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s*g-IRRESOLUTE TOPOLOGICAL VECTOR SPACES

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Abstract. We introduce and study a new class of spaces, namely s^*g-irresolute topological vector spaces via s^*g-open sets. We explore and investigate several properties and characterizations of this new notion of s^*g-irresolute topological vector space.

Keywords: topological space; s^*g-closed set; s^*g-open set; s^*g-irresolute topological vector space.

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1. INTRODUCTION

The concept of topological vector spaces was introduced by Kolmogroff in 1934 [19]. Its properties and characterizations were studied and investigated by many different mathematicians. Due to its large number of exciting and interesting properties and characterizations, it has been used in different advanced branches of mathematics like fixed point theory, operator theory, variational inequalities, differential calculus, etc. The researchers not only make use of topological vector spaces in many other fields to develop new concepts but also stretch and extend this notion in every possible way to make the field of study a more convenient and understandable. In 2008, M. Khan, T. Nori, and M. Hussain [16] introduced s^*g-*closed* sets and s^*g-*open* sets in topological spaces and showed that the family of all s^*g-*open* subsets of a topological space (X,τ) forms a

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topology on X which is finer than τ. Also, they studied some characterizations and basic properties of s^*g-*open* sets and s^*g-*closed* sets. They also used these sets to define and study a new class of functions, namely, s^*g-*continuous* functions as well as s^*g-*Normal* spaces. We introduce s^*g-*irresolute* topological vector spaces by using s^*g-*open* sets and investigate several general properties and characterizations of this notion of s^*g-*irresolute* topological vector space. We also give several characterizations of s^*g-*Hausdorff* spaces. Furthermore, we show that the extreme point of the convex subset of s^*g-*irresolute* topological vector space X lies in the boundary spaces.

2. s*g-OPEN SETS IN TOPOLOGICAL SPACES

The (X,τ) and (Y,σ) $(\text{or simply, } X \text{ and } Y)$ denote topological spaces on which no separation axioms are assumed unless explicitly stated. A subset A of a topological space (X,τ) is said to be open if $A\in\tau$. A subset A of a topological space X is said to be closed if the set $A^c = X - A$ is open. The interior of a subset A of a topological space X is defined as the union of all open sets contained in A. It is denoted by $Int(A)$. The closure of a subset A of a topological space X is defined as the intersection of all closed sets containing A. It is denoted by $Cl(A)$.

We start recalling the following definitions and results from $[16]$, which are necessary for this study in the sequel.

Definition 2.1. A subset A of a topological space (X,τ) is said to be *semi-open* set if $A\subseteq Cl[Int(A)]$. $SO(X)$ represents the collection of all *semi-open* sets in X.

Definition 2.2. A subset A of a topological space (X,τ) is said to be *semi-closed* set if $X-A$ is *semi-open*. $SC(X)$ represents the collection of all *semi-closed* sets in X.

Definition 2.3. A subset A of a topological space (X,τ) is said to be α-*open* set if $A\subseteq Int[Cl(Int(A))]$.

Definition 2.4. A subset A of a topological space (X,τ) is said to be α-*closed* set if $X-A$ `is α-*open*.

Definition 2.5. Let (X,τ) be a topological space. A subset A of X is said to be generalized closed (briefly, g-*closed*) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X. The complement of a *g-closed* set is *g-open*.

Definition 2.6. Let (X,τ) be a topological space. A subset A of X is said to be generalized *semi-closed* (briefly, *gs-closed*) if $sCl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X. The complement of a *gs-closed* set is *gs-open*.

Definition 2.7. Let (X,τ) be a topological space. A subset A of X is said to be generalized α-*closed* (briefly, $g\alpha$-*closed*) if $\alpha\text{-}Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-*open* in X. The complement of a $g\alpha$-*closed* set is $g\alpha$-*open*.

Definition 2.8. Let (X,τ) be a topological space. A subset A of X is said to be s^*g-*closed* if $Cl(A)\subseteq G$ whenever $A\subseteq G$ and G is *semi-open* in X. The collection of all s^*g-*closed* subsets of X is denoted by $S^*GC(X)$.

Definition 2.9. Let (X,τ) be a topological space and $A\subseteq X$. Then the s^*g-*closure* of A, denoted by $s^*g\text{-}Cl(A)$ is the intersection of all s^*g-*closed* subsets of X which contain A.

Definition 2.10. Let (X,τ) be a topological space. A subset A of X is said to be s^*g-*open* if $X-A$ is s^*g-*closed*, or equivalently, if $G\subseteq Int(A)$ whenever $G\subseteq A$ and G is *semi-closed* in X. The collection of all s^*g-*open* subsets of X is denoted by $S^*GO(X)$.

Definition 2.11. Let (X,τ) be a topological space and $A\subseteq X$. Then the s^*g-*interior* of A, denoted by $s^*g\text{-}Int(A)$ is the union of all s^*g-*open* subsets of X which are contained in A.

Definition 2.12. [23] A subset A of a topological space (X,τ) is said to be:

(i) An $\alpha\text{-}s^*g$-*open* set if $A\subseteq s^*g\text{-}Int\left[Cl\left(s^*g\text{-}Int(A)\right)\right]$.

(ii) A *pre*-s^*g-*open* set if $A\subseteq s^*g\text{-}Int\left[Cl(A)\right]$.

(iii) A $b\text{-}s^*g$-*open* set if $A\subseteq s^*g\text{-}Int\left[Cl(A)\right]\cup Cl\left[s^*g\text{-}Int(A)\right]$.

(iv) A $\beta\text{-}s^*g$-*open* set if $A\subseteq Cl\left[s^*g\text{-}Int\left(Cl(A)\right)\right]$.

Theorem 2.13. The union of two s^*g-*closed* sets (and hence the finite union of s^*g-*closed* sets) in a topological space (X,τ) is s^*g-*closed*.

Proof. Let A and B be any two s^*g-*closed* sets in a topological space (X,τ). Let G be a *semi-open* set containing $A\cup B$. Then $Cl(A)\subseteq G$ and $Cl(B)\subseteq G$ implies that $Cl(A\cup B)\subseteq G$. This proves that $A \cup B$ is $s^* g\text{-}closed$.

Theorem 2.14. An arbitrary intersection of $s^* g\text{-}closed$ sets in a topological space X is $s^* g\text{-}closed$.

Proof. Theorem 3.12 in [16].

Corollary 2.15. For any space (X, τ), $S^* GO(X)$ is a topology on X.

Remarks 2.16. (1) We summarize the fundamental relationships between several types of generalized closed sets in the following diagram. None of the implications is reversible.

$$\begin{array}{ccccc}
closed & \longrightarrow & s^* g\text{-}closed & \longrightarrow & g\text{-}closed \\
\downarrow & & \downarrow & & \\
\alpha\text{-}closed & \longrightarrow & g\alpha\text{-}closed & \longrightarrow & gs\text{-}closed
\end{array}$$

(2) The following diagram represents the fundamental relationships between several types of open sets and $s^* g\text{-}open$ sets. None of the implications is reversible.

$$\begin{array}{ccccccccc}
open & \longrightarrow & \alpha-open & \longrightarrow & pre-open & \longrightarrow & b-open & \longrightarrow & \beta-open \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
s^* g\text{-}open & \longrightarrow & \alpha-s^* g\text{-}open & \longrightarrow & pre-s^* g\text{-}open & \longrightarrow & b-s^* g\text{-}open & \longrightarrow & \beta-s^* g\text{-}open
\end{array}$$

Definition 2.17. Let (X, τ) be a topological space and let $x \in X$. A subset A of X is said to be $s^* g\text{-}neighborhood$ of x if there exists an $s^* g\text{-}open$ set G such that $x \in G \subseteq A$.

The set of all $s^* g\text{-}neighborhoods$ of $x \in X$ is called $s^* g\text{-}neighborhood$ system at x and is denoted by $s^* g\text{-}N(x) = \{A \subseteq X : A \text{ is } s^* g\text{-}neighborhood \text{ of } x\}$.

Remark 2.18. Every neighborhood A of $x \in X$ is $s^* g\text{-}neighborhood$ of x. But in general, an $s^* g\text{-}neighborhood$ A of $x \in X$ need not be a neighborhood of x in X.

Theorem 2.19. Let (X, τ) be a topological space and $A, B \subseteq X$. Then the following assertions are true.

(1) $s^* g\text{-}Int(X) = X$ and $s^* g\text{-}Int(\phi) = \phi$.

(2) $Int(A) \subseteq s^* g\text{-}Int(A) \subseteq A$.

(3) $A \subseteq s^* g\text{-}Cl(A) \subseteq Cl(A)$.

(4) If B is any $s^* g\text{-}open$ set contained in A, then $B \subseteq s^* g\text{-}Int(A)$.

(5) A is s^*g-*closed* if and only if $s^*g\text{-}Cl(A)=A$.

(6) If $A\subseteq B$, then $s^*g\text{-}Int(A)\subseteq s^*g\text{-}Int(B)$.

(7) $s^*g\text{-}Int\left[s^*g\text{-}Int(A)\right]=s^*g\text{-}Int(A)$.

(8) A is s^*g-*open* if and only if $s^*g\text{-}Int(A)=A$.

(9) $s^*g\text{-}Cl(X)=X$ and $s^*g\text{-}Cl(\phi)=\phi$.

(10) If B is any s^*g-*closed* set containing A, then $s^*g\text{-}Cl(A)\subseteq B$.

(11) If $A\subseteq B$, then $s^*g\text{-}Cl(A)\subseteq s^*g\text{-}Cl(B)$.

(12) $s^*g\text{-}Cl\left[s^*g\text{-}Cl(A)\right]=s^*g\text{-}Cl(A)$.

(13) $s^*g\text{-}Int(A\cap B)=s^*g\text{-}Int(A)\cap s^*g\text{-}Int(B)$ and $s^*g\text{-}Cl(A\cup B)=s^*g\text{-}Cl(A)\cup s^*g\text{-}Cl(B)$.

(14) $X-s^*g\text{-}Int(A)=s^*g\text{-}Cl(X-A)$.

(15) $X-s^*g\text{-}Cl(A)=s^*g\text{-}Int(X-A)$.

(16) $x\in s^*g\text{-}Cl(A)$ if and only if for every s^*g-*open* set U containing x, $U\cap A\neq\phi$.

(17) $x\in s^*g\text{-}Int(A)$ if and only if there is an s^*g-*open* set U in X such that $x\in U\subseteq A$.

(18) $\bigcup_{\lambda\in\Lambda}s^*g\text{-}Cl(U_\lambda)\subseteq s^*g\text{-}Cl\left(\bigcup_{\lambda\in\Lambda}U_\lambda\right)$ and $\bigcup_{\lambda\in\Lambda}s^*g\text{-}Int(U_\lambda)\subseteq s^*g\text{-}Int\left(\bigcup_{\lambda\in\Lambda}U_\lambda\right)$.

Definition 2.20 A mapping $f:(X,\tau)\to(Y,\sigma)$ is called s^*g-*irresolute* at a point $x\in X$ if for all s^*g-*open* subsets V in Y containing $f(x)$, there is an s^*g-*open* subset U of X such that $x\in U$ and $f(U)$ is a subset of V. The function f will be called s^*g-*irresolute* if f is s^*g-*irresolute* at each point $x\in X$.

Theorem 2.21. Let $f:(X,\tau)\to(Y,\sigma)$ be a function. Then the following statements are equivalent.

(1) f is s^*g-*irresolute*.

(2) For each $x\in X$ and each s^*g-*neighborhood* V of $f(x)$ in Y, there is an s^*g-*neighborhood* U of x such that $f(U)\subseteq V$.

(3) The inverse image of every s^*g-*closed* subset of Y is an s^*g-*closed* subset of X.

(4) The inverse image of every s^*g-*open* subset of Y is an s^*g-*open* subset of X.

Definition 2.22. A function $f:(X,\tau)\to(Y,\sigma)$ is called s^*g-*continuous* if $f^{-1}(V)$ is s^*g-*open* set in X for every open set V in Y.

Proposition 2.23. A function $f:(X,\tau)\to(Y,\sigma)$ is called $s^{*}g$*-continuous* if and only if $f^{-1}(V)$ is $s^{*}g$*-closed* set in X for every closed set V in Y.

Definition 2.24. A function $f:(X,\tau)\to(Y,\sigma)$ is called *Pre-*$s^{*}g$*-open* if and only if the image set $f(U)$ is $s^{*}g$*-open* set in Y for every $s^{*}g$*-open* set U in X.

Proposition 2.25. A bijection function $f:(X,\tau)\to(Y,\sigma)$ is called $s^{*}g$*-homeomorphism* if f is *pre-*$s^{*}g$*-open* and $s^{*}g$*-irreseolute.*

Theorem 2.26. Let (X,τ) be a topological space. Then the family $S^{*}GO(X,\tau)$ of all $s^{*}g$*-open* subsets of X forms a topology on X.

Proposition 2.27. A subset A of a topological space (X,τ) is $s^{*}g$*-open* if and only if it is an $s^{*}g$*-neighborhood* of each of its points.

Proof . $\Rightarrow$: If A is $s^{*}g$*-open* in X, then $x\in A\subseteq A$ for each $x\in A.$ Thus A is an $s^{*}g$*-neighborhood* of each of its points.

Conversely, suppose that A is an $s^{*}g$*-neighborhood* of each of its points. Then for each $x\in A,$ there exists an $s^{*}g$*-open* set U_x in X such that $x\in U_x\subseteq A.$ Hence $\bigcup_{x\in A}U_x\subseteq A.$ Since $A\subseteq\bigcup_{x\in A}U_x,$ therefore $A=\bigcup_{x\in A}U_x.$ Thus A is an $s^{*}g$*-open* set in X, since it is a union of $s^{*}g$*-open* sets.

3. PROPERTIES OF s*g-IRRESOLUTE TOPOLOGICAL VECTOR SPACES

In this section, we define and investigate some basic properties of $s^{*}g$*-irresolute* topological vector spaces.

Definition 3.1. A topological space $(X_{(K)},\tau)$ is called $s^{*}g$*-irresolute* topological vector space $(s^{*}gITVS)$ whenever the following conditions are satisfied.

(1) for each $x,y\in X$ and for each $s^{*}g$*-open* neighborhood W of $x+y$ in X, there exist $s^{*}g$*-open* neighborhoods U and V in X of x and y respectively, such that $U+V\subseteq W.$

(2) for each $x\in X,$ $\lambda\in K$ and for each $s^{*}g$*-open* neighborhood W of λx in X, there exist $s^{*}g$*-open* neighborhoods U of λ in K and V of x in X, such that $U.V\subseteq W.$

Theorem 3.2. Let $(X_{(K)},\tau)$ be an $s^{*}g$*-irresolute* topological vector space. Then the following assertions are true.

(1) The (left) right translation mapping $T_x : X \to X$ defined by $T_x(y) = y + x;$ for all $x, y \in X$, is $s^* g$-*irresolute*.

(2) The multiplication mapping $M_\lambda : X \to X$, defined by $M_\lambda(x) = \lambda x$, for all $x \in X$, is $s^* g$-*irresolute*.

Proof. (1) Let W be an $s^* g$-*open* neighborhood of $T_x(y) = x + y$. Then by definition, there exist $s^* g$-*open* neighborhoods U and V in X containing y and x respectively, such that $U + V \subseteq W$. This gives that $T_x(U) = U + x \subseteq U + V \subseteq W$. This proves that, $T_x : X \to X$ is an $s^* g$-*irresolute* mapping.

(2) Let $x \in X$, $\lambda \in K$. Then $M_\lambda(x) = \lambda x$. Let W be any $s^* g$-*open* neighborhood of λx. Then by definition of $s^* gITVS$, there exist $s^* g$-*open* neighborhoods U in K of λ and V in X of x, such that $U.V \subseteq W$. This gives that $M_\lambda(V) = \lambda V \subseteq U.V \subseteq W$. This proves that $M_\lambda : X \to X$ is an $s^* g$-*irresolute* mapping.

Theorem 3.3. Let $(X_{(K)}, \tau)$ be an $s^* g$-*irresolute* topological vector space. Let $A \in S^* GO(X)$. Then the following statements are true.

(1) $x + A \in S^* GO(X)$, for every $x \in X$.

(2) $\lambda A \in S^* GO(X)$, for every $non-zero$ scalar $\lambda \in K$.

Proof. (1) Let $y \in x + A$. Then $y = x + a$ for some $a \in A$. By definition of $s^* g$-*irresolute* topological vector spaces, there exist $s^* g$-*open* sets U and V in X containing $-x$ and y respectively such that $U + V \subseteq A$. This gives $-x + V \subseteq U + V \subseteq A$. This implies $y \in V \subseteq x + A$. Therefore $y \in s^* g\text{-}Int(x + A)$. Hence, $x + A = s^* g\text{-}Int(x + A)$. This proves that $x + A$ is $s^* g$-*open* in X.

(2) Let $x \in \lambda A$. Then $x = \lambda a$ for some $a \in A$. Thus $a = \frac{1}{\lambda} x \in A$. By definition of $s^* g$-*irresolute* topological vector spaces, there exist $s^* g$-*open* sets U in K containing $\frac{1}{\lambda}$ and V in X containing x such that $U.V \subseteq A$. This implies that $a = \frac{1}{\lambda} x \in \frac{1}{\lambda} V \subseteq U.V \subseteq A$. Hence $x \in V \subseteq \lambda A$. Thus we obtain $x \in s^* g\text{-}Int(\lambda A)$. Therefore, it follows that $\lambda A \subseteq s^* g\text{-}Int(\lambda A)$. Hence, $\lambda A = s^* g\text{-}Int(\lambda A)$. This shows that $\lambda A \in S^* GO(X)$.

Corollary 3.4. Let $(X_{(K)}, \tau)$ be an $s^* g$-*irresolute* topological vector space. Let A be an $s^* g$-*open* subset of X. Then the following statements are true.

(1) $x+A\subseteq s^*g\text{-}Cl\left[s^*g\text{-}Int(x+A)\right]$ for each $x\in X$.

(2) $\lambda A\subseteq s^*g\text{-}Cl\left[s^*g\text{-}Int(\lambda A)\right]$ for any non-zero scalar λ.

Theorem 3.5. Let $\left(X_{(K)},\tau\right)$ be an $s^*g\text{-}irresolute$ topological vector space. Let $A\in S^*GO(X)$ and B be any subset of X. Then prove that $A+B\in S^*GO(X)$.

Proof. Suppose $A\in S^*GO(X)$ and $B\subseteq X$. Then, for each $b\in B$ and by Theorem 3.3 (1), we have $A+b\in S^*GO(X)$. Then $A+B=\bigcup\{A+b:b\in B\}$ is a union of $s^*g\text{-}open$ sets. Since arbitrary union of $s^*g\text{-}open$ sets is $s^*g\text{-}open$, therefore $A+B$ is $s^*g\text{-}open$ in X.

Corollary 3.6. Suppose $\left(X_{(K)},\tau\right)$ is an $s^*g\text{-}irresolute$ topological vector space and let $A\in S^*GO(X)$. Then the set $U=\bigcup_{n=1}^{\infty}(nA)$ is $s^*g\text{-}open$ in X.

Theorem 3.7. Let $\left(X_{(K)},\tau\right)$ be an $s^*g\text{-}irresolute$ topological vector space. Let $A\subseteq X$. Then the following statements are true.

(1) $s^*g\text{-}Int(x+A)=x+s^*g\text{-}Int(A)$, for any $x\in X$.

(2) $s^*g\text{-}Int(\lambda A)=\lambda\left[s^*g\text{-}Int(A)\right]$, for any non-zero scalar $\lambda\in K$.

Proof. (1) By Theorem 3.3 (1), $x+s^*g\text{-}Int(A)$ is $s^*g\text{-}open$. Therefore $x+s^*g\text{-}Int(A)\subseteq x+A$ implies $x+s^*g\text{-}Int(A)\subseteq s^*g\text{-}Int(x+A)$. Now let $z\in s^*g\text{-}Int(x+A)$. Then $z=x+y$ for some $y\in A$. By definition of s^*gITVS, there exist $s^*g\text{-}open$ sets U and V in X containing x and y respectively, such that $U+V\subseteq s^*g\text{-}Int(x+A)$. This gives that $z=x+y\in x+V\subseteq s^*g\text{-}Int(x+A)\subseteq x+A$. Therefore it follows that $V\subseteq -x+s^*g\text{-}Int(x+A)\subseteq -x+(x+A)=A$. Since V is $s^*g\text{-}open$, then $V\subseteq s^*g\text{-}Int(A)$ and therefore $y\in s^*g\text{-}Int(A)$. Thus $-x+z\in s^*g\text{-}Int(A)$. Hence $z\in x+s^*g\text{-}Int(A)$. Therefore, it follows that $s^*g\text{-}Int(x+A)\subseteq x+s^*g\text{-}Int(A)$. Consequently, we conclude that $s^*g\text{-}Int(x+A)=x+s^*g\text{-}Int(A)$.

(2) By Theorem 3.3(2), $\lambda\left[s^*g\text{-}Int(A)\right]$ is $s^*g\text{-}open$. Thus $\lambda\left[s^*g\text{-}Int(A)\right]\subseteq\lambda A$ implies that $\lambda\left[s^*g\text{-}Int(A)\right]\subseteq s^*g\text{-}Int(\lambda A)$. Next, if $y\in s^*g\text{-}Int(\lambda A)$, then $y=\lambda x$ for some $x\in A$. By definition of s^*gITVS, there exist $s^*g\text{-}open$ sets U of λ in K and V of x in X such that $U.V\subseteq s^*g\text{-}Int(\lambda A)$. Therefore, $y=\lambda x\in\lambda V\subseteq U.V\subseteq s^*g\text{-}Int(\lambda A)\subseteq\lambda A$. This implies that $x\in V\subseteq A$. Since V is $s^*g\text{-}open$. Thus $x\in s^*g\text{-}Int(A)$. Consequently, $y=\lambda x\in\lambda\left[s^*g\text{-}Int(A)\right]$. Therefore we obtain

$s^*g\text{-}Int(\lambda A) \subseteq \lambda[s^*g\text{-}Int(A)]$. Hence we conclude that $s^*g\text{-}Int(\lambda A) = \lambda[s^*g\text{-}Int(A)]$.

Theorem 3.8. Let $(X_{(K)}, \tau)$ be an s^*g-*irresolute* topological vector space. Let $A, B \subseteq X$. Then $s^*g\text{-}Int(A) + s^*g\text{-}Int(B) \subseteq s^*g\text{-}Int(A+B)$.

Proof. We know that $s^*g\text{-}Int(A) \subseteq A$ and $s^*g\text{-}Int(B) \subseteq B$. Hence we obtain $s^*g\text{-}Int(A) + s^*g\text{-}Int(B) \subseteq A+B$. By Theorem 3.5, $s^*g\text{-}Int(A) + s^*g\text{-}Int(B)$ is s^*g-*open*. Therefore we have $s^*g\text{-}Int(A) + s^*g\text{-}Int(B) = s^*g\text{-}Int[s^*g\text{-}Int(A) + s^*g\text{-}Int(B)] \subseteq s^*g\text{-}Int(A+B)$. Thus we get $s^*g\text{-}Int(A) + s^*g\text{-}Int(B) \subseteq s^*g\text{-}Int(A+B)$.

Theorem 3.9. Let F be any s^*g-*closed* subset of an s^*g-*irresolute* topological vector space X. Then the following statements are true.

(1) $x + F \in S^*GC(X)$, for every $x \in X$.

(2) $\lambda F \in S^*GC(X)$, for each non – zero scalar $\lambda \in K$.

Proof. (1) Suppose that $y \in s^*g\text{-}Cl(x+F)$. Consider $z = -x + y$ and let W be any s^*g-*open* set in X containing z. Then there exist s^*g-*open* sets U and V in X such that $-x \in U$, $y \in V$ and $U + V \subseteq W$. Since $y \in s^*g\text{-}Cl(x+F)$, $(x+F) \cap V \neq \phi$. So, there is an element $a \in (x+F) \cap V$. Thus $a \in x + F$ and $a \in V$. Hence $-x + a \in F$ and $-x + a \in U + V$. Therefore $-x + a \in F \cap (U+V) \subseteq F \cap W$. Thus $F \cap W \neq \phi$. Therefore $z \in s^*g\text{-}Cl(F) = F$. Hence $y \in x + F$. Thus we conclude that $x + F = s^*g\text{-}Cl(x+F)$. This proves that $x + F$ is s^*g-*closed* set in X.

(2) Assume that $x \in s^*g\text{-}Cl(\lambda F)$. Let W be any s^*g-*open* neighborhood of $y = \frac{1}{\lambda} x$ in X. Since X is s^*gITVS, there exist s^*g-*open* sets U in K containing $\frac{1}{\lambda}$ and V in X containing x such that $U.V \subseteq W$. By hypothesis, $(\lambda F) \cap V \neq \phi$. Therefore, there is an element $a \in (\lambda F) \cap V$. Thus $a \in \lambda F$ and $a \in V$. Hence $\frac{1}{\lambda} a \in F$ and $\frac{1}{\lambda} a \in \frac{1}{\lambda} V \subseteq U.V \subseteq W$. Therefore $F \cap W \neq \phi$. Hence $y \in s^*g\text{-}Cl(F) = F$. Thus $x \in \lambda F$ and thereby, $\lambda F = s^*g\text{-}Cl(\lambda F)$. Hence $\lambda F \in S^*GC(X)$.

Corollary 3.10. Let $(X_{(K)}, \tau)$ be an s^*g-*irresolute* topological vector space and let $A \subseteq X$. Then $s^*g\text{-}Cl[x + s^*g\text{-}Cl(A)] = x + s^*g\text{-}Cl(A)$ for each $x \in X$.

Theorem 3.11. Let $(X_{(K)}, \tau)$ be an s^*g-*irresolute* topological vector space and S be a subspace of

X. If S contains a $non-empty$ $s^*g\text{-}open$ subset of X, then S is $s^*g\text{-}open$ in X.

Proof. Suppose U is a $non-empty$ $s^*g\text{-}open$ subset of X such that $U \subseteq S$. By Theorem 3.3(1), for any $y \in S$, $U + y$ is an $s^*g\text{-}open$ subset of X. Since S is a subspace of X, so also we have $U + y \subseteq S$ for any $y \in S$,Thus $S = \bigcup\{U + y : y \in S\}$ is $s^*g\text{-}open$ in X being a union of $s^*g\text{-}open$ sets.

Theorem 3.12. Let A be any subset of an $s^*g\text{-}irresolute$ topological vector space X. Then the following statements are true.

(1) $x + s^*g\text{-}Cl(A) = s^*g\text{-}Cl(x + A)$, for any $x \in X$.

(2) $s^*g\text{-}Cl(\lambda A) = \lambda[s^*g\text{-}Cl(A)]$, for any non-zero scalar λ.

Proof. (1) By applying Theorem 3.9(1), $x + s^*g\text{-}Cl(A)$ is $s^*g\text{-}closed$. Hence $x + A \subseteq x + s^*g\text{-}Cl(A)$ implies $s^*g\text{-}Cl(x + A) \subseteq x + s^*g\text{-}Cl(A)$. For the reverse inclusion, let $z \in x + s^*g\text{-}Cl(A)$. Then $z = x + y$, for some $y \in s^*g\text{-}Cl(A)$. Let W be any $s^*g\text{-}open$ neighborhood of z in X. Then, there exist $s^*g\text{-}open$ neighborhoods U and V of x and y respectively in X such that $U + V \subseteq W$. Since $y \in s^*g\text{-}Cl(A)$, $A \cap V \neq \phi$. Consider $a \in A \cap V$. Then $x + a \in (x + A) \cap (U + V) \subseteq (x + A) \cap W$. Therefore we have $(x + A) \cap W \neq \phi$. Consequently, $z \in s^*g\text{-}Cl(x + A)$. Thus $x + s^*g\text{-}Cl(A) \subseteq s^*g\text{-}Cl(x + A)$. Hence, $x + s^*g\text{-}Cl(A) = s^*g\text{-}Cl(x + A)$.

(2) By Theorem 3.9 (2), $\lambda[s^*g\text{-}Cl(A)]$ is $s^*g\text{-}closed$. Therefore $\lambda A \subseteq \lambda[s^*g\text{-}Cl(A)]$ implies that $s^*g\text{-}Cl(\lambda A) \subset \lambda[s^*g\text{-}Cl(A)]$. Next, let $x \in s^*g\text{-}Cl(A)$ and let W be any $s^*g\text{-}open$ neighborhood of $z = \lambda x$ in X. Then we get $s^*g\text{-}open$ sets U in K containing λ and V in X containing x such that $U.V \subseteq W$. Since $x \in s^*g\text{-}Cl(A)$, there is an element $a \in A \cap V$ and thus $y = \lambda a \in (\lambda A) \cap (\lambda V) \subseteq (\lambda A) \cap (UV) \subseteq (\lambda A) \cap W$. Hence $(\lambda A) \cap W \neq \phi$. Therefore it follows that $z = \lambda x \in s^*g\text{-}Cl(\lambda A)$. Thus $\lambda[s^*g\text{-}Cl(A)] \subseteq s^*g\text{-}Cl(\lambda A)$. Hence the assertion follows.

Theorem 3.13. Let $(X_{(K)}, \tau)$ be an $s^*g\text{-}irresolute$ topological vector space. Let A and B be subsets of X. Then prove that $s^*g\text{-}Cl(A) + s^*g\text{-}Cl(B) \subseteq s^*g\text{-}Cl(A + B)$.

Proof. Let $x \in s^*g\text{-}Cl(A)$ and $y \in s^*g\text{-}Cl(B)$. Let W be an $s^*g\text{-}open$ neighborhood of $x + y$. Then there exist $s^*g\text{-}open$ neighborhoods U and V of x and y respectively, such that $U + V \subseteq W$. Since, $x \in s^*g\text{-}Cl(A)$, $y \in s^*g\text{-}Cl(B)$, there are $a \in A \cap U$ and $b \in B \cap V$. Then,

$a+b\in(A+B)\cap(U+V)\subseteq(A+B)\cap W.$ Thus we have $(A+B)\cap W\neq\phi.$ This implies that $x+y\in s^*g\text{-}Cl(A+B).$ Hence eventually we obtain $s^*g\text{-}Cl(A)+s^*g\text{-}Cl(B)\subseteq s^*g\text{-}Cl(A+B).$

Theorem 3.14. Let $(X_{(K)},\tau)$ be an s^*g-*irresolute* topological vector space. For given $y\in X$ and $\lambda\in K$ with $\lambda\neq 0,$ each translation mapping $T_y : X\to X$ defined by $T_y(x)=x+y$ and multiplication mapping $M_\lambda : X\to X$ defined by $M_\lambda(x)=\lambda x,$ where $x\in X$, is s^*g-*homeomorphism* onto itself.

Proof. First, we show that $T_y : X\to X$ is s^*g-*homeomorphism.* It is obviously bijective. By Theorem 3.2 (1), T_y is s^*g-*irresolute.* Moreover, T_y is *pre*$-s^*g$-*open* because for any s^*g-*open* set U, by Theorem 3.3 (1), $T_y(U)=U+y$ is s^*g-*open.* Similarly, we can prove that M_λ is s^*g-*homeomorphism.*

Theorem 3.15. Let $(X_{(K)},\tau)$ be an s^*g-*irresolute* topological vector space. Then any s^*g-*open* subspace of X is s^*g-*closed* in X.

Proof. Let G be an s^*g-*open* subspace of X. Then by Theorem 3.3 (1), for any $x\in X-G,$ $G+x$ is s^*g-*open.* We also clearly have $x\in G+x\subseteq X-G.$ Then, $Z=\bigcup\{G+x : x\in X-G\}=X-G$ being a union of s^*g-*open* sets is s^*g-*open.* Therefore, $G=X-Z$ is s^*g-*closed*.

Theorem 3.16. Let $(X_{(K)},\tau)$ be an s^*g-*irresolute* topological vector space and B be an s^*g-*open* set in X. Then for any subset A of X ,we have $A+B=s^*g\text{-}Cl(A)+B.$

Proof. Since we know that $A\subseteq s^*g\text{-}Cl(A),$ so $A+B\subseteq s^*g\text{-}Cl(A)+B.$ Conversely, let $y\in s^*g\text{-}Cl(A)+B$ and write $y=x+b,$ where $x\in s^*g\text{-}Cl(A)$ and $b\in B.$ There exists an s^*g-*open* neighborhood V of zero such that $T_b(V)=V+b\subseteq B.$ Now, V is s^*g-*open* neighborhood of 0 in X, this gives that $-V$ is also s^*g-*open* neighborhood of 0 in X. Then $x-V$ is an s^*g-*open* neighborhood of $x.$ Since $x\in s^*g\text{-}Cl(A),$ so there exists an element $a\in A\cap(x-V).$ We know that $y=x+b=a-a+x+b\in a+V+b\subseteq A+B.$ Therefore, $s^*g\text{-}Cl(A)+B\subseteq A+B.$ Hence, consequently, we obtain $A+B=s^*g\text{-}Cl(A)+B.$

Theorem 3.17. Let $(X_{(K)},\tau)$ be an s^*g-*irresolute* topological vector space. Then the scalar multiple of s^*g-*closed* set is s^*g-*closed*.

Proof. Let B be an s^*g-*closed* set in X and let $\lambda\in K-\{0\}.$ Then $X-B$ is s^*g-*open* set in X. Now $M_\lambda(X-B)=\lambda(X-B)=\lambda X-\lambda B=X-\lambda B\in S^*GO(X).$ Therefore, $\lambda B\in S^*GC(X).$

Definition 3.18. A topological space (X,τ) is said to be s^*g-*compact* if every cover of X by s^*g-*open* sets of X has a finite sub cover. A subset A of X is said to be s^*g-*compact* relative to X if every cover of A by s^*g-*open* sets of X has a finite sub cover.

Theorem 3.19. Let $\left(X_{(K)},\tau\right)$ be an s^*g-*irresolute* topological vector space and let A be any s^*g-*compact* set in X. Then prove that $x+A$ is s^*g-*compact* for each $x\in X$.

Proof. Let $\Omega=\{U_\alpha:\alpha\in\Lambda\}$ be an s^*g-*open* cover of $x+A$. Then $A\subseteq\bigcup\{-x+U_\alpha:\alpha\in\Lambda\}$ and $\{-x+U_\alpha:\alpha\in\Lambda\}\subseteq S^*GO(X)$. By hypothesis, $A\subseteq\bigcup\{-x+U_\alpha:\alpha\in\Lambda_0\}$ for some finite subset $\Lambda_0\subseteq\Lambda$. Whence we find that $x+A\subseteq\bigcup\{U_\alpha:\alpha\in\Lambda_0\}$. This shows that $x+A$ is s^*g-*compact*. Hence, the proof is complete.

Theorem 3.20. Let $\left(X_{(K)},\tau\right)$ be an s^*g-*irresolute* topological vector space. The scalar multiple of s^*g-*compact* set is s^*g-*compact*.

Proof. Let A be an s^*g-*compact* subset of X. If $\lambda=0$ we are nothing to prove. Assume $\lambda\in K-\{0\}$. Let $\Psi=\{U_\alpha:\alpha\in\Lambda\}$ be an s^*g-*open* cover of λA. Then $A\subseteq\left(\frac{1}{\lambda}\right)(\bigcup\Psi)=\left(\frac{1}{\lambda}\right)(\bigcup\{U_\alpha:\alpha\in\Lambda\})=\bigcup\left\{\left(\frac{1}{\lambda}\right)U_\alpha:\alpha\in\Lambda\right\}$. Since $\{U_\alpha:\alpha\in\Lambda\}\subseteq S^*GO(X)$ and $\left(X_{(K)},\tau\right)$ is s^*gITVS, so we obtain $\left\{\left(\frac{1}{\lambda}\right)U_\alpha:\alpha\in\Lambda\right\}\subseteq S^*GO(X)$, By hypothesis A is s^*g-*compact*, therefore there exists a finite subset $\Lambda_0\subseteq\Lambda$ such that $A\subseteq\bigcup\left\{\left(\frac{1}{\lambda}\right)U_\alpha:\alpha\in\Lambda_0\right\}$. This implies that $\lambda A\subseteq\bigcup\{U_\alpha:\alpha\in\Lambda_0\}$. Hence λA is s^*g-*compact*.

Definition 3.21. A mapping $f:\left(X_{(K)},\tau_X\right)\to\left(Y_{(K)},\tau_Y\right)$ is said to be linear if $f(\alpha x+\beta y)=\alpha f(x)+\beta f(y)$, for all $x,y\in X$ and $\alpha,\beta\in K$.

Definition 3.22. A mapping $f:X\to K$ is called linear functional if $f(\alpha x+\beta y)=\alpha f(x)+\beta f(y)$, for all $x,y\in X$ and $\alpha,\beta\in K$. The kernel of f is defined by $Ker(f)=\{x\in X: f(x)=0\}$.

Theorem 3.23. Let $f:\left(X_{(K)},\tau_X\right)\to\left(Y_{(K)},\tau_Y\right)$ be a linear mapping such that f is s^*g-*irresolute* at 0. Then f is s^*g-*irresolute* on X.

Proof. Let x be any non-zero element of X and V be any s^*g-*open* set in Y containing $f(x)$.

Since the translation of a s^*g-*open* set in an s^*g-*irresolute* topological vector spaces is s^*g-*open*, $V-f(x)$ is s^*g-*open* set in Y containing 0. Since f is s^*g-*irresolute* at 0, there exists an s^*g-*open* set U in X containing 0 such that $f(U)\subseteq V-f(x)$. Furthermore, the linearity of f implies that $f(x+U)\subseteq V$. By Theorem 3.3 (1), $x+U$ is s^*g-*open* and hence f is s^*g-*irresolute* at x. By hypothesis, f is s^*g-*irresolute* at 0. This reflects that f is s^*g-*irresolute.*

Corollary 3.24. Let $\left(X_{(K)},\tau\right)$ be an s^*g-*irresolute* topological vector space. Let $f:X\to K$ be a linear function which is s^*g-*irresolute* at 0. Then the set $F=\{x\in X:f(x)=0\}$ is s^*g-*closed*.

4. CHARACTERIZATIONS OF s*g-IRRESOLUTE TOPOLOGICAL VECTOR SPACES

In this section, we give some characterizations of s^*g-*irresolute* topological vector spaces.

Theorem 4.1. Let (X,τ) be an s^*gITVS. For $x\in X$, the following assertions are true:

(1) If $U\in s^*g\text{-}N(x)$, then $x\in U$.

(2) If $U\in s^*g\text{-}N(x)$ and V is a neighborhood of x, then $U\cap V\in s^*g\text{-}N(x)$.

(3) If $U\in s^*g\text{-}N(x)$, then there exists $V\in s^*g\text{-}N(x)$ such that $U\in s^*g\text{-}N(y)$, for all $y\in V$.

(4) If $U\in s^*g\text{-}N(x)$ and $U\subseteq V$, then $V\in s^*g\text{-}N(x)$.

(5) If $U\in s^*g\text{-}N(0)$, then $\lambda U\in s^*g\text{-}N(0)$ for every non-zero element $\lambda\in R$.

(6) If $U\in s^*g\text{-}N(x)$ and V is an s^*g-*neighborhood* of x, then $U\cap V\in s^*g\text{-}N(x)$.

(7) $U\in s^*g\text{-}N(0)$ if and only if $x+U\in s^*g\text{-}N(x)$.

Proof. We will prove (2), (5) and (7) while the proofs of others follow easily.

(2) If U is an s^*g-*neighborhood* of x, and V is a neighborhood of x, then there is an s^*g-*open* subset A and an open set B such that $x\in A\subseteq U$ and $x\in B\subseteq V$. Then $x\in A\cap B\subseteq U\cap V$ and $\tau\subseteq S^*GO(X)$. Thus $A\cap B\in S^*GO(X)$. Therefore $U\cap V$ is an s^*g-*neighborhood* of x.

(5) Let U be an s^*g-*neighborhood* for zero. Then there exists an s^*g-*open* neighborhood V of zero such that $V\subseteq U$. Since the map $M_\lambda:X\to X$, defined by $M_\lambda(x)=\lambda x$, is s^*g-*irresolute.* The inverse map $N_\lambda;X\to X$, defined by $N_\lambda(x)=\frac{1}{\lambda}x$, is also an s^*g-*irresolute.* Thus M_λ is s^*g-*homeomorphism,* for each $\lambda\in R-\{0\}$. Hence $M_\lambda(V)=\lambda V$ is an s^*g-*open* neighborhood of zero. Furthermore, clearly

$\lambda V \subseteq \lambda U$. Thus consequently $\lambda U \in s^* g\text{-}N(0)$.

(7) Suppose U is an $s^*g\text{-}neighborhood$ for zero. Then there exists an $s^*g\text{-}open$ neighborhood V of zero such that $V \subseteq U$. Since the map $T_x : X \to X$, defined by $T_x(y) = y + x$, is $s^*g\text{-}irresolute$. The inverse map $S_x ; X \to X$, defined by $S_x(y) = x - y$, is also an $s^*g\text{-}irresolute$. Thus T_x is $s^*g\text{-}homeomorphism$, for each $x \in X$. Hence $T_x(V) = x + V$ is an $s^*g\text{-}open$ neighborhood for a point x. Clearly $x + V \subseteq x + U$. Thus $x + U \in N_x$. The converse can be proved similarly.

Definition 4.2. A subset A of a topological vector space X is called balanced if and only if $\lambda A \subseteq A$ for each $\lambda \in R$ such that $|\lambda| \leq 1$.

Definition 4.3. A subset A of a topological vector space X is called absorbing if for all $x \in X$ there exists a number $\varepsilon > 0$ such that $\lambda x \in A$ for $|\lambda| \leq \varepsilon$.

Definition 4.4. A set C of a topological vector space X is said to be convex, if and only if it contains all segments between its points: $x \in C$, $y \in C$, for $t \in [0,1]$ implies $tx + (1-t)y \in C$, or equivalently $tC + (1-t)C \subseteq C$, for all $t \in [0,1]$. A set C of a topological vector space X is said to be absolutely convex if it is both convex and balanced.

Theorem 4.5. Let $\left(X_{(K)}, \tau\right)$ be an $s^*g\text{-}irresolute$ topological vector space. If a subset C of X is convex, then $s^*g\text{-}Cl(C)$ is also convex.

Proof. The convexity of C implies $tC + (1-t)C \subseteq C$. By Theorem 3.12 (2), and Theorem 3.13, it follows immediately that $t\left[s^*g\text{-}Cl(C)\right] + (1-t)\left[s^*g\text{-}Cl(C)\right] = s^*g\text{-}Cl(tC) + s^*g\text{-}Cl\left[(1-t)C\right] \subseteq s^*g\text{-}Cl\left[tC + (1-t)C\right] = s^*g\text{-}Cl(C)$. Thus $t\left[s^*g\text{-}Cl(C)\right] + (1-t)\left[s^*g\text{-}Cl(C)\right] \subseteq s^*g\text{-}Cl(C)$. Hence we conclude that $s^*g\text{-}Cl(C)$ is convex.

Theorem 4.6. Let $\left(X_{(K)}, \tau\right)$ be an s^*gITVS. If a subset C of X is convex, then $s^*g\text{-}Int(C)$ is also convex.

Proof. By Theorem 3.7 (2), and Theorem 3.8, $t\left[s^*g\text{-}Int(C)\right] + (1-t)\left[s^*g\text{-}Int(C)\right] = s^*g\text{-}Int(tC) + s^*g\text{-}Int\left[(1-t)C\right] \subseteq s^*g\text{-}Int\left[tC + (1-t)C\right] = s^*g\text{-}Int(C)$. Therefore $s^*g\text{-}Int(C)$ is convex.

Theorem 4.7. Let $\left(X_{(R)}, \tau\right)$ be an s^*gITVS. Then the following statements are equivalent:

(a) Every $s^*g\text{-}neighborhood$ U of zero is absorbing.

(b) For every $s^* g$-*neighborhood* U for zero, there exists a balanced set $V \in s^* g\text{-}N(0)$ such that $V \subseteq U$.

Proof. (a) Suppose U is an $s^* g$-*neighborhood* for zero. Then there exists an $s^* g$-*open* subset $V \in N_0$ such that $V \subseteq U$. By hypothesis X is an $s^* gITVS$. So there exist $s^* g$-*open* sets V_1 of $\mathbb{R}$ containing *zero* and V_2 of X containing zero such that $V_1.V_2 \subseteq V$. The set V_1 contains an open interval of the form $(-\varepsilon, \varepsilon)$ for some $\varepsilon > 0$. Therefore $tx \in V \subseteq U$ for all $t \in (-\varepsilon, \varepsilon)$ and for all $x \in V_2$. This implies U is absorbing.

(b) Let U be an $s^* g$-*neighborhood* of zero. By hypothesis X is an $s^* gITVS$. So there exist $s^* g$-*open* sets V_1 of $\mathbb{R}$ containing zero and V_2 of X containing zero such that $V_1.V_2 \subseteq U$. Then there exists $\varepsilon > 0$ such that $(-\varepsilon, \varepsilon) \subseteq V_1$. Define $W = \cup\{tV_2 : t \in R, |t| < \varepsilon\}$. Since tV_2 is an $s^* g$-*neighborhood* of zero, for $t \neq 0$ and $tV_2 \subseteq U$ for $t \in (-\varepsilon, \varepsilon)$. Thus W is an $s^* g$-*neighborhood* for zero and $W \subseteq U$. Now we have to show that W is balanced. Let $r \in R$ such that $|r| \leq 1$. Let $t \in (-\varepsilon, \varepsilon)$ and $x \in V_2$. Since $|rt| = |r||t| \leq |t| < \varepsilon$. Thus $r(tx) = (rt)x \in (-\varepsilon, \varepsilon).V_2 \subseteq W$. This shows that $rW \subseteq W$. Therefore W is balanced.

Theorem 4.8. Let X be an $s^* g$-*irresolute* topological vector space. Then $s^* g\text{-}Cl(A) = \cap\{A + U : U \in s^* g\text{-}N(0)\}$.

Proof. Assume $x \in s^* g\text{-}Cl(A)$, and let U be an $s^* g$-*neighborhood* of zero. Then by Theorem 4.7(b), there exits a balanced neighborhood V for zero such that $V \subseteq U$. Thus $x + V$ is an $s^* g$-*neoghborhood* for x and $x \in s^* g\text{-}Cl(A)$, so $(x+V) \cap A \neq \phi$. Take $a \in (x+V) \cap A$. Then $a \in x+V$ and $a \in A$. Let $a = x + v$ for some $v \in V$. Since V is balanced, so $A - V = A + V$. Take $x = a + (-v) \in A - V$ implies $x \in A + V \subseteq A + U$. Thus $x \in A + U$, for any $s^* g$-*neighborhood* U of zero. Therefore, we obtain $s^* g\text{-}Cl(A) \subseteq \cap\{A + U : U \in s^* g\text{-}N(0)\}$.

Conversely if $x \notin s^* g\text{-}Cl(A)$, then there exists a balanced neighborhood U for zero such that $(x+U) \cap A = \phi$. Thus $x \notin A - U = A + U$. It follows that $\cap\{A + U : U \in s^* g\text{-}N(0)\} \subseteq s^* g\text{-}Cl(A)$. Thus we get $s^* g\text{-}Cl(A) = \cap\{A + U : U \in s^* g\text{-}N(0)\}$.

Theorem 4.9. Let X be an $s^* gITVS$. Then the following assertions are true.

(a) For every $U \in s^* g\text{-}N(0)$, there exists symmetric set $V \in s^* g\text{-}N(0)$ such that $V + V \subseteq U$.

(b) For every $U \in s^{*}g\text{-}N(0)$, there exits an $s^{*}g\text{-}closed$ balanced set $V \in s^{*}g\text{-}N(0)$ such that $V \subseteq U$.

(c) For every $U \in s^{*}g\text{-}N(0)$, there exists symmetric set $V \in s^{*}g\text{-}N(0)$ such that $V + V + V \subseteq U$.

Proof. (a) Assume $U \in s^{*}g\text{-}N(0)$. By hypothesis X is an $s^{*}gITVS$. There exist $s^{*}g\text{-}open$ neighborhoods V_1 and V_2 for zero in X such that $V_1 + V_2 \subseteq U$. Let $V = V_1 \cap (-V_1) \cap V_2 \cap (-V_2)$. Then V is a symmetric $s^{*}g\text{-}open$ neighborhood of zero and $V + V \subseteq V_1 + V_2 \subseteq U$.

(b) Let U be an $s^{*}g\text{-}neihborhood$ of zero in X. By part (a) there is $s^{*}g\text{-}neihborhood$ V for zero with $V + V \subseteq U$. By Theorem 4.7 (b), there exits $s^{*}g\text{-}neihborhood$ W for zero which is balanced and $W \subseteq V$. By Theorem 4.8, $s^{*}g\text{-}Cl(W) \subseteq W + V \subseteq V + V \subseteq U$. This shows that U contains a $s^{*}g\text{-}closed$ neighborhood of zero.

(c) Follows easily from (a).

Definition 4.10. A topological space (X,τ) is called $s^{*}g\text{-}Hausdorff$, if each two distinct points x and y in X, there exist disjoint $s^{*}g\text{-}open$ sets U, V such that $x \in U$ and $y \in V$.

Now we give some properties of $s^{*}g\text{-}Hausdorff$ space.

Theorem 4.11. Let X be an $s^{*}gITVS$. Then the following statements are equivalent.

(a) X is $s^{*}g\text{-}Hausdorff$.

(b) If $x \in X$, $x \neq 0$, then there exists $U \in s^{*}g\text{-}N(0)$ such that $x \notin U$.

(c) If $x, y \in X$, $x \neq y$, there exists $V \in s^{*}g\text{-}N(x)$ such that $y \notin V$.

Proof. By continuity of translation, it is sufficient to prove the equivalence between (a) and (b) only.

(a)$\Rightarrow$(b): Assume x be a non-zero vector belongs to X. Therefore there are disjoint $s^{*}g\text{-}open$ sets $U, V \subseteq X$ such that $0 \in U$ and $x \in V$. Thus $U \in s^{*}g\text{-}N(0)$, $V \in s^{*}g\text{-}N(x)$ and $x \notin U$.

(b)$\Rightarrow$(a): Let $x, y \in X$ be such that $x - y \neq 0$. Then there exists $U \in s^{*}g\text{-}N(0)$ such that $x - y \notin U$. By Theorem 4.9 (a), there exists $s^{*}g\text{-}neihborhood$ W of zero such that $W + W \subseteq U$. By Theorem 4.7 (b), W can be assumed to be balanced. Let $V_1 = x + W$ and $V_2 = y + W$. We note that $V_1 \in s^{*}g\text{-}N(x)$, $V_2 \in s^{*}g\text{-}N(y)$ and $V_1 \cap V_2 = \phi$, since if $z \in V_1 \cap V_2$, then $z - x \in W$ and $z - y \in W$. Since W is balanced, so $-(z - x) \in W$. It follows that $x - y = (z - y) + [-(z - x)] \in W + W \subseteq U$, which is a contradiction. So, we must have $V_1 \cap V_2 = \phi$. Finally, by the definition of $s^{*}g\text{-}neihborhood$, there exist $V_1^{*}, V_2^{*} \in S^{*}GO(X)$ such that $x \in V_1^* \subseteq V_1$, $y \in V_2^* \subseteq V_2$, and $V_1^* \cap V_2^* = \phi$. This shows that the space X is s^*g-*Hausdorff*. This completes the proof.

The following result follows from Theorem 4.11.

Corollary 4.12. Let X be an s^*gITVS. Then the following statements are equivalent.

(a) X is s^*g-*Hausdorff*.

(b) $\bigcap\{U : U \in s^*g\text{-}N(0)\} = \{0\}$.

(c) $\bigcap\{U : U \in s^*g\text{-}N(x)\} = \{x\}$.

Theorem 4.13. An s^*gITVS X is s^*g-*Hausdorff* if and only if every one-point set in X is s^*g-*closed* in X.

Proof. Let $x \in X$ and $y \in X - \{x\}$. Then $y - x \neq 0$, and by assumption, there exists $U \in s^*g\text{-}N(0)$ such that $y - x \notin U$. By Theorem 4.9 (b), there exists an s^*g-*closed* and balanced set $V \in s^*g\text{-}N(0)$ such that $V \subseteq U$. It follows that $y - x \notin V$ that is $y - x \in X - V$. Thus $y \in (X - V) + \{x\}$. But $(X - V) + \{x\}$ is s^*g-*open*, since V is s^*g-*closed*, and $(X - V) + \{x\} \subseteq X - \{x\}$. This shows that $X - \{x\}$ is s^*g-*open*. For the converse, let $x \in X$ and assume that $\{x\}$ is s^*g-*closed*. Then by Theorem 4.8, $\{x\} = s^*g\text{-}Cl(\{x\}) = \bigcap\{U + \{x\} : U \in s^*g\text{-}N(0)\} = \bigcap\{V : V \in s^*g\text{-}N(x)\}$, where $V = U + \{x\} \in s^*g\text{-}N(x)$. Then by Corollary 4.12, X is s^*g-*Hausdorff*. This completes the proof.

Since translation is an s^*g-*homeomorphism* and as a consequence of Theorem 4.13, we have the following result.

Corollary 4.14. An s^*gITVS X is s^*g-*Hausdorff* if and only if $\{0\}$ is s^*g-*closed* in X.

Theorem 4.15. Let C, K be disjoint sets in an s^*gITVS X with C s^*g-*closed*, K s^*g-*compact*. Then there exists $U \in s^*g\text{-}N(0)$ with $(K + U) \cap (C + U) = \phi$.

Proof. If $K = \phi$, then there is nothing to prove. Otherwise, let $x \in K$ by the invariance with translation, we can assume $x = 0$. Then $X - C$ is an s^*g-*open* neighborhood of zero. Since addition is s^*g-*irresolute* and s^*g-*continuous*, by $0 + 0 + 0 = 0$, there is an s^*g-*open* neighborhood $U \in s^*g\text{-}N(0)$ such that $3U = U + U + U \subseteq X - C$. By defining $W = U \cap (-U) \subseteq U$ we have that W is s^*g-*open* symmetric and $3W = W + W + W \subseteq X - C$. This means that $\phi = \{3x : x \in W\} \cap C = \{2x : x \in W\} \cap \{y - x : y \in C, x \in W\} \supseteq W \cap (C + W)$. This concludes the proof for a single point.

Sine K is s^*g-*compact*, then repeating the above argument for all $x \in K$, we obtain symmetric s^*g-*open* sets V_x such that $(x+2V_x)\cap(C+V_x)=\phi$. The sets $\{V_x : x \in K\}$ are an s^*g-*open* covering of K, but K is s^*g-*compact*. Hence there is a finite number of points $x_i \in K$, $i=1,2,...,n$, such that $K \subseteq \bigcup_{i=1}^{n}(x_i+V_{x_i})$. Define the s^*g-*open* neighborhood V of zero by $V=\bigcap_{i=1}^{n}V_{x_i}$. Then we get $(K+V)\cap(C+V)\subseteq \bigcup_{i=1}^{n}(x_i+V_{x_i}+V)\cap(C+V)\subseteq \bigcup_{i=1}^{n}\left[(x_i+2V_{x_i})(C+V_{x_i})\right]=\phi$. This completes the proof.

Lemma 4.16. If U is an s^*g-*open* set and $U\cap A=\phi$, then $U\cap\left[s^*g\text{-}Cl(A)\right]=\phi$.

Proof. Suppose that there exists an $x\in U\cap\left[s^*g\text{-}Cl(A)\right]$. Then $x\in s^*g\text{-}Cl(A)$ and U is an s^*g-*open* neighborhood of x and $X-U$ is s^*g-*closed* set containing A, hence $s^*g\text{-}Cl(A)\subseteq X-U$ and $x\notin s^*g\text{-}Cl(A)$ which is contradiction, hence $U\cap\left[s^*g\text{-}Cl(A)\right]=\phi$.

Corollary 4.17. Let C,K be disjoint sets in an s^*gITVS X with C s^*g-*closed*, K s^*g-*compact*. Then there exists $U\in s^*g\text{-}N(0)$ with $\left[s^*g\text{-}Cl(K+U)\right]\cap(C+U)=\phi$.

Proof. By Theorem 4.15, there exists $U\in s^*g\text{-}N(0)$ such that $(K+U)\cap(C+U)=\phi$. Now $C+U=\bigcup\{y+U : y\in C\}$ is s^*g-*open* set being a union of s^*g-*open* sets. Then by Lemma 4.16, we obtain $\left[s^*g\text{-}Cl(K+U)\right]\cap(C+U)=\phi$.

Theorem 4.18. Let X be an s^*gITVS. Let $f : X\to\mathbb{R}$ be a non-zero linear map. Then $f(G)$ is s^*g-*open* in $\mathbb{R}$ whenever G is s^*g-*open* in X.

Proof. Let G be a nonempty s^*g-*open* set. Then one can assume that there is $x_0\in X-\{0\}$ such that $f(x_0)=1$. For any $a\in G$, it is required to show that $f(a)\in s^*g\text{-}Int\left[f(G)\right]$. Since $G\in s^*g\text{-}N(a)$ by Theorem 4.1 we have $G-a\in s^*g\text{-}N(0)$. By Theorem 4.7 (a) $G-a$ is absorbing, that is, absorbs x_0, namely there exists an $\varepsilon>0$ such that $\lambda x_0\in G-a$ whenever $\lambda\in\mathbb{R}$ with $|\lambda|\le\varepsilon$. Now for any $\beta\in\mathbb{R}$ with $|\beta-f(a)|<\varepsilon$ we have $(\beta-f(a))x_0\in G-a$, hence $f\left[(\beta-f(a))x_0\right]\in f(G-a)$. Since f is linear. This implies that $(\beta-f(a))f(x_0)\in f(G-a)$. So we get $(\beta-f(a))(1)\in f(G-a)=f(G)-f(a)$. This implies that $\beta\in f(G)$ and $f(a)\in(\beta-\varepsilon,\beta+\varepsilon)$. Thus $f(a)\in Int\left[f(G)\right]\subseteq s^*g\text{-}Int\left[f(G)\right]$; hence consequently $f(G)=s^*g\text{-}Int\left[f(G)\right]$.

Lemma 4.19. [14]. Let X be vector space and $\phi\ne K\subseteq X$. For $a\in K$, the following statements are equivalent.

(1) a is an extreme point of K.

(2) If $x,y\in K$ are such that $a=\frac{1}{2}(x+y)$, then $a=x=y$.

(3) Let $x,y\in K$ be such that $x\neq y$, let $\lambda\in(0,1)$ and $a=\lambda x+(1-\lambda)y$. Then we have either $\lambda=0$ or $\lambda=1$.

Theorem 4.20. Let X be an s^*gITVS and C be a convex subset of X. Then $\left[s^*g\text{-}Int(C)\right]\cap(\partial C)=\phi$.

Proof. If $s^*g\text{-}Int(C)=\phi$, then the result is trivial. Suppose that $s^*g\text{-}Int(C)\neq\phi$ and let $x\in s^*g\text{-}Int(C)$. Then there exists $V\in s^*g\text{-}N(0)$ such that $x+V\subseteq C$. As the map $\Phi:\mathbb{R}\to X$, where $\Phi(\mu)=\mu x$ is continuous at $\mu=1$, for this the $s^*g\text{-}Ineighborhood$ $x+V$, there is an $r>0$ such that $\mu x\in x+V$ whenever $|\mu-1|\leq r$. In particular, we have $(1+r)x\in x+V\subseteq C$ and $(1-r)x\in x+V\subseteq C$. Now consider $x=\lambda(1+r)x+(1-\lambda)(1-r)x$ and take $\lambda=\frac{1}{2}$. Consequently, we have $x=\frac{1}{2}(1+r)x+\left(1-\frac{1}{2}\right)(1-r)x$, which implies that x is not an extreme point of C.

Theorem 4.21. Let X be an s^*gITVS and W an $s^*g\text{-}neighborhood$ of 0. Then there is an $s^*g\text{-}neighborhood$ U of 0 such that $s^*g\text{-}Cl(U)\subseteq W$. Equivalently, if C is a $s^*g\text{-}closed$ subset of X and x a point of X outside C then there are disjoint $s^*g\text{-}open$ sets U_1 and U_2 with $x\in U_1$ and $C\subseteq U_2$.

Proof. Let x be a point outside an $s^*g\text{-}closed$ set $C\subseteq X$. We will produce an $s^*g\text{-}open$ set U containing x with $s^*g\text{-}Cl(U)\cap C=\phi$; then $U_1=U$ and $U_2=X-s^*g\text{-}Cl(U)$, the complement of the $s^*g\text{-}closure$ of U, are disjoint $s^*g\text{-}open$ sets with $x\in U_1$ and $C\subseteq U_2$, as desired. We know that X looks the same everywhere, so we may work with $x=0$. Let W be the complement of C. Then W is an $s^*g\text{-}open$ set with $0\in W$. By hypothesis X is s^*gITVS. So by Theorem 4.9 (a), there exists an $s^*g\text{-}open$ subset U of 0 such that $U+U\subseteq W$. This means that $U+U$ is disjoint from C. Equivalently, U is disjoint from $C-U$. For otherwise there would be an $x\in U$ which could be expressed as $c-y$ with $c\in C$ and $y\in U$, which would imply that $c=x+y\in U+U\subseteq W$ is in W. Now the set $-U$ is $s^*g\text{-}open$ because the map $X\to X:\ x\to(-1)x=-x$ is an $s^*g\text{-}homeomorphism$, and hence so are all its translates $x-U$. So the set $U_2=C-U=\bigcup\{c-U:c\in C\}$ is $s^*g\text{-}open$, being the union of s^*g-*open* sets. Thus we have found an s^*g-*open* set $U_1 = U$ containing 0 and an s^*g-*open* set U_2, disjoint from U_1, with $C \subseteq U_2$.

Theorem 4.22. Let $(X_{(K)}, \tau)$ be an s^*gITVS. Then every s^*g-*open* subspace S of X is also an s^*gITVS.

Proof. Let W be an s^*g-*open* neighborhood of $x+y$ in S where x,y are two distinct points in S. Since S is an s^*g-*open* subspace of X, then W is an s^*g-*open* neighborhood of $x+y$ in X, and by definition of s^*gITVS, there exist s^*g-*open* neighborhoods U of x in X and V of y in X such that $U+V \subseteq W$. Then the sets $G = U \cap S$ and $H = V \cap S$ are s^*g-*open* neighborhoods of x and y in S such that $G + H \subseteq U + V \subseteq W$. Now suppose $\lambda \in K$, $x \in S$ and let W be an s^*g-*open* neighborhood of λx in S. Since S is an s^*g-*open* subspace of X, then W is an s^*g-*open* neighborhood of λx in X. Then there exist s^*g-*open* neighborhoods U of λ in K and V of y in X such that $U.V \subseteq W$. Then the set $G = U \cap S$ is an s^*g-*open* neighborhood of λ in K and the set $H = V \cap S$ is an s^*g-*open* neighborhood of x in S. Also $G.H \subseteq U.V \subseteq W$. Hence S is an s^*gITVS.

Theorem 4.23. Suppose that $(X_{(K)}, \tau)$ is an s^*gITVS. If $S \subseteq X$ is a linear subspace, then so is $s^*g\text{-}Cl(S)$.

Proof. Let S be a linear subspace of X. Thus $S + S \subseteq S$ and for all $\lambda \in K$, $\lambda.S \subseteq S$. By Theorem 3.13, $s^*g\text{-}Cl(S) + s^*g\text{-}Cl(S) \subseteq s^*g\text{-}Cl(S+S) \subseteq s^*g\text{-}Cl(S)$. By Theorem 3.12, for every $\lambda \in K$, $\lambda\left[s^*g\text{-}Cl(S)\right] = s^*g\text{-}Cl(\lambda S) \subseteq s^*g\text{-}Cl(S)$. Therefore, $s^*g\text{-}Cl(S)$ is linear subspace of X.

Definition 4.24. Suppose that $(X_{(K)}, \tau)$ is an s^*gITVS. A subset $E \subseteq X$ is said to be bounded if for all s^*g-*open* sets V containing 0, there exists $s \in R$ such that for all $t > s$, $E \subseteq tV$. That is, every s^*g-*open* neighborhood of zero contains E after being blown up sufficiently.

Theorem 4.25. Suppose that $(X_{(K)}, \tau)$ is an s^*gITVS. If E is a bounded subset of X, then $s^*g\text{-}Cl(E)$ is bounded.

Proof. Let W be an s^*g-*open* set containing 0, then by Theorem 4.21, there exists $U \in s^*g\text{-}N(0)$ such that $s^*g\text{-}Cl(U) \subseteq W$. Since E is bounded, so $E \subseteq tU \subseteq t\left[s^*g\text{-}Cl(U)\right] \subseteq tW$, for sufficiently large values of t. It follows that for large enough t, $s^*g\text{-}Cl(E) \subseteq s^*g\text{-}Cl(tU) \subseteq t\left[s^*g\text{-}Cl(U)\right] \subseteq tW$. Thus, $s^*g\text{-}Cl(E)$

is bounded.

Theorem 4.26. Let (X,τ) be an s^*gITVS. Let V be an s^*g-*open* neighborhood of zero in X. Then for every sequence $\{r_n : n \in N\}$ of positive real numbers such that $r_n \to \infty$, $\bigcup_{n=1}^{\infty} r_n V = X$.

Proof. Let $x \in X$ and consider the sequence $\left\{\frac{x}{r^n} : n \in N\right\}$. This sequence converges to 0 by the s^*g-*irresoluteness* of the scalar multiplication $F \times X \to X$. Thus, for sufficiently large n, $\frac{x}{r^n} \in V$ i.e., $x \in r_n V$.

Theorem 4.27. Let (X,τ) be an s^*gITVS. Then every s^*g-*compact* set is bounded.

Proof. Let C be an s^*g-*compact* subset of X. We need to prove that it is bounded, namely, that for every s^*g-*open* neighborhood V of 0, $C \subseteq tV$ for sufficiently large t. Let V be an s^*g-*open* neighborhood of 0, then by Theorem 4.7(b), there exists a balanced s^*g-*open* neighborhood W of 0 such that $W \subseteq V$. By Theorem 4.26, $C \subseteq \bigcup_{n=1}^{\infty} nW$. Since, C is s^*g-*compact*, therefore there exists a positive integer m such that $C \subseteq \bigcup_{j=1}^{m} n_j W = n_m \bigcup_{j=1}^{m} (n_j / n_m) W \subseteq n_m W$. Thus, for all $t > n$, $C \subseteq n_m W = [t(n_m / t)] W \subseteq tW \subseteq tV$, which implies that C is bounded.

Theorem 4.28. Let (X,τ) be an s^*gITVS. Then every Cauchy sequence in X is bounded.

Proof. Let $\{x_n : n \in N\}$ be a Cauchy sequence in X. Let W be an s^*g-*open* neighborhood of zero, then by Theorem 4.9 (a), there exists an s^*g-*open* neighborhood V of 0 such that $V + V \subseteq W$. By definition of a Cauchy sequence, there exists N such that for all $m,n \geq N$, $x_n - x_m \in V$ and in particular for all $n > N$, $x_n \in x_N + V$. Set $s > 1$ such that $x_N \in sV$, then for all $n > N$, $x_n \in sV + V \subseteq sV + sV \subseteq sW$. Since for balanced sets $sW \subseteq tW$ for $s < t$, and since every s^*g-*open* neighborhood of 0 contains a balanced neighborhood, this proves that the sequence is bounded.

Definition 4.29. Let X be a vector space over $\mathbb{R}$. A non-negative real-valued function p defined on X is a *pseudonorm* if it satisfies the following two conditions.

(i) $p(\lambda x) = |\lambda| p(x)$, for all $x \in X$ and $\lambda \in \mathbb{R}$;

(ii) $p(x+y) \leq p(x) + p(y)$, for all $x, y \in X$.

Now, we introduce the notion of locally convex s^*gITVS. Moreover, we give a necessary and sufficient condition, in terms of convex $s^* g$-*neighborhoods* of 0, for an $s^* gITVS$ to be locally convex.

Definition 4.30. An $s^* gITVS$ (X,τ) is locally convex if for all $x \in X$, every $S \in s^* g\text{-}N(x)$ contains a convex set $U \in s^* g\text{-}N(x)$.

Theorem 4.31. An $s^* gITVS$ (X,τ) is locally convex if and only if every $S \in s^* g\text{-}N(0)$ contains a convex set $U \in s^* g\text{-}N(0)$.

Proof. The sufficiency part is trivial. Let $S \in s^* g\text{-}N(x)$. Then by Theorem 4.1 (7), $S - x \in s^* g\text{-}N(0)$ and by assumption, there exists a convex set $U \in s^* g\text{-}N(0)$ such that $U \subseteq S - x$. Hence by Theorem 4.1 (7) again, $U + x \in s^* g\text{-}N(x)$. As $U + x \subseteq S$ and as $U + x$ is convex, (X,τ) is a locally convex $s^* gITVS$.

Corollary 4.32. In a locally convex $s^* gITVS$ (X,τ), a *pseudonorm* p is $s^* g$-*irresolute* if and only if p is $s^* g$-*irresolute* at zero.

Proof. If p is $s^* g$-*irresolute,* then p is $s^* g$-*irresolute* at zero. Conversely, suppose p is $s^* g$-*irresolute* at 0, and let $x \in X$ and $V \in N_{p(x)}(\mathbb{R})$. Then by Theorem 4.1 (7), $V - p(x) \in N_0(\mathbb{R}) = N_{p(0)}(\mathbb{R})$ and thus $(-\varepsilon,\varepsilon) \subseteq V - p(x)$ for some $\varepsilon > 0$. Clearly $(-\varepsilon,\varepsilon)$ being an open set in $\mathbb{R}$ is $s^* g$-*open* set in $\mathbb{R}$. By assumption, there exists $U \in s^* g\text{-}N(0)$ such that $p(U) \subseteq (-\varepsilon,\varepsilon)$ and as $p(y) \geq 0$ for all $y \in U$, $p(U) \subseteq [0,\varepsilon)$. Then by Theorem 4.1 (7), $U + x \in s^* g\text{-}N(x)$. For all $y \in U$, $0 \leq p(x+y) \leq p(x) + p(y) \leq p(x) + \varepsilon$, $p(x+y) \in [0, p(x)+\varepsilon)$. Therefore it follows that $p(U+x) \subseteq V$.

Definition 4.33. Let A be an absolutely convex subset of a vector space X. Then the functional defined by $p(x) = inf\{\lambda : \lambda > 0, x \in \lambda A\}$ is called the *gauge* of A.

Lemma 4.34. [14]. In a vector space X, the *gauge* of an absolutely convex and absorbent subset is a *pseudonorm.*

Now, we prove the main result in which we characterize absolutely convex and absorbent $s^* g$-*neoghborhoods* of zero in terms of their $s^* g$-*irresolute gauges*.

Theorem 4.35. Let p be a *gauge* of an absolutely convex and absorbent subset U of an $s^* gITVS$ (X,τ). Then p is $s^* g$-*irresolute* if and only if U is an $s^* g$-*neighborhood* of zero.

Proof. If p is $s^* g$-*irresolute,* then as $(-1,1)$ is an $s^* g$-*open* set in $\mathbb{R}$. $V = \{x : p(x) < 1\} = p^{-1}[(-1,1)]$ is an

s^*g-*open* subset of X. Thus as $V \subseteq U$, $U \in s^*g\text{-}N(0)$. Conversely, if $U \in s^*g\text{-}N(0)$ and $\varepsilon > 0$, then by Theorem 4.1 (5), $V = \varepsilon U \in s^*g\text{-}N(0)$ and $p(x) < \varepsilon$ for all $x \in V$. Thus $p(V) \subseteq (-\varepsilon, \varepsilon)$. Hence, p is s^*g-*irresolute* at zero. By Lemma 4.34, p is a *pseudonorm* and by Corollary 4.32, p is s^*g-*irresolute* at each $x \in X$. Therefore p is s^*g-*irresolute.*

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CONFLICT OF INTERESTS

The author declares that there is no conflict of interests.

REFERENCES

[1] M. E. Abd El-Monsef, S. N. El-Deeb, R. A. Mahmoud, β open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12 (1983), 77-90.

[2] R. I. M. Ali, J. H. Hussein, S. K. Hameed, On semi-pre irresolute topological vector spaces, Diyala J. Pure Sci. 14 (3) (2018), 183-191.

[3] T. Al-Hawary, On α-topological vector spaces and α-irresolute gauges, Bol. Mat. 24 (2) (2017), 149-154.

[4] T. Al-Hawary, α-topological vector spaces, Hacettepe J. Math. Stat. 47 (5) (2018), 1102-1107.

[5] D. Andrijevic, Semi-preopen sets, Mat. Vesnik, 38 (1986), 24 – 32.

[6] C. K. Basu, M. K. Ghosh, β-Closed spaces and β-θ-subclosed graphs, Eur. J. Pure Appl. Math. 1 (3) (2008), 40-50.

[7] G. Aslim, A. Guler Caksu, T. Noiri, On πgs-closed sets in topological spaces. Acta Math. Hung. 112 (2006), 275-283.

[8] M. Caldas, Functions with strongly β-θ-closed graphs, J. Adv. Stud. Topol. 3 (2012), 1 – 6.

[9] M. Caldas, On contra βθ-continuous functions, Proyecciones J. Math. 39 (2013), 333-342.

[10] M. Caldas, Other characterizations of β-θ-R0 topological spaces, Tamkang J. Math., 49 (3) (2013), 303-312.

[11] A. Devika, R. Vani, On πg* β-closed sets in topological spaces, J. Appl. Comput. Math. 7 (3) (2018), 1-5.

[12] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac Sci. Kochi. Univ. Ser. A. Math. 16 (1995), 35-48.

[13] H.Z. Ibrahim. α-topological vector spaces. Sci. J. Univ. Zakho. 5 (1) (2017), 107-111.

[14] J. Horvath, Topological vector spaces and distributions, Addison-Wesley, Reading, Mass (1966).

[15] M. Khan and B. Ahmad, On p-regular spaces, Math. Today, 13 (1995), 51-56.

[16] M. Khan, T. Noiri and M. Hussain, On s*g-closed sets and s*-normal spaces, J. Nat. Sci. Math. 48 (2008), 31-41.

[17] M.D. Khan, S. Azam, M.S. Bosan, s-Topological vector spaces. J. Linear Topol. Algebra. 4 (2015), 153-158.

[18] M. Khan, M. I. Iqbal, On irresolute topological vector spaces, Adv. Pure Math. 6 (2016), 105-112.

[19] A. Kolomogroff, Zur Normierbarkeit eines topologichen linearem Raumes, Stud. Math. 5 (1934), 29-33.

[20] R. M. Latif, ωb-Topological Vector Spaces, WSEAS Trans. Math. 19 (2020), Art. #12, 119 –132.

[21] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Mon. 70 (1963), 36 – 41.

[22] N. Levine, Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo, 19 (2) (1970), 89 –96.

[23] S. I. Mahmood, On weak D_{g*s}-sets and associative separation axioms, Ibn Al-Haitham J. Pure Appl. Sci. 27 (1) (2014), 351-367.

[24] S. I. Mahmood and J. S. Tareq, On s*g-α-open sets in topological spaces, Ibn Al-Haitham J. Pure Appl. Sci. 27 (3) (2014), 542 – 555.

[25] T. Noiri, Weak and strong forms of β-irresolute functions, Acta Math. Hung. 99 (2003), 315 – 328.

[26] O. Njastad, On some classes of nearly open sets, Pac. J. Math. 15 (1965), 961 – 970.

[27] M. Ram, S. Sharma, Corrigendum to: on irresolute topological vector spaces. New Trends Math. Sci. 6 (4) (2018), 40-44.

[28] J. H. Park, J. K. Park, On πgp-continuous functions in topological spaces, Chaos Solitons Fractals. 20 (2004), 467–477.

[29] M. Ram, S. Sharma, S. Billawria, A. Hussain. On almost s-topological vector spaces. J. Adv. Stud. Topol. 9 (2) (2018), 139-146.

[30] T. N. Rajesh, T. V. Vijayabhathi, On strongly preirresolute topological vector spaces, Math. Bohem. 138 (1), (2013), 37-42.

[31] A. Robert, S. P. Missier, A new class of nearly open sets, Int. J. Math. Arch. 3 (7) (2012), 1-8.

[32] A. Robert, S. P. Missier, A new class of sets weaker than α-open sets. Int. J. Math. Soft Comput. 4(2) (2014), 197-206.

[33] A. Robert, S. P. Missier, Between α-closed sets and semi α-closed sets, Int. J. Mod. Eng. Res. 4 (6) (2014), 34-41.

[34] A. Robert, S. P, Missier, Semi-star-alpha-open sets and associated functions, Int. J. Comput. Appl. 104 (16) (2014), 24-29.

[35] S. Sharma, M. Ram and S. Billawria, On almost pre-topological vector spaces, Open Access Library J. 5 (2018), 1–10

[36] S. Sharmaa, M. Rama. On β-topological vector spaces. J. Linear Topol. Algebra. 8 (1) (2019), 63- 70.

[37] S. Sharma, S. Billawria, M. Ram, T. Landol, On almost $\beta-\text{topological}$ vector spaces, Open Access Library J. 6 (2019), 1 – 9.

[38] N.V. Velicko, H-closed topological spaces. Amer. Math. Soc. Translat. 78 (1968), 103-118.

[39] S. Willard, R. M. Latif, Semi-open sets and regularly closed sets in compact metric spaces, Math. Japon. 46 (1) (1997), 157 – 161.

[40] S. Willard, General topology, Addison-Wesley. Reading, Mass. 1970.