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# EFFECT OF PRESSURE WORK ON FREE CONVECTION FLOW FROM AN ISOTHERMAL TRUNCATED CONE WITH TEMPERATURE DEPENDENT VISCOSITY

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**Abstract:** The object of this article is to analyze the impact of pressure work on a steady free flow of convection over an isothermal truncated cone with viscosity dependent on temperature. The numerical results of the converted non-similar boundary layer equations are found in this paper by a finite differential method with quasi-linearization scheme. The effects of several physical parameters are represented graphically for changed values of the pressure work ($\epsilon$) and the parameter of viscosity dependent on temperature ($\varepsilon$). Results indicate that both pressure function parameter and temperature dependent viscosity are strongly impacted by skin friction and transferring of heat. It is perceived that the skin friction coefficient decreases and the transferring of heat coefficient rises, increasing the pressure work ($\epsilon$).

**Keywords:** free convection; pressure work; isothermal truncated cone; variable viscosity.

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## 1. INTRODUCTION

Natural convection flow is also used in nuclear reactor cooling, or in the study of star and planet formation. Studying temperature and transferring of heat is of great prominence to the engineers. We usually neglect the study of free convection flows and pressure work results, here we discuss the impact of pressure work on a natural convective flow along an isothermal truncated cone.

A few writers examined natural (free) laminar convection flows, particularly due to the non-uniform surface temperature. The universal relationships on isothermal axisymmetric shapes for similar solutions and demonstrated that a vertical cone has such an answer for the flow past observed by Mark and Prins [1, 2]. Braun et al. [3] examined flows with free convection comparisons on two dimensional axisymmetric bodies. Hering and Grosh [4] have exhausted the free convective from a non-isothermal vertical cone. Roy [5] extended Hering and Grosh's study [4] to include high Prandtl numbers. For an isothermal surface, Na and Chiou [6] introduced the free convective over a frustum of a cone deprived of transverse curvature impact.

Alamgir [7] is analyzed the complete transferring of heat from the vertical cone by integral method for natural laminar convection. The outcome of viscous dissipation and pressure work in free convective flow along a vertical isothermal plate was studied by Pantokratoras[8]. Alam et al. [9] observed the natural convective flow of a vertical porous round cone, sustained with pressure work at non-uniform surface temperature. Recently Elbashbeshy et al. [10] examined the outcome of the natural convective flow around a truncated cone with pressure work.

In the overhead readings it was believed the fluid's viscosity was unchanged. Viscosity, however, is known to change knowingly with temperature. Gary et al. [11] and Mehta et al. [12] have revealed that the flow features may varied considerably when this effect is included equated to the constant viscosity case. The results of free convective from a vertical cone [13- 15] have been studied by several authors. Hossain et al. [16] recently considered a viscous fluid's free convective flow around a truncated cone with viscosity.

In the current research, in the existence of pressure work with temperature dependent viscosity, we examine the steady laminar natural convective flow and transferring of heat over an isothermal truncated cone by motivated. Here, the focus was limited only to their pressure effects operating with viscosity.

## 2. MATHEMATICAL FORMULATION

Figure 1 displays the flow structure of the isothermally truncated cone and the two-dimensional cartesian coordinate scheme. The origin of the coordinate scheme is located at the vertex of the entire cone, where $x$ is the coordinate along the surface of the cone, and y is the usual surface coordinate. It is presumed that the boundary layer forms at the top edge of the truncated cone ($x = x_0$), the temperature at the circular base is believed to be the similar as the $T_\infty$ ambient temperature. The $T_w$ cone surface temperature is constant and higher than the $T_\infty$ ($T_w > T_\infty$) free stream temperature.

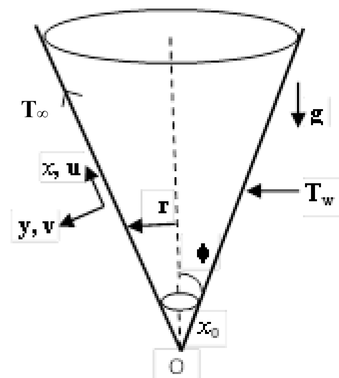


Figure. 1 The Geometry and system of coordinates

The boundary layer equations for free convective flow over an isothermal truncated cone, under the above assumptions, valid in the field $x_0 \leq x \leq \infty$, are as follows

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta cos\gamma(T - T_\infty) + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{T\beta u}{\rho C_p}\frac{\partial p}{\partial x} \tag{3}$$

Here $u$, $v$ represents the velocity components in $x$- and $y$-directions, $g$ denotes the gravitational acceleration, $\mu$ is the fluid viscosity coefficient, $\beta$ is the thermal expansion coefficient, T denotes the temperature inside the boundary layer, $\alpha$ denotes the thermal diffusivity, $\rho$ is the fluid density, $C_p$ is the specific heat and $\partial p/\partial x = \rho g$ be the hydrostatic pressure.

The boundary conditions are known by:

$$u = 0, \quad v = 0, \quad T = T_w \ at \quad y = 0 \tag{4}$$

$$u \to 0, \quad T \to T_\infty \quad as \quad y \to \infty$$

In the current study, the viscosity of a semi-empirical formula is given by

$$\frac{\mu}{\mu_\infty}=\frac{1}{1+\gamma(T-T_\infty)} \tag{5}$$

And as established by Ling and Dybbs [17], have remained accepted, here viscosity of the fluid denoted by $\mu_\infty$.

By representing the stream function $\psi$ defined by

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{6}$$

Here we considered the boundary layer to be thin enough. The local radius to a point in the boundary layer can be changed by the truncated cone radius, $r = x\sin\gamma$, where the cone's semi-vertical angle is $\gamma$.

Applying the following transformations.

$$\xi=\frac{x^*}{x_0}=\frac{x-x_0}{x_0}, \quad \eta=\frac{y}{x^*}(Gr_{x^*})^{1/4} \quad , \quad (Gr_{x^*})^{1/4}=\frac{g\beta\cos\gamma(T_w-T_\infty)x^{*3}}{\vartheta^2}$$

$$\psi=\upsilon r(Gr_{x^*})^{1/4}f(\xi,\eta),\ T-T_\infty=(T_w-T_\infty)G(\xi,\eta),\quad f'=F=\frac{\partial f}{\partial\eta}$$

$$u=\frac{\upsilon(Gr_{x^*})^{1/2}}{x^*}F=U_rF \quad , \quad v=-\frac{\upsilon(Gr_{x^*})^{1/4}}{x^*}\left[\left(\frac{\xi}{\xi+1}+\frac{3}{4}\right)F+\xi\frac{\partial f}{\partial\xi}-\frac{1}{4}\eta F\right] \tag{7}$$

To equation from (1) to (4), the continuity equation (1) is identically contented, then equation (2) and (3) reduced, respectively to

$$F''+(1+\varepsilon G)\left[\left(\frac{\xi}{\xi+1}+\frac{3}{4}\right)fF'-\frac{1}{2}F^2+G\right]-\left(\frac{\varepsilon}{1+\varepsilon G}\right)G'F'-(1+\varepsilon G)\xi\left[FF_\xi-F'f_\xi\right]=0 \tag{8}$$

$$G''+\left(\frac{\xi}{\xi+1}+\frac{3}{4}\right)fG'P_r-\xi\left(FG_\xi-G'f_\xi\right)P_r-\epsilon GFP_r=0 \tag{9}$$

The boundary conditions for the above non dimensional equations (8)-(9) are known by

$$f'=F=0,\quad G=1 \text{ at } \quad \eta=0 \tag{10}$$

$$F=0,\ G=0 \text{ as } \eta\to\infty$$

Here $\psi$ and $f$ are dimensional and dimensionless stream function correspondingly, $F$ denotes the dimensionless velocity and $G$ denotes the dimensionless temperature, $\eta$ denotes the pseudo similarity variable. $\varepsilon$ denotes the variable viscosity parameter, $Gr_{x^*}$ is local Grashof number, $x$ denotes the stream wise coordinate, $\xi$ is the dimensionless distance, $x^*$ denotes the distance measured from the leading edge of the truncated cone, $Pr = \upsilon/\alpha$ is the Prandtl number, and Gebhart [18] is the person introduced first pressure work parameter $\epsilon = g\beta\, x^* / C_p$

The local skin friction $(C_f)$ and transferring of heat coefficient in terms of Nusselt number $(Nu)$ can be stated as

$C_f = \frac{2\tau_w}{\rho U_r^2}$ and $Nu = -\frac{q_w x^*}{k(T_w - T_\infty)}$, Where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$

Using the transformation (7), then $C_f$ and $Nu$ take the form:

$$(Gr_{x^*})^{1/4} C_f = \left(\frac{2}{1+\varepsilon}\right) F'(\xi, 0) \tag{11}$$

$$\frac{Nu}{(Gr_{x^*})^{1/4}} = -G'(\xi, 0) \tag{12}$$

## 3. RESULTS AND DISCUSSION

The scheme of coupled nonlinear partial differential equations (8) and (9) with boundary conditions (10) was resolved numerically by the implicit finite difference system along with the method of quasilinearisation. Having described the method in Inouye and Tate[19] and A. H. Srinivasa et al. [20]. Its definition is neglected here for the sake of shortness. To verify the precision of obtained numerical results, we equated our steady state outcomes with Na & Chiou[6], as shown in Table 1, *Pr* = 0.7, 1.0 and 10.0 nearby the foremost edge $\xi = 0$ and $\epsilon = 0$.

The results obtained for steadiness are presented in the form of graphs for skin friction, transferring of heat coefficient, velocity and temperature outlines in various pressure work values along with the *Pr* = 0.72 and temperature dependent viscosity as shown in figures 2 to 5.

Table 1. Contrast of values of $F'(0,0)$ and – $G'$ (0, 0) for many values of (Pr = 0.7, 1.0 and 10.0 at $\epsilon = 0$ & $\xi = 0$ ) with [6]

| $\epsilon = 0$ | | | | |
|---|---|---|---|---|
| | $F'(0,0)$ | | $-G'(0,0)$ | |
| Pr | Ref.[6] | Present results | Ref.[6] | Present results |
| 0.7 | 0.9584 | 0.9588 | 0.3532 | 0.3539 |
| 1.0 | 0.9081 | 0.9080 | 0.4010 | 0.4014 |
| 10.0 | 0.5930 | 0.5932 | 0.8269 | 0.8261 |

Figure 2 shows that variation of skin friction coefficient $[c_f(Gr_{x^*})^{1/4}]$ and transferring of heat coefficient $[(N_u(Gr_{x^*})^{-1/4})]$ for pressure work ($\epsilon$) at the stream wise location $\xi = 1.0$, for Pr $=$ $0.72$ is presented in figures 2(a) & 2(b) correspondingly. It is observed that $[c_f(Gr_{x^*})^{1/4}]$ found to decline & $[(N_u(Gr_{x^*})^{-1/4}]$ rises with rise of dimensionless distance ($\xi$) in the range ($0 \leq \xi \leq 1.0$). The percentage of decrease in $[c_f(Gr_{x^*})^{1/4}]$ is 28.09% whereas the percentage of increase in $[(N_u(Gr_{x^*})^{-1/4})]$ is 27.38% near $\xi = 1.0$.

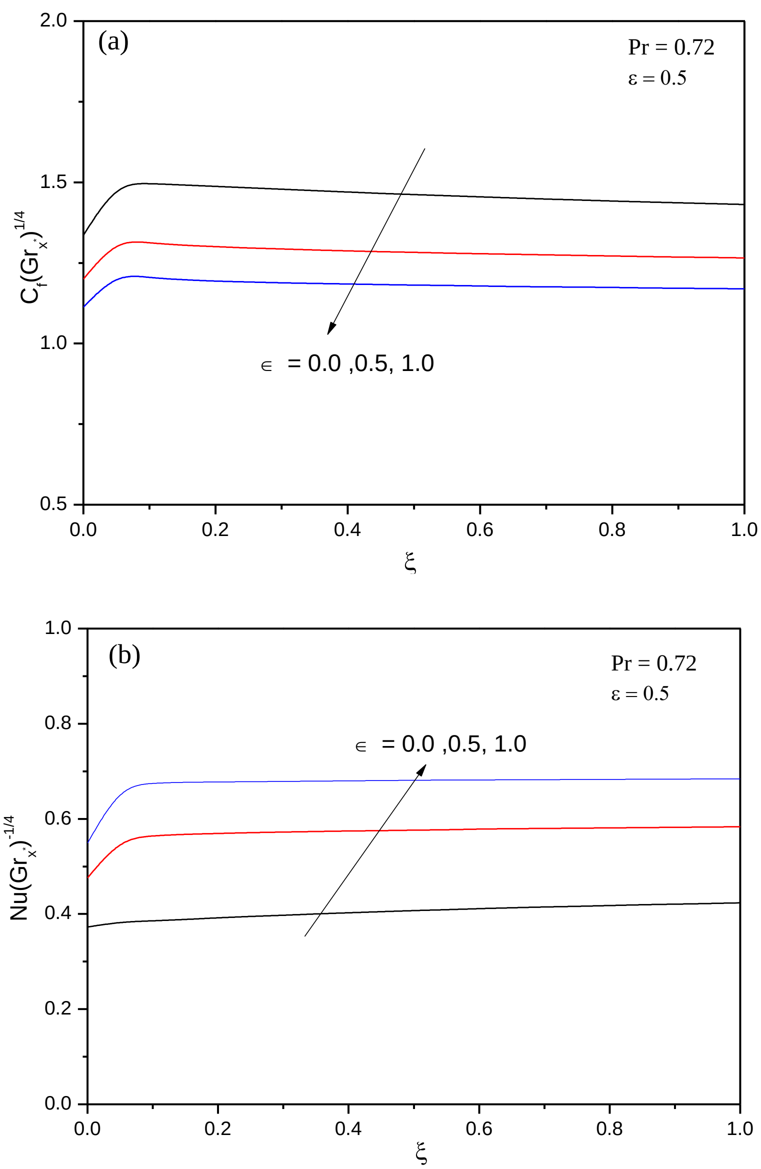


**Figure 2**. (a) illustrates the Skin friction and (b) transferring of heat coefficient for changed data of pressure work with *Pr = 0.72 and* $\varepsilon = 0.5$

The corresponding outlines of velocity ($F$) & temperature ($G$) are revealed in fig 3(a) & 3(b) correspondingly. It is perceived that velocity declines sharply near the wall where, as the temperature declines, with rise of pressure work. Numerical findings from these data show that at rising distances from the leading edge. The percentage of decline in the thickness of the boundary layer of momentum is 10.88 percentages and moreover the thermal boundary layer thickness is almost 14.08 percentage close to $\eta = 1.2$, while the parameter of pressure work ranges from 0.0 to 1.0.

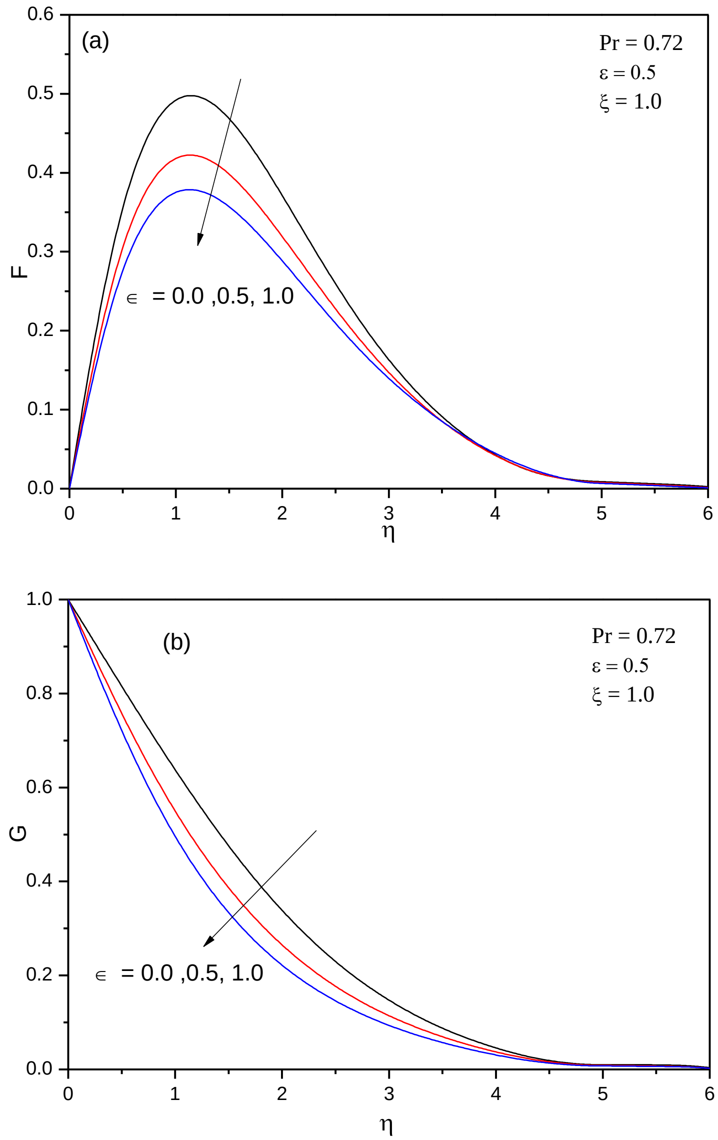


**Figure 3** (a) illustrates the Velocity and (b) Temperature outlines for changed facts of pressure work at $\varepsilon = 0.5$ and $Pr = 0.72$

Figure 4(a) and 4 (b) illustrates the influence of variable viscosity $(\varepsilon)$ for a fixed pressure work $(\epsilon)$ and Prandtl number *(Pr)* . It is perceived that $[c_f(Gr_{x^*})^{1/4}]$ found to decline & $[(N_u(Gr_{x^*})^{-1/4}]$ rises with rise of dimensionless distance $(\xi)$ in the range $(0 \leq \xi \leq 1.0)$. The percentage of decrease in $[c_f(Gr_{x^*})^{1/4}]$ is 50.75% whereas the percentage of increase in $[(N_u(Gr_{x^*})^{-1/4})]$ is 4.89% near $\xi = 1.0$.

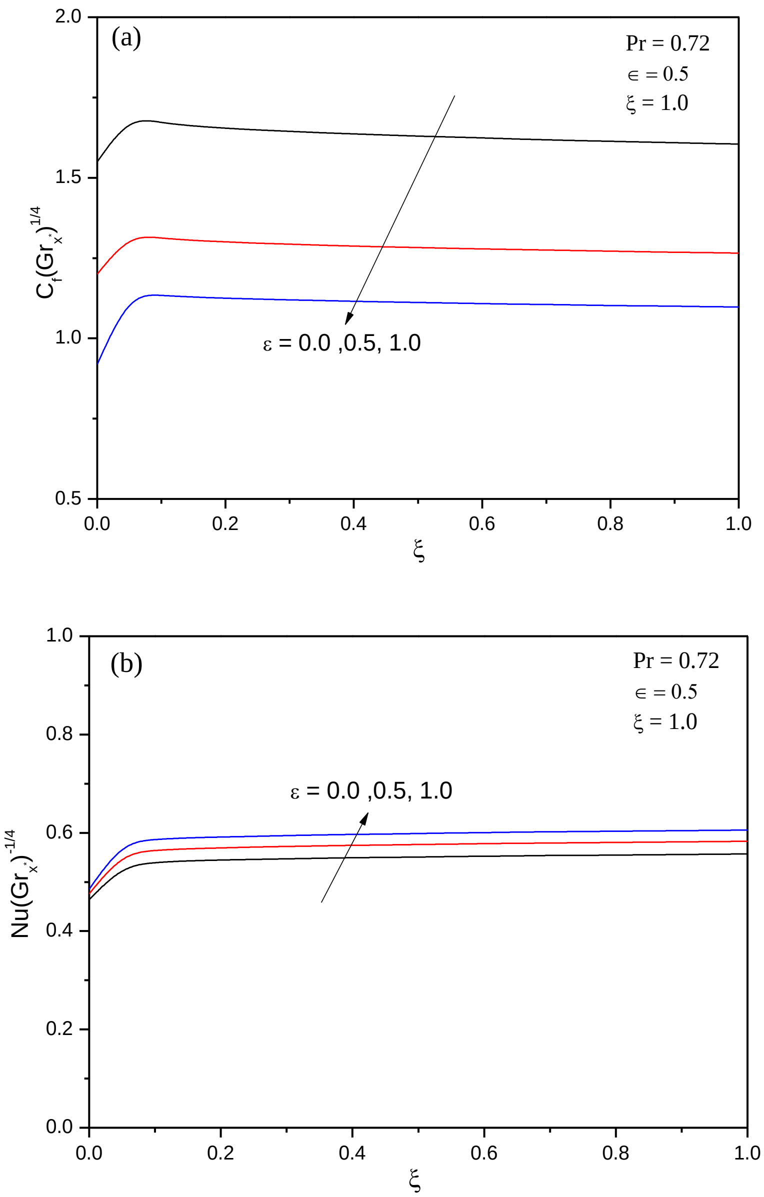


**Figure 4** (a) illustrates the Skin friction $[c_f]$ and (b) transferring of heat$[N_u]$ factor, for changed facts of $\varepsilon$ with $Pr = 0.72$ *and* $\epsilon = 0.5$

Figure 5 (a) and 5 (b) highlights the influence of variable viscosity $(\varepsilon)$ for a fixed pressure work $(\epsilon)$ as well as Prandtl number $(Pr)$ . The impact of variable viscosity on the corresponding velocity and temperature outlines for a fixed facts of $Pr$ = 0.72, and $\epsilon = 1.0$. It is perceived that velocity found to increase and temperature falls with increase of $\eta$ in the range ($0 \leq \eta \leq 6.0$). The percentage of increase in thickness of momentum boundary layer is 8.17% and thickness of thermal boundary layer is approximately declines 3.22 % near $\eta = 0.8$, when variable viscosity parameter varies from 0.0 to 1.0.

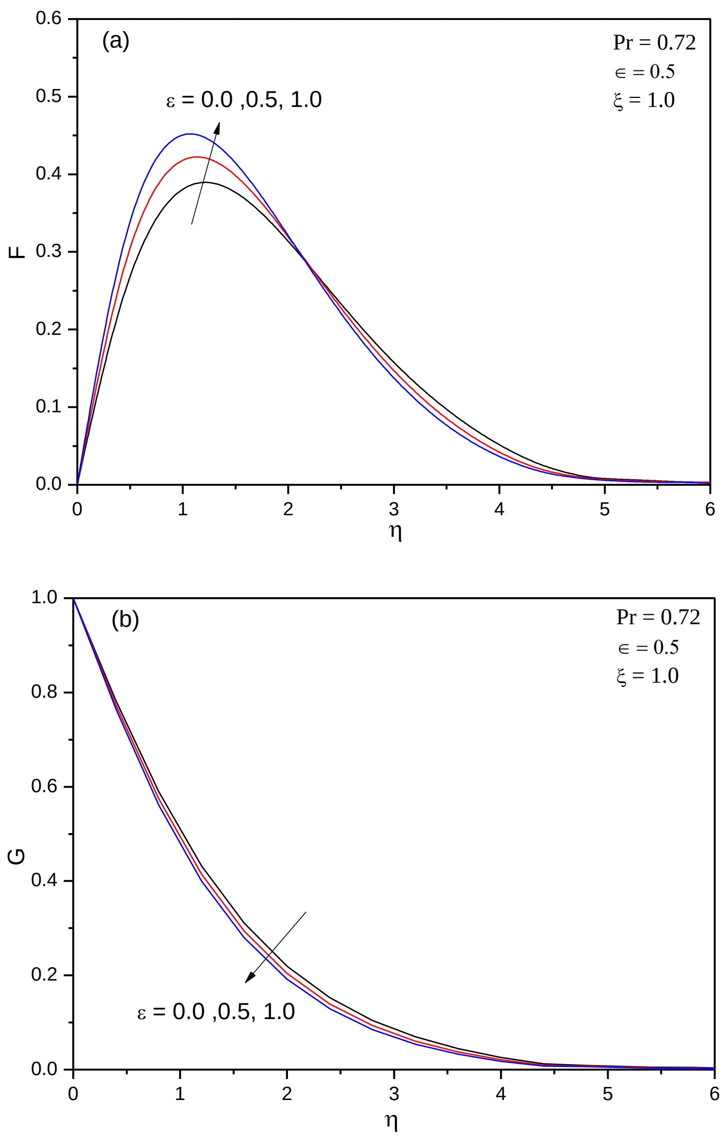


**Figure 5**(a) illustrates the velocity and (b) temperature outlines, for changed value of $\varepsilon$ with $Pr = 0.72$ and $\epsilon = 0.5$

## 4. CONCLUSIONS

The existing study have observed the impact of pressure work parameter with variable viscosity, thus concludes the following

i. As the pressure work rises, the skin- friction coefficient declines but transferring of heat coefficient rises.

ii. Both the velocity and temperature outlines decline with a rise of pressure work$(\epsilon)$.

iii. With rising values of the viscosity $(\varepsilon = 0, 0.5, 1.0)$ the skin friction declines and transferring of heat coefficient rises for the fixed values of Prandtl number *(Pr =0.72)* and pressure work $(\epsilon = 0.5)$.

iv. With an increase in the viscosity $(\varepsilon = 0, 0.5, 1.0)$ the velocity profiles rises and temperature outlines decreases for the fixed pressure work $(\epsilon = 0.5)$ and Prandtl number (*Pr =0.72*).

## CONFLICT OF INTERESTS

The authors confirm that this article contents have no conflict of interest.

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