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COMPUTATIONAL APPROACH FOR TRANSIENT BEHAVIOUR OF M/M (a, b)/1 BULK SERVICE QUEUEING SYSTEM WITH WORKING VACATION

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Abstract: A new computational technique is used to evaluate the Transient behaviour of Single Server Bulk Service Queueing System with Working Vacation with arrival rate λ which follows a Poisson process and the service will be in bulk. In this model the server provides two types of services namely normal service and lower service. The normal service time follows an exponential distribution with parameter μ_1. The lower service rate follows an exponential distribution with parameter μ_2. The vacation time follows an exponential distribution with parameter α. According to Neuts, the server begins service only when a minimum of 'a' customers in the waiting room and a maximum service capacity is 'b'. An infinitesimal generator matrix is formed for all transitions. Time dependent solutions and Steady state solutions are acquired by using Cayley Hamilton theorem. Numerical studies have been done for Time dependent average number of customers in the queue, Transient probabilities of server in vacation and server busy for several values of t, λ, μ_1, μ_2, α, a and b.

Keywords: bulk service; working vacation; infinitesimal matrix; direct truncation method; Cayley Hamilton.

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1. INTRODUCTION

The main objective of this research paper is to analyze the transient behaviour of Bulk service queueing system with working vacation by computational approach. Bulk Queues defined as queues with batch arrivals or batch services or both in batches. The batch size may be fixed or a random variable. In this model, arrival is single but service is in batch. Bulk service queues have potential applications in many areas e.g. in traffic signal systems, in restaurants, cinema halls, in transportation processes involving buses, airplanes and so on.

In the study of queueing systems, determination of transient solution is very much essential to analyze the behaviour of the system. Even in the case of a simple M/M/1 queue, analytical approach to obtain transient behaviour is very difficult. M.F. Neuts (1967) explained about general class of bulk queues with Poisson input. M.F. Neuts (1981) discussed about Matrix Geometric Solutions in Stochastic Models.

L.D. Servi and S.G. Finn (2002) introduced the concept of working vacation. Mian Zhang, Zhenting Hou (2012) has explained the general queue with single working vacation. Cosmika Goswami, N Selvaraju (2013) have examined working vacation queue with priority customers and vacation interruptions.VM Chandrasekaran, K Indhira, MC Saravanarajan, P Rajadurai (2016) have studied survey on working vacation queueing models.

Rakesh Kumar (2017) discussed about transient solution to the M/M/c queueing model. S. I. Ammar (2017) has examined about Transient solution of an M / M /1 vacation queue with a waiting server and impatient customers.

Shanthi, Muthu Ganapathi Subramanian and Gopal Sekar (2019) have studied the Transient behaviour of bulk service queueing model with multiple vacations. Further they have analyzed the transient behaviour of bulk service queueing model with optional services (2020) and also Bernoulli vacation with reneging of customer (2020).

2. THE MATHEMATICAL MODEL AND ITS SOLUTIONS

A new computational method is used to estimate the Transient behaviour of Single server Bulk service queueing system with working vacation whose arrival rate λ follows a Poisson process and the service will be in bulk with normal service time follows an exponential distribution with parameter μ_1 and the lower service rate in working vacation follows an exponential distribution with parameter μ_2. The vacation time follows an exponential distribution with parameter α. Assuming that there are 'a' customers in the system at time t = 0. The general considerations for bulk service queueing system with working vacation are

- After completion of the normal service if the number of customers in the queue is less than 'a' then the server goes for working vacations and becomes idle and start the service with lower service rate only if the batch size reaches 'a'. If the number of customers in the queue lies between 'a' and 'b' then all the customers in the queue will be taken for service and queue becomes empty and server starts normal service. If there are more than 'b' customers are waiting in the queue then the first 'b' customers are taken for service and the remaining customers will have to wait for service.
- During the working vacation period, if he finds the number of customers in the queue is between 'a' and 'b' then all the customers in the queue will be taken for service with lower service rate and queue becomes empty. If he finds the number of customers in the queue is more than b then the first b customers will be taken from the queue for service with lower service rate and remaining customers will be in the queue.
- After completion of the vacation period the server comes back to the system and starts normal service only if there are a minimum of 'a' customers in the queue, if the server finds less than 'a' customers in the queue then he will be waiting in the system for normal service. If he finds the number of customers in the queue is between 'a' and 'b' then all the customers in the queue will be taken for normal service and queue becomes empty and server starts normal service. If he finds more than 'b' customers are waiting in the the

queue then the first ‘b’ customers are taken for normal service and the remaining customers will have to wait for normal service.

3. DESCRIPTION OF RANDOM PROCESS

Let N(t) be the random variable which represents the number of customers in the queue at time t and C(t) be the random variable which represents the server status at time t. The random process is described as

$\{<N(t), C(t)> / N(t) = 0,1,2,3,\ldots,a-1 ; C(t) = 0,2\} \cup \{<N(t), C(t)> / N(t) = 0,1,2,3,\ldots ; C(t) = 1,3\}$

C (t) = 0 if the server is idle at time t

C (t) = 1if the server is busy with normal service at time t

C (t) = 2 if the server is idle in working vacation at time t

C (t) = 3 if the server is busy in working vacation with lower service at time t

We define,

$P_{n0}(t)$: Probability that there are no customers in the queue when the server is idle at time t

$P_{n1}(t)$: Probability that there are n customers in the queue when the server is busy at time t

$P_{n2}(t)$: Probability that the server is idle in working vacation when there are n customers in the queue at time t

$P_{n3}(t)$: Probability that the server is busy in working vacation when there are n customers in the queue at time t

The Chapman-Kolmogorov equations are

$$\left.\begin{aligned}
P_{00}'(t) &= -\lambda P_{00}(t)+\alpha P_{02}(t)\\
P_{01}'(t) &= -(\lambda+\mu_1)P_{01}(t)+\alpha P_{03}(t)+\lambda P_{a-10}(t)+\mu_1\sum_{k=a}^{b}P_{k1}(t)\\
P_{02}'(t) &= -(\alpha+\lambda)P_{02}(t)+\mu_1 P_{01}(t)+\mu_2 P_{03}(t)\\
P_{03}'(t) &= -(\alpha+\lambda+\mu_2)P_{03}(t)+\lambda P_{a-12}(t)+\mu_2\sum_{k=a}^{b}P_{k3}(t)
\end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned}
P_{n0}'(t) &= -\lambda P_{n0}(t)+\lambda P_{n-10}(t)+\alpha P_{n2}(t)\\
P_{n1}'(t) &= -(\lambda+\mu_1)P_{n1}(t)+\alpha P_{n3}(t)+\lambda P_{n-11}(t)+\mu_1 P_{b+n1}(t)\\
P_{n2}'(t) &= -(\alpha+\lambda)P_{n2}(t)+\lambda P_{n-12}(t)+\mu_1 P_{n1}(t)+\mu_2 P_{n3}(t)\\
P_{n3}'(t) &= -(\alpha+\lambda+\mu_2)P_{n3}(t)+\lambda P_{n-13}(t)+\mu_2 P_{b+n3}(t)
\end{aligned}\right\}\text{for } n = 1,2,3,\ldots(a-1) \tag{2}$$

$$\left.\begin{aligned}
P_{n1}'(t) &= -(\lambda+\mu_1)P_{n1}(t)+\alpha P_{n3}(t)+\lambda P_{n-11}(t)+\mu_1 P_{b+n1}(t)\\
P_{n3}'(t) &= -(\alpha+\lambda+\mu_2)P_{n3}(t)+\lambda P_{n-13}(t)+\mu_2 P_{b+n3}(t)
\end{aligned}\right\}\text{for } n = a,a+1,a+2,.. \tag{3}$$

The infinitesimal generator matrix **Q** for this model is given below

$$Q=\begin{pmatrix}
A_{00} & A_{01} & A_{02} & A_{03} & A_{04} & \ldots..\\
A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & \ldots..\\
A_{20} & A_{21} & A_{22} & A_{23} & A_{24} & \ldots..\\
A_{30} & A_{31} & A_{32} & A_{33} & A_{34} & \ldots..\\
. & . & . & . & . & \ldots..\\
. & . & . & . & . & \ldots..
\end{pmatrix}$$

The matrices $A_{00,}A_{01},A_{10},A_{11},A_{20},A_{02},\ldots$ are described in the Infinitesimal generator matrix **Q** can be obtained from the following infinitesimal transition rates of process **X** as follows

$$q_{(0,j)(l,m)}=\begin{cases}
-\lambda & if \quad (l,m)=(i,j) \quad for \quad j=0\\
\lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=0\\
-(\lambda+\mu_1) & if \quad (l,m)=(i,j) \quad for \quad j=1\\
\lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=1\\
\mu_1 & if \quad (l,m)=(i,j+1) \quad for \quad j=1\\
-(\alpha+\lambda) & if \quad (l,m)=(i,j) \quad for \quad j=2\\
\lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=2\\
\alpha & if \quad (l,m)=(i,0) \quad for \quad j=2\\
-(\alpha+\lambda+\mu_2) & if \quad (l,m)=(i,j) \quad for \quad j=3\\
\lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=3\\
\mu_2 & if \quad (l,m)=(i,2) \quad for \quad j=3\\
\alpha & if \quad (l,m)=(i,1) \quad for \quad j=3\\
0 & otherwise
\end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -\lambda & if \quad (l,m)=(i,j) \quad for \quad j=0 \text{ and } i=1,2,3,\dots,a-2 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=0 \text{ and } i=1,2,3,\dots,a-2 \\ -(\lambda+\mu_1) & if \quad (l,m)=(i,j) \quad for \quad j=1 \text{ and } i=1,2,3,\dots,a-2 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=1 \text{ and } i=1,2,3,\dots,a-2 \\ \mu_1 & if \quad (l,m)=(i,j+1) \quad for \quad j=1 \text{ and } i=1,2,3,\dots,a-2 \\ -(\alpha+\lambda) & if \quad (l,m)=(i,j) \quad for \quad j=2 \text{ and } i=1,2,3,\dots,a-2 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=2 \text{ and } i=1,2,3,\dots,a-2 \\ \alpha & if \quad (l,m)=(i,0) \quad for \quad j=2 \text{ and } i=1,2,3,\dots,a-2 \\ -(\alpha+\lambda+\mu_2) & if \quad (l,m)=(i,j) \quad for \quad j=3 \text{ and } i=1,2,3,\dots,a-2 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=3 \text{ and } i=1,2,3,\dots,a-2 \\ \mu_2 & if \quad (l,m)=(i,2) \quad for \quad j=3 \text{ and } i=1,2,3,\dots,a-2 \\ \alpha & if \quad (l,m)=(i,1) \quad for \quad j=3 \text{ and } i=1,2,3,\dots,a-2 \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -\lambda & if \quad (l,m)=(i,j) \quad for \quad j=0 \text{ and } i=a-1 \\ \lambda & if \quad (l,m)=(0,j+1) \quad for \quad j=0 \text{ and } i=a-1 \\ -(\lambda+\mu_1) & if \quad (l,m)=(i,j) \quad for \quad j=1 \text{ and } i=a-1 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=1 \text{ and } i=a-1 \\ \mu_1 & if \quad (l,m)=(i,j+1) \quad for \quad j=1 \text{ and } i=a-1 \\ -(\alpha+\lambda) & if \quad (l,m)=(i,j) \quad for \quad j=2 \text{ and } i=a-1 \\ \lambda & if \quad (l,m)=(0,j+1) \quad for \quad j=2 \text{ and } i=a-1 \\ \alpha & if \quad (l,m)=(i,0) \quad for \quad j=2 \text{ and } i=a-1 \\ -(\alpha+\lambda+\mu_2) & if \quad (l,m)=(i,j) \quad for \quad j=3 \text{ and } i=a-1 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=3 \text{ and } i=a-1 \\ \mu_2 & if \quad (l,m)=(i,2) \quad for \quad j=3 \text{ and } i=a-1 \\ \alpha & if \quad (l,m)=(i,1) \quad for \quad j=3 \text{ and } i=a-1 \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -(\lambda+\mu_1) & if \quad (l,m)=(i,j) \quad for \quad j=1 \text{ and } i=a \text{ } to \text{ } b \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=1 \text{ and } i=a \text{ } to \text{ } b \\ \mu_1 & if \quad (l,m)=(0,j) \quad for \quad j=1 \text{ and } i=a \text{ } to \text{ } b \\ -(\alpha+\lambda+\mu_2) & if \quad (l,m)=(i,j) \quad for \quad j=3 \text{ and } i=a \text{ } to \text{ } b \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=3 \text{ and } i=a \text{ } to \text{ } b \\ \mu_2 & if \quad (l,m)=(0,j) \quad for \quad j=3 \text{ and } i=a \text{ } to \text{ } b \\ \alpha & if \quad (l,m)=(i,1) \quad for \quad j=3 \text{ and } i=a \text{ } to \text{ } b \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -(\lambda+\mu_1) & if \quad (l,m)=(i,j) \quad for \quad j=1 \text{ and } i=b+1,b+2,... \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=1 \text{ and } i=b+1,b+2,... \\ \mu_1 & if \quad (l,m)=(i-b,j) \quad for \quad j=1 \text{ and } i=b+1,b+2,... \\ -(\alpha+\lambda+\mu_2) & if \quad (l,m)=(i,j) \quad for \quad j=3 \text{ and } i=b+1,b+2,... \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \quad j=3 \text{ and } i=b+1,b+2,... \\ \mu_2 & if \quad (l,m)=(i-b,j) \quad for \quad j=3 \text{ and } i=b+1,b+2,... \\ \alpha & if \quad (l,m)=(i,1) \quad for \quad j=3 \text{ and } i=b+1,b+2,... \\ 0 & otherwise \end{cases}$$

Remaining all other entries are zero.

Further, we can write the above equations (1), (2) and (3) as

$X'(t)=AX(t)$ where $A=Q^T$

Where $[X(t)]^T=(P_{00},P_{01},P_{02},P_{03},P_{10},P_{11},P_{12},P_{13},...,P_{a-10},P_{a-11},P_{a-12},P_{a-13},P_{a1},P_{a3},...,P_{b1},P_{b3},...)$

Solving the above set of equation we get,

$$X(t)=e^{tA}\ X_0$$

When t=0, $X_0=X(0)=\begin{pmatrix}1\\0\\0\\.\\.\\.\end{pmatrix}$

4. DESCRIPTION OF COMPUTATIONAL METHOD

The following effective computational procedure is used to find the Time dependent probabilities of number of customers in the queue at time t. The time dependent probabilities vector is denoted by

$[X(t)]^T=(P_{00},P_{01},P_{02},P_{03},P_{10},P_{11},P_{12},P_{13},...,P_{a-10},P_{a-11},P_{a-12},P_{a-13},P_{a1},P_{a3},...,P_{b1},P_{b3},...)$

Step 1: Assume that the matrix Q is finite that is the number of customers in the queue at time t is M (sufficiently large). The value of M can be chosen so that the loss probability is small. Due to the intrinsic nature of the system, the only choice available for studying M is through algorithmic methods. While a number of approaches are available for determining the cut-off point, M, the one that seems to perform well is to increase M until the largest individual change in the elements of **X (t)** for successive values is less than ε a predetermined infinitesimal value.

Step 2: Find the Eigen values of this finite order matrix tQ^T.

Step 3: Let $d_1, d_2, d_3, \ldots d_{2(M+a+1)}$ be 2(M+a+1) Eigen values.

Step 4: Use these Eigen values in the Vandermonde's matrix

$$V = \begin{pmatrix} 1 & d_1 & d_1^2 & . & \ldots & d_1^{2(M+a)+1} \\ 1 & d_2 & d_2^2 & . & \ldots & d_2^{2(M+a)+1} \\ 1 & d_3 & d_3^2 & . & \ldots & d_3^{2(M+a)+1} \\ . & . & . & . & \ldots & . \\ . & . & . & . & \ldots & . \\ 1 & d_{2(M+a+1)} & d_{2(M+a+1)}^2 & . & \ldots & d_{2(M+a+1)}^{2(M+a)+1} \end{pmatrix}.$$

Step 5: Let $C = \begin{pmatrix} e^{d_1} & e^{d_2} & . & . & . & e^{d_{2(M+a+1)}} \end{pmatrix}^T$ and $\alpha = \begin{pmatrix} \alpha_0 & \alpha_1 & . & . & . & \alpha_{2(M+a)+1} \end{pmatrix}^T$

Step 6: Find $\alpha = V^{-1}C$ and we get $\alpha = \begin{pmatrix} \alpha_0 & \alpha_1 & \alpha_2 & . & . & . & \alpha_{2(M+a)+1} \end{pmatrix}^T$.

Step 7: Using α in $e^{tA} = \alpha_0 I + \alpha_1 \left(tQ^T\right) + \alpha_2 \left(tQ^T\right)^2 + \ldots + \alpha_{2(M+a)+1} \left(tQ^T\right)^{2(M+a)+1}$

Step 8: Extract the first column of this Exponential matrix tQ^T and store in X (t).

Step 9: This probability vector **X (t)** provides time dependent probabilities of number of customers in the queue at time t.

5. SYSTEM PERFORMANCE MEASURES

The following system measures are used to bring out the Transient behaviour of bulk service queueing model with working vacation under study. Numerical study has been dealt in very large scale to study the following measures for several values of t, λ, μ_1, μ_2, α, a and b.

a. Probability that there are n customers in the queue when the server is idle at time t = $P_{n0}(t)$

b. Probability that there are n customers in the queue when the server is busy at time t = $P_{n1}(t)$

c. Probability that there are n customers in the queue when the server is idle in working vacation period at time t = $P_{n2}(t)$

d. Probability that there are n customers in the queue when the server is busy in working vacation period at time t = $P_{n3}(t)$

e. Probability that the server is idle at time t = $P_{idle}(t) = \sum_{n=0}^{a-1} P_{n0}(t)$

f. Probability that the server is busy at time t = $P_{busy}(t) = \sum_{n=0}^{\infty} P_{n1}(t)$

g. Probability that the server is idle in working vacation period at time t =

$$P_{\text{idle in wor vac}}(t) = \sum_{n=0}^{a-1} P_{n2}(t)$$

h. Probability that the server is busy in working vacation period at time t =

$$P_{\text{busy in wor vac}}(t) = \sum_{n=0}^{\infty} P_{n3}(t)$$

i. Average number of customers in the queue =

$$L_q(t) = \sum_{n=0}^{a-1} nP_{n0}(t) + \sum_{n=0}^{\infty} nP_{n1}(t) + \sum_{n=0}^{a-1} nP_{n2}(t) + \sum_{n=0}^{\infty} nP_{n3}(t)$$

6. NUMERICAL COMPUTATIONS

The Time dependent System performance measures and Transient probabilities of this model have been done and expressed in the form of tables, which are shown below for several values of t, λ, μ_1, μ_2, α, a and b.

Table 1 to Table 4 show Transient probabilities of number of customers in the queue when the server is idle for several values of t, λ, μ_1, μ_2, α, a and b. We infer the following

- As the value of t increases the Transient Probabilities $P_{n0}(t) \to P_{n0}$
- The sequence $\{P_{n0}(t)\} \to 0$ as n $\to \infty$ for all values of t

Table 5 to Table 8 show Transient probabilities of number of customers in the queue when the server is busy for several values of t, λ, μ_1, μ_2, α, a and b. We infer the following

- As the value of t increases the Transient Probabilities $P_{n1}(t) \to P_{n1}$
- The sequence $\{P_{n1}(t)\} \to 0$ as n $\to \infty$ for all values of t

Table 9 to Table 12 show Transient probabilities of number of customers in the queue when the server is idle in working vacation period for several values of t, λ, μ_1, μ_2, α, a and b. We infer the following

- As the value of t increases the Transient Probabilities $P_{n2}(t) \to P_{n2}$
- The sequence $\{P_{n2}(t)\} \to 0$ as n $\to \infty$ for all values of t

Table 13 to Table 16 show Transient probabilities of number of customers in the queue when the server is busy in working vacation period for several values of t, λ, μ_1, μ_2, α, a and b. We infer the following

- As the value of t increases the Transient Probabilities $P_{n3}(t) \to P_{n3}$
- The sequence $\{P_{n3}(t)\} \to 0$ as n $\to \infty$ for all values of t

Table 17 to Table 20 show Time dependent System performance measures for several values of t, λ, μ_1, μ_2, α, a and b. We infer the following

- As the value of t increases and for several values of t, λ, μ_1, μ_2, α, a and b.

  $P_{\text{idle}}(t) \to P_{\text{idle}}, P_{\text{busy}}(t) \to P_{\text{busy}}, P_{\text{idle in vac}}(t) \to P_{\text{idle in vac}},$
  $P_{\text{busy in vac}}(t) \to P_{\text{busy in vac}}$ and $L_{\text{q}}(t) \to L_{\text{q}}$

Table 1: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$
1.0	0.1228	0.2172	0.2738
1.2	0.1230	0.2166	0.2721
1.4	0.1230	0.2166	0.2718
1.6	0.1230	0.2166	0.2717
1.8	0.1230	0.2166	0.2717
2.0	0.1230	0.2166	0.2717
2.2	0.1230	0.2166	0.2717
2.4	0.1230	0.2166	0.2717
2.6	0.1230	0.2166	0.2717
2.8	0.1230	0.2166	0.2717
3.0	0.1230	0.2166	0.2717

Table 2: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$
1	0.0917	0.1602	0.2056	0.2326
1.2	0.0933	0.1623	0.2034	0.2276
1.4	0.0933	0.1634	0.2040	0.2264
1.6	0.0931	0.1635	0.2045	0.2265
1.8	0.0930	0.1634	0.2046	0.2267
2	0.0930	0.1633	0.2046	0.2267
2.2	0.0930	0.1633	0.2045	0.2267
2.4	0.0930	0.1633	0.2045	0.2267
2.6	0.0930	0.1633	0.2045	0.2267
2.8	0.0930	0.1633	0.2045	0.2267
3	0.0930	0.1633	0.2045	0.2267

Table 3: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$
1	0.0916	0.1760	0.2360
1.2	0.0915	0.1758	0.2355
1.4	0.0915	0.1758	0.2354
1.6	0.0915	0.1758	0.2354
1.8	0.0915	0.1758	0.2354
2	0.0915	0.1758	0.2354
2.2	0.0915	0.1758	0.2354
2.4	0.0915	0.1758	0.2354
2.6	0.0915	0.1758	0.2354
2.8	0.0915	0.1758	0.2354
3	0.0915	0.1758	0.2354

Table 4: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$
1	0.0701	0.1336	0.1780	0.2073
1.2	0.0698	0.1336	0.1782	0.2067
1.4	0.0697	0.1334	0.1782	0.2067
1.6	0.0697	0.1334	0.1781	0.2067
1.8	0.0697	0.1334	0.1781	0.2067
2	0.0697	0.1334	0.1781	0.2067
2.2	0.0697	0.1334	0.1781	0.2067
2.4	0.0697	0.1334	0.1781	0.2067
2.6	0.0697	0.1334	0.1781	0.2067
2.8	0.0697	0.1334	0.1781	0.2067
3	0.0697	0.1334	0.1781	0.2067

Table 5: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$
1.0	0.0732	0.0192	0.0050	0.0013	0.0003
1.2	0.0728	0.0191	0.0050	0.0013	0.0003
1.4	0.0727	0.0191	0.0050	0.0013	0.0003
1.6	0.0727	0.0191	0.0050	0.0013	0.0003
1.8	0.0727	0.0191	0.0050	0.0013	0.0003
2.0	0.0727	0.0191	0.0050	0.0013	0.0003
2.2	0.0727	0.0191	0.0050	0.0013	0.0003
2.4	0.0727	0.0191	0.0050	0.0013	0.0003
2.6	0.0727	0.0191	0.0050	0.0013	0.0003
2.8	0.0727	0.0191	0.0050	0.0013	0.0003
3.0	0.0727	0.0191	0.0050	0.0013	0.0003

Table 6: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$
1	0.0600	0.0154	0.0039	0.0010	0.0002
1.2	0.0587	0.0150	0.0039	0.0010	0.0003
1.4	0.0583	0.0149	0.0038	0.0010	0.0002
1.6	0.0583	0.0149	0.0038	0.0010	0.0002
1.8	0.0583	0.0149	0.0038	0.0010	0.0002
2	0.0583	0.0149	0.0038	0.0010	0.0002
2.2	0.0583	0.0149	0.0038	0.0010	0.0002
2.4	0.0583	0.0149	0.0038	0.0010	0.0002
2.6	0.0583	0.0149	0.0038	0.0010	0.0002
2.8	0.0583	0.0149	0.0038	0.0010	0.0002
3	0.0583	0.0149	0.0038	0.0010	0.0002

Table 7: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$
1	0.0845	0.0289	0.0098	0.0033	0.0011	0.0004
1.2	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
1.4	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
1.6	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
1.8	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
2	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
2.2	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
2.4	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
2.6	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
2.8	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004
3	0.0844	0.0289	0.0098	0.0033	0.0011	0.0004

Table 8: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$
1	0.0698	0.0231	0.0076	0.0025	0.0008
1.2	0.0695	0.0230	0.0076	0.0025	0.0008
1.4	0.0695	0.0230	0.0076	0.0025	0.0008
1.6	0.0695	0.0231	0.0076	0.0025	0.0008
1.8	0.0695	0.0230	0.0076	0.0025	0.0008
2	0.0695	0.0230	0.0076	0.0025	0.0008
2.2	0.0695	0.0230	0.0076	0.0025	0.0008
2.4	0.0695	0.0230	0.0076	0.0025	0.0008
2.6	0.0695	0.0230	0.0076	0.0025	0.0008
2.8	0.0695	0.0230	0.0076	0.0025	0.0008
3	0.0695	0.0230	0.0076	0.0025	0.0008

Table 9: Transient probability distribution of number of customers in the queue when the server is idle in working vacation for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$
1.0	0.1231	0.0929	0.0536
1.2	0.1230	0.0935	0.0548
1.4	0.1230	0.0936	0.0551
1.6	0.1230	0.0936	0.0551
1.8	0.1230	0.0936	0.0551
2.0	0.1230	0.0936	0.0551
2.2	0.1230	0.0936	0.0551
2.4	0.1230	0.0936	0.0551
2.6	0.1230	0.0936	0.0551
2.8	0.1230	0.0936	0.0551
3.0	0.1230	0.0936	0.0551

Table 10: Transient probability distribution of number of customers in the queue when the server is idle in working vacation for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$	$P_{32}(t)$
1	0.0945	0.0700	0.0393	0.0195
1.2	0.0938	0.0709	0.0412	0.0216
1.4	0.0931	0.0705	0.0414	0.0221
1.6	0.0930	0.0703	0.0413	0.0222
1.8	0.0930	0.0702	0.0412	0.0222
2	0.0930	0.0703	0.0412	0.0222
2.2	0.0930	0.0703	0.0412	0.0222
2.4	0.0930	0.0703	0.0412	0.0222
2.6	0.0930	0.0703	0.0412	0.0222
2.8	0.0930	0.0703	0.0412	0.0222
3	0.0930	0.0703	0.0412	0.0222

Table 11: Transient probability distribution of number of customers in the queue when the server is idle in working vacation for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$
1	0.1281	0.1178	0.0829
1.2	0.1280	0.1180	0.0833
1.4	0.1280	0.1180	0.0834
1.6	0.1280	0.1180	0.0834
1.8	0.1280	0.1180	0.0834
2	0.1280	0.1180	0.0834
2.2	0.1280	0.1180	0.0834
2.4	0.1280	0.1180	0.0834
2.6	0.1280	0.1180	0.0834
2.8	0.1280	0.1180	0.0834
3	0.1280	0.1180	0.0834

Table 12: Transient probability distribution of number of customers in the queue when the server is idle in working vacation for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$	$P_{32}(t)$
1	0.0978	0.0893	0.0624	0.0392
1.2	0.0975	0.0891	0.0626	0.0399
1.4	0.0976	0.0891	0.0626	0.0400
1.6	0.0976	0.0891	0.0626	0.0400
1.8	0.0976	0.0891	0.0626	0.0400
2	0.0976	0.0891	0.0626	0.0400
2.2	0.0976	0.0891	0.0626	0.0400
2.4	0.0976	0.0891	0.0626	0.0400
2.6	0.0976	0.0891	0.0626	0.0400
2.8	0.0976	0.0891	0.0626	0.0400
3	0.0976	0.0891	0.0626	0.0400

Table 13: Transient probability distribution of number of customers in the queue when the server is busy in working vacation for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{03}(t)$	$P_{13}(t)$	$P_{23}(t)$	$P_{33}(t)$	$P_{43}(t)$
1.0	0.0133	0.0033	0.0008	0.0002	0.0000
1.2	0.0138	0.0034	0.0008	0.0002	0.0001
1.4	0.0139	0.0035	0.0009	0.0002	0.0001
1.6	0.0139	0.0035	0.0009	0.0002	0.0001
1.8	0.0139	0.0035	0.0009	0.0002	0.0001
2.0	0.0139	0.0035	0.0009	0.0002	0.0001
2.2	0.0139	0.0035	0.0009	0.0002	0.0001
2.4	0.0139	0.0035	0.0009	0.0002	0.0001
2.6	0.0139	0.0035	0.0009	0.0002	0.0001
2.8	0.0139	0.0035	0.0009	0.0002	0.0001
3.0	0.0139	0.0035	0.0009	0.0002	0.0001

Table 14: Transient probability distribution of number of customers in the queue when the server is busy in working vacation for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{03}(t)$	$P_{13}(t)$	$P_{23}(t)$	$P_{33}(t)$	$P_{43}(t)$
1	0.0046	0.0011	0.0002	0.0001	0.0000
1.2	0.0053	0.0013	0.0003	0.0001	0.0000
1.4	0.0055	0.0014	0.0003	0.0001	0.0000
1.6	0.0056	0.0014	0.0003	0.0001	0.0000
1.8	0.0056	0.0014	0.0003	0.0001	0.0000
2	0.0056	0.0014	0.0003	0.0001	0.0000
2.2	0.0056	0.0014	0.0003	0.0001	0.0000
2.4	0.0056	0.0014	0.0003	0.0001	0.0000
2.6	0.0056	0.0014	0.0003	0.0001	0.0000
2.8	0.0056	0.0014	0.0003	0.0001	0.0000
3	0.0056	0.0014	0.0003	0.0001	0.0000

Table 15: Transient probability distribution of number of customers in the queue when the server is busy in working vacation for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{03}(t)$	$P_{13}(t)$	$P_{23}(t)$	$P_{33}(t)$	$P_{43}(t)$	$P_{53}(t)$
1	0.0268	0.0085	0.0027	0.0008	0.0003	0.0001
1.2	0.0271	0.0086	0.0027	0.0009	0.0003	0.0001
1.4	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
1.6	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
1.8	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
2	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
2.2	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
2.4	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
2.6	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
2.8	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001
3	0.0271	0.0086	0.0028	0.0009	0.0003	0.0001

Table 16: Transient probability distribution of number of customers in the queue when the server is busy in working vacation for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{03}(t)$	$P_{13}(t)$	$P_{23}(t)$	$P_{33}(t)$	$P_{43}(t)$
1	0.0124	0.0039	0.0012	0.0004	0.0001
1.2	0.0127	0.0040	0.0013	0.0004	0.0001
1.4	0.0128	0.0041	0.0013	0.0004	0.0001
1.6	0.0128	0.0041	0.0013	0.0004	0.0001
1.8	0.0128	0.0041	0.0013	0.0004	0.0001
2	0.0128	0.0041	0.0013	0.0004	0.0001
2.2	0.0128	0.0041	0.0013	0.0004	0.0001
2.4	0.0128	0.0041	0.0013	0.0004	0.0001
2.6	0.0128	0.0041	0.0013	0.0004	0.0001
2.8	0.0128	0.0041	0.0013	0.0004	0.0001
3	0.0128	0.0041	0.0013	0.0004	0.0001

Table 17: System performance measures for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	Pidle(t)	Pbusy(t)	Pw-idle(t)	Pw-busy(t)	Lq(t)
1	0.6138	0.0991	0.2695	0.0176	0.9993
1.2	0.6117	0.0986	0.2713	0.0183	0.9988
1.4	0.6113	0.0985	0.2716	0.0185	0.9987
1.6	0.6112	0.0985	0.2717	0.0186	0.9988
1.8	0.6112	0.0985	0.2717	0.0186	0.9988
2	0.6112	0.0985	0.2717	0.0186	0.9988
2.2	0.6112	0.0985	0.2717	0.0186	0.9988
2.4	0.6112	0.0985	0.2717	0.0186	0.9988
2.6	0.6112	0.0985	0.2717	0.0186	0.9988
2.8	0.6112	0.0985	0.2717	0.0186	0.9988
3	0.6112	0.0985	0.2717	0.0186	0.9988

Table 18: System performance measures for various values of t, $\lambda = 5$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	Pidle(t)	Pbusy(t)	Pw-idle(t)	Pw-busy(t)	Lq(t)
1	0.6901	0.0806	0.2233	0.0060	1.5042
1.2	0.6866	0.0789	0.2275	0.0070	1.4979
1.4	0.6872	0.0783	0.2271	0.0074	1.4983
1.6	0.6876	0.0783	0.2267	0.0074	1.4993
1.8	0.6877	0.0783	0.2266	0.0074	1.4996
2	0.6876	0.0784	0.2266	0.0074	1.4996
2.2	0.6875	0.0784	0.2267	0.0074	1.4996
2.4	0.6875	0.0784	0.2267	0.0074	1.4996
2.6	0.6875	0.0784	0.2267	0.0074	1.4996
2.8	0.6875	0.0784	0.2267	0.0074	1.4996
3	0.6875	0.0784	0.2267	0.0074	1.4996

Table 19: System performance measures for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 3 and b = 5.

t	Pidle(t)	Pbusy(t)	Pw-idle(t)	Pw-busy(t)	Lq(t)
1	0.5036	0.1283	0.3288	0.0392	0.9973
1.2	0.5028	0.1282	0.3293	0.0397	0.9973
1.4	0.5026	0.1282	0.3295	0.0398	0.9973
1.6	0.5026	0.1281	0.3295	0.0398	0.9973
1.8	0.5026	0.1281	0.3295	0.0398	0.9973
2	0.5026	0.1281	0.3295	0.0398	0.9973
2.2	0.5026	0.1281	0.3295	0.0398	0.9973
2.4	0.5026	0.1281	0.3295	0.0398	0.9973
2.6	0.5026	0.1281	0.3295	0.0398	0.9973
2.8	0.5026	0.1281	0.3295	0.0398	0.9973
3	0.5026	0.1281	0.3295	0.0398	0.9973

Table 20: System performance measures for various values of t, $\lambda = 7$, $\mu_1 = 15$, $\mu_2 = 10$, $\alpha = 5$, a = 4 and b = 12.

t	Pidle(t)	Pbusy(t)	Pw-idle(t)	Pw-busy(t)	Lq(t)
1	0.58895	0.10429	0.28883	0.01793	1.49735
1.2	0.58834	0.10393	0.28908	0.01865	1.49850
1.4	0.58804	0.10397	0.28922	0.01877	1.49863
1.6	0.58790	0.10397	0.28933	0.01879	1.49856
1.8	0.58787	0.10397	0.28936	0.01880	1.49855
2	0.58787	0.10396	0.28936	0.01880	1.49855
2.2	0.58787	0.10396	0.28936	0.01880	1.49855
2.4	0.58787	0.10396	0.28936	0.01880	1.49855
2.6	0.58787	0.10396	0.28936	0.01880	1.49855
2.8	0.58787	0.10396	0.28936	0.01880	1.49855
3	0.58787	0.10396	0.28936	0.01880	1.49855

7. SPECIAL CASE

- If $\mu_1 = \mu_2$ this model coincides with transient behaviour of bulk service queueing system.
- If α tends to ∞ this model coincides with transient behaviour bulk service queueing system.

8. CONCLUSION

Cayley Hamilton theorem was used to evaluate the Transient behaviour of Bulk service queueing system with working vacation model using infinite generator matrix. Numerical studies have been analysed in elaborate manner. In this model we have provided transient probability distribution of number of customers in the queue at time t and also time dependent system measures.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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