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AN EXTENDED IMAGE ENCRYPTION WITH MARKOV PROCESSES IN SOLUTIONS IMAGES DYNAMICAL SYSTEM OF NON-LINEAR DIFFERENTIAL EQUATIONS

MAHWISH BANO[1], SALEEM ABDULLAH[2], T.M. SHAH[3], THAMMARAT PANITYAKUL[4,*], RONNASON CHINRAM[4,5]

[1]Department of Mathematics, Air University, Islamabad 44000, Pakistan

[2]Department of Mathematics, Abdul Wali Khan University, Mardan 23200, Pakistan

[3]Department of Mathematics, Preston University, Islamabad 44000, Pakistan

[4]Division of Computational Science, Prince of Songkla University, Hat Yai, Songkhla 90110, Thailand

[5]Centre of Excellence in Mathematics, Si Ayuthaya Road, Bangkok 10400, Thailand

Abstract. The process of hiding texts or images into images and signals has been remained contemporary topics over more than decades in Information Sciences. An attempt has been made to hide information or images into the solutions of non-linear differential equations of some especial type like Lorezn and Rossler's equations, these equations have great importance in almost all practical applications from engineering, geography (weather forecasting) and even in stock markets due to its chaotic behavior of solutions which is dependent on initial conditions. The chaotic nature and the aperiodicity of such solutions have become the main source of attraction for encryptions. The 3D parametric (time) series of solutions of Lorenz and Rossler's equations are produced phase vectors, and one of these phase vectors is chosen for the purpose of text or image embedding into its error (noise). The error (noise) is a part of chaotic solutions behaved aperiodicity in phase vectors. This attracted the embedding of manipulated text or image into it. Modified or reconstructed phase vector embedded with text or image when reproduced in the form of phase surfaces are quite similar to the original solutions surfaces. So Lorenz and Rossler solutions

*Corresponding author

E-mail address: thammarat.p@psu.ac.th

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are used for embedding benchmark images like Lena, Fruits and Vegetables. Having embedded these images into series solutions, we found Lorenz solutions are more appropriate over the Rossler solutions. A thorough statistical analysis together with histograms of both original images and embedded images are reported. Experiments with RGB values of embedded images are also reported. In the present research, we have further extended this technique by using Markov processes to our target images and converted into encrypted images before embedding into solution phase vector of Lorenz and Rossler's equations. Markov processes are made more confusing/diffusing the target images and has become difficult for intruders to attack for retrieval.

Keywords: cryptography; encryption/decryption; image processing; Markov processes; Lorenz equations; Rossler's equations.

2010 AMS Subject Classification: 94A08, 94A60.

1. Introduction

The idea of embedding images into images has been a very versatile and hot topic over a decade. The chaotic behavior of many mathematical functions has attracted for such embedding. This phenomena is inherited non-repeating and non-periodicity. An encryption tool is an effective approach to protect such information when sending and receiving data through multiple ways of communications [5, 13, 16]. Techniques based on chaos theory were developed encryption with confusion and diffusion into multiple rounds ([6, 8, 9, 11, 15, 19, 20, 22, 24]). Non-linear dynamical systems have been based on Chaos [10]. A non-linear chaotic map together with genetic algorithm was employed for text and images encryption [20, 21]. The analysis of such embedding was presented in [4]. Various techniques were developed in order to secure these digital images. Researchers are working to develop new and innovative techniques using chaotic behavior of solutions of non-linear differential equations. In the present analysis, an idea is presented to use solutions of dynamical system for image encryption. Image encryption algorithm based on chaotic economic model were considered in [3]. Image encryption based on Lorenz and Rossler was studied in [1] and [2]. Solutions of Lorenz and Rossler are highly non-linear parametric and sensitive to initial conditions. These solutions behaves like series solutions of multiple equations, in which fixed and bifurcation points can be plotted accordingly. Solutions have become chaotic for certain parameter values. When it is chaotic, called Lorenz or Rossler attractors and have certain chaotic properties. The attractors can effectively be used for text and image encryption through mathematical data manipulations. The purpose of this paper is to provide analytical expressions for the variables involved in Lorenz and Rossler's equations. An investigation is carried out for the influence of parameters over the state variables in these equations. Further to our previous attempt, this extended work is embedded pictures into phase solutions of Lorenz and Rossler together with Markov processes on image matrices. Markov processes have applied to image matrices of Lena, Vegetable and Fruits and obtained encrypted images which are absolutely unrecognizable. These encrypted images are then embedded into solution matrices of Lorenz and Rossler. Decryption process is obtained by decrypt the encrypted pictures from solution matrices followed by the inverse Markov processes. Markov processes have confused/diffused target images and made difficult for intruders to attack for retrieval. Statistical analysis has shown reliability of the method with no extra computational time.

2. A Classical Solution set of Non-Linear Ordinary Differential Equations

Non-linear ordinary differential equations have great importance while modeling physical problems. The solutions of these equations have in close agreement with experimental but sometimes they behaved strangely. A series of solutions are obtained with different initial conditions. Lorenz [14] has found strange chaotic behavior in solutions of differential equations. He then successfully has implemented in weather forecasting as the weather continuously changes with various initial conditions. Rossler has also obtained the similar behavior in his solutions. He has implemented these solutions to explain problems in engineering, geography and even in stock market.

2.1. Lorenz Equations.

Lorenz has introduced ordinary differential equations whose solutions can be modeled the simplest example of deterministic non periodic flow and finite amplitude convection. He has explained the work of meteorologist and physicists while incorporating several physical phenomena ([21]). He has found variables to construct a simple model based on the 2-dimensional representation of the earth's atmosphere. Consider the following differential equations,

(1) $$\frac{dx}{dt} = -ax + ay,$$

(2) $$\frac{dy}{dt} = bx - y - xz,$$

(3) $$\frac{dz}{dt} = -cz + xy,$$

where $a = 15, b = 28, c = 8/3$ are parameters. Initial conditions for above equations at $t = 0$, are $x(t) = y(t) = z(t) = 1$.

2.2. Rossler's Equations. Similar to Lorenz equations, Rossler has developed the nonlinear ordinary differential equations in 3-dimensions with a different combination of variables and initial conditions. He has also found chaotic behavior in solutions which can be used in various chaotic problems. The Rossler's ordinary differential equations are as follows:

(4) $$\frac{dx}{dt} = -y - z,$$

(5) $$\frac{dy}{dt} = x - dy,$$

(6) $$\frac{dz}{dt} = -e + z(x - f),$$

where $d = 0.15, e = 0.2, f = 10$ are parameters. Initial conditions for above equations at $t = 0$, are $x(t) = 0.5, y(t) = z(t) = 0$.

2.3. Numerical Solutions of Lorenz and Rossler's Equations. For numerical solutions, we reconsider the Lorenz equations after linearization about the origin as

(7) $$\frac{dx}{dt} = \sigma(y - z),$$

(8) $$\frac{dy}{dt} = \gamma x - y,$$

(9) $$\frac{dz}{dt} = -bz,$$

which decoupled the z-motion. In a special case with $\sigma = 15, b = 8/3, \gamma = 28$ and the initial conditions as $(x_0, y_0, z_0) = (0, 1, 0)$, the resulting solutions $y(t)$ is given in Figure 1.

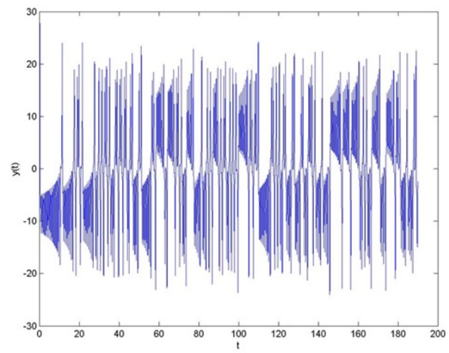

FIGURE 1. Phase vector component, $y(t)$ against time t

Figure 1 shows that after an initial transients, the solutions are settled down into an irregular shape that continued as time approaches to infinity but never repeats exactly. The motion is aperiodic. The nature of this aperiodicity is an asset for encryption applications. The Lorenz has found a wonderful structure emerged if this solution is visualized as a trajectory in phase space. If $x(t)$ is plotted against $z(t)$, the famous butterfly wing pattern appeared Figure 2.

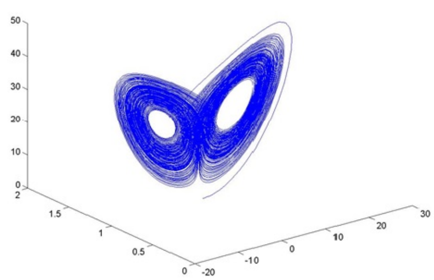

FIGURE 2. Phase vector, $x(t)$ vs $z(t)$, keeping $y(t) = 1.0$, Butterfly structure

In Figure 2, it is seen that the trajectory to cross itself repeatedly, but that is just an artifact of projecting the three dimensional trajectory onto a two dimensional plane. In 3-D no crossing has occurred. The number of circuits appeared on each side varies unpredictably from one cycle to the next. The sequence of cycles is random and useful from encryption purposes. The 3-D plot of these trajectories are appeared to be a pair of butterfly wings known as Attractor (Figure 2). The unique theorem means trajectories can not be crossed or merged, surfaces of the Attractor can only be appeared to merge [14]. The infinite complex surfaces is called Fractal.

Similar articulation has appeared in the solution of Rossler's equations with parameters $d = 0.15, e = 0.2, f = 10$ and initial conditions (0.5, 0, 0). The surfaces appeared with solutions of Rossler's equations have also no crossings. This can also be used for encryption purposes.

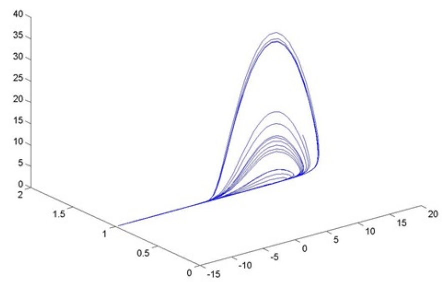

FIGURE 3. Phase vector space in Rossler solution

2.4. Results and Discussions. The images or text are in the form of matrices. The elements of these matrices are pixel values can easily be manipulated to hide in the solution sets of Lorenz and Rossler's equations. The solutions set of Lorenz and Rossler's equations are given in the form of time series phase vectors as

$$X_{i+1} = F(X_i), \tag{10}$$

where $i = 1, 2, \ldots, n$ and X_i is the time series solutions.

Images and text can be embedded into this time series solutions by the reconstruction of phase spaces such that the original series structure must not be changed. Embedding has done through time delay procedure in the series solutions and reconstructed phase space, a feature of Lorenz or Rossler's equations. The delay reconstruction has derived with the solutions of Lorenz equations as

$$S_n = S(X(n\delta t)) + \eta_n \tag{11}$$

where, S_n is the measurement function and, η_n is the measurement noise. The delay reconstruction in m dimensions is formed by a vector, β_n as

$$\beta_n = (S_{n-(m-1)\tau}, S_{n-(m-2)\tau}, \dots, S_{n-\tau}, S_n).$$

In practice, the necessary dimensions of reconstructed phase space (minimum embedding dimension) is unknown. The embedding theorem guaranteed, that for an ideal noise free data, there exists an embedding dimensions, m such that, β_n is equivalent to the original phase space vectors and the attractor formed by β_n is equivalent to the attractor in the original phase space if $m > 2D$. Details can be found in [14]. An interesting conclusion from the embedding point of view is that if a time series comes from a dynamical system that is on an Attractor, the trajectories constructed from the time series by embedding must have the same topological properties as the original system. The requirement is that if the original Attractor has dimension D, then an embedding dimension, $m = D + 1$ would be adequate for reconstructing the Attractor. This idea has implemented in the present paper in a simplest form of embedding into attractors.

3. Proposed Image Encryption in Attractors Model

Lorenz and Rossler's equations in its non-linear forms are solved numerically using RK 4^{th} order methods and obtained solutions, $X(t) = (x(t), y(t), z(t))$ for a given time interval $T = \partial t$ for which the solutions becomes stable. The famous pictures like butterfly and sliding mountains have obtained with the various combinations of phase vectors (Figure 2 and 3).

These pictures are generated with the original phase space vectors (time series solutions), for embedding purposes, we have selected one of the phase space vector say $x(t)$. We described the embedding algorithm in the following section.

3.1. Markov Processes. The simplest form of the Markov process is the future prediction using the current input conditions. The current input conditions are evolved in the form of a sizeable matrix called Markov matrix whose sum of the elements in row or column is equal to unity. The use of this Markov matrix together with initial input vectors can easily be predicted the future as $P_n = (M)P_{n-1}$ where M is the Markov matrix based on current input conditions and P_0 is the initial guess vector. This is an iterative method equally applicable to weather prediction, stock market, or any statistical processes. In the present analysis we have evolved a Markov matrix of the same size as target pictures (Lena, Vegetable and Fruit). The Markov matrices of same size as the original pictures have produced unrecognizable images which we can be used for encryption/decryption processes.

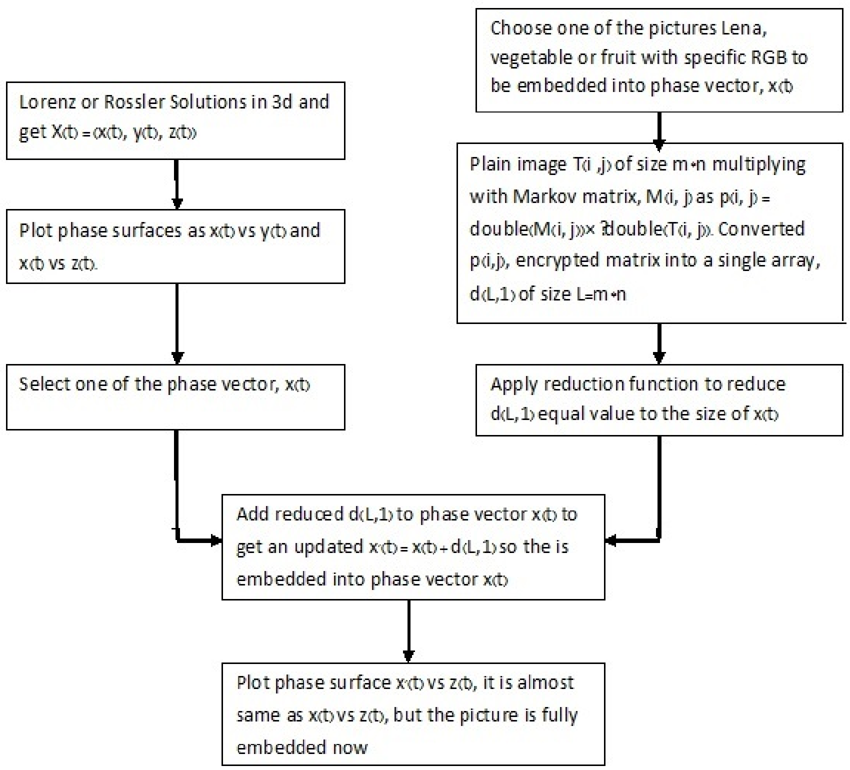

FIGURE 4. Flow chart of Image Encryption Algorithm

3.2. Image Encryption/Decryption. The size of a plain image, $T(i,j)$ is $M \times N$, where $T(i,j)$ is the pixel value in RGB format at i^{th} column and j^{th} row. The proposed scheme is described in figure 4 and 5 as follows

- Consider a plain image, $T(i,j)$ of size $M \times N$.
- Obtained a Markov image, $M(i,j)$ of the same size as $M \times N$.

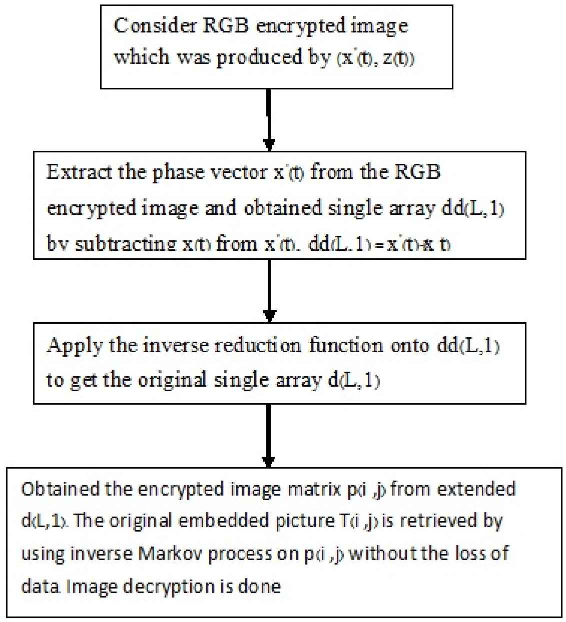

FIGURE 5. Flow chart of Image Decryption Algorithm

- Multiply the original image with Markov image to get an unrecognizable image as $p(i,j) = double(M(i,j))\dot{\times}double(T(i,j))$. This step is called confusion/diffusion process which turned the original picture into completely unrecognizable image.
- Convert this image, $p(i,j)$ of size $L = M \times N$ into single array, $d(L,1)$ of size $L = M \times N$.
- Apply reduction function to reduce the size $d(L,1)$ to a value close to one of the phase vector values obtained by solving Lorenz or Rossler equations, $x(t)$.
- Obtained Lorenz or Rossler solutions, $X(t)$.
- Plot phase surfaces as $(x(t), y(t))$ or $(x(t), z(t))$ etc.
- Consider one of the phase vector say, $x(t)$, add unrecognizable image of single array reduced function $dd(L,1)$ to get a new phase vector $x^{'}(t)$. It is mandatory that the size of $dd(L,1)$ must be less than or equal to the size of original phase vector $x(t), x^{'}(t) = x(t) + dd(L,1)$.
- Plot phase surfaces $(x'(t), z(t))$ which must be the same as $(x(t), z(t))$.

- The new phase surfaces have encapsulated the unrecognizable image $p(i,j)$ under consideration (original image processed with Markov image).

3.3. Decryption.

- Read RGB encrypted image which was produced by $(x'(t), z(t))$.
- Extract the phase vector $x'(t)$ from the RGB encrypted image and obtained $dd(L,1)$ by subtracting $x(t)$ from $x'(t)$.
- Apply the inverse reduction function onto $dd(L,1)$ to get the original single array $d(L,1)$.
- Finally the single array $d(L,1)$ extended into unrecognizable matrix.

The matrix, $p(i,j)$, has embedded the original plain image through Markov processes. Having used the inverse Markov processes we have got back the original target matrix, $T(i,j)$. Care must be taken in transformations from single array to full matrix without the loss of data. Unlike our previous simple algorithm, this is more complicated non-linear algorithm which have embedded a Markov processed plain image or text in to the solutions of Lorenz or Rossler's equations in the form of reconstructed phase vectors.

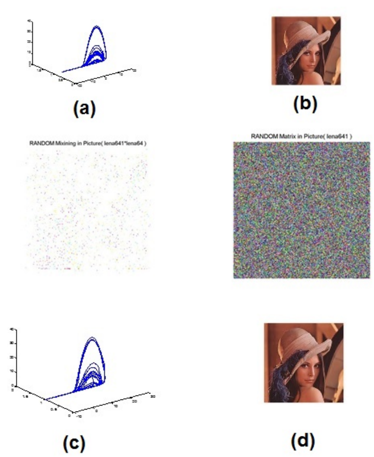

FIGURE 6. Rossler solutions Images and Encrypted Images

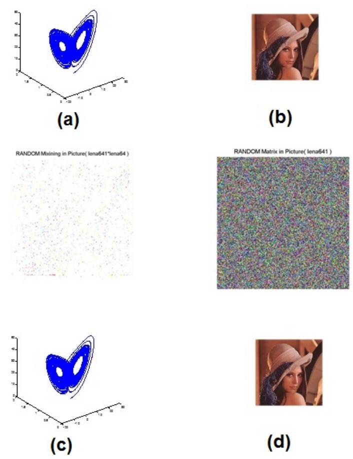

FIGURE 7. Lorenz solutions Images and Encrypted Images

4. NUMERICAL EXPERIMENT OF PROPOSED ALGORITHM

As an experiment, we have taken the famous benchmark pictures like Lena, Fruits and Vegetables of lower dimension (64×64) than usual dimension 512×512. We obtained numerical solutions of Lorenz and Rossler's equations in the form of time series phase vectors $X(t) = (x(t), y(t), z(t))$. The algorithm described in section 3 is applied on a phase vector $x(t)$ and encrypted all three pictures. we have also calculated PSNR for all sub steps given in Table 2.

5. PERFORMANCE ANALYSIS OF INDIVIDUAL ALGORITHMIC STEPS

In order to analyze the proposed algorithm, the conventional standards have been analyzed in detail in the following sub section.

5.1. Randomness Test for Cipher System. The essential part of this algorithm is a random test. In every crypto system, the security of the system must have some random tests, for instance, periodicity, uniform distribution, high intricacy and productivity. In order to fulfill the prerequisites, NIST SP 800-22 for testing the haphazardness of digital images is used. A number of tests are included in the testing tool but some of the tests are performed over the chosen digital images (Lena, Vegetable and fruit). The effects of these tests are reported in Table

2. It is shown in the table that our anticipated digital image encryption mechanism effectively pass the NIST tests. In the light of accomplished outcomes, the proposed random cipher in our encryption algorithm can be taken as irregular.

5.2. Histograms Uniformity of Cipher System. A histogram is an accurate representation of the distribution of numerical data which is an estimate of the probability distribution of a continuous variables introduced by Karl Pearson [17]. To construct a histogram (one variable only) different from bar graph are related two variables. The range of the values is divided the entire range of values into a series of intervals and then count the number of variables falls into each interval. This is called a bin, usually specified as a consecutive non overlapping intervals of a variable adjacent and usually of equal size. The variable bin width is usually displayed. Mathematically it is given as

$$n = \sum_{i=1}^{k} m_i \tag{12}$$

where n is total number of observations and k is the total number of bins, m_i is the histogram. The histogram of butterfly, embedded Lena into butterfly is given in Figure 8(a,b), similarly the histograms of solution of Rossler ODEs and Lena embedded into Rossler are given in Figure 8 (c,d). The histograms of embedded vegetable into Lorenz and Rossler's solutions are given in Figure 8 (e,f). The histograms showed that the embedding images into ODEs solutions have not disturbed the original solutions.

5.3. Pixels Correlation Test. This test is performed on adjoining pixels in horizontal, vertical and diagonal direction. To test the relationship between neighboring pixels in both plain and encrypted images, the correlation coefficients of each combined pairs are ascertained utilizing the following mathematical expression

$$\gamma_{x,y} = \frac{\sigma_{x,y}}{\sqrt{\sigma_x^2 \sigma_y^2}} \tag{13}$$

where x and y are values of two adjacent pixels at grey scale in the image. The correlation coefficients of a plain and cipher images have different contents shown in Table 1 which shows

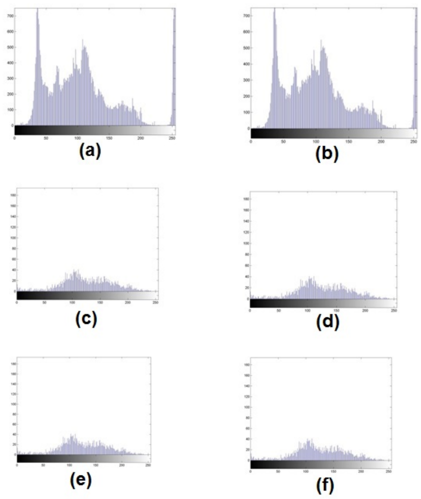

FIGURE 8. Histograms of Images and Encrypted Images

correlation distribution of original and encrypted images in horizontal vertical and diagonal directions.

5.4. PSNR Analysis. To evaluate the image quality based on pixels, we have to calculate PSNR and MSE value as follows

$$MSE = \frac{\sum_i^M \sum_j^N (P_{i,j} - C_{i,j})}{M \times N} \tag{14}$$

$$PSNR = 20 \log_{10} \frac{I_{MAX}}{\sqrt{MSE}} \tag{15}$$

where P and C are pixels situated at row and column location of a unique digital and encrypted image separately. High MSE is represented better encryption security. The MSE and PSNR for all three digital images are presented in Table 2.

5.5. Robustness. Having considered the RGB of Lena it was found that all three colours are appreciably embedded into Lorenz solutions. This experiment showed that our encryption scheme is quite robust and have given no clue to invaders to access or estimate any information from the encrypted images.

Standard Images	Plain	Plain	Plain	Encrypted (Proposed Scheme)
-	Horizontal	Vertical	Diagonal	Horizontal
Lena	0.9738	0.9866	0.9611	-0.0112
Fruits	0.9752	0.9757	0.9563	-0.0128
Vegetable	0.9748	0.9436	0.9259	-0.0107

Standard Images	Encrypted (Proposed Scheme)	Encrypted (Proposed Scheme)
-	Vertical	Diagonal
Lena	-0.0092	0.0026
Fruits	-0.0154	0.0011
Vegetable	-0.0140	0.0055

Standard Images	Reference	Reference	Reference
-	Horizontal	Vertical	Diagonal
Lena	0.0141	0.0107	0.0086
Fruits	-	-	-
Vegetable	-	-	-

TABLE 1. Correlation coefficients of plain and cipher images

Standard Images	MSE	PSNR
Lena	0.9923	0.9822
Fruits	0.9266	0.9677
Vegetable	0.9133	0.9233

TABLE 2. MSE and PSNR values of images

6. CONCLUSION

We have invented an idea of encryption technique based on solutions of non-linear ODEs. The idea was presented which used solutions of dynamical systems for image or text encryption. Solutions of dynamical systems like Lorenz and Rossler are highly non-linear parametric and dependent on initial conditions. These solutions behaves like series solutions of multiple equations, in which fixed and bifurcation points can be plotted accordingly. Solutions have become chaotic for certain parameter values. These values are categorized Lorenz or Rossler Attractors. These Attractors have effectively used for encryption purposes. Earlier, we have used these Attractors for image encryption through data manipulations. We have considered three benchmark images to be encrypted in the proposed encryption technique. These images are Lena, Vegetable and Fruit.

Having obtained the chaotic solutions of Lorenz and Rossler's equations for specific initial conditions, images of Lena, Vegetable and Fruit are first confused / diffused with Markov processes and then placed in Lorenz and Rossler solutions. Unlike our previous work the encrypted images of all three target pictures are then placed into one of the solutions of Lorenz phase vector say, $x(t)$. This embedding has been done by simply converted the encrypted images into single array say, $x'(t)$ of any RGB values of the same length as $x(t)$. The embedded chaotic solutions of Lorenz and Rossler are then plotted with this new extended variable, $x'(t)$. All three encrypted images are placed and plotted the solutions in figures. We have concluded that Lorenz solutions are more suitable over Rossler solutions, therefore, further experiments and analysis have been carried out with Lorenz chaotic solutions only. Original images, encrypted images and embedded images are given in figures. A thorough statistical and data analysis together with histograms of both original images, encrypted images and embedded images are given tables.

The two famous non-linear ODEs Lorenz and Rosseler's equations are considered. These equations are solved for a stable solution in the form of known figures as butterfly and water fall. The three bench mark images Lena, vegetable and fruit are embeded into solution surfaces of Lorenz and Rossler. Having considered all three colors (RGB) of such images, it was found that all three colors of images have behaved equally in embedding.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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