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NEW CLASS OF SECOND ORDER ROTATABLE DESIGNS USING BALANCED TERNARY DESIGNS

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Abstract: In this paper, following works of Tyagi and Rizwi (1979), Kanna et al. (2018), a new class of second order rotatable designs (SORD) using balanced ternary designs are suggested. Here we develop a method of balanced ternary designs by combining the methods of Tyagi and Rizwi. Here we observed that the number of design point is lesser than the other designs. Table 3 provides the complete idea regarding the smaller number of designs points when we compare with the balanced incomplete block designs, Tyagi and Rizwi design points.

Keywords: second order rotatable designs; balanced incomplete block designs (BIBD); balanced ternary designs.

2010 AMS Subject Classification: 62K10.

1. INTRODUCTION

The concept of rotatability was proposed by Box and Hunter (1957). Das and Narasimham (1962) constructed second order rotatable designs through balanced incomplete block designs.

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Tyagi (1964) constructed second order rotatable designs through pairwise balanced designs.

In this paper, following the works of Tyagi and Rizwi (1979), Kanna et.al (2018), second order rotatable designs though balanced ternary design is suggested. Here we develop a method balanced ternary designs by combining both Tyagi and Rizwi, Kanna et.al and also observed the variance function of estimated response for different parameters of balanced ternary designs.

2. PRELIMINARIES

Method of construction of Second Order Rotatable Designs (SORD) using BIBD (cf. Box and Hunter (1957), Das and Narasimham (1962)):

A second order response surface as D = ((X_{iu})) for fitting

$$y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum\sum_{i \leq j}^{v} b_{ij} x_{iu} x_{ju} + e_u \tag{2.1}$$

where x_{iu} denotes the level of the i^{th} factor (i=1,2, …,v) in the u-th run (u=1,2, …, N=2n) of the experiment, e_u's are uncorrelated random errors is said to be a SORD, if the variance of the estimated response of $\widehat{Y}_u$ from the fitted surface is only a function of the distance, $(d^2 = \sum x_i^2)$ of the point $(x_{1u}, x_{2u}, \dots, x_{vu})$ from the origin (centre) of the design.

Let (v,b,r,k,λ) denote a BIBD, $2^{t(k)}$ denote a fractional replicate of 2^k in +1 and -1 levels, in which no interaction with less than five factors is confounded. [1-(v,b,r,k,λ)] denote the design points generated from the transpose of incidence matrix of BIBD. [1-(v,b,r,k,λ)] $2^{t(k)}$ are the $b2^{t(k)}$ design points generated from the BIBD by "multiplication" (cf. Raghavarao (1971), pp. 298-300), (a,0,0, …,0) 2^1 denote the design points generated from (a,0,0, …,0) point set, and ∪ denotes combination of the design points generated from different sets of points. n_0 denote the number of central points.

Theorem (i): when r<3λ

The design points, [1-(v,b,r,k,λ)] $2^{t(k)}$ ∪ (a,0,0, …,0) 2^1∪ (n_0) will give a v-dimensional SORD in N design points, with $a^4 = (3\lambda - r)2^{t(k)-1}$.

Proof: For the design points generated from the BIBD, simple symmetry conditions are true.

Further we have the reduced conditions,

$$\sum_{u=1}^{N} x_{iu}^{2} = r2^{t(k)} + 2a^{2} = \text{ constant} = N\lambda_2 \tag{2.2}$$

$$\sum_{u=1}^{N} x_{iu}^{4} = r2^{t(k)} + 2a^{4} = \text{constant} = 3N\,\lambda_4, \tag{2.3}$$

$$\sum_{u=1}^{N} x_{iu}^{2}x_{ju}^{2} = \lambda 2^{t(k)} = \text{constant} = N\lambda_4, \tag{2.4}$$

From (2.3) and (2.4) we get $a^4 = (3\lambda - r)2^{t(k)-1}$.

where, N= $b2^{t(k)} + 2v$.

Substitute 'a' value in (2.2) and (2.3), we get the λ_2 and λ_4 values.

Theorem (ii): when r=3λ

The design points [1-(v, b, r, k, λ)] $2^{t(k)} \cup$ (n_0) will give a v-dimensional SORD in N design points.

Proof: For the design points generated from the BIBD, simple symmetry conditions are true. Further we have

$$\sum_{u=1}^{N} x_{iu}^{2} = r2^{t(k)} = \text{ constant} = N\lambda_2 \tag{2.5}$$

$$\sum_{u=1}^{N} x_{iu}^{4} = r2^{t(k)} = \text{constant} = 3\,N\lambda_4, \tag{2.6}$$

$$\sum_{u=1}^{N} x_{iu}^{2}x_{ju}^{2} = \lambda 2^{t(k)} = \text{constant} = N\lambda_4, \tag{2.7}$$

where, N = $b2^{t(k)}$.

From (2.5), (2..6) and (2.7) we get λ_2, λ_4 values.

Theorem (iii): when r>3λ

The design points, [1-(v,b,r,k,λ)] $2^{t(k)} \cup$ (a,a,a, …,a) $2^{t(v)} \cup$ (n_0) will give a v-dimensional SORD in N design points, with $a^4 = (r - 3\lambda)2^{t(k)-t(v)-1}$.

Proof: For the design points generated from the BIBD, simple symmetry conditions are true. Further we have

$$\sum_{u=1}^{N} x_{iu}^{2} = r2^{t(k)} + 2^{t(v)}a^{2} = \text{ constant} = N\lambda_{2} \tag{2.8}$$

$$\sum_{u=1}^{N} x_{iu}^{4} = r2^{t(k)} + 2^{t(v)}a^{4} = \text{constant} = 3N\,\lambda_{4}, \tag{2.9}$$

$$\sum_{u=1}^{N} x_{iu}^{2}x_{ju}^{2} = \lambda 2^{t(k)} + 2^{t(v)}a^{4} = \text{constant} = N\lambda_{4}, \tag{2..10}$$

where, N= $b2^{t(k)} + 2^{t(v)}$.

From (2.9) and (2.10) we get $a^{4} = (r - 3\lambda)2^{t(k)-t(v)-1}$.

Substitute ‘a’ value in (2.8) and (2.9), we get the λ_2 and λ_4 values.

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{v+2}(\text{Non – singularity condition}) \tag{2.11}$$

The variance and covariance of the estimated parameters are

$$V(\hat{b}_0) = \frac{(v+2)\lambda_4}{N[(v+2)\lambda_4 - v\lambda_2^2]}\sigma^2$$

$$V(\hat{b}_i) = \frac{1}{N\lambda_2}\sigma^2$$

$$V(\hat{b}_{ij}) = \frac{1}{N\lambda_4}\sigma^2$$

$$V(\hat{b}_{ii}) = \frac{1}{2N\lambda_4}\left[\frac{(v+1)\lambda_4 - (v-1)\lambda_2^2}{(v+2)\lambda_4 - v\lambda_2^2}\right]\sigma^2$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = -\frac{\lambda_2}{N[(v+2)\lambda_4 - v\lambda_2^2]}\sigma^2$$

$$Cov(\hat{b}_{ii}, \hat{b}_{jj}) = \frac{(\lambda_2^2 - \lambda_4)}{2N\lambda_4\,[(v+2)\lambda_4 - v\lambda_2^2]}\sigma^2 \tag{2.12}$$

and other co-variances vanish.

Further,

$$V(\hat{y}) = V(\hat{b}_0) + [V(\hat{b}_i) + 2Cov(\hat{b}_0, \hat{b}_{ii})]d^2 + V(\hat{b}_{ii})d^4 \tag{2.13}$$

3. MAIN RESULTS

Construction of second order rotatable designs through Balanced Ternary Designs (c.f. Tyagi and Rizwi (1979))

Let N_1 be the incidence matrix of balanced incomplete block design with parameters (v, b, r, k, λ) (assume $9\lambda^2 \geq r\lambda(v-1)$) and let $N_2 = I_v$, where I_v is the identity matrix of order v. The balanced ternary designs are derived by adding the elements of j^{th} row of N_2 those rows of N_1 which contain unity in the j^{th} column, then vr blocks so formed constitute a balanced Ternary Designs with parameters V=v, B= vr, R=(k+1)r, K=k+1 and $\pi = \lambda(k+2)$.

Replace the elements 2 with α and 1 with β, then associate each block with an appropriate fraction of factorials (say $2^{t(k)}$) with levels ±1 such that no lower order interaction effects are confounded. Add n_0 $(n_0 > 0)$ central design points (0, 0, … , 0) to the resulting design, then total number of design points are

$$N = vr\ 2^{t(k)} + n_0.$$

For the design points generated from the balanced ternary designs, simple symmetry conditions (2.2) to (2.6) are true. Further, from (2.2) to (2.6), we have,

$$\sum_{u=1}^{N} x_{iu}^2 = 2^{t(k_1)}(r\alpha^2 + \lambda(v-1)\beta^2) = \text{constant} = N\lambda_2 \tag{3.1}$$

$$\sum_{u=1}^{N} x_{iu}^4 = 2^{t(k_1)}(r\alpha^4 + \lambda(v-1)\beta^4) = \text{constant} = 3N\lambda_4 \tag{3.2}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 2^{t(k_1)}(2\lambda\alpha^2\beta^2) = \text{constant} = N\lambda_4 \tag{3.3}$$

From (3.2) and (3.3), we can obtain

$$r\alpha^4 + \lambda(v-1)\beta^4 - 6\lambda\alpha^2\beta^2 = 0. \tag{3.4}$$

Let t = α^2/β^2, then (3.4) can be expressed in the quadratic form as

$$r\,t^2 - 6\,\lambda t + \lambda(v-1) = 0 \tag{3.5}$$

Solve (3.5), we get t and choose any real value for β, then the real value of α can be obtained.

Substituting α^2, β^2 in (3.1) and (3.3), we get λ_2 and λ_4.

The variance of estimated response for different parameters of balanced ternary design is calculated. The numerical calculations are appended in the Appendix in Table 1.

Example:

Consider a balanced incomplete block design (BIBD) with parameters (4, 4, 3, 3, 2). Let N_1 be the incidence matrix of balanced incomplete block design with parameters ($v = 4, b = 4, r = 3, k = 3, \lambda = 2$) and $N_2 = I_v$.

Plan of BIBD

1	2	3
1	2	4
1	3	4
2	3	4

Incidence Matrix N_1

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

N_2 is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resulting balanced ternary design is with parameters

$(V = 4, B = 12, R = 12, K = 4, \pi = 10)$.

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

Replace the elements 2 with α and 1 with β,

$$\begin{bmatrix} \pm\alpha & \pm\beta & \pm\beta & 0 \\ \pm\alpha & \pm\beta & 0 & \pm\beta \\ \pm\alpha & 0 & \pm\beta & \pm\beta \\ \pm\beta & \pm\alpha & \pm\beta & 0 \\ \pm\beta & \pm\alpha & 0 & \pm\beta \\ 0 & \pm\alpha & \pm\beta & \pm\beta \\ \pm\beta & \pm\beta & \pm\alpha & 0 \\ \pm\beta & 0 & \pm\alpha & \pm\beta \\ 0 & \pm\beta & \pm\alpha & \pm\beta \\ \pm\beta & \pm\beta & 0 & \pm\alpha \\ \pm\beta & 0 & \pm\beta & \pm\alpha \\ 0 & \pm\beta & \pm\beta & \pm\alpha \end{bmatrix}$$

From (3.1), (3.2) and (3.3), we have

$$\sum_{u=1}^{N} x_{iu}^2 = 8\,(3\alpha^2 + 6\beta^2) = N\lambda_2 \tag{3.6}$$

$$\sum_{u=1}^{N} x_{iu}^4 = 8\,(3\alpha^4 + 6\beta^4) = 3N\lambda_4 \tag{3.7}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 8\,(4\alpha^2\beta^2) = N\lambda_4 \tag{3.8}$$

From (3.7) and (3.8), we can obtain

$$3\alpha^4 + 6\beta^4 - 12\alpha^2\beta^2 = 0. \tag{3.9}$$

Let t = α^2/β^2, then (3.9) can be expressed in the quadratic form as

$$3t^2 - 12t + 6 = 0 \tag{3.10}$$

Solve (3.10), we get t=2.4142 and t=-0.4142

For t=2.4142 and β^2=1, then get $\alpha^2 = 2.4142$

Substituting α^2, β^2 in (3.6) and (3.8), we get $\lambda_2 = 1.09217$ and $\lambda_4 = 0.79644$.

For $n_0 = 1$, non-singularity condition (2.11) is also satisfied.

The variance of estimated response is given by

$$V(\hat{y}) = V(\hat{b}_0) + [V(\hat{b}_i) + 2\mathrm{Cov}(\hat{b}_0, \hat{b}_{ii})]d^2 + V(\hat{b}_{ii})d^4$$

$$= 6.79176\,\sigma^2 - 1.87957\,\sigma^2 d^2 + 3.55320\,\sigma^2 d^4.$$

4. New Class of SORD Using Balanced Ternary Designs

Following Tyagi and Rizwi (1979), Kanna et.al. (2018), construction of second order rotatable designs (SORD) through balanced ternary designs studied.

Let N_1 be the incidence matrix of balanced incomplete block design with parameters (v, b, r, k, λ) (r < 3λ) and let $N_2 = 2I_v$, where I_v is the identity matrix of order v. the balanced ternary designs are derived by adding the elements of j^{th} row of N_2 those rows of N_1 which contain zero in the j^{th} column, then v(b-r) blocks so formed constitute Balanced Ternary Designs with parameters V = v, B = v(b-r), R=(k+2)(b-r), K=k+2 and π = (r-λ)(k+3).

Replace the elements 2 with α and 1 with β, then associate each block with an appropriate fraction of factorials (say $2^{t(k)}$) with levels ±1 such that no lower order interaction effects are confounded. Add n_0 $(n_0 > 0)$ central design points (0, 0, … , 0) to the resulting design, then total number of design points are

$$N = v(b-r)\ 2^{t(k)} + n_0$$

For the design points generated from the balanced ternary design, simple symmetry conditions (2.2) to (2.6) are true. Further, from (2.2) to (2.6), we have,

$$\sum_{u=1}^{N} x_{iu}^2 = 2^{t(k_1)}((b-r)\alpha^2 + (r-\lambda)(v-1)\beta^2) = \text{constant} = N\lambda_2 \tag{4.1}$$

$$\sum_{u=1}^{N} x_{iu}^4 = 2^{t(k_1)}((b-r)\alpha^4 + (r-\lambda)(v-1)\beta^4) = \text{constant} = 3N\lambda_4 \tag{4.2}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 2^{t(k_1)}(\lambda(v-k)\beta^4 + (b-\lambda)\alpha^2\beta^2) = \text{constant} = N\lambda_4 \tag{4.3}$$

From (4.2) and (4.3), we can obtain

$$(b-r)\alpha^4 + \{(r-\lambda)(v-1) - 3\lambda(v-k)\}\beta^4 - 3(b-\lambda)\alpha^2\beta^2 = 0. \tag{4.4}$$

Let t = α^2/β^2, then (4.4) can be expressed in the quadratic form as

$$(b-r)\,t^2 - 3(b-\lambda)t + \{(r-\lambda)(v-1) - 3\lambda(v-k)\} = 0 \tag{4.5}$$

Solve (4.5), we get t and choose any real value for β, then the real value of α can be obtained.

Substituting α^2, β^2 in (4.1) and (4.3), we get λ_2 and λ_4.

The variance of estimated response for different parameters of balanced ternary design is calculated. The numerical calculations are appended in the Appendix in Table 2.

Example:

Let N_1 be the incidence matrix of balanced incomplete block design with parameters $(v = 4, b = 4, r = 3, k = 3, \lambda = 2)$ and $N_2 = 2I_v$.

Plan of BIBD

1	2	3
1	2	4
1	3	4
2	3	4

Incidence Matrix N_1

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

N_2 is

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

The resulting balanced ternary design is with parameters

$(V = 4, B = 4, R = 5, K = 5, \pi = 6)$.

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \pm\alpha & \pm\beta & \pm\beta & \pm\beta \\ \pm\beta & \pm\alpha & \pm\beta & \pm\beta \\ \pm\beta & \pm\beta & \pm\alpha & \pm\beta \\ \pm\beta & \pm\beta & \pm\beta & \pm\alpha \end{bmatrix}$$

From (4.1), (4.2) and (4.3), we have

$$\sum_{u=1}^{N} x_{iu}^2 = 8\,(\alpha^2 + 3\beta^2) = N\lambda_2 \tag{4.6}$$

$$\sum_{u=1}^{N} x_{iu}^4 = 8\,(\alpha^4 + 3\beta^4) = 3N\lambda_4 \tag{4.7}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 8\,(2\beta^4 + 2\alpha^2\beta^2) = N\lambda_4 \tag{4.8}$$

From (4.7) and (4.8), we can obtain

$$\alpha^4 - 3\beta^4 - 6\alpha^2\beta^2 = 0. \tag{4.9}$$

Let t = α^2/β^2, then (4.9) can be expressed in the quadratic form as

$$t^2\text{-}6t\text{-}3 = 0 \tag{4.10}$$

Solve (4.10), we get t=6.4641 and t=-0.4641

For t=6.4641 and β^2=1, then$\alpha^2 = 6.4641$.

Substituting α^2, β^2 in (4.6) and (4.8), we get $\lambda_2 = 2.2943$ and $\lambda_4 = 3.6190.$

For $n_0 = 1$, non-singularity condition (2.11) is also satisfied.

The variance of estimated response is given by

$$V(\hat{y}) = V(\hat{b}_0) + [V(\hat{b}_i) + 2Cov(\hat{b}_0, \hat{b}_{ii})]d^2 + V(\hat{b}_{ii})d^4$$

$$= 1.0000\sigma^2 - 0.1981\sigma^2 d^2 + 0.0147\sigma^2 d^4.$$

APPENDIX

Table 1:

Balanced Incomplete Block Design	Balanced Ternary Design	n_0	N	$V(\hat{y})$
(3, 3, 2, 2, 1)	(3, 6, 6, 3, 4)	1	25	$0.9999\sigma^2 - 0.5182\sigma^2 d^2 + 0.0955\sigma^2 d^4$
(4, 4, 3, 3, 2)	(4, 12, 12, 4, 10)	1	97	$6.7918\sigma^2 - 1.8796\sigma^2 d^2 + 3.5532\sigma^2 d^4$
(4, 6, 3, 2, 1)	(4, 12, 9, 3, 4)	1	49	$1\sigma^2 - 0.9583\sigma^2 d^2 + 0.3021\sigma^2 d^4$
(5, 5, 4, 4, 3)	(5, 20, 20, 5, 18)	1	321	$0.9990\sigma^2 - 0.1129\sigma^2 d^2 + 0.2463\sigma^2 d^4$

Table 2:

Balanced Incomplete Block Design	Balanced Ternary Design	n_0	N	V(ŷ)
(3, 3, 2, 2, 1)	(3, 3, 4, 4, 5)	1	13	$1.0000\sigma^2 - 0.2144\sigma^2d^2 + 0.0225\sigma^2d^4$
(4, 4, 3, 3, 2)	(4, 4, 5, 5, 6)	1	33	$1.0000\sigma^2 - 0.1981\sigma^2d^2 + 0.0147\sigma^2d^4$
(4, 6, 3, 2, 1)	(4, 12, 12, 4, 10)	1	49	$1.0000\sigma^2 – 0.1206\sigma^2d^2 + 0.2836\sigma^2d^4$
(5, 5, 4, 4, 3)	(5, 5, 6, 6, 7)	1	81	$1.0002\sigma^2 - 0.1804\sigma^2d^2 + 0.0103\sigma^2d^4$
(5, 10, 6, 3, 3)	(5, 20, 20, 5, 18)	0	160	$0.0885\sigma^2 - 0.0156\sigma^2d^2 + 0.0023\sigma^2d^4$
(6, 6, 5, 5, 4)	(6, 6, 7, 7, 8)	1	97	$1.0000\sigma^2 - 0.1615\sigma^2d^2 + 0.0085\sigma^2d^4$
(6, 10, 5, 3, 2)	(6, 30, 25, 5, 18)	0	240	$0.0307\sigma^2 - 0.0036\sigma^2d^2 + 0.0016\sigma^2d^4$
(7, 7, 4, 4, 2)	(7, 21, 18, 6, 14)	0	336	$0.0273\sigma^2 - 0.0030\sigma^2d^2 + 0.0011\sigma^2d^4$
(8, 14, 7, 4, 3)	(8, 56, 42, 6, 28)	0	896	$0.0071\sigma^2 - 0.0003\sigma^2d^2 + 0.0005\sigma^2d^4$

Table 3: Comparison of design points

BIBD			BTD (Tyagi & Rizwi)			BTD (New Class)		
Parameters	n_0	N	Parameters	n_0	N	Parameters	n_0	N
(3, 3, 2, 2, 1)	0	18	(3, 3, 2, 2, 1)	1	25	(3, 3, 2, 2, 1)	1	13
(4, 4, 3, 3, 2)	0	40	(4, 4, 3, 3, 2)	1	97	(4, 4, 3, 3, 2)	1	33
(4, 6, 3, 2, 1)	0	32	(4, 12, 9, 3, 4)	1	49	(4, 12, 12, 4, 10)	1	49
(5, 5, 4, 4, 3)	0	90	(5, 5, 4, 4, 3)	1	49	(5, 5, 4, 4, 3)	1	81

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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