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A FIXED POINT THEOREM USING E.A PROPERTY ON MULTIPLICATIVE METRIC SPACE

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Abstract: The emphasis of this paper is to establish a common fixed point theorem on a multiplicative metric space using the conditions weakly compatible mappings and EA-property. Further some examples are discussed to substantiate our result.

Keywords: common fixed point; multiplicative metric space; weakly compatible mappings and EA- property.

2010 AMS Subject Classification: 54H25, 47H10.

1. INTRODUCTION

In the recent past, the notion of multiplicative metric space (MMS) was introduced by Bashirove et.al. [1]. Many authors [3], [4], [5], [7], [8] and [9] proved fixed point theorems on multiplicative metric space. Jungck and Rhoades [10] defined the weaker class of mappings as weakly compatible mappings. Aamri and Moutawakil [2] developed the notion of E.A

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property .Further Ozavsar et.al. [7] designed the notion of convergence and proved unique common fixed point results in multiplicative metric space. In this paper we generate a common fixed point theorem using the concept of weakly compatible mappings with EA property. Our presentation is also supported by the provision of a suitable example.

2. PRELIMINARIES

2.1 Definition:

Let X $\neq \phi$, an MMS is a mapping $\delta: X \times X \to \mathbb{R}+$ holding the conditions below:

(i) $\delta(\alpha,\beta) \geq 1$, $\delta(\alpha,\beta) = 1 \Leftrightarrow \alpha = \beta$,

(ii) $\delta(\alpha,\beta) = \delta(\beta,\alpha)$,

(iii) $\delta(\alpha,\beta) \leq \delta(\alpha,\gamma).\delta(\gamma,\beta)$ $\forall$ $\alpha,\beta,\gamma \in$ X.

Mapping together with X, (X,δ) is called MMS.

2.2 Definition:

In a MMS a sequence $\{\alpha_k\}$ is assumed as

i. a multiplicative convergent if for any multiplicative open ball $B_\in(\alpha) = \{\beta / \delta(\alpha,\beta) < \in\}, \in > 1$, *then* $\exists N \epsilon \mathbb{N}$ such that $\alpha_k \epsilon B_\in$ (X) $\forall k \geq \mathbb{N}$ holds. That is $d(\alpha_k,\alpha) \to 1$ as $k \to \infty$.

ii. A multiplicative Cauchy sequence is one if $\forall \epsilon > 1$, N$\in \mathbb{N}$ such that $\delta(\alpha_k,\alpha_l) < \epsilon$ $\forall k,l \geq \mathbb{N}$ holds. That is $\delta(\alpha_k,\alpha_l) \to 1$ as $k,l \to \infty$.

iii. An MMS is complete if every multiplicative Cauchy sequence is convergent in it.

2.3 Definition:

Let f be a mapping of MMS and if the existence of a number $\lambda \epsilon [0, 1)$ such that $\delta(G\alpha, G\beta) \leq \delta^\lambda(\alpha,\beta)$ $\forall$ $\alpha,\beta \epsilon$ X holds, then G is known as multiplicative contraction.

2.4 Definition:

We define mappings G and I of a MMS as compatible if $\delta(GI\alpha_k, IG\alpha_k) = 1$ as $k \to \infty$, when ever $\{\alpha_k\}$ is a sequence in X such that $G\alpha_k = I\alpha_k = \mu$ as $k \to \infty$ for some $\mu \in X$.

2.5 Definition:

The mappings G and Hof a MMS in which if $G\mu = I\mu$ for some $\mu \epsilon X$ such that $GI\mu = IG\mu$ holds then we say that G and I are weakly compatible mappings.

2.6 Definition:

Mappings G and I of a MMS (X,d) are said to hold EA property if

$$\lim_{k\to\infty} Gx_k = \lim_{k\to\infty} Ix_k = \mu \text{ some } \mu \in X.$$

Now we discuss an example for E.A property.

Example:

Suppose $X = [2,4]$ with $\delta(\alpha,\beta) = e^{|\alpha-\beta|}$ for all $\alpha, \beta \in X$

Define $G(\alpha) = \begin{cases} 2 & if\ \alpha = 2 \\ \dfrac{2\alpha}{3} & if\ 3 < \alpha \le 4 \end{cases}$

and $I(\alpha) = \begin{cases} 2 & if\ 2 \le \alpha < 3 \\ \dfrac{\alpha+3}{3} & \text{if } 3 \le \alpha < 4 \end{cases}$

Take a sequence $\{\alpha_k\}$ $as\ \alpha_k = 3 + \frac{1}{k}$ for $k \ge 0.$

$$Then\ G\alpha_k = G\left(3+\frac{1}{k}\right) = \frac{2\left(3+\frac{1}{k}\right)}{3} = 2 + \frac{1}{k} = 2 \text{ as } k \to \infty\ and$$

$$I\alpha_k = I(3+\frac{1}{k}) = \frac{(3+\frac{1}{k}+3)}{3} = (\frac{6}{3}+\frac{1}{3k}) = 2 + \frac{1}{k} = 2\ as\ k \to \infty.$$

This gives $G\alpha_k = I\alpha_k = 2 \in X$ as $k \to \infty$.

This gives (G,I) satisfies EA-property.

$$Then\ GI\alpha_k = G(2+\frac{1}{k}) = \frac{4}{3}$$

and $IG\alpha_k = I(2+\frac{1}{k}) = 2$.

Therefore $GI\alpha_k \ne IG\alpha_k$, this shows the pair (G, I) is not compatible.

Also G(2)=I(2)=2,and GI(2)=IG(2),this shows the pair (G,I) is weakly compatible.

3. MAIN RESULTS

Now we prove our main theorem on MMS.

3.1. Theorem

Suppose in a complete MMS (X, δ), there are four mappings G, H, I and J holding the conditions

(C1) $G(X) \subseteq J(X)$ *and* $H(X) \subseteq I(X)$

(C2) $$\delta(G\alpha, H\beta) \leq \left[\max, \left\{ \begin{array}{l} \frac{\delta(G\alpha, I\alpha)\delta(H\beta, J\beta)}{1+\delta(I\alpha, J\beta)}, \frac{\delta(G\alpha, J\beta)\delta(I\alpha, H\beta)}{1+\delta(J\beta, I\alpha)}, \\ \frac{\delta(G\alpha, J\beta)\delta(H\beta, J\beta)}{1+\delta(I\alpha, J\beta)}, \frac{\delta(G\alpha, I\alpha)\delta(H\beta, I\alpha)}{1+\delta(I\alpha, J\beta)} \end{array} \right\} \right]^{\lambda}$$

for all $\alpha,\beta \in X$, where $\lambda \in \left(0, \frac{1}{3}\right)$

(C3) the pairs (G, I) and (H,J) are satisfying the E.A property

(C4) the pair of mappings(G,I) and (H,J) are weakly compatible.

Then the above mappings will be having a common fixed point.

Proof:

Begin with using the condition (C1), there is a point $\propto_0 \epsilon X$ such that $G\propto_0 = J\propto_1 = \beta_0$ (Say).

For this point $\propto_1$ then there exists $\propto_2 \epsilon X$ such that $H\propto_1 = I\propto_2 = \beta_1$ (say).

Continuing this process, it is possible to construct a Sequence $\{\beta_k\}$ in X

Such that $\beta_{2k} = G\alpha_{2k} = J\alpha_{2k+1}$ and $\beta_{2k+1} = H\alpha_{2k+1} = I\alpha_{2k+2}$ for $k \geq 0$.

We now prove $\{\beta_k\}$ is a Cauchy sequence in MMS.

Consider $\delta(\beta_{2k}, \beta_{2k+1.}) =$

$$\delta(G\alpha_{2k}, H\alpha_{2k+1}) \leq \left[\max, \left\{ \begin{array}{l} \frac{\delta(G\alpha_{2k}, I\alpha_{2k+1})\delta(H\alpha_{2k+1}, J\alpha_{2k+1})}{1+\delta(I\alpha_{2k}, J\alpha_{2k+1})}, \frac{\delta(G\alpha_{2k}, J\alpha_{2k+1})\delta(I\alpha_{2k}, H\alpha_{2k+1})}{1+\delta(J\alpha_{2k+1}, I\alpha_{2k})}, \\ \frac{\delta(G\alpha_{2k}, J\alpha_{2k+1})\delta(H\alpha_{2k+1}, J\alpha_{2k+1})}{1+\delta(I\alpha_{2k}, J\alpha_{2k+1})}, \frac{\delta(G\alpha_{2k}, I\alpha_{2k})\delta(H\alpha_{2k+1}, I\alpha_{2k})}{1+\delta(I\alpha_{2k}, J\alpha_{2k+1})} \end{array} \right\} \right]^{\lambda}$$

$$\delta(\beta_{2k}, \beta_{2k+1}) \leq \left[\max, \left\{ \begin{array}{l} \frac{\delta(\beta_{2k}, \beta_{2k-1})\delta(\beta_{2k-1}, \beta_{2k-1})}{1+\delta(\beta_{2k-1}, \beta_{2k-1})}, \frac{\delta(\beta_{2k}, \beta_{2k-1})\delta(\beta_{2k-1}, \beta_{2k+1})}{1+\delta(\beta_{2k-1}, \beta_{2k-1})}, \\ \frac{\delta(\beta_{2k}, \beta_{2k-1})\delta(\beta_{2k+1}, \beta_{2k-1})}{1+\delta(\beta_{2k-1}, \beta_{2k-1})}, \frac{\delta(\beta_{2k}, \beta_{2k})\delta(\beta_{2k+1}, \beta_{2k-1})}{1+\delta(\beta_{2k-1}, \beta_{2k-1})} \end{array} \right\} \right]^{\lambda}$$

$$\delta(\beta_{2k}, \beta_{2k+1}) \leq \left[\max, \left\{ \delta(\beta_{2k}, \beta_{2k-1}), \delta(\beta_{2k-1}, \beta_{2k+1}), \delta(\beta_{2k+1}, \beta_{2k-1}), \delta(\beta_{2k-1}, \beta_{2k+1}) \right\} \right]^{\lambda}$$

on simplification

$$\delta(\beta_{2k}, \beta_{2k+1}) \leq \left[\delta(\beta_{2k-1}, \beta_{2k+1}) \right]^{\lambda}$$

$$\delta(\beta_{2k}, \beta_{2k+1}) \leq \left[\delta(\beta_{2k-1}, \beta_{2k}), \delta(\beta_{2k}, \beta_{2k+1}) \right]^{\lambda}$$

$$\delta^{1-\lambda}(\beta_{2k}, \beta_{2k+1.}) \leq \delta^{\lambda}(\beta_{2k-1}, \beta_{2k})$$

$$\delta(\beta_{2k}, \beta_{2k+1.}) \leq \delta^{\frac{\lambda}{1-\lambda}}(\beta_{2k-1}, \beta_{2k})$$

$$\delta(\beta_{2k}, \beta_{2k+1.}) \leq \delta^{h}(\beta_{2k-1}, \beta_{2n}) \text{ where } h = \frac{\lambda}{1-\lambda} \in (0,1) - - - - - - - - (1)$$

Now it gives

$$\left[\delta(\beta_k, \beta_{k+1}) \right] \leq \delta^{h}(\beta_{k-1}, \beta_k) \leq \delta^{h^2}(\beta_{k-2}, \beta_{k-1}) \leq - - - \leq \delta^{h^k}(\beta_0, \beta_1).$$

Hence for k<l, on using the multiplicative triangle inequality we get

$$\left[\delta(\beta_k, \beta_l) \right] \leq \left[\delta^{h^k}(\beta_0, \beta_1) \right] \left[\delta^{h^{k+1}}(\beta_0, \beta_1) \right] - - - - - \left[\delta^{h^{l-1}}(\beta_0, \beta_1) \right]$$

$$\left[\delta(\beta_k,\beta_l)\right] \le \left[\delta^{\frac{1}{1-h}(h^k)}(\beta_0,\beta_1)\right].$$

This shows $\{\beta_k\}$ as a cauchy sequence in MMS.

Since on using (C3), the pair (G, I) satiesfies EA - property, $\exists$ a sequence $\{\alpha_k\} \in X$ such that

$\lim_{k\to\infty} G\alpha_k = \lim_{k\to\infty} I\alpha_k = \mu$ for some $\mu \in X$.---------(2)

Since $G(X) \subseteq J(X)$ then $\exists$ sequence $\{\beta_k\}$ in X such that $G\alpha_k = J\beta_k$.

Hence $\lim_{k\to\infty} J\beta_k = \mu$.---------(3)

From (2) and (3) it gives

$\lim_{k\to\infty} G\alpha_k = \lim_{k\to\infty} I\alpha_k = \lim_{k\to\infty} J\beta_k = \mu$ for some $\mu \in X$-------(4)

We now show that $\lim_{k\to\infty} H\beta_k = \mu$.

In the inequality (C2), by putting $\alpha = \alpha_k$ and $\beta = \beta_k$ then we have

$$\delta(G\alpha_k, H\beta_k) \le \left[\max\left\{\begin{array}{l}\dfrac{\delta(G\alpha_k, I\alpha_k)\delta(H\beta_k, J\beta_k)}{1+\delta(I\alpha_k, J\beta_k)}, \dfrac{\delta(G\alpha_k, J\beta_k)\delta(I\alpha_k, H\beta_k)}{1+\delta(I\alpha_k, J\beta_k)}, \\ \dfrac{\delta(G\alpha_k, J\beta_k)\delta(H\beta_k, J\beta_k)}{1+\delta(I\alpha_k, J\beta_k)}, \dfrac{\delta(G\alpha_k, I\alpha_k)\delta(H\beta_k, I\alpha_k)}{1+\delta(I\alpha_k, J\beta_k)}\end{array}\right\}\right]^\lambda$$

$$\delta(\mu, H\beta_k) \le \left[\max\left\{\frac{\delta(\mu,\mu)\delta(H\beta_k,\mu)}{1+\delta(\mu,\mu)}, \frac{\delta(\mu,\mu)\delta(\mu,H\beta_k)}{1+\delta(\mu,\mu)}, \frac{\delta(\mu,\mu)\delta(H\beta_k,\mu)}{1+\delta(\mu,\mu)}, \frac{\delta(\mu,\mu))\delta(\mu,H\beta_k)}{1+\delta(\mu,\mu))}\right\}\right]^\lambda$$

$$\delta(\mu, H\beta_k) \le \left[\max\left\{\delta(H\beta_k,\mu), \delta(H\beta_k,\mu), \delta(H\beta_k,\mu), \delta(H\beta_k,\mu)\right\}\right]^\lambda$$

This gives $\delta(\mu, H\beta_k) \le \left[\delta(H\beta_k,\mu)\right]^\lambda \Rightarrow H\beta_k = \mu$.

This gives $\lim_{k\to\infty} G\alpha_k = \lim_{k\to\infty} I\alpha_k = \lim_{k\to\infty} J\beta_k = \lim_{k\to\infty} H\beta_k = \mu$ for some $\mu \in X$---------(5)

Nowthe pair (G,I) is weakly compatible with $G\alpha_k = I\alpha_k$ gives $GI\alpha_k = IG\alpha_k$ and this inturn implies Gμ=Iμ.

Now we show that Gμ = μ.

Putting α=μ and $\beta=\beta_k$ in the inequality (C2) we have

$$\delta(G\mu, H\beta_k) \le \left[\max,\left\{\begin{array}{l}\dfrac{\delta(G\mu,I\mu)\delta(H\beta_k,J\beta_k)}{1+\delta(I\mu,J\beta_k)},\dfrac{\delta(G\mu,J\beta_k)\delta(I\mu,H\beta_k)}{1+\delta(I\mu,J\beta_k)},\\ \dfrac{\delta(G\mu,J\beta_k)\delta(H\beta_k,Jv)}{1+\delta(I\mu,J\beta_k)},\dfrac{\delta(G\mu,I\beta_k)\delta(H\beta_k,I\mu)}{1+\delta(I\mu,J\beta_k)}\end{array}\right\}\right]^{\lambda}$$

$$\delta(G\mu,\mu) \le \left[\max,\left\{\dfrac{\delta(G\mu,I\mu)\delta(\mu,\mu)}{1+\delta(I\mu,\mu)},\dfrac{\delta(G\mu,\mu)\delta(I\mu,\mu)}{1+\delta(I\mu,\mu)},\dfrac{\delta(G\mu,\mu)\delta(\mu,\mu)}{1+\delta(I\mu,\mu)},\dfrac{\delta(G\mu,I\mu)\delta(\mu,I\mu)}{1+\delta(I\mu,\mu)}\right\}\right]^{\lambda}$$

$$\delta(G\mu,\mu) \le \left[\max,\left\{\begin{array}{l}\dfrac{\delta(G\mu,G\mu)\delta(\mu,\mu)}{1+\delta(G\mu,\mu)},\dfrac{\delta(G\mu,\mu)\delta(G\mu,\mu)}{1+\delta(G\mu,\mu)},\\ \dfrac{\delta(G\mu,\mu)\delta(\mu,\mu)}{1+\delta(G\mu,\mu)},\dfrac{\delta(G\mu,G\mu)\delta(\mu,G\mu)}{1+\delta(G\mu,\mu)}\end{array}\right\}\right]^{\lambda} \quad [\because G\mu = I\mu]$$

$$\delta(G\mu,\mu) \le \left[\max,\left\{\dfrac{1}{\delta(G\mu,\mu)},\delta(G\mu,\mu),\delta(G\mu,\mu),1\right\}\right]^{\lambda}$$

either $\delta(G\mu,\mu) \le \left[\delta^{\lambda}(G\mu,\mu)\right]$ or $\delta(G\mu,\mu) \le 1,$

this gives $G\mu = \mu$, which implies $G\mu = I\mu = \mu.$ -------- (6)

Since (H,J) is weakly compatible mapping with $H\beta_k=J\beta_k$ and $HJ\beta_k=JH\beta_k$ and this

inturnimplies Hμ=Jμ.

Now, we show that Hμ = μ.

Putting α=μ and β= μ in the inequality (C2) we have

$$\delta(G\mu,H\mu) \le \left[\max,\left\{\begin{array}{l}\dfrac{\delta(G\mu,I\mu)\delta(H\mu,J\mu)}{1+\delta(I\mu,J\mu)},\dfrac{\delta(G\mu,J\mu)\delta(I\mu,H\mu)}{1+\delta(I\mu,J\mu)},\\ \dfrac{\delta(G\mu,J\mu)\delta(H\mu,J\mu)}{1+\delta(I\mu,J\mu)},\dfrac{\delta(G\mu,I\mu)\delta(H\mu,I\mu)}{1+\delta(I\mu,J\mu)}\end{array}\right\}\right]^{\lambda}$$

$$\delta(\mu,H\mu)\leq\left[\max,\left\{\frac{\delta(\mu,\mu)\delta(H\mu,J\mu)}{1+\delta(\mu,J\mu)},\frac{\delta(\mu,J\mu)\delta(\mu,H\mu)}{1+\delta(\mu,J\mu)},\frac{\delta(\mu,J\mu)\delta(H\mu,J\mu)}{1+\delta(\mu,J\mu)},\frac{\delta(\mu,\mu)\delta(H\mu,\mu)}{1+\delta(\mu,J\mu)}\right\}\right]^{\lambda}$$

$$\delta(\mu,H\mu)\leq\left[\max,\left\{\begin{array}{l}\frac{\delta(\mu,\mu)\delta(H\mu,H\mu)}{1+\delta(\mu,H\mu)},\frac{\delta(\mu,H\mu)\delta(\mu,H\mu)}{1+\delta(\mu,H\mu)},\\ \frac{\delta(\mu,H\mu)\delta(H\mu,H\mu)}{1+\delta(\mu,H\mu)},\frac{\delta(\mu,\mu)\delta(H\mu,\mu)}{1+\delta(\mu,H\mu)}\end{array}\right\}\right]^{\lambda}\ [\because H\mu=J\mu]$$

$$\delta(\mu,H\mu)\leq\left[\max,\left\{\frac{1}{\delta(\mu,H\mu)},\delta(\mu,H\mu),1,1\right\}\right]^{\lambda}$$

$$\delta(\mu,H\mu)\leq[\delta(\mu,H\mu)]^{\lambda}\Rightarrow\delta(\mu,H\mu)\leq\left[\delta^{\lambda}(\mu,H\mu)\right]\Rightarrow H\mu=\mu$$

Hence $H\mu=J\mu=\mu$ ----------(7)

From (6) and (7) we have $I\mu=H\mu=J\mu=G\mu=\mu$. - - - - - (8) .

This shows that μ is a common fixed point of G, H, I and J.

For uniqueness:

consider $\phi\ (\mu\neq\phi)$ as an another common fixed point of four mappings G, H, I and J.

Substitute $\alpha=\mu\ \&\ \beta=\phi$ in the inequality(C2) then we have

$$\delta(G\mu,H\phi)\leq\left[\max,\left\{\begin{array}{l}\frac{\delta(G\mu,I\mu)\delta(H\phi,J\phi)}{1+\delta(I\mu,J\phi)},\frac{\delta(G\mu,J\phi)\delta(I\mu,H\phi)}{1+\delta(I\mu,J\phi)},\\ \frac{\delta(G\mu,J\phi)\delta(H\phi,J\phi)}{1+\delta(I\mu,J\phi)},\frac{\delta(G\mu,Iv)\delta(H\phi,I\mu)}{1+\delta(I\mu,J\phi)}\end{array}\right\}\right]^{\lambda}$$

$$\delta(\mu,\phi)\leq\left[\max,\left\{\frac{\delta(\mu,\mu)\delta(\phi,\phi)}{1+\delta(\mu,\phi)},\frac{\delta(\mu,\phi)\delta(\mu,\phi)}{1+\delta(\mu,\phi)},\frac{\delta(\mu,\phi)\delta(\phi,\phi)}{1+\delta(\mu,\phi)},\frac{\delta(\mu,\phi)\delta(\phi,\mu)}{1+\delta(\mu,\phi)}\right\}\right]^{\lambda}$$

$$\delta(\mu,\phi)\leq\left[\max,\left\{\frac{1}{\delta(\mu,\phi)},\delta(\mu,\phi),1,\delta(\mu,\phi)\right\}\right]^{\lambda}$$

$$\delta(\mu,\varphi)\leq\left[\{\delta(\mu,\varphi)\}\right]^{\lambda}\text{ which implies }\mu=\varphi,\text{ where }\lambda\in(0.\frac{1}{3})$$

This assures the uniqueness of the common fixed point.

Now we substantiate our result with an example.

3.2 Example:

Suppose $X=[0,1]$ with $\delta(\alpha,\beta)=e^{|\alpha-\beta|}$ for all $\alpha,\beta\in X$.

Define $G(\alpha)=H(\alpha)=\begin{cases}\frac{3\alpha+1}{3} & if\ \alpha\in[0,\frac{2}{3}) \\ \frac{2\alpha+2}{5} & if\ \alpha\geq\frac{2}{3}\end{cases}$ and $I(\alpha)=J(\alpha)=\begin{cases}1-\alpha & if\ \alpha\in[0,\frac{2}{3}) \\ \alpha & \text{if } \alpha\geq\frac{2}{3}\end{cases}$

Then $G(X)=H(X)=[\frac{1}{3},1]\cup(\frac{2}{3})$ while $I(X)=J(X)=[1,\frac{1}{3})\cup(\frac{2}{3})$

the condition $G(X)\subseteq J(X)$ and $H(X)\subseteq I(X)$, (C1) is satisfied.

Take a sequence $\{\alpha_k\}$ *as* $\alpha_k=\frac{1}{3}-\frac{1}{k}$ for $k\geq 0$.

Now

$$G\alpha_k=G\left(\frac{1}{3}-\frac{1}{k}\right)=\frac{3\left(\frac{1}{3}-\frac{1}{k}\right)+1}{3}=\frac{\left(1-\frac{3}{k}\right)+1}{3}=\left(\frac{2}{3}-\frac{1}{k}\right)=\frac{2}{3}\ \text{as } k\to\infty\ \textit{and}$$

$$I\alpha_k=I(\frac{1}{3}-\frac{1}{k})=(1-(\frac{1}{3}-\frac{1}{k})=(\frac{2}{3}+\frac{1}{k})=\frac{2}{3}\quad as\ k\to\infty.$$

This gives $G\alpha_k=I\alpha_k=\frac{2}{3}$ as $k\to\infty$.

Similarly $H\alpha_k=J\alpha_k=\frac{2}{3}$ as $k\to\infty$.

Hence the pairs (G,I),(H,J) satisfy EA- property.

Also

$$G\left(\frac{2}{3}\right)=\frac{2\left(\frac{2}{3}\right)+2}{5}=\left(\frac{10}{15}\right)=\frac{2}{3}\text{ and } I\left(\frac{2}{3}\right)=\frac{2}{3}\text{ which implies } G(\frac{2}{3})=I(\frac{2}{3}).$$

Simillarly $H\left(\frac{2}{3}\right)=\frac{2\left(\frac{2}{3}\right)+2}{5}=\left(\frac{10}{15}\right)=\frac{2}{3}$ and $J\left(\frac{2}{3}\right)=\frac{2}{3}$, which gives that $H\left(\frac{2}{3}\right)=J\left(\frac{2}{3}\right)$

$$GI(\frac{2}{3}) = G(\frac{2}{3}) = \frac{2(\frac{2}{3})+2}{5} = \frac{10}{15} = \frac{2}{3} \ \ and$$

$$IG(\frac{2}{3}) = I\,\frac{2(\frac{2}{3})+2}{5} = I(\frac{10}{15}) = I(\frac{2}{3}) = \frac{2}{3}.$$

Hence $GI(\frac{2}{3}) = IG(\frac{2}{3})$ and $HJ(\frac{2}{3}) = JH(\frac{2}{3})$ which gives (G, I) , (H, J) are weakly compatible mappings.

$$But\ \ GI\alpha_k = GI\left(\frac{1}{3}-\frac{1}{k}\right) = G(1-(\frac{1}{3}-\frac{1}{k}) = G(\frac{2}{3}+\frac{1}{k}) = \frac{2(\frac{2}{3}+\frac{1}{k})+2}{5} = (\frac{10}{15}+\frac{2}{5k}) = \frac{2}{3} \ \text{ as k} \to \infty.$$

$$and\ \ IG\alpha_k = IG\left(\frac{1}{3}-\frac{1}{k}\right) = \text{I}[\frac{3\left(\frac{1}{3}-\frac{1}{k}\right)+1}{3}] = \text{I}\left(\frac{2}{3}-\frac{1}{k}\right) = 1-\left(\frac{2}{3}-\frac{1}{k}\right) = \frac{1}{3} \ \text{ as k} \to \infty.$$

Therefore

$$\lim_{k\to\infty}\delta(GI\alpha_k, IG\alpha_k) = \delta(\frac{2}{3},\frac{1}{3}) \neq 1, \text{similarly } \lim_{k\to\infty}\delta(HJ\alpha_k, JH\alpha_k) = \delta(\frac{2}{3},\frac{1}{3}) \neq 1.$$

Showing that the compatibility condition is not fulfilled.

We now establish that the mappings G,H,I and J satisfy the Condition(C2) .

Case (i):

$$If\ \ \alpha,\beta \in [0,\frac{2}{3}) \ \ then\ we\ have\ \ \delta(G\alpha, H\beta) = e^{|G\alpha - H\beta|}$$

Putting $\alpha = \frac{1}{3}$ and $\beta = \frac{1}{2}$, then the inequality (C2) gives

$$d(\frac{2}{3},\frac{5}{6}) \le \left[\max\left\{\frac{d(\frac{2}{3},\frac{2}{3})d(\frac{5}{6},\frac{1}{2})}{1+d(\frac{2}{3},\frac{1}{2})}, \frac{d(\frac{2}{3},\frac{1}{2})d(\frac{2}{3},\frac{5}{6})}{1+d(\frac{2}{3},\frac{1}{2})}, \frac{d(\frac{2}{3},\frac{1}{2})d(\frac{5}{6},\frac{1}{2})}{1+d(\frac{2}{3},\frac{1}{2})}, \frac{d(\frac{2}{3},\frac{2}{3})d(\frac{5}{6},\frac{2}{3})}{1+d(\frac{2}{3},\frac{1}{2})}\right\}\right]^{\lambda}$$

$$e^{0.16} \le \left[\max\left\{\frac{e^{0}e^{0.33}}{1+e^{0.16}}, \frac{e^{0.16}e^{0.16}}{1+e^{0.16}}, \frac{e^{0.38}e^{0.33}}{1+e^{0.16}}, \frac{e^{0}e^{0.16}}{1+e^{0.16}}\right\}\right]^{\lambda}$$

$$e^{0.16} \leq \left[\max\left\{ \frac{e^{0.33}}{1+e^{0.16}}, \frac{e^{0.32}}{1+e^{0.16}}, \frac{e^{0.71}}{1+e^{0.16}}, \frac{e^{0.16}}{1+e^{0.16}} \right\} \right]^{\lambda}$$

$$e^{0.16} \leq \left[\max\left\{ e^{0.17}, e^{0.16}, e^{0.55}, e^{0} \right\} \right]^{\lambda}$$

$$e^{0.16} \leq e^{0.55\lambda}$$

Thus we have $e^{0.16} \leq e^{0.55\lambda} \Rightarrow \lambda = 0.3$, where $\lambda \in (0, \frac{1}{3})$.

Hence the condition (C2) is satisfied.

Case (ii):

If $\alpha, \beta \in [\frac{2}{3}, 1]$ then we have $\delta(G\alpha, H\beta) = e^{|G\alpha - H\beta|}$

putting $\alpha = \frac{4}{5}$ and $\beta = 1$, in the inequality (C-2) gives

$$\delta(\frac{18}{25}, \frac{4}{5}) \leq \left[\max\left\{ \frac{\delta(\frac{18}{25}, \frac{4}{5})\delta(\frac{4}{5}, 1)}{1+\delta(\frac{4}{5}, 1)}, \frac{\delta(\frac{18}{25}, 1)\delta(\frac{4}{5}, \frac{4}{5})}{1+\delta(\frac{4}{5}, 1)}, \frac{\delta(\frac{18}{25}, 1)\delta(\frac{4}{5}, 1)}{1+\delta(\frac{4}{5}, 1)}, \frac{\delta(\frac{18}{25}, \frac{4}{5})\delta(\frac{4}{5}, \frac{4}{5})}{1+\delta(\frac{4}{5}, 1)} \right\} \right]^{\lambda}$$

$$e^{0.08} \leq \left[\max\left\{ \frac{e^{0.08}e^{0.2}}{1+e^{0.2}}, \frac{e^{0.28}e^{0}}{1+e^{0.2}}, \frac{e^{0.28}e^{0.2}}{1+e^{0.2}}, \frac{e^{0.08}e^{0}}{1+e^{0.2}} \right\} \right]^{\lambda}$$

$$e^{0.08} \leq \left[\max\left\{ \frac{e^{0.28}}{1+e^{0.2}}, \frac{e^{0.28}}{1+e^{0.2}}, \frac{e^{0.48}}{1+e^{0.2}}, \frac{e^{0.08}}{1+e^{0.2}} \right\} \right]^{\lambda}$$

$$e^{0.08} \leq \left[\max\left\{ e^{0.08}, e^{0.08}, e^{0.28}, e^{-0.12} \right\} \right]^{\lambda}$$

$$e^{0.08} \leq e^{0.28\lambda}.$$

Therefore $e^{0.08} \leq e^{0.28\lambda} \Rightarrow \lambda = 0.28$, where $\lambda \in (0, \frac{1}{3})$.

Hence the condition (C2) is satisfied.

Similarly we can prove other cases.

It can be observed that $\frac{2}{3}$ is the common unique fixed point for the four self mappings H, G, I and J.

CONCLUSION

In this paper we established a result in multiplicative metric space using the set of conditions weakly compatible mappings and EA-property and also an example is given to justify our theorem.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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