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INTUITIONISTIC L-FUZZY GRAPH

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Abstract. In this paper we introduce intuitionistic L-fuzzy graph as a generalization of intuitionistic fuzzy graph. The operations on intuitionistic L-fuzzy graph such as union, intersection, complement are discussed. Also, we try to define matrices associated with Intuitionistic L-fuzzy graph.

Keywords: intutionistic L-fuzzy graph; index matrix representation; incidence matrix representation; union and intersection of intuitionistic L-fuzzy graph; complement of intuitionistic L-fuzzy graph.

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1. Introduction

In the classical paper by Rosenfeld [1] the concept of fuzzy graphs as a means to model several real life situations was introduced. Ever since then, fuzzy graph theory has witnessed tremendous growth. Graphs are models of relations between objects. The objects are represented by vertices and relations by edges. When the description of objects, or relationships, or both happens to possess uncertainty, we design a fuzzy graph. Fuzzy graph theory has numerous applications in modern science and technology especially in the field of information theory,

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neural networks, expert systems, cluster analysis, medical diagnosis and control theory. Several papers in areas related to fuzzy graph theory are available in literature.

According to Klir and Yuan [2] an L-Fuzzy set is a fuzzy set in which the range [0,1] is replaced by a lattice. Pramada Ramachandran and K. V. Thomas introduced the concept of L-Fuzzy graph. They studied isomorphism and associated matrices of L-Fuzzy graph.

In 1983, Atanassov [3] introduced the concept of Intuitionistic fuzzy sets as a generalization of fuzzy sets. Atanassov added a new component which determines the degree of non-membership in the definition of fuzzy sets. Atanassov also introduced the concept of Intuitionistic fuzzy graph. M. G. Karunambigai and R. Parvathi [4] [5] introduced the concept of fuzzy graph elaborately and analysed its components. Akram et al. discussed the properties of strong Intuitionistic fuzzy graphs and also the properties of Intuitionistic fuzzy cycle and Intutionistic fuzzy trees [6] [7].

In this paper, we (try to) introduce Intuitionistic L-fuzzy graph as a generalization of Intuitionistic fuzzy graph. We discuss about its matrix representation. We also try to study the properties of Intuitionistic L-fuzzy graph. In Section 2, we review several basic concepts of Intuitionistic fuzzy graphs.

2. Preliminaries

2.1. Definition. An Intuitionistic Fuzzy Graph is of the form G = (V, E) where
V $= \{v1, v2, v3, ...vn\}$ such that (i) $\mu_1 : V \longrightarrow$[0,1] and $\gamma_1 : V \longrightarrow$[0,1] denote the degree of membership and non membership of the element v_i in V respectively and
0$\leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every v_i in V (i = 1,2,3,...n)
(ii)E $\subseteq$ V$\times$ V where μ_2 :V$\times$ V $\longrightarrow$[0,1] and γ_2: V$\times$ V $\longrightarrow$[0,1] are such that
μ_2 $(v_i, v_j) \leq$ min $(\mu_1(v_i), \mu_1(v_j))$, γ_2 $(v_i, v_j) \leq$ max($\gamma_1(v_i), \gamma_1(v_j))$ and
$0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i.v_j) \leq 1$ for every $(v_i.v_j$) in E

2.2. Definition. An Intuitionistic fuzzy graph G = (V, E) is said to be complete Intuitionistic fuzzy graph if $\mu_{2ij} =$ min (μ_{1i}, μ_{1j}) and $\gamma_{2ij} =$ max $(\gamma_{1i}, \gamma_{1j})$ for every v_i, v_j in V.

The triple $< v_i, \mu_{1i}, \gamma_{1i} >$ denote the degree of membership and non membership of the vertex v_i.The triple $< e_{ij}, \mu_{2ij}, \gamma_{2ij} >$ denote the degree of membership and non membership of the edge relation $e_{ij} = (v_i, v_j)$ on V

2.3. Definition. The complement of an Intuitionistic fuzzy graph G =(V,E) is an Intuitionistic fuzzy graph $\overline{\mathrm{G}} = (\overline{\mathrm{V}}, \overline{\mathrm{E}})$ where

(1)$\overline{\mathrm{V}} = \mathrm{V}$, (2) $\overline{\mu_{1\mathrm{i}}} = \mu_{1i}$,$\overline{\gamma_{1\mathrm{i}}} = \gamma_{1i}$ for all i=1,2,...n

$\overline{\mu_{2\mathrm{ij}}} = min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ and

$\overline{\mu_{2\mathrm{ij}}} = max(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij}$ for all i,j=1,2,3,...n

2.4. Definition. Let (L,$\leq$) be a complete lattice with an Involutive order reversing operation N:L $\longrightarrow$ L.Let a set E be fixed.An Intuitionistic L-fuzzy set A* in E is defined as an object having the form A*= $\{< x, \mu_A(x), \nu_A(x) |$x inE$\}$ where the function

μ_A : E $\longrightarrow$ L and ν_A: E$\longrightarrow$ L define the degree of membership and degree of non membership respectively of the elements x in E and for every x in E :μ_A (x)$\leq$ N(ν_A(x)).

3. Intuitionistic L-Fuzzy Graph

3.1. Definition. An Intuitionistic L-Fuzzy graph is of the form $G_L = (V_L, E_L)$ where

$V_L = \{v1, v2, v3....vn\}$ such that

(1)$\mu_1 : V \longrightarrow$ L and $\gamma_1 : V \longrightarrow$ L denote the degree of membership and non membership grade of the element v_i in V respectively and μ_1(v)$\leq$ N(γ_1(v)) for all v in V where N(v) is an involutive order reversing operation.

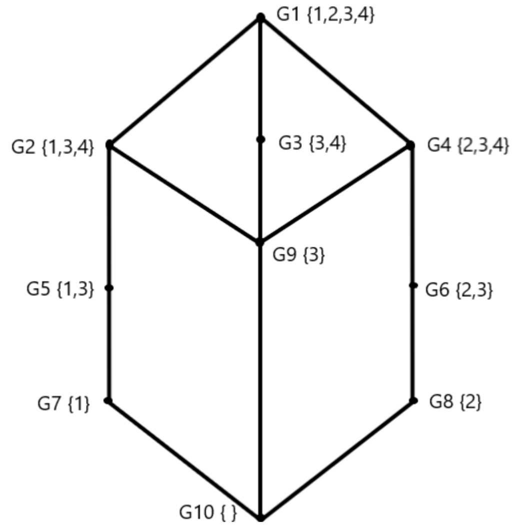
(2) E $\subseteq$ V $\times$ V where μ_2 :V$\times$ V $\longrightarrow$ L and γ_2 :V$\times$ V $\longrightarrow$L such that

$\mu_2\ (v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j))$ and

$\gamma_2\ (v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j))$, $\mu_2\ (v_i, v_j) \leq N(\gamma_2\ (v_i, v_j))$

denote the membership and non membership of an edge (vi,vj) in E respectively.

3.2. Example. Consider the lattice



The index matrix representation of an Intuitionistic L-Fuzzy graph is given below

$$E=\{\mu_{ij},\gamma_{ij}\}=\begin{bmatrix} <G_{10},G_1> & <G_7,G_8> & <G_8,G_7> \\ <G_7,G_8> & <G_1,G_{10}> & <G_{10},G_1> \\ <G_8,G_7> & <G_{10},G_1> & <G_1,G_{10}> \end{bmatrix}$$

3.3. Definition. Let $G_L=(V_L,E_L)$ be an Intuitionistic L-fuzzy graph. The index matrix representation of Intuitionistic L-Fuzzy graph is of the form [V, $E\subset V\times V$] where V= {v1,v2,..vn} and E= $\{\mu_{ij},\nu_{ij}\}$

$$\begin{bmatrix} <\mu_{11},\nu_{11}> & <\mu_{12},\nu_{12}> & \ldots\ldots <\mu_{1n},\nu_{1n}> \\ \ldots\ldots\ldots\ldots & & \\ <\mu_{n1},\nu_{n1}> & <\mu_{n2},\nu_{n2}>\ldots & \ldots\ldots <\mu_{nn},\nu_{nn}> \end{bmatrix}$$

where $<\mu_{ij},\nu_{ij}>$ in L× L $(1\leq I,j\leq n)$ the edge between two vertices vi and vj is indexed by $<\mu_{ij},\nu_{ij}>$

3.4. Definition. Let $G_L=(V_L,E_L)$ be an Intuitionistic L-Fuzzy graph.The incidence matrix representation of Intuitionistic L-Fuzzy graph is of the form

$$B=\{b_{\mu i},b_{\nu j}\}=\begin{cases} <\mu(ej),\nu(ej)> & \text{if an edge ej is incident on the vertex vj} \\ <0,1> & \text{otherwise} \end{cases}$$

Notes. 1. Index matrix representation is always a symmetric Intuitionistic matrix

2.Exactly two entries in each column of incidence matrix is same.

4. Operations on Intuitionistic L-Fuzzy Graph

4.1. Definition. Let G_{1L} and G_{2L} be any two Intuitionistic L-Fuzzy Graphs.We define the intersection of G_{1L} and G_{2L} be $G_{1L} \cap G_{2L} = (V_1 \cap V_2, E_1 \cap E_2)$

$(\mu_{11} \cap \mu_{12})(v) = \mu_{11}(v) \wedge \mu_{12}(v)$

$(\gamma_{11} \cap \gamma_{12})(v) = \gamma_{11}(v) \vee \gamma_{12}(v)$

$(\mu_{21} \cap \mu_{22})(v_i, v_j) = \mu_{21}(v_i, v_j) \wedge \mu_{22}(v_i, v_j)$

$(\gamma_{21} \cap \gamma_{22})(v_i, v_j) = \gamma_{21}(v_i, v_j) \vee \gamma_{22}(v_i, v_j)$

4.2. Theorem. Let G_{1L} and G_{2L} be two Intuitionistic L-Fuzzy Graphs. Then their intersection is again an Intuitionistic L-Fuzzy Graph.

Proof. For that we need to prove that

(i)$(\mu_{11} \cap \mu_{12})(v) \leq N((\gamma_{11} \cap \gamma_{12})(v))$ $\quad \forall v \in V_1 \cap V_2$

(ii) $(\mu_{21} \cap \mu_{22})(v_i, v_j) \leq N((\gamma_{21} \cap \gamma_{22})(v_i, v_j))$ $\quad \forall (v_i, v_j) \in E_1 \cap E_2$

(iii)$(\mu_{21} \cap \mu_{22})(v_i, v_j) \leq (\mu_{11} \cap \mu_{12})(v_i) \wedge (\mu_{11} \cap \mu_{12})(v_j)$

(iv)$(\gamma_{21} \cap \gamma_{22})(v_i, v_j) \leq (\gamma_{11} \cap \gamma_{12})(v_i) \vee (\gamma_{11} \cap \gamma_{12})(v_j)$

For,

(i)$(\mu_{11} \cap \mu_{12})(v) = \mu_{11}(v) \wedge \mu_{12}(v)$ (definition of intersection)

$\leq N(\gamma_{11}(v)) \wedge N(\gamma_{12}(v))$ (definition of Intuitionistic L-Fuzzy graph)

$\leq N(\gamma_{11}(v) \vee \gamma_{12}(v))$ (property of involutive order reversing operation) $= N(\gamma_{11} \cap \gamma_{12})(v))$ (definition of intersection)

(2)$(\mu_{21} \cap \mu_{22})(vi, vj) = \mu_{21}(vi, vj) \wedge \mu_{22}(vi, vj)$

$\leq N(\gamma_{21}(vi, vj)) \wedge N(\gamma_{22}(vi, vj))$

$\leq N(\gamma_{21}(vi, vj) \vee \gamma_{22}(vi, vj))$

$= N(\gamma_{21} \cap \gamma_{22})(vi, vj)$

(3) $(\mu_{21} \cap \mu_{22})(vi, vj) = \mu_{21}(vi, vj) \wedge \mu_{22}(vi, vj)$

$\leq [\mu_{11}(vi) \wedge \mu_{11}(vj)] \wedge [(\mu_{12}(vi) \wedge \mu_{12}(vj)]$

$\leq [\mu_{11}(vi) \wedge \mu_{12}(vi)] \wedge [(\mu_{11}(vj) \wedge \mu_{12}(vj)]$

$\leq (\mu_{11} \cap \mu_{12})(vi) \wedge (\mu_{11} \cap \mu_{12})(vj)$

(4) $(\gamma_{21} \cap \gamma_{22})(vi, vj) = \gamma_{21}(vi, vj) \vee \gamma_{22}(vi, vj)$

$\leq [\gamma_{11}(vi) \vee \gamma_{11}(vj)] \vee [\gamma_{12}(vi) \vee \gamma_{12}(vj)]$

$\leq [\gamma_{11}(vi) \vee \gamma_{12}(vi)] \vee [\gamma_{11}(vj) \vee \gamma_{12}(vj)]$

$= (\gamma_{11} \cap \gamma_{12})(vi) \vee (\gamma_{11} \cap \gamma_{12})(vj)$

4.3. Definition. LetG_{1L} and G_{2L} be two Intuitionistic L-Fuzzy graphs .We define the union of G_{1L} and G_{2L} be $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

$(\mu_{11} \cup \mu_{12})(v) = \mu_{11}(v) \vee \mu_{12}(v)$

$(\gamma_{11} \cup \gamma_{12})(v) = \gamma_{11}(v) \wedge \gamma_{12}(v)$

$(\mu_{21} \cup \mu_{22})(v_i, v_j) = \mu_{21}(v_i, v_j) \vee \mu_{22}(v_i, v_j)$

$(\gamma_{21} \cup \gamma_{22})(v_i, v_j) = \gamma_{21}(v_i, v_j) \wedge \gamma_{22}(v_i, v_j)$

4.4. Theorem. Let G_{1L} and G_{2L} be two Intuitionistic L-Fuzzy Graphs. Then their union is again an Intuitionistic L-Fuzzy Graph.

The Proof is similar as above

4.5. Definition. Let $G_L = (V_L, E_L)$be an Intuitionistic L-Fuzzy graph. We define its complement as ${G_L}^c = (V_L^c, E_L^c)$ where $V_L^c = \{v, \mu_1^c(v), \gamma_1^c(v)\}$ and

$E_L^c = \{(vi, vj), \mu_2^c(vi, vj), \gamma_2^c(vi, vj)\}$

where $\mu_1^c(v) = \gamma_1$(v),$\gamma_1^c(v) = \mu_1(v)$

$\mu_2^c(vi, vj) = \mu_1(vi) \wedge \mu_1(vj) \wedge \mu_2(vi, vj)^c$

$\gamma_2^c(vi, vj) = \gamma_1(vi) \vee \gamma_1(vj) \vee \gamma_2(vi, vj)^c$ for all v in V and (vi,vj) in $V \times V$ and $\mu_2(vi, vj)^c$,$\gamma_2(vi, vj)^c$ are the complement of $\mu_2(vi, vj)$ for all v in V , (vi,vj) in$V \times V$

5. Conclusion

In this paper we defined Intuitionistic L-Fuzzy Graph and complement of Intuitionistic L-Fuzzy graph. Then we defined union and intersection of Intuitionistic L-fuzzy graphs. We proved that the union of two Intuitionistic L-fuzzy graph is also an Intuitionistic L-fuzzy graph. Similarly, we proved that intersection of two Intuitionistic L-Fuzzy graphs is also an Intuitionistic L-Fuzzy graph. We have also discussed matrices associated with Intuitionistic L- Fuzzy graph.There is a scope to introduce more concepts related to these types of graphs.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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