiction. Therefore, $\lim_{n\to\infty} v_n = p$. Hence $\{p\} \subset F_m(p)$. Since F_m is arbitrary, $\{p\} \subset \bigcap_{i=1}^{\infty} F_i(p)$.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

AUTHORS' CONTRIBUTIONS

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

ACKNOWLEDGEMENTS

The authors express deep gratitude to the referee for his/her valuable comments and suggestions.

REFERENCES

[1] S. Banach, Sur les operations dans les ensemlesabstraits et leur application aux equations integrales, Fund. Math. 3 (1922), 133-181.

[2] L.A. Zadeh, Fuzzy sets, Inform. Control, 8 (1965), 338-353.

[3] M.D. Weiss, Fixed points and induced fuzzy topologies for fuzzy sets, J. Math. Anal. Appl. 50 (1975), 142-150.

[4] D. Butnariu, Fixed point theorems for fuzzy mappings, Fuzzy Sets Syst. 7 (1982), 191-207.

[5] K.L. Singh, R. Talwar, Fixed point of fuzzy mappings, Soochow J. Math. 19 (1993), 95-102.

[6] D. Mihet, On fuzzy contractive mappings in fuzzy metric spaces, Fuzzy Sets Syst. 158 (2007), 915-921.

[7] D. Qiu, L. Schu, J. Guan, Common fixed point theorems for fuzzy mappings under F-contraction condition, Chaos Solitons Fractals, 41 (2009), 360-367.

[8] I. Beg and M. Abbas, Invariant approximation for fuzzy nonexpansive mappings, Math. Bohem. 136 (2011), 51-59.

[9] S. Heilpern, Fuzzy maps and fixed point theorem, J. Math.Anal. Appl. 83 (1981), 566-569.

[10] S. B. Nadler Jr., Multivalued contraction mappings, Pac. J. Math., 30 (1969), 475-487.

[11] R.K. Bose, D. Sahani, Fuzzy mappings and fixed point theorems, Fuzzy Sets Syst. 21 (1987), 53-58.

[12] P. Vijayaraju, M. Marudai, Fixed point theorems for fuzzy mappings, Fuzzy Sets Syst. 135 (2003), 401-408.

[13] B. Watson, B. E. Rhoades, Generalized contraction and fixed points in metric space, Math. Japonica, 34 (1989), 975-982.

[14] B. Watson, B. E. Rhoades, Fixed point for set valued mappings in metric space, Math. Japonica, 35 (1990), 735-743.

[15] B. E. Rhoades, Fixed points of some fuzzy mappings, Soochow J. Math. 22 (1996), 111-115.

[16] B.E. Rhoades, A common fixed point theorem for a sequence of fuzzy mappings, Int. J. Math. Math. Sci. 3 (1995), 447-450.

[17] B.S. Lee, G.M. Lee, S.J. Cho, D.S. Kim, Generalized common fixed point theorems for a sequence of fuzzy mappings, Int. J. Math. Math. Sci., 17 (1994), 437-440.

[18] A.S. Saluja, P.K. Jhade, S. Manro, A Common Fixed Point Theorem for Fuzzy Maps under Nonexpansive type Condition, Appl. Math. Inf. Sci. Lett. 3 (2015), 19-23.

[19] B.K. Dass, S. Gupta, An extension of Banach contraction principle through rational expressions, Indian J. Pure Appl. Math. 6 (1975), 1455-1458.