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ECONOMIC ORDER QUANTITY MODEL FOR A PRODUCT WITH EXPIRATION DATE, LIMITED SHELF SPACE AND FRESHNESS DEPENDENT SELLING PRICE UNDER THE EFFECT OF TRADE CREDIT FINANCING

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Abstract. In practise, supplier offers the retailer a credit/delay period for settling the account and no interest is charged on the outstanding account if the account is settled by the end of the delay period. With the consideration of trade credit, we develop an economic order quantity model for a product with expiration date and freshness dependent selling price. Objectives of our model are twofold. The initial one is the consideration of the circumstance that the demand of the product is dependent upon the freshness condition as well as selling price of the product and the last one is relaxation of inventory level at the end of cycle. The solution procedure of proposed optimization model is illustrated analytically and numerically by a couple of examples. Concavity of the average profit function is shown by plotting graphs. To study the effect of fluctuating the value of all parameters in the proposed maximization model a sensitivity analysis is carried out.

Keywords: inventory; trade credit; freshness dependent selling price; limited shelf space; freshness dependent demand; price dependent demand.

2010 AMS Subject Classification: 91B02.

1. INTRODUCTION

In the conventional economic order quantity model, to encourage sales and diminish inventory, seller generally offers to his/her customer a delay in payments. There is no interest charge if the

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unpaid amount is paid with in this permitted delay period. However, if the payment is unpaid in full by termination of the allowable delay period, interest is charged on the unpaid amount. This policy is acknowledged as trade credit policy. Goyal [1] was the inventor who merged this policy in his inventory model with constant demand. Later Goyal [1], many research- works in this field have been jumping up. Considering deteriorating product Aggarwal & Jaggi [2] stretched Goyal's model. Again Aggarwal & Jaggi's model was generalised by Jamal et al. [3] by considering shortages. Many researchers like Teng ang Goyal [4], De and Goswami [5], Taleizadeh et al. [6], Chang et al. [7], Zhang et al. [8] developed their respective inventory models by considering different type of trade credit policy (partial or full or two level etc.). In this projected work we incorporate trade credit policy to promote sales and decrease inventory level.

Most of the products of grocery shop, dairy industry, medicine shop, alcohol shop etc., are perishable and they have certain expiration date. At the end of life time/ expiration date, the product spoils totally and has no utility for consumers. In this work, we formulate and analyse an inventory model with demand decreasing in the age of the product. The model is motivated by the increased customers interest in fresh product in the aforesaid sector. It is commonly known from observation and empirical study in marketing (Tsiros and Heliman [9]) that, product becomes less attractive for consumer when they lose their freshness. To best of our knowledge, there are only a few papers in the perishable inventory literature that take into account the decreasing effectiveness of perishable goods throughout their life time. Fujiwara and Perera [10] was the pioneer who considered decreasing utility of perishable goods linked with lifetime. However, they use a constant demand rate. A multiitem production model for perishable products having age dependent demand was studied by Amorim et al. [11]. Chen et al. [12] developed an inventory model with positive inventory level at the end of cycle with stock dependent and linearly decreasing demand function with age of the product. Dobson et al. [13] formulated an EOQ model with deterministic life time, using an only age dependent demand function that declines linearly with age of the inventory until it vanishes.

It is obvious that the lesser price, the higher the demand. Therefore, price is an important factor in a consumer's purchasing decision. Buyers like to buy from a shop which has less selling price. If the retailer rises the retailing price of the product, the consumers would move other shopping places to fulfil their demand. As a result, demand for perishable goods is influenced by the combined effect of selling price and product freshness. There are numerous research works have been done on the effect of price variations. Kotler [14] incorporated marketing strategies into inventory decisions and debated the relationship between economic order quantity and pricing decision. Ladany and Sterleib [15] studied the effect of selling price variation on EOQ. Goyal & Gunasekaran [16], Bhunia and Shaikh [17] developed EOQ models considering the effect of price variations on EOQ. Ranganayaki et al. [18] invented an inventory model with demand dependent on price under fuzzy environment.

Another major factor of demand is variable selling price. Alturki and Alfares [19] developed a warehouse selection model with time dependent selling price. Generally, demand of a product decreases with increase in selling-price and vice versa. Also, lifetime of such products affects the selling-price. To promote to retail products of short life retailer uses low selling price for short life products. Although in this field not so many research papers have been published. Iqbal and Sarkar [20] developed a supply chain model with linearly life time dependent selling price. In this study, we consider that selling price has a reverse relationship with the freshness of the product to increase the demand of the product remaining in stock.

The current work is established under the subsequent considerations: i) selling price and freshness sensitive demand, after exceeding life time there is no demand i.e., cycle time should be less than life time of the product ii) selling price is dependent upon the freshness of the product iii) permissible delay in payments for the retailer iv) inventory level at the end of cycle may be positive or zero v) limited shelf space of the retailer. Our goal is to determine the maximum profit of this model. The following part of the paper is designed to organize as cited. The assumptions and notations of the model are presented in section 2. Following the section 3, we have established a mathematical optimization problem of this model. In section 4, theoretical result for optimality of the profit function is presented. In section 5, we give numerical solution procedure and algorithm for the proposed model. In section 6, some numerical examples and graphical representation are carried out. The sensitivity analysis is recorded in section 7. In the last, we conclude and give some future research scope in section 8.

2. Assumptions and Notations

In order to develop the projected model, the following assumptions and symbolizations are used all over this paper.

2.1 Assumptions:

i) Single perishable item inventory model is considered. We assume that there will be no deterioration of the product before reaching its expiration date.

ii) The replenishment rate is infinite and lead time is zero.

iii) Shortages are not allowed

iv) Selling price p of the product depends upon age of the product i.e., $p = p_0\left(1 - \frac{t}{L}\right)$, where p_0 & L are initial selling price and life-time of the product respectively.

v) The demand rate D of this model is assumed to be a function of selling price and freshness or age of the product i.e., $D = (\alpha - \beta p)\left(1 - \frac{t}{L}\right)$, where $\alpha, \beta > 0$. At time $t = 0$, the product is fresh and there is no age effect on the demand. Then the product loses its freshness with time, so the demand for the product decreases. After reaching the maximum life-time of the product demand becomes zero.

vi) Positive or zero inventory level is allowed at end of the cycle and the remaining inventory is disposed of.

vii) Under a trade credit period $M,$ retailer would settle the account at time $t = M$ and pay for interest charges on items in stock with rate I_p over the interval $[M, T]$ as $T \geq M$. On the other hand, when the retailer settles the account at time $t = M$, it is not required to pay any interest charge for items in stock throughout the entire cycle as $T \leq M$.

viii) The retailer can collect and receive interest from the commencement of the inventory cycle until the termination of the trade credit period offered by the supplier. That is, the retailer can collect revenue and earn interest during the interval $[0. M]$ with interest rate I_e under trade credit conditions.

2.2 Notations:

i) K: replenishment cost per cycle

ii) α: constant part of the demand rate$(\alpha > 0)$

iii) β: price dependent demand rate parameter $(\beta > 0)$

iv) p: selling price per unit item

v) p_0: initial selling price per unit item

vi) L: life /expiration time of the product

vii) W: maximum shelf space of the retailer

viii) C_h: holding cost for per unit item per unit time

ix) C_p: purchase cost per unit item

x) Q: number of ordering quantity per replenishment cycle

xi) q: stock level at the end of cycle

xii) s: salvage value of the per unit disposed product

xiii) M: retailer's trade credit period offered by the supplier

xiv) I_e: interest earned from the customer per dollar per unit time

xv) I_p: interest paid per dollar in stocks per unit time to the supplier.

xvi) $I(t)$: Inventory level at time t

xvii) AP: average profit per unit time

Decision Variables

i) T: Replenishment time per cycle, it should be less than product life i.e., $T \leq L$

ii) q: inventory level at the end of cycle $(q \geq 0)$.

3. Mathematical Model Formulation

The aim of the model is to find out the maximum profit for items taking above mentioned demand. The level of inventory depletes as a result of demand.

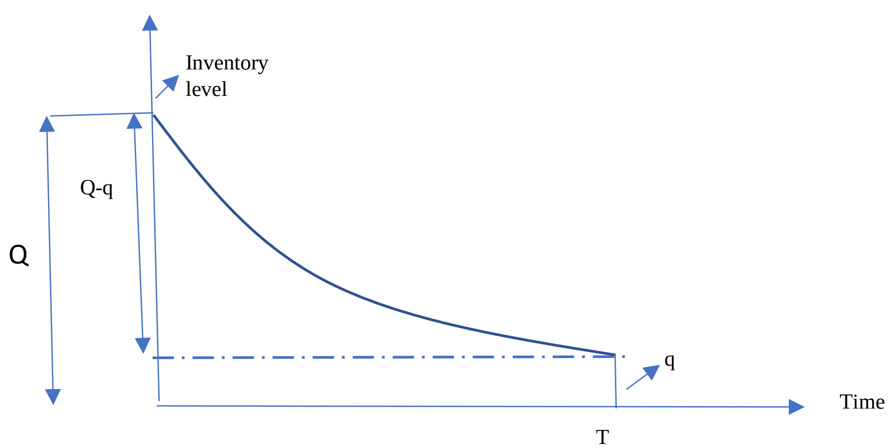

Fig 1. Pictorial representation of the inventory system

The governing differential equation is as follows:

$$\frac{dI(t)}{dt} = -(\alpha - \beta p)\left(1 - \frac{t}{L}\right), \qquad 0 \leq t \leq T \leq L$$

$$\Rightarrow \frac{dI(t)}{dt} = -(\alpha - \beta p_0(1 - t/L))\left(1 - \frac{t}{L}\right), \qquad 0 \leq t \leq T \leq L \tag{1}$$

With boundary conditions $I(T) = q \geq 0$ and $I(0) = Q \leq W$.

The solution of equation (1) is

$$I(t) = A(T-t) - \frac{B}{2}(T-t)^2 - \frac{C}{3}(T-t)^3 + q \tag{2}$$

Where $A = (\alpha - \beta p_0)$, $B = \frac{(\alpha - 2\beta p_0)}{L}$, $C = \frac{b p_0}{L^2}$

Using the initial condition $I(0) = Q$, from equation (2) we get

$$Q = AT - \frac{B}{2}T^2 - \frac{C}{3}T^3 + q \tag{3}$$

Based on the above expressions and assumptions, the profit function in each cycle consists following terms:

Total sales revenue (SR) per cycle $= \int_0^T p\,(\alpha - \beta p)\left(1 - \frac{t}{L}\right) dt$

$$= p_0 \left\{AT - \left(\frac{A}{L} + B\right)\frac{T^2}{2} + \left(\frac{B}{L} - C\right)\frac{T^3}{3} + \frac{C}{L}\frac{T^4}{4}\right\} \tag{4}$$

Replenishment cost (RC) per cycle$=K$ (5)

Total inventory cost (IHC)per cycle$=C_h \int_0^T I(t)dt = C_h\left[A\frac{T^2}{2} - B\frac{T^3}{3} - C\frac{T^4}{4} + qT\right]$ (6)

Total purchase cost (PC) per cycle $=C_p Q = C_p\left[AT - \frac{B}{2}T^2 - \frac{C}{3}T^3 + q\right]$ (7)

Total salvage value for the disposed product (SV) per cycle$=sq$ (8)

Earned and payable interest:

For calculating earned and payable interest, there exists two potential cases:

Case 1. $M \le T$

Since the credit period M is shorter than or equal to the cycle time T, so the retailer begins to pay interest for the items in stock after time M with rate I_p. Therefore, interest payable per cycle is

$$IP_1 = C_p I_p \int_M^T I(t)dt$$

$$= C_p I_p\left[A\frac{T^2}{2} - ATM + A\frac{M^2}{2} - B\frac{T^3}{3} + B\frac{T^2}{2}M - \frac{B}{2}\frac{M^3}{3} - C\frac{T^4}{4} + C\frac{T^3}{3}M - \frac{C}{3}\frac{M^4}{4} + q(T-M)\right] \tag{9}$$

Since $M \le T$, so the retailer can earn interest in the interval$[0, M]$(see Fig 2)

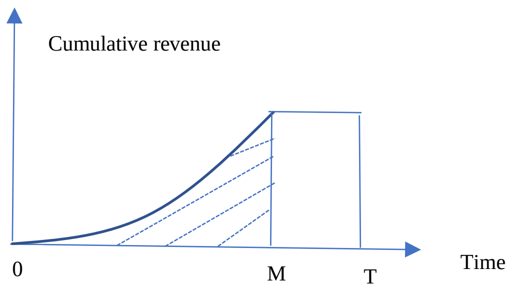

Fig 2. Total accumulation of interest earned when $M \le T$

Therefore, interest earned per cycle is

$$IE_1 = I_e \int_0^M pDdt = p_0 I_e \left[A\frac{M^2}{2} - \left(\frac{A}{L}+B\right)\frac{M^3}{3} + \left(\frac{B}{L}-C\right)\frac{M^4}{4} + \frac{C}{5L}M^5\right] \quad (10)$$

Case 2. $M \geq T$

Since the cycle time is shorter than or equal to credit period M, so the retailer need not to pay any interest to the supplier.

Therefore, interest payable per cycle is $IP_2 = 0$ (11)

Since $M \geq T$, on the interval $[0, T]$ retailer sells items and continuous to accumulate the sales revenue to earn interest with rate I_e, and on the interval $[T, M]$, the retailer can use all the sales revenue to earn interest with rate I_e(see Fig 3).

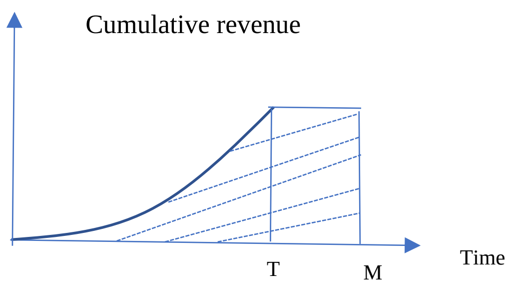

Fig 3. Total accumulation of interest earned when $M \geq T$

Therefore, interest earned per cycle is $IE_2 = I_e \int_0^T pDtdt + I_e \left(\int_0^T pDdt\right)(M-T)$

$$= p_0 I_e \left[AT\left(M-\frac{T}{2}\right) - \left(\frac{A}{L}+B\right)\frac{T^2}{2}\left(M-\frac{T}{3}\right) + \left(\frac{B}{L}-C\right)\frac{T^3}{3}\left(M-\frac{T}{4}\right) + \frac{C}{L}\frac{T^4}{4}\left(M-\frac{T}{5}\right)\right] \quad (12)$$

So for case 1($M \leq T$), average profit($AP_1(q,T)$) per unit time is $=\frac{1}{T}(SR + SV + IE_1 - RC - IHC - PC - IP_1)$

$$= \frac{1}{T}\begin{bmatrix} A\left(p_0 I_e - C_p I_p\right)\frac{M^2}{4} - \left\{\frac{p_0 I_e}{3}\left(\frac{A}{L}+B\right) - B\frac{C_p I_p}{6}\right\}M^3 + \left\{\frac{p_0 I_e}{4}\left(\frac{B}{L}-C\right) - C\frac{C_p I_p}{12}\right\}M^4 + C\frac{p_0 I_e}{5L}M^5 + \left(s - C_p + M\right)q - K + \\ T\left\{A\left(p_0 - C_p - C_p I_p M\right) - \left(C_h + C_p I_p\right)q\right\} + \frac{T^2}{2}\left\{B\left(C_p - C_p I_p M - p_0\right) - A\left(\frac{p_0}{L} + C_h\right)\right\} \\ + \frac{T^3}{3}\left\{B\left(\frac{p_0}{L} + C_h + C_p I_p\right) + CC_p\left(1 - I_p M\right)\right\} + \frac{T^4}{4}C\left\{\frac{p_0}{L} + C_h + C_p I_p\right\} \end{bmatrix}$$

(13)

and for case 1($M \geq T$), average profit($AP_2(q,T)$) per unit time is $=\frac{1}{T}(SR + SV + IE_2 - RC - IHC - PC - IP_2)$

$$= \frac{1}{T}\left[\begin{array}{c}(s-C_p)q-K+T\{A(p_0(1+I_eM)-C_p)-C_hq\}+\frac{T^2}{2}\left\{B\left(C_p-p_0(1+I_eM)\right)-A\left(\frac{p_0}{L}(1+I_eM)+p_0I_e+C_h\right)\right\}\\ +\frac{T^3}{3}\left\{B\left(\frac{p_0}{L}(1+I_eM)+C_h+\frac{p_0I_e}{2}\right)+C\left(C_p-\frac{p_0}{L}-p_0I_eM\right)+A\frac{p_0I_e}{2L}\right\}+\frac{T^4}{4}\left\{C\left(\frac{p_0}{L}(1+I_eM)+C_h+\frac{p_0I_e}{2}\right)-B\frac{p_0I_e}{3L}\right\}\\ -\frac{T^5}{20L}Cp_0I_e\end{array}\right] \tag{14}$$

Now our objective is to obtain optimal cycle time T^* and inventory remaining at end of cycle q^* in order to maximize the average profit per unit time.

4. THEORETICAL RESULT FOR OPTIMALITY

Our optimization problem in the proposed work is $\max\limits_{q,T} AP(q,T)=\begin{cases}AP_1(q,T), \text{if } M\le T\\ AP_2(q,T), \text{if } M\ge T\end{cases}$

Subject to $0\le q\le Q\le W,\ T\le L.$

Case1. $M\le T$

Taking 1st and 2nd derivatives of $AP_1(q,T)$ in (13) with respect to q, we find

$$\frac{\partial(AP_1(q,T))}{\partial q}=\frac{(s-C_p+M)}{T}-T(C_h+C_pI_p) \tag{15}$$

and $\frac{\partial^2(AP_1(q,T))}{\partial q^2}=0$ (16)

Here we see that $AP_1(q,T)$ is linear function of q. So $AP_1(q,T)$ is either increasing or decreasing function in q. Therefore, two cases may arise.

Subcase 1. When $\frac{\partial(AP_1(q,T))}{\partial q}>0$, $AP_1(q,T)$ is strictly increasing function in q. Thus $AP_1(q,T)$ attains its maximum when q reaches its maximum.

Now as $Q\le W$, so by the help of equation (3), maximum value of q is

$$W-AT+\frac{B}{2}T^2+\frac{C}{3}T^3 \tag{17}$$

Putting the expression $W-AT+\frac{B}{2}T^2+\frac{C}{3}T^3$ in place of q in equation (13), we obtain the following expression of the profit function as a function of T only:

$$AP_{11}(T)=\frac{1}{T}\left[\begin{array}{c}A(p_0I_e-C_pI_p)\frac{M^2}{4}-\left\{\frac{p_0I_e}{3}\left(\frac{A}{L}+B\right)-B\frac{C_pI_p}{6}\right\}M^3+\left\{\frac{p_0I_e}{4}\left(\frac{B}{L}-C\right)-C\frac{C_pI_p}{12}\right\}M^4+C\frac{p_0I_e}{5L}M^5\\ +(s-C_p+M)\left(W-AT+\frac{B}{2}T^2+\frac{C}{3}T^3\right)-K+T\left\{A(p_0-C_p-C_pI_pM)-(C_h+C_pI_p)\left(W-AT+\frac{B}{2}T^2+\frac{C}{3}T^3\right)\right\}\\ +\frac{T^2}{2}\left\{B(C_p-C_pI_pM-p_0)-A\left(\frac{p_0}{L}+C_h\right)\right\}+\frac{T^3}{3}\left\{B\left(\frac{p_0}{L}+C_h+C_pI_p\right)+CC_p(1-I_pM)\right\}+\frac{T^4}{4}C\left\{\frac{p_0}{L}+C_h+C_pI_p\right\}\end{array}\right] \tag{18}$$

Again Hessian matrix for $AP_1(q,T)$ is $\begin{bmatrix} \frac{\partial^2 AP_1}{\partial q^2} & \frac{\partial^2 AP_1}{\partial q \partial T} \\ \frac{\partial^2 AP_1}{\partial T \partial q} & \frac{\partial^2 AP_1}{\partial T^2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial^2 AP_1}{\partial q \partial T} \\ \frac{\partial^2 AP_1}{\partial T \partial q} & \frac{\partial^2 AP_1}{\partial T^2} \end{bmatrix} = -\left(\frac{\partial^2 AP_1}{\partial q \partial T}\right)^2 < 0$

So, there exists unique T^* at which the profit function $AP_{11}(T)$ attains a maximum.

Subcase 2. When $\frac{\partial(AP_1(q,T))}{\partial q} \leq 0$, $AP_1(q,T)$ is non- increasing function in q. Thus $AP_1(q,T)$ attains its maximum when q reaches its minimum i.e., when $q = 0$.

Putting $q = 0$ in equation (13), we obtain the following expression of the profit function as a function of T only:

$$AP_{12}(T) = \frac{1}{T}\begin{bmatrix} A(p_0 I_e - C_p I_p)\frac{M^2}{4} - \left\{\frac{p_0 I_e}{3}\left(\frac{A}{L} + B\right) - B\frac{C_p I_p}{6}\right\}M^3 + \left\{\frac{p_0 I_e}{4}\left(\frac{B}{L} - C\right) - C\frac{C_p I_p}{12}\right\}M^4 + C\frac{p_0 I_e}{5L}M^5 \\ -K + T\{A(p_0 - C_p - C_p I_p M)\} + \frac{T^2}{2}\left\{B(C_p - C_p I_p M - p_0) - A\left(\frac{p_0}{L} + C_h\right)\right\} \\ + \frac{T^3}{3}\left\{B\left(\frac{p_0}{L} + C_h + C_p I_p\right) + CC_p(1 - I_p M)\right\} + \frac{T^4}{4}C\left\{\frac{p_0}{L} + C_h + C_p I_p\right\} \end{bmatrix} \tag{19}$$

As Hessian matrix for $AP_1(q,T)$ is less than zero, so there exists unique T^* at which the profit function $AP_{12}(T)$ attains a maximum.

Case2. $M \geq T$

Similarly, taking 1st and 2nd derivatives of $AP_2(q,T)$ in (14) with respect to q, we find

$$\frac{\partial(AP_2(q,T))}{\partial q} = \frac{(s - C_p)}{T} - C_h T \tag{20}$$

and $\frac{\partial^2(AP_2(q,T))}{\partial q^2} = 0$ (21)

Here also we see that $AP_2(q,T)$ is linear function of q. So $AP_2(q,T)$ is either increasing or decreasing function in q.

Subcase 1. $\frac{\partial(AP_1(q,T))}{\partial q} > 0$.

Similarly, putting the expression $W - AT + \frac{B}{2}T^2 + \frac{C}{3}T^3$ in place of q in equation (14), we obtain the following expression of the profit function as a function of T only:

$$AP_{21}(T) = \frac{1}{T}\begin{bmatrix} (s - C_p)\left(W - AT + \frac{B}{2}T^2 + \frac{C}{3}T^3\right) - K + T\left\{A(p_0(1 + I_e M) - C_p) - C_h\left(W - AT + \frac{B}{2}T^2 + \frac{C}{3}T^3\right)\right\} \\ + \frac{T^2}{2}\left\{B(C_p - p_0(1 + I_e M)) - A\left(\frac{p_0}{L}(1 + I_e M) + p_0 I_e + C_h\right)\right\} \\ + \frac{T^3}{3}\left\{B\left(\frac{p_0}{L}(1 + I_e M) + C_h + \frac{p_0 I_e}{2}\right) + C\left(C_p - \frac{p_0}{L} - p_0 I_e M\right) + A\frac{p_0 I_e}{2L}\right\} \\ + \frac{T^4}{4}\left\{C\left(\frac{p_0}{L}(1 + I_e M) + C_h + \frac{p_0 I_e}{2}\right) - B\frac{p_0 I_e}{3L}\right\} - \frac{T^5}{20L}Cp_0 I_e \end{bmatrix}$$

(22)

Again, as Hessian matrix for $AP_2(q,T)$ is less than zero, so there exists unique T^* at which the profit function $AP_{21}(T)$ attains a maximum.

Subcase 2. $\frac{\partial(AP_1(q,T))}{\partial q} \leq 0$

For this case, putting $q = 0$ in equation (14), we obtain the following expression of the profit function as a function of T only:

$$AP_{22}(T) = \frac{1}{T}\left[\begin{array}{c} -K + T\{A(p_0(1+I_eM) - C_p)\} + \frac{T^2}{2}\left\{B(C_p - p_0(1+I_eM)) - A\left(\frac{p_0}{L}(1+I_eM) + p_0I_e + C_h\right)\right\} \\ + \frac{T^3}{3}\left\{B\left(\frac{p_0}{L}(1+I_eM) + C_h + \frac{p_0I_e}{2}\right) + C\left(C_p - \frac{p_0}{L} - p_0I_eM\right) + A\frac{p_0I_e}{2L}\right\} + \frac{T^4}{4}\left\{C\left(\frac{p_0}{L}(1+I_eM) + C_h + \frac{p_0I_e}{2}\right) - B\frac{p_0I_e}{3L}\right\} \\ - \frac{T^5}{20L}Cp_0I_e \end{array}\right] \tag{23}$$

Here also there exists unique T^* at which the profit function $AP_{22}(T)$ attains a maximum.

5. ALGORITHM

Here we draft the algorithm for finding optimal solution of the proposed model.

Step1. Find solution of the equation $\frac{d(AP_{11}(T))}{dT} = 0$ and if the solution satisfies $M \leq T$ then denote it as T_{11}^* and find $AP_{11}(T_{11}^*)$

Step 2. Find solution of the equation $\frac{d(AP_{12}(T))}{dT} = 0$ and if the solution satisfies $M \leq T$ then denote it as T_{12}^* and find $AP_{12}(T_{12}^*)$

Step 3. Find solution of the equation $\frac{d(AP_{21}(T))}{dT} = 0$ and if the solution satisfies $M \geq T$ then denote it as T_{21}^* and find $AP_{21}(T_{21}^*)$

Step 4. Find solution of the equation $\frac{d(AP_{22}(T))}{dT} = 0$ and if the solution satisfies $M \geq T$ then denote it as T_{22}^* and find $AP_{22}(T_{22}^*)$.

Step 5. Set $AP(T^*) = \text{Max}\,\{AP_{11}(T_{11}^*), AP_{12}(T_{12}^*), AP_{21}(T_{21}^*), AP_{22}(T_{22}^*)\}$

Step 6. Calculate the corresponding value of q^* (either 0 or $W - AT + \frac{B}{2}T^2 + \frac{C}{3}T^3$) and Q^* (from equation (3))

6. NUMERICAL ILLUSTRATIONS

To exemplify different cases of the established model, three numerical examples are taken with their appropriate values.

Example 1. Let us take the following parameters in appropriate units as follows:

$L = 2$ yrs., $M = 0.25$ yrs., $W = 500$, $\alpha = 350$, $\beta = 0.04$, $K = 1200$ \$, $C_h = 0.5$ \$, $C_p = 5$\$, $p_0 = 15.75$\$, $s = 5.5$\$, $I_e = 0.2$\$, $I_p = 0.1$.

Step1. Solution of $\frac{d(AP_{11}(T))}{dT} = 0$ is 0.753514 which satisfies $M(= 0.25) \leq T$. Hence T_{11}^*=0.753514 and $AP_{11}(T_{11}^*)$=540.991

Step 2. Solution of $\frac{d(AP_{12}(T))}{dT} = 0$ is 0.860006 which satisfies $M \leq T$. Hence T_{12}^*=0.0.860006 and $AP_{12}(T_{11}^*)$=560.694

Step 3. Solution of $\frac{d(AP_{21}(T))}{dT} = 0$ is 0.687526 which does not satisfy $M \geq T$. Hence T_{21}^* does not exists.

Step 4. Solution of $\frac{d(AP_{22}(T))}{dT} = 0$ is 0.782859 which does not satisfy $M \geq T$. Hence T_{22}^* does not exists

Hence optimum cycle time T^* =0.860006($> M$) and optimum profit is 560.694. Correspondingly q^*=0, Q^*=235.944

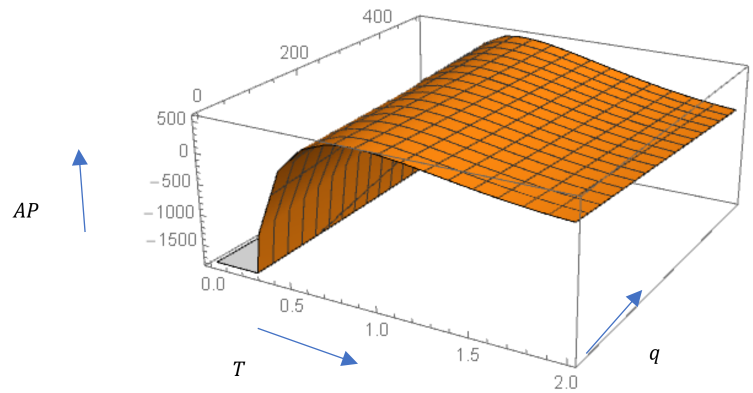

Fig 4. Profit per unit time vs T and q of example 1

Example 2. Here take the parameters as follows:

$L = 2$ yrs., $M = 0.25$ yrs., $W = 500$, $\alpha = 350$, $\beta = 0.04$, $K = 1200$ \$, $C_h = 0.5$ \$, $C_p = 5$\$, $p_0 = 20.75$\$, $s = 5.5$\$, $I_e = 0.2$\$, $I_p = 0.1$.

Using these values of the parameters of the model and applying the algorithm we find T^* =0.59856($> M$) and optimum profit is 1774.81. Correspondingly q^*=322.231, Q^*=500

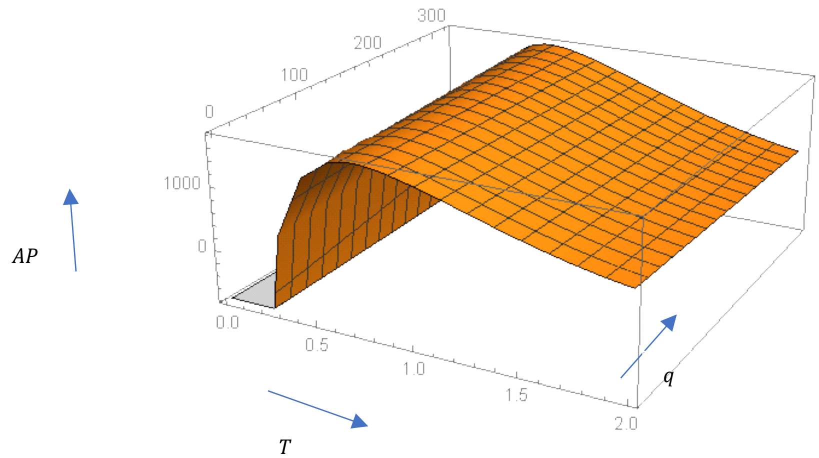

Fig 5. Profit per unit time vs T and q of example 2

Example 3. Here take the parameters as follows:
$L = 2$ yrs., $M = 0.25$ yrs., $W = 500$, $\alpha = 350$, $\beta = 0.04$, $K = 1200$ \$, $C_h = 0.5$ \$, $C_p = 5$\$, $p_0 = 15.75$\$, $s = 7.1$\$, $I_e = 0.2$\$, $I_p = 0.1$.

Using these values of the parameters of the model and applying the algorithm we find T^* =0.248442($< M$) and optimum profit is 1805.25. Correspondingly q^*=418.278, Q^*=500

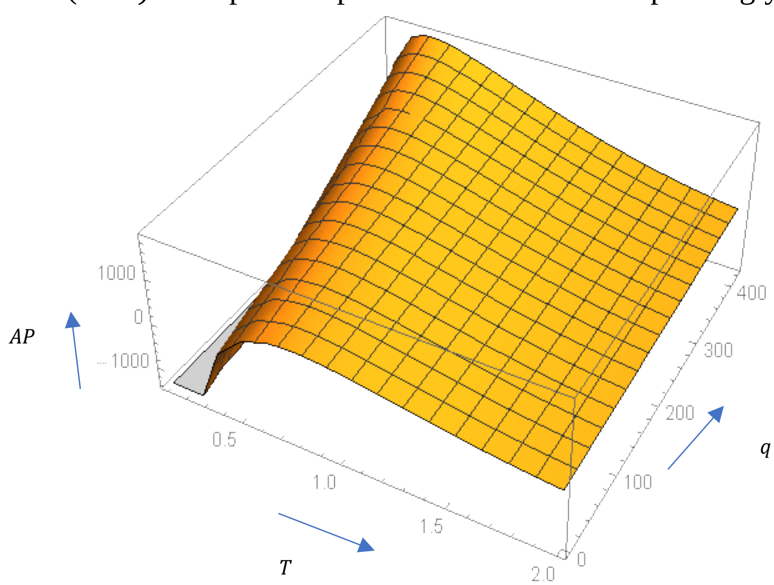

Fig 6. Profit per unit time vs T and q of example 3

7. SENSITIVITY ANALYSIS

To test the flexibility of the model, we study the impact of changes in different parameters against optimal solutions (T, q), optimal order quantities and average profit for the example1. Changing the value on one parameter at a time and fixing other remaining parameters, the analysis has been done. Table 1 presents the observed results with various parameters.

Parameter	Original value	New value	T^*	q^*	Q^*	AP
K	1200	1600	1.0678537	0	273.585	143.316
		1400	0.961184	0	255.213	340.763
		1000	0.63212	314.011	500	830.223
		800	0.50148	346.73	500	1183.93
α	350	400	0.691451	271.532	500	853.304
		375	0.820109	0	244.151	702.971
		325	0.905892	0	227.387	424.487
		300	0.951511	0	218.754	291.811
β	0.04	0.12	0.863161	0	235.886	556.057
		0.08	0.861578	0	235.915	558.372
		0.02	0.859223	0	235.959	561.858
		0.01	0.858833	0	236.011	562.440
C_h	0.50	0.7	0.854524	0	234.848	539.292
		0.6	0.857259	0	235.396	549.983
		0.4	0.749126	287.229	500	579.508
		0.3	0.744818	288.173	500	618.082
C_p	5	7	0.928546	0	249.192	-26.7693
		6	0.891264	0	242.087	265.622
		4	0.426873	366.754	500	1452.930
		3	0.136229	454.024	500	2860.065
p_0	15.75	35.75	0.411643	371.228	500	5851.234
		25.75	0.511696	344.222	500	3088.17
		20.75	0.786542	0	267.543	1087.452
		13.75	0.973959	0	257.566	130.637
s	5.5	6.5	0.441874	362.651	500	1087.63
		6	0.610982	319.100	500	763.082
		5	0.860060	0	235.944	560.694
		4.5	0.860060	0	235.944	560.694
I_p	0.1	0.4	0.813442	0	226.475	235.25
		0.2	0.843810	0	232.693	451.496
		0.05	0.743723	288.411	500	664.967
		0.025	0.738961	289.459	500	727.052
I_e	0.2	0.8	0.842463	0	232.421	602.059
		0.4	0.854153	0	234.775	574.388
		0.1	0.862934	0	236.527	553.882
		0.05	0.864399	0	236.818	550.485
W	500	950	0.543876	785.784	950	614.318
		800	0.616472	617.769	800	570.769
		650	0.860006	0	235.944	560.694
		350	0.860006	0	235.944	560.694
L	2	2.5	0.918867	0	262.108	835.409
		2.25	0.889607	0	249.441	708.631
		1.75	0.831199	0	221.515	385.431
		1.5	0.676441	316.892	500	178.010
M	0.25	0	0.888727	0	231.675	533.107
		0.15	0.864217	0	236.782	575.915
		0.35	0.721946	293.235	500	566.441
		0.50	0.675899	393.707	500	605.263

Table 1. Sensitivity Analysis

From Table 1, the following observations can be made.

(i) There is positive effects on the average profit per unit time (AP) with respect to the value of the parameter $\alpha,\ p_0, s, I_e, W, L, M$ that is AP increases when the values of $\alpha,\ p_0, s, I_e, W, L, M$ increase, while for the parameters K, β, C_h, C_p, I_p there is negative impact on AP.

(ii) The optimal cycle time T^* depends on parameters K, β, C_h, L, C_p and M in a positive way but it depends on parameters $p_{0,}\alpha, s, I_p, I_e, W$ in negative way.

(iii) It is not easy to say that the effects on the optimal ordering quantity Q^* and end inventory level q^* with respect to the value of all parameters are positive or negative. Sometimes optimal solution will occur by taking zero end inventory model sometimes positive end inventory model. But it can be certainly concluded that relaxation of end inventory level produces better result.

8. CONCLUSIONS

Most of existing published papers on inventory control with selling price and freshness dependent demand did not consider the freshness dependent selling price as well as the effect of trade credit policy. In today's world, both trade credit policy as well as freshness dependent selling price are most realistic assumptions. The current study extends the previous existing work by incorporating these two above mentioned concepts. The traditional assumption of zero ending inventory is relaxed in our model to any positive amount of inventory. Limited shelf space for holding inventory and salvage revenue of disposal items are been introduced. After formulating the model, solution procedure has been developed to determine optimal cycle length and stock of disposal items. With the help of MATHEMATICA 12 software three different numerical example are demonstrated for illustration purpose. The concave nature of the profit function is justified by drawing graphs in three dimensions. To check the changes in the decision variables for changes in different parameters, a sensitivity analysis is also carried out.

This model can be further developed by considering several realistic features such as two-level trade credit, shortages and delay freshness degradation.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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