Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 5, 2070-2093
https://doi.org/10.28919/jmcs/4817
ISSN: 1927-5307

SPREAD OF COVID-19 IN MOROCCO DISCRETE MATHEMATICAL MODELING: OPTIMAL CONTROL STRATEGIES AND COST-EFFECTIVENESS ANALYSIS

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Abstract. In this article, we study the transmission of (COVID-19) in the human population. We use the compartments model to describe the spread of this infectious disease. We divide the infected people with Covid-19 disease into three groups because the patients go through different stages, which are: infection, symptoms and serious or critical complications. We propose a discrete mathematical model with control strategies using three variables of controls u, v and w that represent respectively: Urging people to wash their hands with water and soap, cleaning and disinfecting surfaces frequently, urging people to use masks to cover the sensitive body parts and the treatment of patients infected with (COVID-19) by taking them to hospitals and quarantine sites. Pontryagin's Maximum Principle, in discrete time, is used to characterize the optimal controls and the optimality system is solved by an iterative method. Finally, Numerical simulations are presented with and without controls. Using cost-effectiveness

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Received June 30, 2020

analysis, we will show that the control that represents treatment of patients infected with (COVID-19) by taking them to hospitals and quarantine sites is the most cost-effective strategy to control the disease.

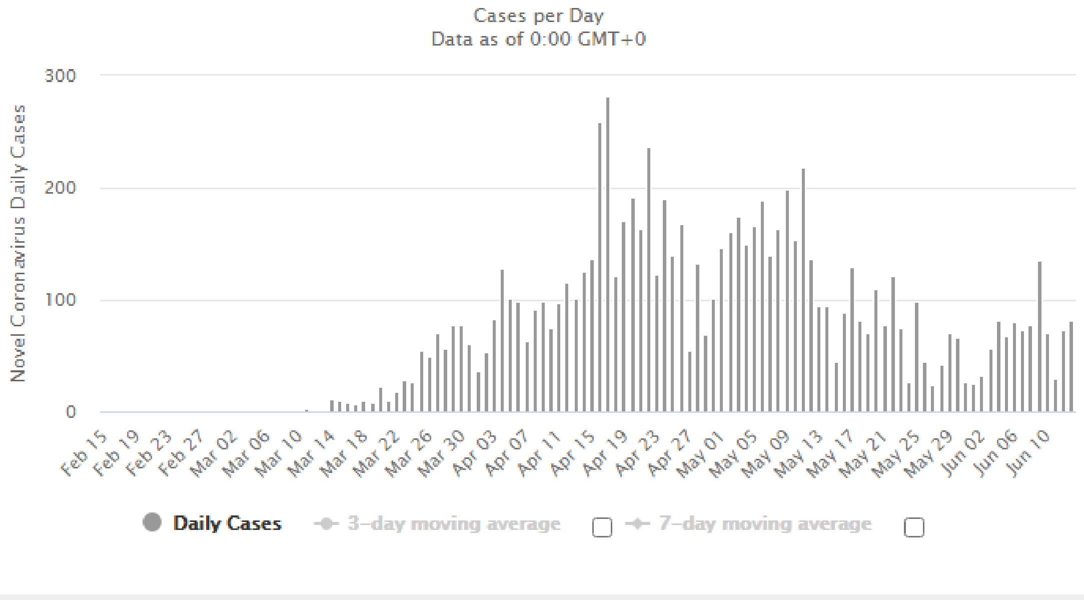
Keywords: COVID-19; discrete mathematical model; Pontryagin's maximum principle; optimal control; cost-effectiveness analysis.

2010 AMS Subject Classification: 93A30, 92B05.

1. Introduction

A new type of coronavirus has emerged in China and has been designated with several names related to the time and place of the epidemic spread in China such as the new coronavirus 2019–2020, the Corona Wuhan virus. The disease was lately branded (COVID-19) by the World Health Organization (WHO). COVID-19 is one of the most harmful and pathogenic viruses for humans and animals affecting the respiratory system. COVID-19 is an infectious virus, which can be transmitted from an animal to a human or from a human to a human. The virus is transmitted from the infected person to other people through direct contact and by touching surfaces contaminated with the disease. Then, it affects parts of the body such as the eyes, nose and mouth. After its spread to China, it moved to the rest of the world, where it spread widely to Asia, America and Europe. On 11 March, the World Health Organization declared the new coronavirus a global pandemic. The new coronavirus is a major threat to the health and safety of people all over the world due to its potentially harmful spreading power. The Moroccan government announces the first case of coronavirus on March 2, 2020 of a person who came from the Italy. Immediately after the emergence of this case, the Moroccan authorities closed borders so quickly, suspended travels with all countries, suspended studies in all educational institutions and imposed a state of health emergency. In addition, other measures were implemented such as creating the Corona Solidarity Fund, establishing field hospitals and adopting the health protocol proposed by the World Health Organization [1]. Registered cases continue to increase rapidly in early 2020, with a total of 7,940,451 COVID-19 cases reported worldwide, including 433,931 deaths [2][4]. In Morocco 8,793 cases, including 212 deaths, which is home to more than 36,839,833 people, according to the Worldometer elaboration of the latest United Nations data and WHO [3][4]. The picture according to watch the number of infections a day cross in Morocco until June 14, 2020 (Figure1)[3].

FIGURE 1. Daily New Cases in Morocco



Over the past few years, a large number of mathematical models have been developed to simulate, analyse and understand the dynamics of the Coronavirus [6][7][8][9][10][11]. According to the characteristics of transmission of the epidemic at different stages, so this paper uses compartment model to describe the spread of this infectious disease, we divide infected people with Covid-19 disease into three groups because the patients go through steps, which are the step of infection, the step of the symptoms and step the serious or critical complications. The patient can move to the recovery step immediately after one of the previous steps. In this work, we propose a mathematical model that describes the dynamics of citizens who have COVID-19. Also, we propose an optimal strategy for the treatment of patients infected with COVID-19 by taking them to hospitals and designated quarantine sites, urging people to wash their hands with water and soap frequently, cleaning and disinfecting surfaces and using masks to cover the sensitive body parts. The population is divided in our model into five compartments. The susceptible individuals (S), the infected individuals without symptoms (Iw), the infected individuals with symptoms (I), the infected individuals with complications (C) and the recovered individuals (R). In order to decrease the number of the infected population, we applied the theory of optimal control for our proposed model. The theory of optimal control and the analysis dynamic systems are a field current research that continues to arouse the interest of scientists.

It has been widely used in different fields such as engineering, biology, mechanics, medicine, robotics, and biomedicine. The aim of this theory is to model processes that evolve over time and to study their behaviours. This study makes it possible, among other things, to predict the behaviour of the system and to control it in order to get the desired results. In recent years, it is noted that more and more attention has been paid to the discrete-time models (see, [12] [13] [14] [15] [16] [17] [18] [19] [20] and the references cited therein). The reasons are as the statistic data is collected at discrete times (day, week, month, or year). So it is more direct, more convenient, and more accurate to describe the phenomena by using the discrete-time models than the continuous-time models. Kermack et all were the first researchers on mathematical epidemiology to suggest the susceptible infected-removed (SIR) model that describes the rapid explosion of an infectious disease for a short time [22]. In this context, many researchers have developed specific mathematical models which represent dynamics. In this work and based on the model they proposed, we offer a new approach taking into account the use of theoretical results provided by Balatif et al [19], where authors implemented a discrete-time model that describes the dynamics of voters and they proposed an optimal control strategy. The same idea and strategy were applied by Labzai et al [20], in order to model and control smoking. Kouidere et al [21], suggested a model of evolution from prediabetes to diabetes with an optimal control approach. Other models from optimal control problems, population dynamics and Discrete Dynamics in Nature and Society can be found in [23–27]. This paper is organized as follows. In Section 2, we explain the methods of disease transmission. In Section 3, we present our discrete mathematical model that describes the dynamics of a population infected with (COVID-19). In Section 4, we present the optimal control problem for the proposed model where we give some results concerning the existence of the optimal controls and the characterization of these optimal controls using Pontryagin's Maximum Principle in discrete time. Numerical simulations through MATLAB and the cost-effectiveness analysis are given in Section 5. Finally, we conclude the paper in Section 6.

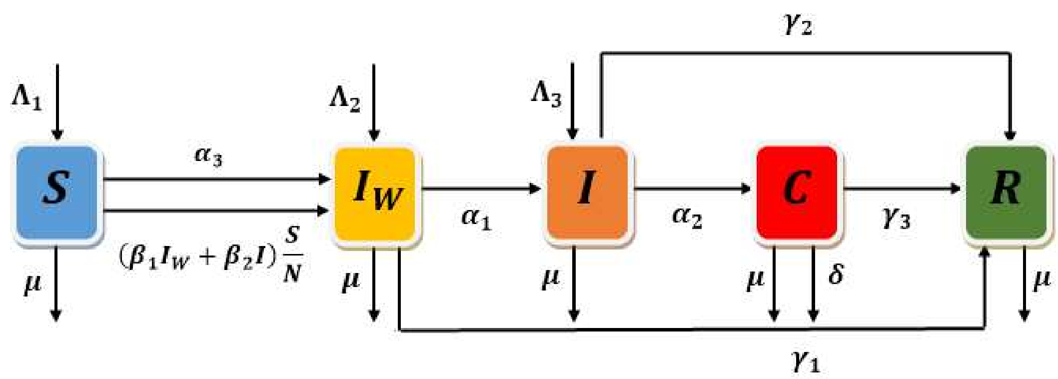
2. METHODS OF SPREAD OF CORONAVIRUS IN HUMANS, ACCORDING TO THE WORLD HEALTH ORGANIZATION

People can get COVID-19 from others who have the virus. The disease spreads mainly from person to person through small droplets of the nose or mouth, which are expelled when a person with COVID-19 coughs, sneezes or speaks. These droplets are relatively heavy, do not travel far and sink rapidly to the ground. Studies have shown that the COVID-19 virus can survive up to 72 hours on plastic and stainless steel, less than 4 hours on copper and less than 24 hours on cardboard. People can catch COVID-19 if they breathe these droplets from a person infected with the virus. Some reports have indicated that people without symptoms can transmit the virus. That's why it's important to stay at least one metre away from the others. These droplets can land on objects and surfaces around the person such as tables, handles, etc. People can become infected by touching these objects or surfaces and then touching their eyes, nose or mouth. WHO is evaluating ongoing research on the subject and will continue to share updated [5]. Based on this data, we will discuss in this work three factors of transmission of the virus to a healthy person by touching infected surfaces or approaching an infected symptomless person or approaching an infected person showing symptoms.

3. A MATHEMATICAL MODEL OF COVID-19

3.1. Description of the Model. We consider a discrete mathematical model SIwICR that describes the dynamics of a population having COVID-19 disease. We divide the population into five compartments. The following illustration will illustrate disease trends the type of COVID-19 disease in the compartments make statements. These trends will be represented by vector arrows in Figure 2.

FIGURE 2. Illustration of movement between compartments



The susceptible people subjected to COVID-19

"S" refers to people who are likely to have COVID-19 disease. This compartment is increased by the recruitment rate denoted Λ_1. It is decreased by a natural mortality rate μ. Also it is decreased by an effective contact with "Iw" at rate β_1 (the rate of patients who become infected with COVID-19 due to contact with the infected people who do not show symptoms) and with "I" at rate β_2 (the rate of patients who become infected with COVID-19 due to contact with the infected people with symptoms). Also it is decreased by α_3 (the rate of people who have been infected with the virus as a result of touching infected areas).

The infected people without symptoms

The compartment of "Iw" is refers to the infected people with COVID-19 without symptoms. It is increased by the incidence rate of immigrants and carriers of the disease without symptoms denoted Λ_2, and also this compartment is increased by β_1, β_2 and α_3.
the compartment "Iw" decreased by natural mortality rate μ and by α_1 which represent a rate of the infected people without symptoms. Also it is decreased by γ_1 the rate of the infected people without symptoms and who become recovered.

The infected people with symptoms

The compartment "I" refers to the infected people with symptoms of COVID-19 disease. It is increased by the incidence rate of infected immigrants and carriers without symptoms denoted

Λ_3. Also it is increased by α_1. This compartment is decreased by natural mortality rate μ and α_2 which represent the rate of the infected people with symptoms who have become infected with complications. Also, it is decreased by γ_2 that represents the rate of the infected people with symptoms who have become recovered individuals.

The infected people with complications

The compartment ”C” refers to people who have infected with complications of (COVID-19) disease. It is increased by α_2. The compartment ”C” decreased by natural mortality rate μ and mortality rate due to COVID-19 disease denoted δ. Also it is decreased by the rate of the infected people with complications who have become recovered individuals denoted γ_3.

The recovered individual population

”R” is the number of recovered individuals. It is increased by γ_3 ,γ_2 and γ_1 and decreased by natural mortality rate μ.

3.2. Model Equations. By adding the rates at which the steps of COVID-19 disease enters the compartment and also by subtracting the rates at which people leave a compartment, we obtain a difference equation for the rate at which patients change in each compartment during separate times. Therefore, we present the COVID-19 disease model with the following system of difference equations:

$$
(1)\qquad \begin{cases}
S(k+1) = \Lambda_1 + (1-\mu-\alpha_3)S(k) - \beta_1\dfrac{S(k)I_w(k)}{N} - \beta_2\dfrac{S(k)I(k)}{N} \\
I_w(k+1) = \Lambda_2 + \alpha_3 S(k) + (1-\mu-\alpha_1-\gamma_1)I_w(k) + \beta_1\dfrac{S(k)I_w(k)}{N} + \beta_2\dfrac{S(k)I(k)}{N} \\
I(k+1) = \Lambda_3 + (1-\mu-\alpha_2-\gamma_2)I(k) + \alpha_1 I_w(k) \\
C(k+1) = \alpha_2 I(k) + (1-\mu-\delta-\gamma_3)C(k) \\
R(k+1) = \gamma_3 C(k) + \gamma_2 I(k) + \gamma_1 I_w(k) + (1-\mu)R(k)
\end{cases}
$$

where $S(0) \geq 0, I_w(0) \geq 0, I(0) \geq 0, C(0) \geq 0$, and $R(0) \geq 0$ are given initial states.

4. THE OPTIMAL CONTROL PROBLEM

So far, there is no treatment or vaccination for COVID-19. For this reason, scientists insist some strategies for combating this disease and to reduce the risk of infection of this virus. These strategies aim at preventing infection and avoiding exposure to this virus by following a prevention protocol: covering the mouth and nose, washing hands with water and soap,cleaning and disinfecting surfaces, objects and goods frequently, also, putting people in quarantine areas to reduce the risk of infection for a new population and to subject these people to specific programs especially the immunodeficiency people.

Our objective in this proposed control strategy is to minimize the number of infected people without symptoms "Iw"; the Infected people with symptoms "I" and infected people with complications "C". So, in the model (1), we include controls $u = (u_0, u_1, ..., u_{T-1})$ which represents the effort to comply with appropriate sanitary controls, which are washing hands with soap and water and disinfecting surfaces and objects frequently, $v = (v_0, v_1, ..., v_{T-1})$ which represents the effort to urge people useding masks to cover the sensitive parts of the body and $w = (w_0, w_1, ..., w_{T-1})$ which represents the treatment of patients infected with COVID-19 by taking them to hospitals and designated quarantine sites.

Thus, the controlled mathematical system is given by the following system of difference equations:

$$(2) \quad \begin{cases} S(k+1) = \Lambda_1 - \alpha_3(1-u_k)S(k) + (1-\mu)S(k) - \beta_1(1-v_k)\dfrac{S(k)I_w(k)}{N} - \beta_2(1-v_k)\dfrac{S(k)I(k)}{N} \\ I_w(k+1) = \Lambda_2 + \alpha_3(1-u_k)S(k) + (1-\mu-\alpha_1-\gamma_1-w_k)I_w(k) + \beta_1(1-v_k)\dfrac{S(k)I_w(k)}{N} \\ +\beta_2(1-v_k))\dfrac{S(k)I(k)}{N} \\ I(k+1) = \Lambda_3 + (1-\mu-\alpha_2-\gamma_2-w_k)I(k) + \alpha_1 I_w(k) \\ C(k+1) = \alpha_2 I(k) + (1-\mu-\delta-\gamma_3-w_k)C(k) \\ R(k+1) = \gamma_3 C(k) + \gamma_2 I(k) + \gamma_1 I_w(k) + (1-\mu)R(k) + (I_w(k)+I(k)+C(k))w_k \end{cases}$$

where $S(0) \geq, I_w(0) \geq, I(0) \geq, C(0) \geq$, and $R(0) \geq$ are given initial states. Then, the problem is to minimize the objective functional:

(3)

$$J(u,v,w) = A_T I_w(T) + B_T I(T) + F_T C(T) + \Sigma_{k=0}^{T-1}(A_k I_w(k) + B_k I(k) + F_k C(k) + \frac{D_k u_k^2}{2} + \frac{E_k v_k^2}{2} + \frac{G_k w_k^2}{2})$$

where the parameters:

$A_k > 0, B_k > 0, F_k > 0, D_k > 0$, $E_k > 0$ and $G_k > 0$ for $k \in 0, 1, 2, ..., T$ are the cost coefficients. They are selected to weigh the relative importance of $I_w(k)$, $I(k)$, $C(k)$, u_k, v_k and w_k at time k .

T is the final time. In other words, we seek the optimal controls u, v and w such that :

$$J(u^*,v^*,w^*) = \min_{(u,v,w)\in U_{ad}} J(u,v,w) \tag{4}$$

where U_{ad} is the set of admissible control defined by $U_{ad} = \{(u,v,w) : u = (u_0,u_1,........,u_{T-1}),$ $v = (v_0,v_1,........,v_{T-1})$ and $w = (w_0,w_1,........,w_{T-1}) /\ 0 \leqslant u_k \leqslant 1$, $0 \leqslant v_k \leqslant 1$ and $0 \leqslant w_k \leqslant 1$; $k \in \{0,1,2,......,T-1\}\}$ the sufficient condition for the existence of optimal controls u, v and w for problems (2) and (3) come from the following theorem .

Theorem 1: there exist the optimal controls u^*, v^* and w^* such that

$$J(u^*,v^*,w^*) = \min_{(u,v,w)\in U_{ad}} J(u,v,w) \tag{5}$$

subject to the controls system (2) with initial conditions.

Proof : Since the coefficients of the state equations are bounded and there are a finite number of time steps. $S = (S(0),S(1),.......,S(T))$; $I_w = (I_w(0),I_w(1),.......,I_w(T)$; $I = (I(0),I(1),.......,I(T))C = (C(0),C(1),.......,C(T))$ and R=(R(0),R(1),.......,R(T)) are uniformly bounded for all (u,v,w) in the controls set U_{ad} and thus J(u,v,w) is bounded for all $(u,v,w) \in U_{ad}$, since J(u,v,w) is bounded, $\inf_{(u,v,w)\in U_{ad}} J(u,v,w)$ is finite, and there exists a sequence $(u^n,v^n,w^n), \in U_{ad}$ such that $\lim_{n\longmapsto+\infty} J(u^n,v^n,w^n) = \inf_{(u,v,w)\in U_{ad}} J(u,v,w)$ and corresponding sequences of states $S^n, I^n_w, I^n, C^n, R^n$,since there is a finile number of uniformly bounded sequences, there exist $(u^*,v^*,w^*) \in U_{ad}$ and S^*, I^*_w, I^*, C^* and $R^* \in \mathbb{R}^{T+1}$ such that an a sequences $u^n \longrightarrow u^*, v^n \longrightarrow v^*, w^n \longrightarrow w^*, S^n \longrightarrow S^*, I^n_w \longrightarrow I^*_w, I^n \longrightarrow I^*, C^n \longrightarrow C^*$ and $R^n \longrightarrow R^*$ .finally, due to the finite dimensional structure of system (2) and the objective function J(u,v,w), u^*, v^* and w^* is an optimal controls with corresponding states S^*, I^*_w, I^*, C^* and R^*. therefore $\inf_{(u,v,w)\in U_{ad}} J(u,v,w)$ is achieved.

In order to derive the necessary condition for optimal controls, the discrete version of pontryagin's maximum principle[12-20] the idea is introducing the adjoint function to attach the system of difference equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts into a problem of minimizing à Hamiltonian H(k) at time step k defined by :

$$H(k) = A_k I_w(k) + B_k I(k) + F_k C(k) + \frac{D_k u_k^2}{2} + \frac{E_k v_k^2}{2} + \frac{G_k w_k^2}{2} + \Sigma_{i=1}^{5} \lambda_{i,k+1} f_{i,k+1} \tag{6}$$

where $f_{i,k+1}$ is the right side of the system of difference equations (2) of the i^{th} state variable at time step k+1.

Theorem 2 : Given the optimal controls u^*, v^*, w^* and the solutions S^*, I_w^*, I^*, C^* and R^* of the corresponding state system (2) there exists adjoint variables $\lambda_{1,k}, \lambda_{2,k}, \lambda_{3,k}, \lambda_{4,k}$ and $\lambda_{5,k}$ satisfying :

$$(7)\qquad \begin{cases} \lambda_{1,k} = \lambda_{1,k+1}(1-\mu-\alpha_3(1-u_k)-\beta_1(1-v_k)\dfrac{I_w(k)}{N}-\beta_2(1-v_k)\dfrac{I(k)}{N}) \\ +\lambda_{2,k+1}(\alpha_3(1-u_k)+\beta_1(1-v_k)\dfrac{I_w(k)}{N}+\beta_2(1-v_k)\dfrac{I(k)}{N}) \\ \\ \lambda_{2,k} = A_k - \lambda_{1,k+1}\beta_1(1-v_k)\dfrac{S(k)}{N}+\lambda_{2,k+1}(1-\mu-\alpha_1-\gamma_1-w_k+\beta_1(1-v_k)\dfrac{S(k)}{N}) \\ +\lambda_{3,k+1}\alpha_1+\lambda_{5,k+1}(\gamma_1+w_k) \\ \\ \lambda_{3,k} = B_k - \lambda_{1,k+1}\beta_2(1-v_k)\dfrac{S(k)}{N}+\lambda_{2,k+1}\beta_2(1-v_k)\dfrac{S(k)}{N}+\lambda_{3,k+1}(1-\mu-\alpha_2-\gamma_2-w_k) \\ +\lambda_{4,k+1}\alpha_2+\lambda_{5,k+1}(\gamma_2+w_k) \\ \\ \lambda_{4,k} = F_k+\lambda_{4,k+1}(1-\mu-\delta-\gamma_3-w_k)+\lambda_{5,k+1}(\gamma_3+w_k) \\ \\ \lambda_{5,k} = (1-\mu)\lambda_{5,k+1} \end{cases}$$

With the tranversality conditions at time T : $\lambda_{1,T}=0$; $\lambda_{2,T}=A_T$; $\lambda_{3,T}=B_T$; $\lambda_{4,T}=F_T$; $\lambda_{5,T}=0$ Furthermore for $k=0,1,2,.......,T-1$, the optimal controls u_k^* , v_k^* and w_k^* are given by

$$(8)\qquad u_k^* = \min(u_{max}, \max(u_{min}, \frac{(\lambda_{2,k+1}-\lambda_{1,k+1})\alpha_3 S(k)}{D_k}))$$

$$(9)\qquad v_k^* = \min(v_{max}, \max(v_{min}, \frac{(\lambda_{2,k+1}-\lambda_{1,k+1})(\beta_1 S(k)I_w(k)+\beta_2 S(k)I(k))}{NE_k}))$$

(10)

$$w_k^* = \min(u_{max}, \max(u_{min}, \frac{(\lambda_{2,k+1}-\lambda_{5,k+1})I_w(k)+(\lambda_{3,k+1}-\lambda_{5,k+1})I(k)+(\lambda_{4,k+1}-\lambda_{5+1})C(k)}{G_k}))$$

Proof : The hamiltonian at time step k is given by

$$H(k)=A_kI_w(k)+B_kI(k)+F_kC(k)+\frac{D_ku_k^2}{2}+\frac{E_kv_k^2}{2}+\frac{G_kw_k^2}{2}+\lambda_{1,k+1}(\Lambda_1+(1-\mu)S(k)-\alpha_3(1-u_k)S(k)-\beta_1(1-v_k)\frac{S(k)I_w(k)}{N}-\beta_2(1-v_k)\frac{S(k)I(k)}{N})+\lambda_{2,k+1}(\Lambda_2+(1-\mu-\alpha_1-\gamma_1-w_k)I_w(k)+\alpha_3(1-u_k)S(k)+\beta_1(1-v_k)\frac{S(k)I_w(k)}{N}+\beta_2(1-v_k)\frac{S(k)I(k)}{N})+\lambda_{3,k+1}(\Lambda_3+(1-\mu-\alpha_2-\gamma_2-w_k)I(k)+\alpha_1I_w(k))+\lambda_{4,k+1}(\alpha_2I(k)+(1-\mu-\delta-\gamma_3-w_k)C(k))+\lambda_{5,k+1}(\gamma_3C(k)+\gamma_2I(k)+\gamma_1I_w(k)+(1-\mu)R(k)+(I_w(k)+I(k)+C(k))w_k)$$

for $k=0,1,2,.......,T-1$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum principle, in discrete time given in [12-20] suth that

$$(11)\quad \begin{cases} \lambda_{1,k}=\frac{dH(k)}{dS(k)}=\lambda_{1,k+1}(1-\mu-\alpha_3(1-u_k)-\beta_1(1-v_k)\frac{Iw(k)}{N}-\beta_2(1-v_k)\frac{I(k)}{N}) \\ +\lambda_{2,k+1}(\alpha_3(1-u_k)+\beta_1(1-v_k)\frac{I_w(k)}{N}+\beta_2(1-v_k)\frac{I(k)}{N}) \\ \\ \lambda_{2,k}=\frac{dH(k)}{dI_w(k)}=A_k-\lambda_{1,k+1}\beta_1(1-v_k)\frac{S(k)}{N}+\lambda_{2,k+1}(1-\mu-\alpha_1-\gamma_1-u_k+\beta_1(1-v_k)\frac{S(k)}{N}) \\ +\lambda_{3,k+1}\alpha_1+\lambda_{5,k+1}(\gamma_1+w_k) \\ \\ \lambda_{3,k}=\frac{dH(k)}{dI(k)}=B_k-\lambda_{1,k+1}\beta_2(1-v_k)\frac{S(k)}{N}+\lambda_{2,k+1}\beta_2(1-v_k)\frac{S(k)}{N} \\ +\lambda_{3,k+1}(1-\mu-\alpha_2-\gamma_2-w_k)+\lambda_{4,k+1}\alpha_2+\lambda_{5,k+1}(\gamma_2+w_k) \\ \\ \lambda_{4,k}=\frac{dH(k)}{dC(k)}=F_k+\lambda_{4,k+1}(1-\mu-\delta-\gamma_3-w_k)+\lambda_{5,k+1}(\gamma_3+w_k) \\ \\ \lambda_{5,k}=\frac{dH(k)}{dR(k)}=(1-\mu)\lambda_{5,k+1} \end{cases}$$

with the tranversality conditions at time T : $\lambda_{1,T}=0$; $\lambda_{2,T}=A_T$; $\lambda_{3,T}=B_T$; $\lambda_{4,T}=F_T$; $\lambda_{5,T}=0$. For $k\in 0,1,2,.......,T-1$, the optimal controls u_k^*,v_k^* and w_k^* can be solved from the optimality condition: $\frac{dH(k)}{du_k}=0$, $\frac{dH(k)}{dv_k}=0$ and $\frac{dH(k)}{dw_k}=0$ that is:

$$\frac{dH(k)}{du_k}=D_ku_k+(\lambda_{1,k+1}-\lambda_{2,k+1})\alpha_3S(k)=0$$

$$\frac{dH(k)}{dv_k} = E_k v_k + \lambda_{1,k+1}(\beta_1 \frac{S(k)I_w(k)}{N} + \beta_2 \frac{S(k)I(k)}{N}) - \lambda_{2,k+1}(\beta_1 \frac{S(k)I_w(k)}{N} + \beta_2 \frac{S(k)I(k)}{N}) = 0$$

$$\frac{dH(k)}{dw_k} = G_k w_k - \lambda_{2,k+1} I_w(k) - \lambda_{3,k+1} I(k) - \lambda_{4,k+1} C(k) + \lambda_{5,k+1}(I_w(k) + I(k) + C(k)) = 0$$

so we have

$$u_k = \frac{(\lambda_{2,k+1} - \lambda_{1,k+1})\alpha_3 S(k)}{D_k}$$

$$v_k = \frac{(\lambda_{2,k+1} - \lambda_{1,k+1})(\beta_1 S(k) I_w(k) + \beta_2 S(k) I(k))}{N E_k}$$

$$w_k = \frac{(\lambda_{2,k+1} - \lambda_{5,k+1}) I_w(k) + (\lambda_{3,k+1} - \lambda_{5,k+1}) I(k) + (\lambda_{4,k+1} - \lambda_{5,k+1}) C(k)}{G_k}$$

by the bounds in U_{ad} of the controls, it easy to obtain u_k^*, v_k^* and w_k^* in the form (8), (9), (10).

5. Numerical Simulation and Cost-effectiveness Analysis

5.1. Algorithm. In this section we present the results obtained by solving numerically the optimality system. There were initial conditions for the state variables and terminal conditions for the adjoints. That is the optimality system is a two-point boundary value problem with separated boundary conditions at times step k=0 and k=T. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration we update the controls by using the characterization. We continue until convergence of successive iterates is achieved. A code is written and compiled in Matlab using the following data (Table1).

Table1: The description of parameters used for the definition of discrete time systems .

Parameter	Description	Estimated value	Source
s_0	Moroccan population	37000000	[3]
iw_0	initial Number of infected people without symptoms	100	Assumed
i_0	initial Number of infected people with symptoms	50	Assumed
c_0	initial Number of infected people with complications	0	Assumed
r_0	initial Number of recovered people	0	Assumed
Λ_1	The incidence of susceptible	2000000	[28]
Λ_2	The incidence of Immigrants with covid19 without symptoms	2000	[28]
Λ_3	The incidence of Immigrants with covid19 with symptoms	500	[28]
μ	Human natural death rate	0.02	[28]
δ	Mortality rate due to covid-19 in Morocco	0.035	[3]
α_1	The rate of infected people with symptoms	0.8	Assumed
α_2	The rate of infected people with complications	0.4	Assumed
α_3	The rate of infected people due to contact with infected surfaces	0.05	Assumed
β_1	The rate of infec people due to contact with infec without sympt	0.2	[28]
β_2	The rate of infec people due to contact with infec with sympt	0.1	[28]
γ_1	The rate of those recovering from covid-19 without symptoms	0.4	Assumed
γ_2	The rate of those recovering from covid-19 with symptoms	0.3	Assumed
γ_3	The rate of those recovering from covid-19 with complications	0.2	Assumed

5.2. Discussion. In this section, we analyse numerically the effects of controls such as the treatment of patients infected with (COVID-19) by subjecting them to quarantine within hospitals and designated places for that, urging people to wash their hands with water and soap, cleaning and disinfecting surfaces frequently and using masks to cover the sensitive body parts. Different simulations can be carried out using various values of parameters. We use parameters values as shown in Table1. Furthermore we investigate numerically the impact of each of the following optimal control strategies:

strategy 1. Urging people to wash their hands with water and soap , clean and disinfect surfaces frequently control u.

strategy 2. Forcing people to use masks to cover the sensitive body parts control v.

strategy 3. The treatment of patients infected with COVID-19 by subjecting them to quarantine within hospitals and designated places for that control w.

strategy 4. Combining of strategy1 and strategy2, by using controls u and v .

strategy 5. Combining of strategy1 and strategy3, by using controls u and w .

strategy 6. Combining of strategy 2 and strategy3, by using controls v and w .

strategy 7. Combining of strategy1, strategy2 and strategy3, by using controls u, v and w.

5.2.1. *Strategy 1.* In this strategy, we apply the control (u) by urging people to wash their hands with water and soap, cleaning and disinfecting surfaces frequently. We observed the in Figures 3 the number of infected people without symptoms "I_w" decreased from $1.233*10^6$ (without controls) to $0.96*10^6$(with controls). Figure 4 demonstrates that the number of the infected people with symptoms "I" decreases from $1.371*10^6$ (without controls) to $1.05*10^6$ (with controls).

Figure 5 demonstrates that the number of the infected people with complications "C" decreases from $2.15*10^6$ (without controls) to $1.65*10^6$ (with controls), at the end of the implementation of the proposed strategy.

5.2.2. *Strategy 2.* In this strategy applying control (v) by forcing people to use masks to cover the sensitive body parts, we observed in Figures 3, 4 and 5, the number of infected humans without symptoms I_w, with symptoms I, and with complications C all decreases slightly compared to cases without control.

5.2.3. *Strategy 3.* In this strategy, we apply the control (w) by treating the patients infected with COVID-19 within quarantined hospitals and designated places for that. We observed that in Figures 3 the number of infected people without symptoms "I_w" decreased from $1.233*10^6$ (without controls) to $0.91*10^6$(with controls). Figure 4 demonstrates that the number of the infected people with symptoms "I" decreases from $1.371*10^6$ (without controls) to $0.63*10^6$ (with controls). Figure 5 demonstrates that the number of the infected people with complications "C" decreases from $2.15*10^6$ (without controls) to $0.45*10^6$ (with controls), at the end of the implementation of the proposed strategy.

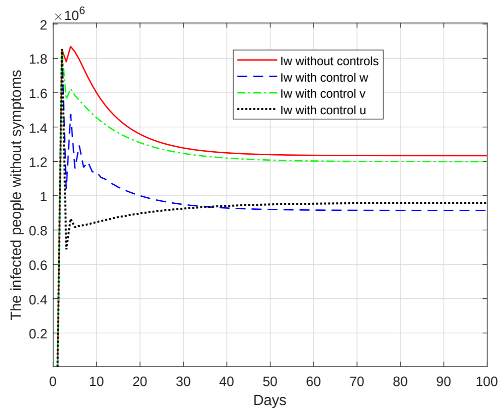


FIGURE 3

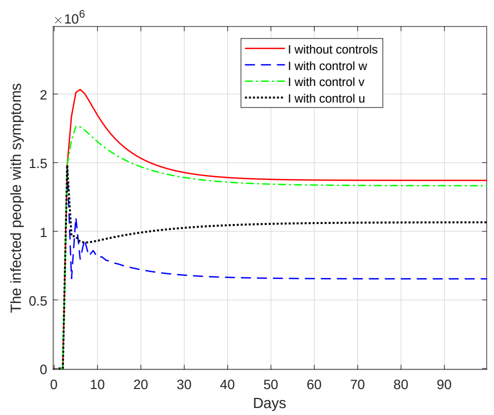


FIGURE 4

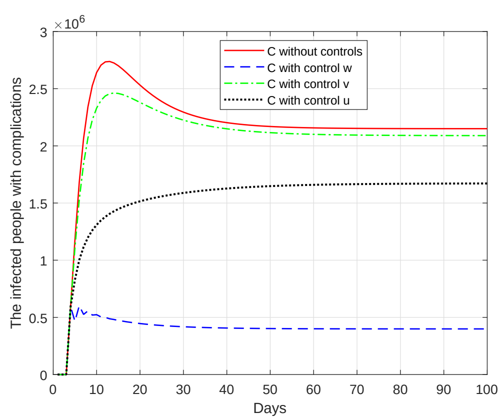


FIGURE 5

5.2.4. *Strategy 4.* Here, using this strategy, we urge people to wash their hands with water and soap, clean and disinfect surfaces frequently (control u) and forcing citizens to use masks to cover the sensitive body parts (control v). We observe that in Figure 6, the number of infected people without symptoms "I_w" decreased from $1.233 * 10^6$ (without controls) to $0.99 * 10^5$(with controls). Figure 7 demonstrates that the number of the infected people with symptoms "I" decreases from $1.371 * 10^6$ (without controls) to $0.135 * 10^6$ (with controls). Figure 8 demonstrates that the number of The infected people with complications "C" decreases from $2.15 * 10^6$ (without controls) to $0.21 * 10^6$ (with controls), at the end of the implementation of the proposed strategy.

5.2.5. *Strategy 5.* Using this strategy, we urge people to wash their hands with water and soap, clean and disinfect surfaces frequently (control u) and by treatment of patients infected with (COVID-19) by subjecting them to quarantine within hospitals and designated places for that (control w). In Figure 6, we observe that the number of infected people without symptoms "I_w" decreased from $1.233 * 10^6$ (without controls) to $0.91 * 10^5$(with controls). Figure 7 demonstrates that the number of the infected people with symptoms "I" decreases from $1.371 * 10^6$ (without controls) to $0.093 * 10^6$ (with controls). Figure 8 demonstrates that the number of The infected people with complications "C" decreases from $2.15 * 10^6$ (without controls) to $0.089 * 10^6$ (with controls), at the end of the implementation of the proposed strategy.

5.2.6. *Strategy 6.* Using, this strategy we force people to use masks to cover the sensitive body parts (control v), and using treatment for patients infected with (COVID-19) by subjecting them to quarantine within hospitals and designated places for that (control w). In Figure 6, we observed that the number of infected people without symptoms "I_w" decreased from $1.233 * 10^6$ (without controls) to $0.98 * 10^5$(with controls). Figure 7 demonstrates that the number of the infected people with symptoms "I" decreases from $1.371 * 10^6$ (without controls) to $0.0105 * 10^6$ (with controls). Figure 8 demonstrates that the number of The infected people with complications "C" decreases from $2.15 * 10^6$ (without controls) to $0.093 * 10^6$ (with controls), at the end of the implementation of the proposed strategy.

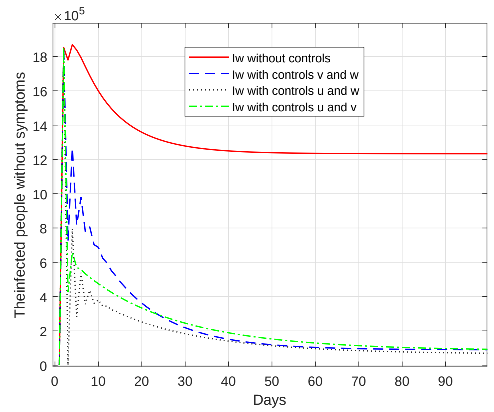


FIGURE 6

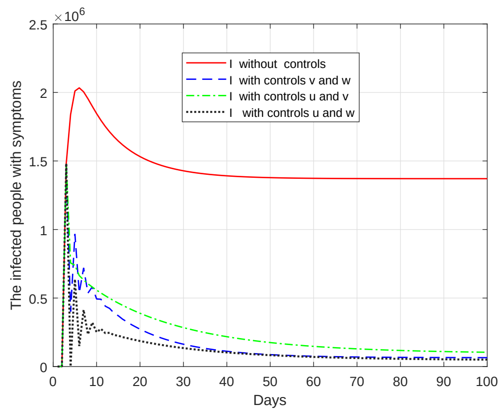


FIGURE 7

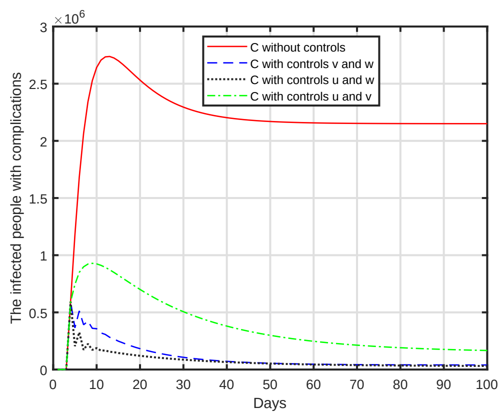


FIGURE 8

5.2.7. *Strategy 7.* In this strategy, we use all the three controls u, v, and w to optimize the objective funtion $J(u,v,w)$. We observed that in Figure 9 the number of infected people without symptoms "I_w" decreased from $1.233*10^6$ (without controls) to 7694(with controls). Figure 10 demonstrates that the number of the infected people with symptoms "I" decreases from $1.371*10^6$ (without controls) to 5700 (with controls). Figure 11 demonstrates that the number of The infected people with complications "C" decreases from $2.15*10^6$ (without controls) to 3708(with controls), at the end of the implementation of the proposed strategy.

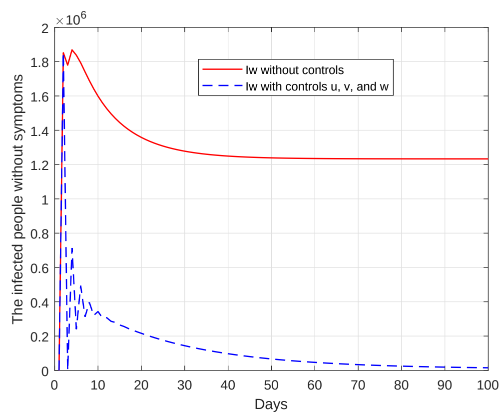


FIGURE 9

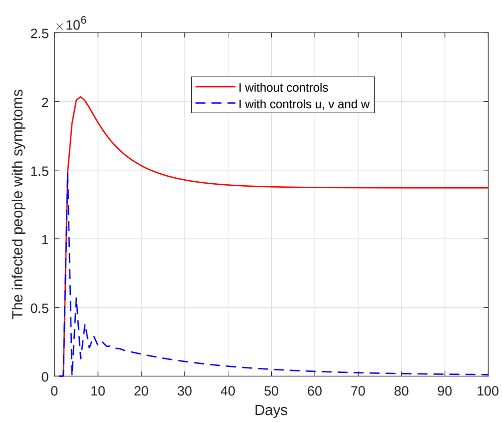


FIGURE 10

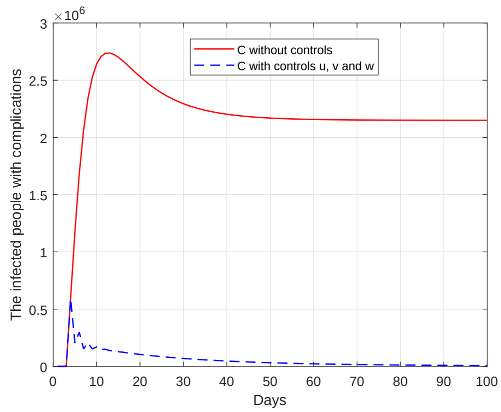


FIGURE 11

5.3. Cost-effectiveness Analysis. Following the method as applied in several studies [29–32], we evaluate the costs using the incremental Cost-Effectiveness Ratio (ICER). This ratio is used to compare the differences between the costs and health outcomes of two alternative intervention strategies. The ICER numerator includes the differences in intervention costs, averted disease costs, costs of prevented cases and averted productivity losses if applicable. While, ICER's denominator is the difference in health outcomes. Given two competing strategies i and j, where strategy j has higher effectiveness than strategy i $(TA(i) < TA(j))$, the ICER values are calculated as follow:

$$ICER(i) = \frac{TC(i)}{TA(i)} \tag{12}$$

$$ICER(j) = \frac{TC(j) - TC(i)}{TA(j) - TA(i)} \tag{13}$$

Where the total costs (TC) and the total cases averted (TA) are defined, in our study, during a given period for strategy i for i = 1, 2, 3, 4, 5, 6, 7 by:

$$TC(i) = \Sigma_{k=0}^{T-1}((D_k u_k^* + E_k v_k^*)S^*(k) + G_k w_k^*(I_w^*(k) + I^*(k) + C^*(k))) \tag{14}$$

$$TA(i)\Sigma_{k=0}^{T}((I_w(k) + I(k) + C(k)) - (I_w^*(k) + I^*(k) + C^*(k))) \tag{15}$$

Where D_k, E_k and G_k corresponds to the person unit cost of the three possible interventions, while $I_w^*(k)$, $I^*(k)$ and $C^*(k)$ is the optimal solution associated to the optimal controls u_k^*, v_k^* and w_k^*. Based on the model simulation results, we ranked, in Table 2 our control strategies in order of increased numbers of averted infections.

Table2 : Total costs and total averted infections for all strategies.

strategy	Total averted infections (TA)	Total cost (TC)
2	$0.1949*10^8$	$1.6541*10^{10}$
1	$1.4027*10^8$	$2.3955*10^{10}$
4	$1.7729*10^8$	$5.2238*10^{10}$
3	$2.8319*10^8$	$8.1219*10^8$
6	$2.8886*10^8$	$1.7626*10^{10}$
5	$3.4718*10^8$	$2.5745*10^{10}$
7	$3.5726*10^8$	$5.3690*10^{10}$

First, we compared the cost-effectiveness of strategy 2 and strategy 1 :

$$ICER(2) = \frac{1.6541*10^{10}}{0.1949*10^8} = 848.691$$

$$ICER(1) = \frac{2.3955*10^{10} - 1.6541*10^{10}}{1.4027*10^8 - 0.1949*10^8} = 61.384$$

Note that ICER(2) higher than ICER(1). This means that strategy2 is dominated by strategy1. Therefore, strategy2 is excluded from the set of alternatives.

Second, we compared the cost-effectiveness of strategy 1 and strategy 4 :

$$ICER(1) = \frac{2.3955*10^{10}}{1.4027*10^8} = 170.777$$

$$ICER(4) = \frac{5.2238*10^{10} - 2.3955*10^{10}}{1.7729*10^8 - 1.4027*10^8} = 763.992$$

By comparing between strategy1 and strategy4 , the lower ICER for strategy1 indicates that strategie4 is strongly dominated. That is, strategy4 is more costly and less effective than strategie1. Therefore, strategy4 is excluded from the set of alternatives.

Next, strategy 1 is compared with strategy 3 :

$$ICER(1) = \frac{2.3955*10^{10}}{1.4027*10^8} = 170.777$$

$$ICER(3) = \frac{8.1219*10^8 - 2.3955*10^{10}}{2.8319*10^8 - 1.4027*10^8} = -161.928$$

Since $ICER(3) < ICER(1)$, then strategy 1 is less effective than strategy 3. Therefore, strategy1 is excluded from the set of alternatives.

Next, we compare the cost-effectiveness of strategy 3 and strategy 6:

$$ICER(3) = \frac{8.1219 * 10^8}{2.8319 * 10^8} = 2.868$$

$$ICER(6) = \frac{1.7626 * 10^{10} - 8.1219 * 10^8}{2.8886 * 10^8 - 2.8319 * 10^8} = 2965.398$$

The comparison of ICER (3) and ICER (6) reveals a cost savings of 2.868 for Strategy 3 over Strategy 6. This means that Strategy6 more expensive and less effective than Strategy 3. Therefore, Strategy 6 is excluded from the set of alternatives.

Now, we compare the cost-effectiveness of strategy 3 and strategy 5 :

$$ICER(3) = \frac{8.1219 * 10^8}{2.8319 * 10^8} = 2.868$$

$$ICER(5) = \frac{2.5745 * 10^{10} - 8.1219 * 10^8}{3.4718 * 10^8 - 2.8319 * 10^8} = 389.636$$

The lower ICER obtained for Strategy 3 is an indication that Strategy 3 strongly dominate Strategy 5, this simply indicates that Strategy5 is more costly to implement compare to Strategy3. Therefore, it is best to exclude Strategie5 from the set of control strategies and alternative interventions to implement in order to preserve limited resources.

Finally, strategy3 is compared with strategy7:

$$ICER(3) = \frac{8.1219 * 10^8}{2.8319 * 10^8} = 2.868$$

$$ICER(7) = \frac{5.369 * 10^{10} - 8.1219 * 10^8}{3.5726 * 10^8 - 2.8319 * 10^8} = 713.889$$

The comparison reveals that strategy3 is cheaper than strategy7 by saving 2.868 .Therefore, strategy3 is the best strategy from all compared strategies due to its cost-effectiveness and healthy benefit. Moreover, Figures 3, 4 and 5 show that the application of the intervention (The treatment of patients infected with COVID-19 by subjecting them to quarantine within hospitals and designated places for that control w) alone is the cheapest. But we do not consider this because a single intervention is not effective in eradicating the disease. The combination of all the three interventions(urging people to wash their hands with water and soap, clean and disinfect surfaces frequently, forcing people to use masks to cover the sensitive body parts, treating patients infected with COVID-19 by taking them to hospitals and designated quarantine sites) is the most expensive strategy compared to other strategies. But this strategy is yielding important results in fighting the spread of the COVID-19 epidemic, See the figures 9, 10 and 11 .

6. CONCLUSION

In this research a mathematical model of the COVID-19 disease is considered to investigate the effect of three optimal controls strategies that respectively represent urging people to wash their hands with water and soap frequently, clean and disinfect surfaces, forcing people to use masks to cover the sensitive body parts, treating patients infected with COVID-19 by submitting them to hospitals and designated quarantine sites. After introducing the work and related literature in the beginning, we formed a mathematical discrete model that describes the dynamics of a population infected by COVID-19 virus, without symptoms, with symptoms and with the serious or critical complications in order to minimize the number of infected people in all steps for COVID-19. We applied the results of the control theory and obtain the characterizations of the optimal controls. Finally, we have a numerical solution obtained from the mathematical model through the maximum principle of Pontryagin's in discrete time. The system of optimality is solved by an iterative method. Also we investigated the cost-effectiveness of the controls to determine the most effective strategy to eliminate COVID-19 with minimum costs. Using ICER cost-effectiveness analysis, we showed that strategy 3 is the most effective strategy but not effective in eliminating the disease. The combination of the three interventions u, v and w (strategie7) is the most costly strategy compared to other strategies. But, this strategy has impressive results in coping with the spread of the disease and reducing the number of infections, see figures 9, 10 and 11. The Moroccan state, since the first day of the emergence of infections by this disease, has committed to implementing this strategy gradually, despite its cost and its impact on the national economy. By simulating the propagation process of COVID-19, we found that the proposed strategie7 (see figures 9, 10 and 11) corresponded closely to official data from Morocco [3].

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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