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COUPLED FIXED POINT THEOREM FOR WEAK COMPATIBLE MAPPINGS IN MENGER SPACES

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Abstract: In this paper, we prove coupled fixed point theorems for a pair of weakly compatible mappings under ϕ-contractive conditions in Menger spaces without appeal to continuity of mappings. We support our result by providing a suitable example. At the end, we give an application of our result.

Keywords: Weakly compatible maps; Menger space, t-norm of H-type.

2000 AMS Subject Classification: 47H10; 54H25

1. Introduction

In 1942 Menger [7] introduced the notion of a probabilistic metric space (PM-space) which is in fact, a generalization of metric space. The idea in probabilistic metric space is to associate a distribution function with a point pair, say (p, q), denoted by F(p, q, t) where t > 0 and interpret this function as the probability that distance between p and q is less than t, whereas in the metric space the distance function is a single positive number. Sehgal [9] initiated the study of fixed points in probabilistic metric spaces. The study of these spaces was expanded rapidly with the pioneering works of Schweizer-Sklar [11].

In 1991, Mishra[8] introduced the notion of compatible mappings in the

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setting of probabilistic metric space. In 1996, Jungck [5] introduce the notion of weakly compatible mappings as follows:

Two self mappings S and T are said to be weakly compatible if they commute at their coincide points, i.e., Tu = Su for some u ∈ X , then TSu = STu.

Further, Singh and Jain [10] proved some results for weakly compatible in Menger spaces.

Fang [3] defined ϕ-contractive conditions and proved some fixed point theorems under ϕ-contractions for compatible and weakly compatible maps in Menger PM-spaces using t-norm of H-type, introduced by Hadžíc [4].

Recently, Bhaskar and Lakshmikantham [2], Lakshmikantham and Ćirić [6] gave some coupled fixed point theorems in partially ordered metric spaces.

Now, we prove a coupled fixed point theorem for a pair of weakly compatible maps satisfying ϕ-contractive conditions in Menger PM-space with a continuous t-norm of H-type. We support our result by an example. At the end, we give an application of our result.

2. Preliminaries

First, recall that a real valued function f defined on the set of real numbers is known as a distribution function if it is non-decreasing, continuous and inf. f(x) = 0, sup. f(x) = 1. In what follows H(x) denotes the distribution function defined as follows:

$$H(x) = \begin{cases} 0 & if \ x \le 0, \\ 1 & if \ x > 0. \end{cases}$$

Definition 2.1. A probabilistic metric space (PM-space) is a pair (X, F) where X is a set and F is a function defined on X × X into the set of distribution functions such that if x, y and z are points of X, then

(F-1) F(x, y; 0) = 0,

(F-2) F(x, y; t) = H(t) iff x = y,

(F-3) F(x, y; t) = F(y, x; t),

(F-4) if F(x, y; s) = 1 and F(y, z; t) = 1, then F(x, z; s+t) = 1 for all x, y, z ∈ X and s, t ≥ 0.

For each x and y in X and for each real number t ≥ 0, F(x, y; t) is to be thought of as the probability that the distance between x and y is less than t.

It is interesting to note that, if (X, d) is a metric space, then the distribution function F(x, y; t) defined by the relation F(x, y; t) = H(t – d(x, y)) induces a PM-space.

Definition 2.2. A t-norm t is a 2-place function, t : [0,1] × [0,1] → [0,1] satisfying the following:

(i) t(0,0) = 0,

(ii) t(0,1) = 1,

(iii) t(a,b) = t(b,a),

(iv) if a ≤ c, b ≤ d, then t(a,b) ≤ t(c,d),

(v) t(t(a,b),c) = t(a,t(b,c)) for all a, b, c in [0,1].

Definition 2.3. A Menger PM-space is a triplet (X, F, t) where (X, F) is a PM-space and t is a t-norm with the following condition:

(F-5) F(x, z; s + t) ≥ t(F(x, y; s), F(y, z; t)), for all x, y, z in X and s, t ≥ 0.

This inequality is known as Menger's triangle inequality.

In our theory, we consider (X, F, t) to be a Menger PM-space along with the following condition:

(F-6) $\lim_{t \to \infty} F(x, y, t)$ = 1, for all x, y in X.

Definition 2.4[4]. Let $\sup._{0 < t < 1}$ Δ(t, t) = 1. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at t = 1, where

$$\Delta^1(t) = t,\ \Delta^{m+1}(t) = t\ \Delta\ (\Delta^m(t)),\ m = 1, 2\ldots,\ t \in [0, 1].$$

The t-norm Δ_M = min. is an example of t-norm of H-type.

Remark 2.1. Δ is a H-type t-norm iff for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Delta^m(t) > (1-\lambda)$ for all m ∈ N, when t > (1-δ).

Definition 2.5. A sequence $\{x_n\}$ in a Menger PM space (X, F, t) is said

(i) to converge to a point x in X if for every $\epsilon > 0$ and $\lambda > 0$, there is an integer n_0 such that $F(x_n, x, \epsilon) > 1 - \lambda$, for all $n \geq n_0$.

(ii) to be Cauchy if for each $\epsilon > 0$ and $\lambda > 0$, there is an integer n_0 such that $F(x_n, x_m, \epsilon) > 1 - \lambda$, for all n, m $\geq n_0$.

(iii) to be complete if every Cauchy sequence in it converges to a point of it.

Definition 2.6[3]. Define $\boldsymbol{\Phi} = \{ \phi : R^+ \to R^+\}$, where $R^+ = [0, +\infty)$ and each $\phi \in \boldsymbol{\Phi}$ satisfies the following conditions:

(ϕ-1) ϕ is non-decreasing;

(ϕ-2) ϕ is upper semicontinuous from the right;

(ϕ-3) $\sum_{n=0}^{\infty} \phi^n(t) < +\infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$, $n \in N$.

Clearly, if $\phi \in \boldsymbol{\Phi}$, then $\phi(t) < t$ for all $t > 0$.

Definition 2.7[3]. An element $x \in X$ is called a common fixed point of the mappings

f: X × X → X and g: X → X if

$$x = f(x, x) = g(x).$$

Definition 2.8[6]. An element $(x, y) \in X \times X$ is called a

(i) coupled fixed point of the mapping f: X × X → X if

$$f(x, y) = x , \quad f(y, x) = y.$$

(ii) coupled coincidence point of the mappings f: X × X → X and g: X → X if

$$f(x, y) = g(x), \quad f(y, x) = g(y).$$

(iii) common coupled fixed point of the mappings f: X × X → X and g: X→ X if

$$x = f(x, y) = g(x), \quad y = f(y, x) = g(y).$$

Definition 2.9[3]. The mappings f: X × X → X and g: X → X are called commutative if

$$gf(x, y) = f(gx, gy), \text{for all } x, y \in X.$$

Abbas, Khan and Redenović [1] introduced the notion of w-compatible mappings as follows:

The mappings f : X × X → X and g : X → X are called w-compatible if

$$g(f(x, y)) = f(gx, gy) \text{ whenever } g(x) = f(x, y) \text{ and } g(y) = f(y,x).$$

Definition 2.10. The maps f: X × X → X and g: X → X are called weakly compatible if $f(x, y) = g(x)$, $f(y, x) = g(y)$ implies $gf(x, y) = f(gx, gy)$, $gf(y, x) = f(gy, gx)$, for all x, y in X.

3. Main results

For convenience, we denote

(3.1) $$[F(x,y,t)]^{n} = \underbrace{F(x,y,t) * F(x,y,t) * \ldots * F(x,y,t)}_{n}, \text{ for all } n \in N.$$

Now we prove our main result.

Theorem 3.1. Let (X, F, $*$) be Menger PM-Space, * being continuous t – norm of H-type. Let f: X × X → X and g: X → X be two mappings and there exists $\phi \in \Phi$ such that

(3.2) F(f(x, y), f(u, v), ϕ(t)) ≥ ψ[F(gx, gu, t) $*$ F(gy, gv, t)], for all x, y, u, v in X and t > 0, where ψ: [0, 1] → [0, 1] is a continuous function such that ψ(t) ≥ t for all t ∈ [0, 1].

Suppose that f(X × X) ⊆ g(X), f and g are weakly compatible, range space of one of the maps f or g is complete. Then f and g have a coupled coincidence point.

Moreover, there exists a unique point x in X such that x = f(x, x) = g(x).

Proof.

Let x_0, y_0 be two arbitrary points in X. Since f(X × X) ⊆ g(X), we can choose x_1, y_1 in X such that $g(x_1) = f(x_0, y_0)$, $g(y_1) = f(y_0, x_0)$.

Continuing in this way we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $g(x_{n+1}) = f(x_n, y_n)$ and $g(y_{n+1}) = f(y_n, x_n)$, for al n ≥ 0.

Step 1. We first show that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Since * is a t-norm of H-type, for any ε> 0, there exists $\delta > 0$ such that

(3.3) $$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} F(x,y,t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$F(gx_0, gx_1, t_0) \geq (1-\delta)$ and $F(gy_0, gy_1, t_0) \geq (1-\delta)$.

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any t > 0, there exists $n_0 \in N$ such that

(3.4) $$t > \sum_{k=n_0}^{\infty} \phi^k(t_0).$$

Using condition (3.2), we have

$$F(gx_1, gx_2, \phi(t_0)) = F(f(x_0, y_0), f(x_1, y_1), \phi(t_0))$$
$$\geq \psi[F(gx_0, gx_1, t_0) * F(gy_0, gy_1, t_0)]$$

$\geq F(gx_0, gx_1, t_0) * F(gy_0, gy_1, t_0)$.

$F(gy_1, gy_2, \phi(t_0)) = F(f(y_0, x_0), f(y_1, x_1), \phi(t_0))$

$\geq \psi[F(gy_0, gy_1, t_0) * F(gx_0, gx_1, t_0)]$

$\geq F(gy_0, gy_1, t_0) * F(gx_0, gx_1, t_0)$.

Similarly, we can also get

$F(gx_2, gx_3, \phi^2(t_0)) = F(f(x_1, y_1), f(x_2, y_2), \phi^2(t_0))$

$\geq \psi[F(gx_1, gx_2, \phi(t_0)) * F(gy_1, gy_2, \phi(t_0))]$

$\geq F(gx_1, gx_2, \phi(t_0)) * F(gy_1, gy_2, \phi(t_0))$

$\geq [F(gx_0, gx_1, t_0)]^2 * [F(gy_0, gy_1, t_0)]^2$.

$F(gy_2, gy_3, \phi^2(t_0)) = F(f(y_1, x_1), f(y_2, x_2), \phi^2(t_0))$

$\geq [F(gy_0, gy_1, t_0)]^2 * [F(gx_0, gx_1, t_0)]^2$.

Continuing in this way, we can get

$F(gx_n, gx_{n+1}, \phi^n(t_0)) \geq [F(gx_0, gx_1, t_0)]^{2^{n-1}} * [F(gy_0, gy_1, t_0)]^{2^{n-1}}$.

$F(gy_n, gy_{n+1}, \phi^n(t_0)) \geq [F(gy_0, gy_1, t_0)]^{2^{n-1}} * [F(gx_0, gx_1, t_0)]^{2^{n-1}}$.

So, from (3.3) and (3.4), for $m > n \geq n_0$, we have

$F(gx_n, gx_m, t)$

$\geq F(gx_n, gx_m, \sum_{k=n_0}^{\infty} \phi^k(t_0))$

$\geq F(gx_n, gx_m, \sum_{k=n}^{m-1} \phi^k(t_0))$

$\geq F(gx_n, gx_{n+1}, \phi^n(t_0)) * F(gx_{n+1}, gx_{n+2}, \phi^{n+1}(t_0)) * \ldots * F(gx_{m-1}, gx_m, \phi^{m-1}(t_0))$

$\geq \left\{[F(gx_0, gx_1, t_0)]^{2^{n-1}} * [F(gy_0, gy_1, t_0)]^{2^{n-1}}\right\} *$

$* \left\{[F(gx_0, gx_1, t_0)]^{2^{n}} * [F(gy_0, gy_1, t_0)]^{2^{n}}\right\} *$

$\ldots$

$* \left\{[F(gx_0, gx_1, t_0)]^{2^{m-2}} * [F(gy_0, gy_1, t_0)]^{2^{m-2}}\right\}$

$= [F(gx_0, gx_1, t_0)]^{2^{n-1}(2^{m-n}-1)} * [F(gy_0, gy_1, t_0)]^{2^{n-1}(2^{m-n}-1)}$

$\geq \underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^n(2^{m-n}-1)} \geq (1-\epsilon)$, which implies that

$F(gx_n, gx_m, t) \geq (1-\epsilon)$, for all $m, n \in N$ with $m > n \geq n_0$ and $t > 0$.

Therefore, $\{gx_n\}$ is a Cauchy sequence. Similarly, we can get that $\{gy_n\}$ is a Cauchy sequence.

Step 2. To show that f and g have a coupled coincidence point.

Without loss of generality, one can assume that g(X) is complete, then there exists points x, y in g(X) so that $\lim_{n\to\infty} g(x_{n+1})$ = x, $\lim_{n\to\infty} g(y_{n+1})$ = y.

For x, y ∈ g(X) implies the existence of p, q in X such that g(p) = x, g(q) = y and hence $\lim_{n\to\infty} g(x_{n+1}) = \lim_{n\to\infty} f(x_n, y_n)$ = g(p) = x,

$\lim_{n\to\infty} g(y_{n+1}) = \lim_{n\to\infty} f(y_n, x_n)$ = g(q) = y.

From (3.2),we have

$F(f(x_n, y_n), f(p, q), \phi(t)) \geq \psi[F(gx_n, g(p), t) * F(gy_n, g(q), t)]$

$\geq F(gx_n, g(p), t) * F(gy_n, g(q), t)$.

Taking limit as n → ∞, we get

F(g(p), f(p, q), ϕ(t)) = 1 that is, f(p, q) = g(p) = x.

Similarly, f(q, p) = g(q) = y.

But f and g are weakly compatible, so that f(p, q) = g(p) = x and f(q, p) = g(q) = y implies gf(p, q) = f(g(p), g(q)) and gf(q, p) = f(g(q), g(p)), that is g(x) = f(x, y) and g(y) = f(y, x).

Hence f and g have a coupled coincidence point.

Step 3. To show that g(x) = x and g(y) = y.

Since * is a t-norm of H-type, for any ϵ > 0, there exists δ > 0 such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} F(x, y, t)$ = 1, for all x, y in X, there exists t_0 > 0 such that

$F(gx, x, t_0) \geq (1-\delta)$ and $F(gy, y, t_0) \geq (1-\delta)$.

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$.

Then for any t > 0, there exists n_0 ∈ N such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0).$$

From (3.2), we have

$F(gx, x, \phi(t_0)) = F(f(x, y), f(p, q), \phi(t_0))$

$\geq \psi[F(gx, gp, t_0) * F(gy, gq, t_0)]$

$\geq F(gx, gp, t_0) * F(gy, gq, t_0)$

$= F(gx, x, t_0) * F(gy, y, t_0)$.

Similarly, $F(gy, y, \phi(t_0)) \geq F(gy, y, t_0) * F(gx, x, t_0)$.

Continuing in a same way, we have for all $n \in N$,

$F(gx, x, \phi^n(t_0)) \geq [F(gx, x, t_0)]^{2^{n-1}} * [F(gy, y, t_0)]^{2^{n-1}}$.

Thus, we have

$$F(gx, x, t) \geq F(gx, x, \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0))$$
$$\geq F(gx, x, \phi^{n_0}(t_0))$$
$$\geq [F(gx, x, t_0)]^{2^{n_0-1}} * [F(gy, y, t_0)]^{2^{n_0-1}}$$
$$\geq \underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^{n_0}} \geq (1-\epsilon).$$

So, for any $\epsilon > 0$, we have $F(gx, y, t) \geq (1-\epsilon)$, for all $t > 0$.

This implies $g(x) = x$. Similarly, $g(y) = y$.

Step 4. Next we shall show that $x = y$.

Since $*$ is a t-norm of H-type, for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} F(x, y, t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$$F(x, y, t_0) \geq (1-\delta).$$

Since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0).$$

From (3.2), we have

$$F(x, y, \phi(t_0)) = F(f(p, q), f(q, p), \phi(t_0))$$
$$\geq \psi[F(gp, gq, t_0) * F(gq, gp, t_0)]$$
$$\geq F(gp, gq, t_0) * F(gq, gp, t_0)$$
$$= [F(x, y, t_0)]^2.$$

Continuing likewise, we have for all $n \in N$, that

$F(x, y, \phi^{n_0}(t_0)) \geq [F(x, y, t_0)]^{2^{n_0}}$.

Thus, we have

F(x, y, t) ≥ F(x, y, $\sum_{k=n_0}^{\infty} \phi^k(t_0)$)

≥ F(x, y, $\phi^{n_0}(t_0)$)

≥ $[F(x, y, t_0)]^{2^{n_0}}$

≥ $\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^{n_0}}$ ≥ (1- ϵ), which implies that x = y.

Thus, we have proved that f and g have a common fixed point x in X.

Step 5. We now prove the uniqueness of x.

Let z be any point in X such that z ≠ x with g(z) = z = f(z, z).

Since * is a t-norm of H-type, for any ϵ > 0, there exists δ > 0 such that

$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p}$ ≥ (1- ϵ), for all p ∈ N.

Since $\lim_{t\to\infty} F(x, y, t)$ = 1, for all x, y in X, there exists t_0 > 0 such that

F(x, z, t_0) ≥ $(1-\delta)$.

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any t > 0, there exists n_0 ∈ N such that

t > $\sum_{k=n_0}^{\infty} \phi^k(t_0)$.

Using condition (3.2), we have

F(x, z, $\phi(t_0)$) = F(f(x, x), f(z, z), $\phi(t_0)$)

≥ $\psi[F(g(x), g(z), t_0) * F(g(x), g(z), t_0)]$

≥ F(g(x), g(z), t_0) * F(g(x), g(z), t_0)

= F(x, z, t_0) * F(x, z, t_0)

= $[F(x, z, t_0)]^2$.

Thus, we have

F(x, z, t) ≥ F(x, z, $\sum_{k=n_0}^{\infty} \phi^k(t_0)$)

≥ F(x, z, $\phi^{n_0}(t_0)$)

≥ $\left([F(x, z, t_0)]^{2^{n_0-1}}\right)^2$

= $\left(F(x, z, t_0)\right)^{2^{n_0}}$

≥ $\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^{n_0}}$ ≥ (1- ϵ), which implies that x = y.

Hence, f and g have a unique common fixed point in X.

Next, we give an example in support of the Theorem 3.1.

Example 3.1. Let X = [-2, 2), a * b = ab for all a, b ϵ [0, 1] and $\varphi(t) = \frac{t}{t+1}$. Then (X, F, *) is a Menger space, where

$$F(x, y, t) = [\varphi(t)]^{|x-y|}, \text{ for all } x, y \text{ in } X \text{ and } t > 0.$$

Let $\psi(t) = t$, $\phi(t) = \frac{t}{2}$, g(x) = x and the mapping f : X × X → X be defined by $f(x, y) = \frac{x^2}{16} + \frac{y^2}{16} - 2$.

It is easy to check that f(X × X) ⊆ X = g(X). Further, f(X × X) is complete and the pair (f, g) is weakly compatible. We now check the condition (3.2),

$F(f(x, y), f(u, v), \phi(t))$

$= F(f(x, y), f(u, v), \frac{t}{2})$

$= \left[\varphi\left(\frac{t}{2}\right)\right]^{|f(x,y) - f(u,v)|}$

$= \left[\frac{t}{t+2}\right]^{|x^2 + y^2 - u^2 - v^2|/16}$

$\geq \left[\frac{t}{t+2}\right]^{|x^2 + y^2 - u^2 - v^2|/8}$

$\geq \left[\frac{t}{t+1}\right]^{|x-u| + |y-v|}$

$= \left[\frac{t}{t+1}\right]^{|x-u|} \left[\frac{t}{t+1}\right]^{|y-v|}$

$= \psi[F(gx, gu, t) * F(gy, gv, t)]$, for every t > 0. Hence, all the conditions of Theorem 3.1, are satisfied. Thus f and g have a unique common coupled fixed point in X. Indeed, $x = 4(1 - \sqrt{2})$ is a unique common coupled fixed point of f and g.

Theorem 3.2. Let (X, F, $*$) be Menger PM - Space, * being continuous t – norm of H-type. Let f: X × X → X and g: X → X be two mappings and there exists $\phi \in \Phi$ satisfying (3.2)

Suppose that f(X × X) ⊆ g(X), f and g are w-compatible, range space of one of the mappings f or g is complete. Then there exists a unique point x in X such that x = f(x, x) = g(x).

Proof.

It follows immediately from Theorem 3.1.

Next we give an application of Theorem 3.1.

4. An Application

Theorem 4.1. Let (X, F, *) be a Menger PM - space, * being continuous t-norm defined by a * b = min.{a, b} for all a, b in X. Let M, N be weakly compatible self maps on X satisfying the following conditions:

(4.1) $M(X) \subseteq N(X)$,

(4.2) there exists $\phi \in \Phi$ such that

$$F(Mx, My, \phi(t)) \geq \psi[F(Nx, Ny, t)]$$ for all x, y in X and t > 0, where $\psi: [0, 1] \to [0, 1]$ is continuous and $\psi(t) \geq t$ for all $t \in [0, 1]$.

If range space of any one of the maps M or N is complete, then M and N have a unique common fixed point in X.

Proof.

By taking f(x, y) = M(x) and g(x) = N(x) for all $x, y \in X$ in theorem (3.1), we get the desired result.

Taking ϕ (t) = kt, $k \in (0, 1)$ and $\psi(t) = t$ we have the following:

Corollary 4.2. Let (X, F, *) be a Menger PM - space, * being continuous t-norm defined by a * b = min.{a, b} for all a, b in X. Let M, N be weakly compatible self maps on X satisfying (4.1) and the following condition:

(4.3) there exists $k \in (0, 1)$ such that

$$F(Mx, My, kt) \geq F(Nx, Ny, t)$$ for all x, y in X and t > 0.

If range space of any one of the maps M or N is complete, then M and N have a unique common fixed point in X.

Taking $N = I_X$ (the identity map on X) in Corollary 4.2, we have the following:

Corollary 4.3. Let (X, F, *) be a Menger PM - space, * being continuous t-norm defined by a * b = min.{a, b} for all a, b in X. Let M, N be weakly compatible self maps on X satisfying (4.1) and the following condition:

(4.4) there exists $k \in (0, 1)$ such that

$$F(Mx, My, kt) \geq F(x, y, t)$$ for all x, y in X and t > 0.

If range space of the map M is complete, then M and N have a unique common fixed point in X.

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