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EFFECT OF STRETCHING PARAMETER IN A NON-LINEAR MHD FLOW CLOSE TO STAGNATION POINT

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Abstract: A parametric study exploring the fall out of MHD and stretching parameter on a viscous incompressible fluid through a vertical porous plate with heat absorption is described. The MHD boundary layer equations with low pressure gradient, controlled by a system of non-linear partial differential equations are work out by adopting Homotopy Perturbation method (HPM). The influence of different relevant physical characteristics are presented and discussed graphically.

Keywords: MHD; homotopy perturbation method; stretching sheet; stagnation point.

2010 AMS Subject Classification: 76W05.

1. INTRODUCTION

MHD is the science of movement in which all the characteristics of fluid with the magnetic benefits under the conduction of electric current. There are lots of applications of MHD principles in Engineering, Plasma Physics, and Biotechnology etc. The effects of MHD in different problems of energy and mass in transit with mixed convection are applied by numerous authors initiated in refs. [1] and [2].

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Due to stretching of an elastic flat sheet the flow moves in its plane with velocity varies with the distance from a fixed point due to the implementation of a stress are termed as stretching flow. *Glass blowing*, *continuous casting* and *spinning of fibers* are cases in point so far as the flow of a stretching surface is concerned. The pioneer work regarding the study of *heat generation* on a stretching surface was introduced in refs. [3] to [5] by taking into account of different aspects of the problem.

In fluid mechanics, the discussion of flow related problems to stagnation point is of great scientific importance due to its numerous applications in technology and engineering. The pioneer work has been made in this area which is shown in refs. [6] to [8], [11].

In the present paper we have introduce a semi-exact method which is called *Homotopy Perturbation Method* (HPM) and applied in MHD boundary layer flow with low pressure gradient over a flat plat. The initial work in HPM was studied by *J. H. He* [9]. This investigation inspired many researchers to solve *nonlinear differential equations.* The main objective of the present investigation is to study the effect of *stretching parameter* of MHD flow related problems to stagnation point by *Homotopy Perturbation Method* (HPM).

2. FORMULATION OF THE PROBLEM

The model which describes the substantial circumstances with initial velocity u_w, T_w be the temperature whereas $u_w(x)$, $T_\infty(x)$ are the velocity and temperature of the flow external to the boundary layer is given in Figure 1.

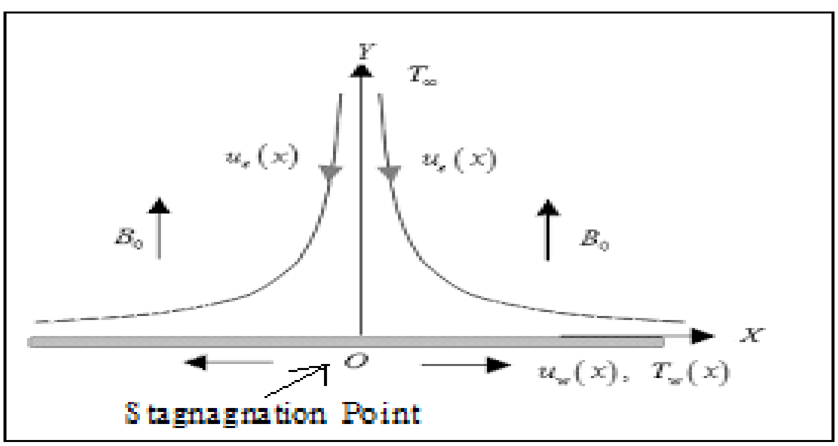

Figure 1: Physical model of the problem

Choudhary *et.al*. [10] prepared some standard assumptions with the help of those, the following governing equations are considered, describing the physical situations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\upsilon}{K}\left[U(x)-u\right]-\frac{\sigma B_0^{\,2}u}{\rho} \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty) \tag{3}$$

Where, symbols used above have their usual meaning expressed in [10].

The boundary conditions of the present problem are given by:

$$\left.\begin{array}{ll} u=u_w(x)=cx, v=0;\ T=T_w & for\ \ y=0 \\ u=u_e(x)=ax; \qquad T=T_\infty & for\ \ y\to\infty \end{array}\right\} \tag{4}$$

With reference to the *stream function* $\psi(x,y)$ in the continuity equation (1), we obtain

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial y} \tag{5}$$

To normalize the flow model the following non dimensional terms are introduced:

$$\psi(x,y)=\sqrt{c\upsilon}\ \ x\,f(\eta),\ \eta=\sqrt{\frac{c}{\upsilon}}\,y,\ T=T_\infty+(T_w-T_\infty)\,\theta(\eta) \tag{6}$$

The following *nonlinear coupled differential equations* are produced by applying (5), (6) in (2), (3):

$$f'''(\eta)+f(\eta).f''(\eta)-f'^2(\eta)-(\lambda+M)\,f'(\eta)+C.(C+\lambda)=0 \tag{7}$$

$$\theta''(\eta)+\Pr.f(\eta).\theta'(\eta)+\Pr.B.\theta=0 \tag{8}$$

With reference to the following qualifying limitations:

$$\left.\begin{array}{ll} f(\eta)=0,\ f'(\eta)=1, \theta(\eta)=1 & as\ \ \eta=0 \\ f'(\eta)\to C, \theta(\eta)\to 0 & as\ \eta\to\infty \end{array}\right\} \tag{9}$$

Where $\lambda=\dfrac{\upsilon}{cK}$ is the *porosity parameter*, $M=\dfrac{B_0^{\,2}\sigma}{\rho c}$ is the *magnetic parameter*, $\Pr=\dfrac{\mu C_p}{k}$ is the *Prandtl number*, $C=\dfrac{c}{a}$ is the *stretching parameter* and $B=\dfrac{Q}{c\rho C_p}$ is the *heat absorption parameter*.

The *nonlinear coupled differential equations* can be rewritten as:

$$f''' + f.f'' - f'^2 - M_1.f' + M_2 = 0 \tag{10}$$

$$\theta'' + \Pr.f.\theta' - M_3\theta = 0 \tag{11}$$

Applying HPM, the equations (10), (11) can take the following form:

$$(1-p)(f''' - M_1.f') + p\left(f''' + f.f'' - f'^2 - M_1.f'\right) = -M_2 \tag{12}$$

$$(1-p)(\theta'' - M_3\theta) + p\left(\theta'' + \Pr.f.\theta' - M_3\theta\right) = 0 \tag{13}$$

Let us consider *"f"* and " θ " as under:

$$\left.\begin{aligned} f &= f_0 + pf_1 + p^2 f_2 + \ldots\ldots\ldots\ldots \\ \theta &= \theta_0 + p\theta_1 + p^2\theta_2 + \ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{14}$$

Using (14) in (12) and (13) we obtain:

$$\begin{aligned} &(1-p)\left[\left(f_0''' + pf_1''' + p^2 f_2''' + \ldots\right) - M_1(f_0' + pf_1' + p^2 f_2' + \ldots)\right] \\ &+p\left[\left(f_0''' + pf_1''' + p^2 f_2''' + \ldots\right) + \left(f_0 + pf_1 + p^2 f_2 + \ldots\right)\left(f_0'' + pf_1'' + p^2 f_2'' + \ldots\right)\right. \\ &\qquad \left. -\left(f_0 + pf_1 + p^2 f_2 + \ldots\right)^2 - M_1(f_0' + pf_1' + p^2 f_2' + \ldots)\right] = -M_2 \end{aligned} \tag{15}$$

And

$$\begin{aligned} &(1-p)\left[\left(\theta_0'' + p\theta_1'' + p^2\theta_2'' + \ldots\right) - M_3(\theta_0 + p\theta_1 + p^2\theta_2 + \ldots)\right] + p\left[\left(\theta_0'' + p\theta_1'' + p^2\theta_2'' + \ldots\right)\right. \\ &\quad \left. + \Pr\left(f_0 + pf_1 + p^2 f_2 + \ldots\right)\left(\theta_0' + p\theta_1' + p^2\theta_2' + \ldots.\right) - M_3(\theta_0 + p\theta_1 + p^2\theta_2 + \ldots) = 0\right. \end{aligned} \tag{16}$$

Equating the terms free from 'p' and containing 'p', equations (15), (16) reformed as:

$$f_0(\eta) = C_1 + C_2 e^{\sqrt{M_1}\eta} + C_3 e^{-\sqrt{M_1}\eta} + \frac{M_2}{M_1}\eta \tag{17}$$

$$\theta_0(\eta) = C_4 e^{\sqrt{M_3}\eta} + C_5 e^{-\sqrt{M_3}\eta} \tag{18}$$

$$\begin{aligned} f_1(\eta) = C_6 + C_7 e^{\sqrt{M_1}\eta} + C_8 e^{-\sqrt{M_1}\eta} + A_{10}\eta^2 + A_{11}\eta^3 + A_{12}\eta - \left[A_{15}e^{2\sqrt{M_1}\eta} - A_{16}e^{-2\sqrt{M_1}\eta}\right] \\ -\eta\left[E_1 e^{\sqrt{M_1}\eta} + E_2 e^{-\sqrt{M_1}\eta}\right] + \eta^2\left[A_{17}e^{\sqrt{M_1}\eta} + A_{18}e^{-\sqrt{M_1}\eta}\right] \end{aligned} \tag{19}$$

$$\begin{aligned} \theta_1(\eta) = C_9 e^{\sqrt{M_3}\eta} + C_{10}e^{-\sqrt{M_3}\eta} + \eta\left[A_{39}e^{\sqrt{M_3}\eta} + A_{40}e^{-\sqrt{M_3}\eta}\right] - \eta^2\left[A_{35}e^{\sqrt{M_3}\eta} - A_{36}e^{-\sqrt{M_3}\eta}\right] \\ -\left[A_{31}e^{M_4\eta} - A_{32}e^{-M_4\eta}\right] + \left[A_{33}e^{M_5\eta} - A_{34}e^{-M_5\eta}\right] \end{aligned} \tag{20}$$

With the location to the subsequent qualifying restrictions:

$$\left.\begin{aligned} &f_0(0) = 0,\ f_0'(0) = 1,\ f_0'(6) = C; \quad \theta_0(0) = 1, \theta_0(6) = 0 \\ &f_1(0) = 0,\ f_1'(0) = 0,\ f_1'(6) = 0; \quad \theta_1(0) = 0, \theta_1(6) = 0 \end{aligned}\right\} \tag{21}$$

($\eta \to \infty$ *were replaced by those at* η *=6 in concurrence with standard practice in the boundary layer theory*).

Neglecting higher order perturbed terms we finally obtain:

$$f(\eta) = f_0 + pf_1$$

$$\theta(\eta) = \theta_0 + p\theta_1$$

The terminologies for *viscous drage* in terms of *skin friction* (τ) and the coefficient of rate of heat transfer (Nu) are articulated as:

$$\tau = \left(\frac{\partial f}{\partial \eta}\right)_{\eta=0} = \sqrt{M_1}(C_2 - C_3) + \frac{M_2}{M_1}$$
$$+p\left[\sqrt{M_1}(C_7 - C_8) + A_{12} - (A_{13} + A_{14}) - 2\sqrt{M_1}(A_{15} + A_{16}) + (A_{19} - A_{20})\right]$$

$$Nu = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} = -\sqrt{M_3}(C_4 - C_5) - p\left[\sqrt{M_3}(C_9 - C_{10}) + A_{39} + A_{40} - M_4(A_{31} - A_{32})\right]$$

3. FINDINGS

The present analysis reveals the expressions of boundary layer equations with *viscous drag* and *co-efficient of rate of heat transfer* are piled up to get a variety of graphs with their substantial interprtations by taking some random values of different parameters implicated in the problem.

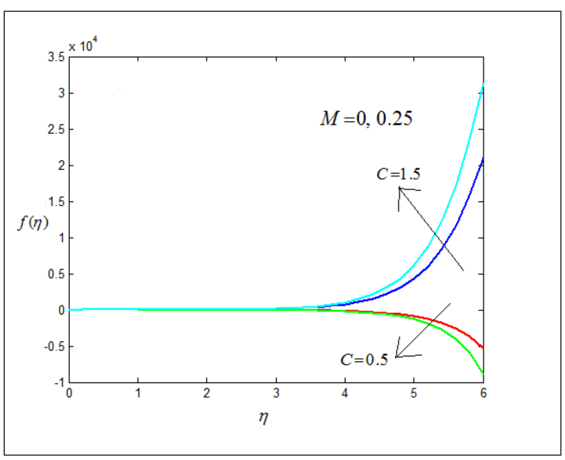

Figure 2: Velocity versus η under λ=3, P=0.0002

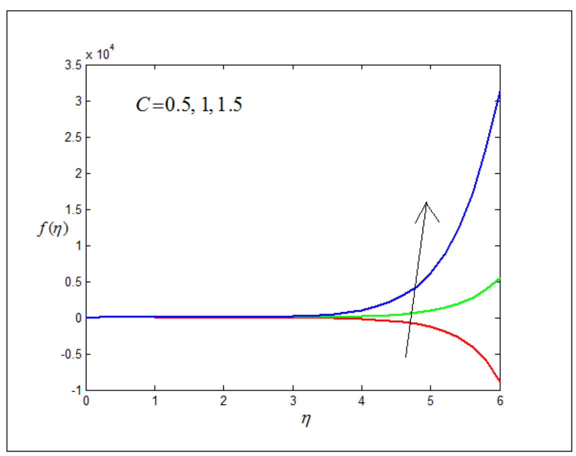

Figure 3: Velocity versus η under M=.25, λ=3, P=0.0002

Figures 2 and 3 depicts the variation of magnetic intensity and homotopy parameter on fluid velocity. It is seen from figure 2 that the motion of the fluid gets its highest and lowest values for C=1.5 and C=0.5 due to Hartmann number far away from the plate. Furthermore, accleration of velocity profile on account of stretching parameter is observed in figure 3. In addition, it is found from both the figures that the fluid velocity is nullified initially but it shows distinct variation as we move far away from the plate.

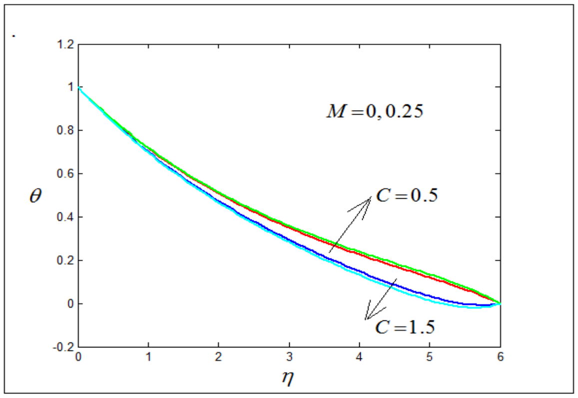

Figure 4: Temperature versus η under λ=3, Pr = 1, $|B|$ =0.1, P =0.0002

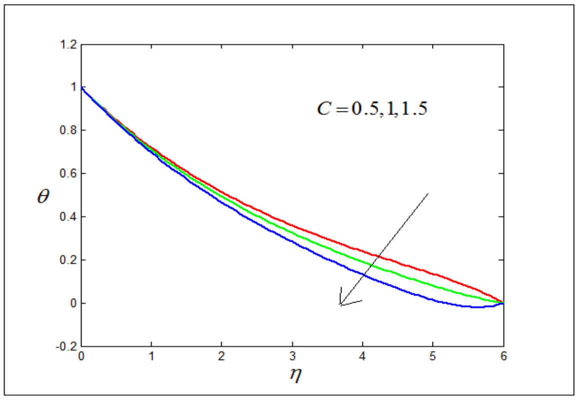

Figure 5: Temperature versus η under M=0.25, λ=3, Pr =1, $|B|$=0.1, P =0.0002

Figures 4 and 5 illustrate the effects of *Hartman number* and stretching parameter on temperature distribution. It is noticed that the temperature distribution of the flow is raised and reduced for varying the stretching parameter under magnetic intensity and stretching parameter brings down the fluid temperature. The two figures also demonstrated the fact that the temperature distribution is not disturbed initially but the variation of the distribution is observed as we move further away from the plate.

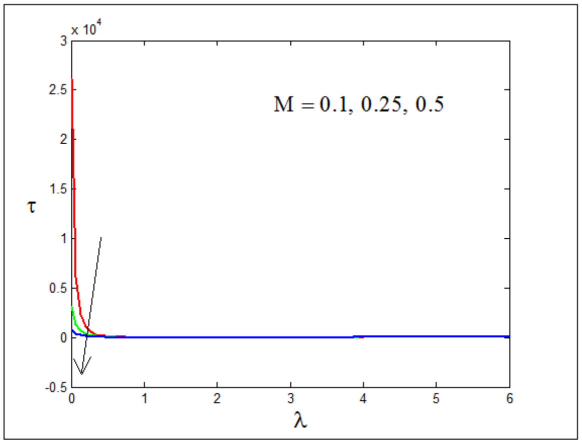

Figure 6: Skin friction versus λ under C=1.5, P=1

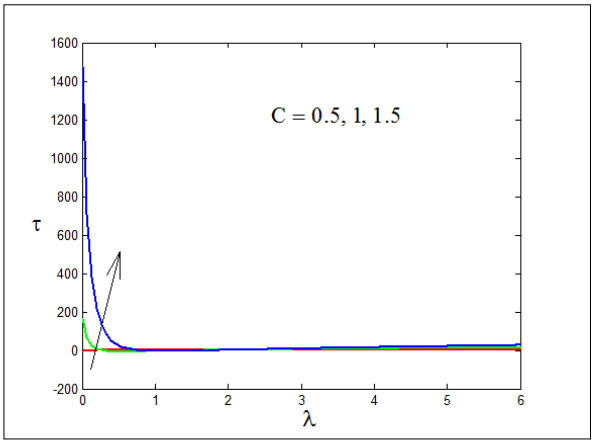

Figure 7: Skin friction versus λ under M=0.25, P=1

The variation of viscous drag against magnetic field and stretching parameter are demonstrated in figures 6 and 7. It is explained from both the figures that the magnetic parameter reduces the friction and it is raised by virtue of stretching parameter.

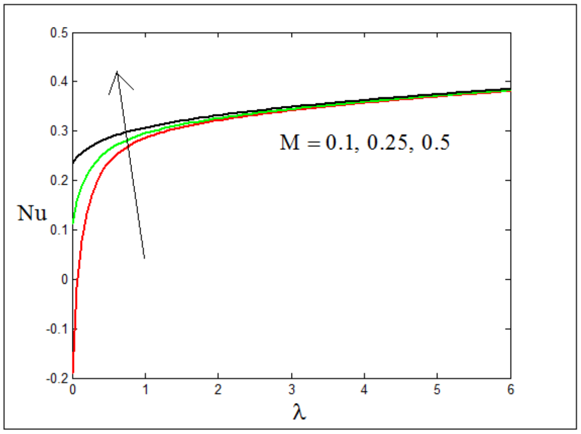

Figure 8: Nusset number versus λ under C=1.5, P=1, Pr =0.1, $|B|$=0.1, P =1

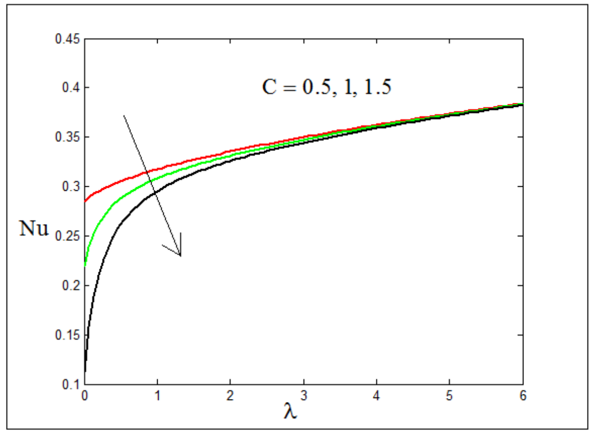

Figure 9: Nusset number versus λ under M=0.25, P=1, Pr =0.1, $|B|$=0.1, P =1

Figures 8 and 9 elaborate that the *Nusselt Number* gets enhanced due to the strength of the applied magnetic field and minimized on account of stretching parameter.

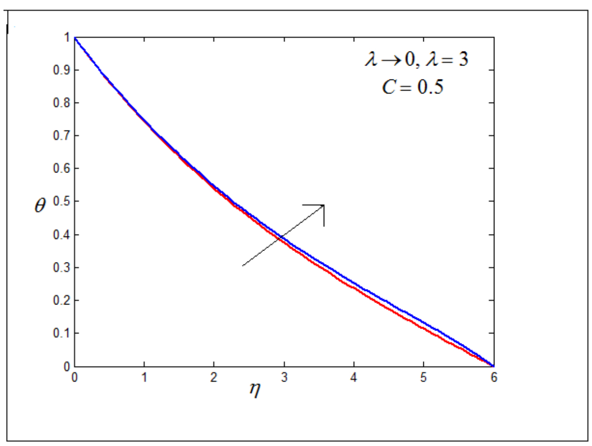

Figure 10: Temperature versus η under M=0, Pr = 0.7, $|B|$ =0.1, P =0.0002

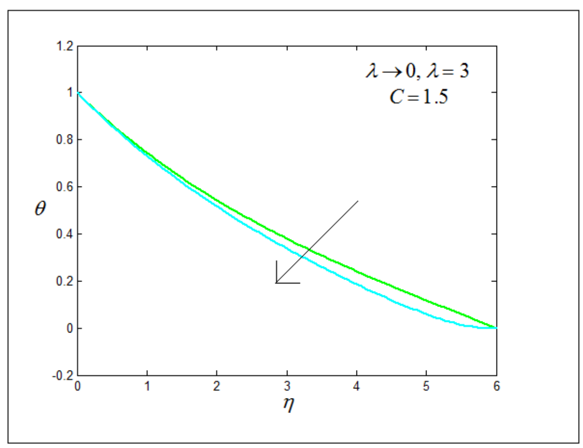

Figure 11: Temperature versus η under M=0, Pr = 0.7, $|B|$ =0.1, P =0.0002

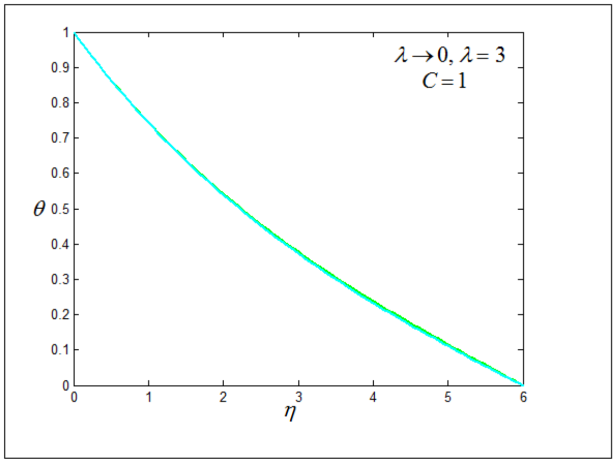

Figure 12: Temperature versus η under
M=0, Pr = 0.7, $|B|$ =0.1, P =0.0002

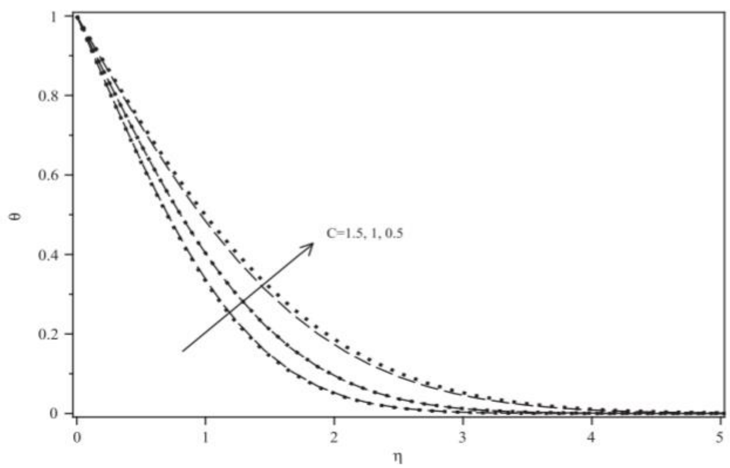

Fig. 4. Temperature profiles $\theta(\eta)$ for various values of M and C when $B=0$ and $Pr=0.7$, (Dashed lines: $M=0$; filled circle: $M=2$).

Figure 13(Fig 4 of Kazem S. et al.[11]): Temperature versus η under Pr = 0.7, B=0

4. COMPARISON OF RESULTS

To compare the results of the present paper, the work of Kazem S. et al.[11] is considered.

Comparing figures 10, 11and 12 with figure 13 (Figure 11 of the work done by Kazem S. et al.[11]), we observe that the effect of λ (which is symbolized as M in the work done by Kazem S. et al.[11]) on temperature profile depends on C. As we can see, increasing λ decreases θ for C >1 while, the inverse behavior is shown for C < 1. The figures are almost identical in nature as the behavior of the fluid temperature versus normal coordinate η is concerned. That is there is an excellent agreement between the results obtained by Kazem S. et al. [11] and the present authors.

5. CONCLUDING REMARKS

1. The motion of the fluid gets its highest and lowest values for the stretching parameter greater than and less than one due to Hartmann number far away from the plate. whereas the fluid motion is accelerated due to stretching parameter.
2. The temperature distribution of the fluid flow is raised and reduced for varying the stretching parameter under magnetic intensity and stretching parameter brings down the fluid temperature.
3. The magnetic parameter reduces the viscous drag and it is enhanced by virtue of stretching parameter.
4. *Nusselt number* gets elevated due to the strength of the applied magnetic field and minimized on account of stretching parameter.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

APPENDIX

$M_1 = \lambda + M$ $\quad M_2 = C.(C+\lambda)$ $\quad M_3 = -\Pr B$

$M_4 = \sqrt{M_1} + \sqrt{M_3}$ $\quad M_5 = \sqrt{M_1} - \sqrt{M_3}$ $\quad M_6 = \frac{M_2}{M_1} - 1$

$C_1 = -C_2 - C_3$ $\quad C_2 = C_3 - \frac{M_6}{\sqrt{M_1}}$ $\quad C_3 = \frac{M_6 e^{6\sqrt{M_1}} - \frac{M_2}{M_1} + C}{\sqrt{M_1}\left(e^{6\sqrt{M_1}} - e^{-6\sqrt{M_1}}\right)}$

$C_4 = \frac{1}{1 - e^{12\sqrt{M_3}}}$ $\quad C_5 = -C_4 e^{12\sqrt{M_3}}$ $\quad A_1 = 2C_2C_3M_1 - 2C_2C_3 - C_1^{\,2}$

$A_2 = -2C_1\frac{M_2}{M_1}$ $\quad A_3 = -\frac{M_2^{\,2}}{M_1^{\,2}}$ $\quad A_4 = C_1C_2(M_1 - 2)$

$A_5 = C_1C_3(M_1 - 2)$ $\quad A_6 = C_2^{\,2}(M_1 - 1)$ $\quad A_7 = C_3^{\,2}(M_1 - 1)$

$A_8 = C_2M_2(1 - \frac{2}{M_1})$ $\quad A_9 = C_3M_2(1 - \frac{2}{M_1})$ $\quad A_{10} = \frac{A_2}{2M_1}$

$A_{11} = \frac{A_3}{3M_1}$ $\quad A_{12} = \frac{1}{M_1}(A_1 + \frac{2A_3}{M_1})$ $\quad A_{13} = \frac{A_4}{2M_1}$

$A_{14} = \frac{A_5}{2M_1}$ $\quad A_{15} = \frac{A_6}{6M_1\sqrt{M_1}}$ $\quad A_{16} = \frac{A_7}{6M_1\sqrt{M_1}}$

$A_{17} = \frac{A_8}{4M_1}$ $\quad A_{18} = \frac{A_9}{4M_1}$ $\quad A_{19} = \frac{3A_8}{4M_1\sqrt{M_1}}$

$A_{20} = \frac{3A_9}{4M_1\sqrt{M_1}}$ $\quad E_1 = A_{13} - A_{19}$ $\quad E_2 = A_{14} + A_{20}$

$E_3 = A_{15} - A_{16}$ $\quad E_4 = \frac{-A_{12} + 2\sqrt{M_1}(A_{15} + A_{16}) + (E_1 + E_2)}{\sqrt{M_1}}$

$$E_5 = -\frac{12A_{10} + 108A_{11} + A_{12}}{\sqrt{M_1}} + 2(A_{15}e^{12\sqrt{M_1}} + A_{16}e^{-12\sqrt{M_1}}) + 6(E_1e^{6\sqrt{M_1}} - E_1e^{-6\sqrt{M_1}})$$
$$+ \frac{1}{\sqrt{M_1}}(E_1e^{6\sqrt{M_1}} + E_2e^{-6\sqrt{M_1}}) - 36(A_{17}e^{6\sqrt{M_1}} - A_{18}e^{-6\sqrt{M_1}}) - \frac{12}{\sqrt{M_1}}(A_{17}e^{6\sqrt{M_1}} + A_{18}e^{-6\sqrt{M_1}})$$

$$C_6 = E_3 - C_7 - C_8 \qquad C_7 = \frac{E_5 - E_4 e^{-6\sqrt{M_1}}}{e^{6\sqrt{M_1}} - e^{-6\sqrt{M_1}}}$$

$$C_8 = \frac{E_5 - E_4 e^{6\sqrt{M_1}}}{e^{6\sqrt{M_1}} - e^{-6\sqrt{M_1}}} \qquad A_{21} = \Pr .C_1.C_4.\sqrt{M_3} \qquad A_{22} = \Pr .C_1.C_5.\sqrt{M_3}$$

$$A_{23} = \Pr .C_2.C_4.\sqrt{M_3} \qquad A_{24} = \Pr .C_3.C_5.\sqrt{M_3} \qquad A_{25} = \Pr .C_2.C_5.\sqrt{M_3}$$

$$A_{26} = \Pr .C_3.C_4.\sqrt{M_3} \qquad A_{27} = \Pr .C_4.\frac{M_2\sqrt{M_3}}{M_1} \qquad A_{28} = \Pr .C_5.\frac{M_2\sqrt{M_3}}{M_1}$$

$$C_9 = \frac{B_2 - B_1 e^{-6\sqrt{M_3}}}{e^{6\sqrt{M_3}} - e^{-6\sqrt{M_3}}} \qquad C_{10} = \frac{B_1 e^{6\sqrt{M_3}} - B_2}{e^{6\sqrt{M_3}} - e^{-6\sqrt{M_3}}} \qquad B_1 = A_{31} - A_{32} - A_{33} + A_{34}$$

$$B_2 = -6(A_{39}e^{6\sqrt{M_3}} + A_{40}e^{-6\sqrt{M_3}}) + 36(A_{35}e^{6\sqrt{M_3}} - A_{36}e^{-6\sqrt{M_3}}) + (A_{31}e^{6M_4} - A_{32}e^{-6M_4}) - (A_{33}e^{6M_5} - A_{34}e^{-6M_5})$$

$$A_{29} = \frac{A_{21}}{2\sqrt{M_3}} \qquad A_{30} = \frac{A_{22}}{2\sqrt{M_3}} \qquad A_{31} = \frac{A_{23}}{M_4{}^2 - M_3}$$

$$A_{32} = \frac{A_{24}}{M_4{}^2 - M_3} \qquad A_{33} = \frac{A_{25}}{M_5{}^2 - M_3} \qquad A_{34} = \frac{A_{26}}{M_5{}^2 - M_3}$$

$$A_{35} = \frac{A_{27}}{4\sqrt{M_3}} \qquad A_{36} = \frac{A_{28}}{4\sqrt{M_3}} \qquad A_{37} = \frac{A_{27}}{4M_3}$$

$$A_{38} = \frac{A_{28}}{4M_3} \qquad A_{39} = -A_{29} + A_{37} \qquad A_{40} = -A_{30} - A_{38}$$

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