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STEADY STATE QUEUE SIZE DISTRIBUTION OF AN $M/M/1$ QUEUE WITH DISASTERS AND REPAIRS UNDER BERNOULLI WORKING VACATION SCHEDULE

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Abstract: An $M/M/1$ queueing model with disasters and repairs under Bernoulli working vacation schedule is considered. In this model, after every completion of service the server has the choice to choose the normal busy state with probability p or he may choose the working vacation state with probability q. Also, disasters are allowed to occur in the busy state. In this paper, the stationary PGF of the number of customers in the system and some performance measures are derived.

Keywords: disaster; repair; Bernoulli working vacation.

2010 AMS Subject Classification: 60E05.

1. INTRODUCTION

Queues with disasters are extensively discussed by various researchers. As disaster occurs all customers in the system are removed. This type of situations are seen to prevail in the

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computer networks (where arrival of virus can be considered as disaster), ATM in a bank, manufacturing systems and so on.

Gelenbe [4] was the first to introduce the concept of arrival of negative customers in the queue. For better understanding the reader may refer to Gelenbe [5], Harrison and Pitel [6], Chao [2], Atencia and Bocharov [1], Kumar and Arivudainambi [10], Kumar and Madheswari [11], Yang et al [15].

Yechiali [16] analysed queue with disaster and impatience. Sudesh [13], Dimou and Economou [3] were some of the remarkable papers in queue with disasters and impatience.

Queue with vacations were studied by many researchers since the late 70's. Reader may look into the survey paper of Ke et al [9] for recent developments in vacation queueing models. But there are only few articles related to queue with disasters and vacations. Queue with disasters and vacations were first introduced by Mytalas and Zazanis [12]. Also, reader may refer Ye et al [7], Kalidass et al [8], Suranga Sampath [14], for better understanding of queues with disasters and vacations.

Due to wide spread applications as well as due to flexibility, Bernoulli vacation was analyzed by many researchers. Practically, the server may opt working vacation after every completion of service depending upon his physical condition. More elaborately, a driver can opt long trip or short trip depending upon his physical condition. Motivated by the above example, in this paper we derived stationary PGF of the number of customers in the system of an $M/M/1$ queue with disasters and repairs under Bernoulli working vacation Schedule. Also some performance measures are derived.

The sections of the paper are given below.

- Section 2 –Description of the model
- Section 3 - Queue size distribution of the model under the steady state
- Section 4 - Performance measures
- Section 5 - Conclusion and Future scope of the model

2. Model Description

A single server queue with disasters and repairs under Bernoulli vacation schedule is considered. Here arrival follows Poisson distribution with parameter λ and service follows exponential distribution with parameter μ. Whenever the server completes the service to a customer, the server may choose a working vacation with probability q or the server may continue the service to the next waiting customer with probability p. Also the duration of vacation times follow exponential distribution with parameter η. Disaster occurs during the busy period. After the occurrence of disaster all customers in the system are flushed out and system becomes empty. Subsequently repair time starts. Also it is assumed that disaster and repair times are exponentially distributed with parameter α and r respectively.

Notations for number of customers in the system and system states are $\chi(t)$ and $\mathcal{J}(t)$ respectively. Mathematically,

$$\mathcal{J}(t) = \begin{cases} 1; \text{server is in busy state} \\ 0; \text{server is in working vacation state} \\ 2; \text{server is in repair state} \\ 3; \text{server being idle} \end{cases}$$

Hence $(\mathcal{J}(t), \chi(t))$ is a Markov process with state space $\Omega = \{(0,0) \cup (3,0) \cup (2,0) \cup (j,n);\, j = 0,1,2,3 \;\; n = 1,2,....\}$. The system is consider to be stable as long as $\alpha > 0$.

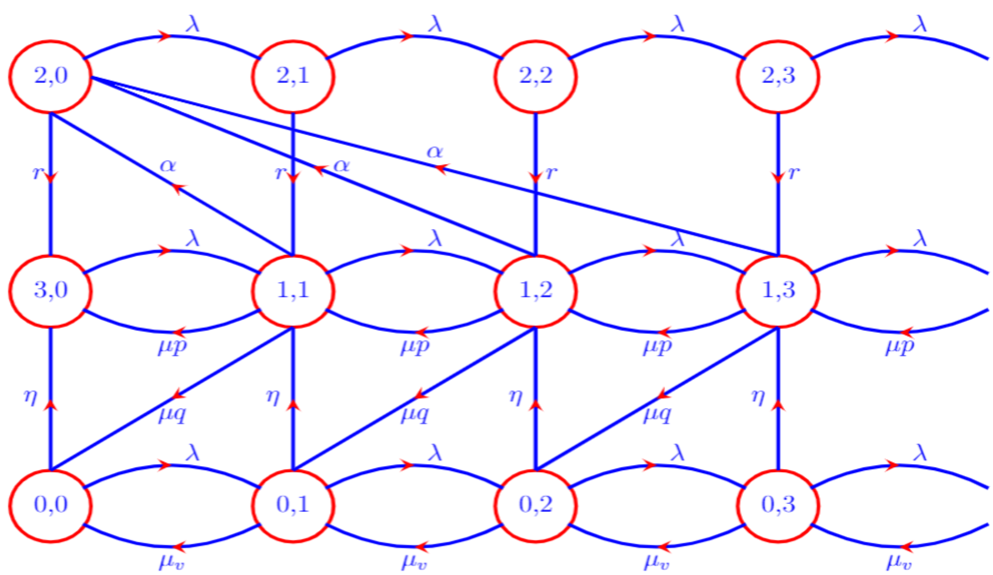

Figure 2.1: State transition diagram of a Single Server Queue with Disasters and Repair under Bernoulli Working Vacation Schedule

3. QUEUE SIZE DISTRIBUTION UNDER STEADY STATE

Let $P_{0,n}(n = 0,1,2, \dots)$, $P_{1,n}(n = 1,2, \dots)$, $P_{2,n}(n = 0,1,2, \dots)$ and $P_{3,0}$ denote the steady state probabilities and satisfy the following equations.

$$(\lambda + \eta)\mathrm{P}_{0,0} = \mu q \mathrm{P}_{1,1} + \mu_v \mathrm{P}_{0,1}, \tag{1}$$

$$(\lambda + \eta + \mu_v)\mathrm{P}_{0,n} = \mu q \mathrm{P}_{1,n+1} + \lambda \mathrm{P}_{0,n-1} + \mu_v \mathrm{P}_{0,n+1}, n = 1,2, \dots, \tag{2}$$

$$(\lambda + \mu + \alpha)\mathrm{P}_{1,1} = \lambda \mathrm{P}_{3,0} + \mu p \mathrm{P}_{1,2} + \eta \mathrm{P}_{0,1} + r \mathrm{P}_{2,1}, \tag{3}$$

$$(\lambda + \mu + \alpha)\mathrm{P}_{1,n} = \lambda \mathrm{P}_{1,n-1} + \mu p \mathrm{P}_{1,n+1} + r \mathrm{P}_{2,n} + \eta \mathrm{P}_{0,n}, n = 2,3, \dots, \tag{4}$$

$$\lambda \mathrm{P}_{3,0} = \mu p \mathrm{P}_{1,1} + \eta \mathrm{P}_{0,0} + r \mathrm{P}_{2,0}, \tag{5}$$

$$(\lambda + r)\mathrm{P}_{2,0} = \alpha \sum_{n=1}^{\infty} \mathrm{P}_{1,n}, \tag{6}$$

and

$$(\lambda + r)\mathrm{P}_{2,n} = \lambda \mathrm{P}_{2,n-1}, n = 1,2, \dots . \tag{7}$$

Normality condition is

$$\mathrm{P}_{3,0} + \sum_{n=0}^{\infty} \mathrm{P}_{0,n} + \sum_{n=1}^{\infty} \mathrm{P}_{1,n} + \sum_{n=0}^{\infty} \mathrm{P}_{2,n} = 1. \tag{8}$$

Let

$$H_0(z) = \sum_{n=0}^{\infty} \mathrm{P}_{0,n} z^n,$$
$$H_1(z) = \sum_{n=1}^{\infty} \mathrm{P}_{1,n} z^n,$$

and

$$H_2(z) = \sum_{n=0}^{\infty} \mathrm{P}_{2,n} z^n.$$

Therefore, the normality condition become,

$$\mathrm{P}_{3,0} + H_0(1) + H_1(1) + H_2(1) = 1. \tag{9}$$

By multiplying the equation (1) and (2) by 1 and z^n respectively and summing up over n yields,

$$H_0(z) = \frac{\mu q H_1(z) - \mu_v \mathrm{P}_{0,0}(1 - \mathrm{z})}{(\eta + \mu_v)z - \mu_v + \lambda z(1 - z)}. \tag{10}$$

By multiplying the equation (6) and (7) by 1, z^nrespectively and summing up over n yields,

$$H_2(z) = \frac{\alpha H_1(1)}{\lambda(1-z)+r} \ . \tag{11}$$

By multiplying the equation (3) and (4) by z, z^n respectively and summing over n yields

$$H_1(z) = \frac{\eta z H_0(z) + rzH_2(z) - \lambda z(1-z)\mathrm{P}_{3,0}}{\lambda z(1-z) + \alpha z + \mu z - \mu p}. \tag{12}$$

After substituting $H_0(z)$and $H_2(z)$,the equation (12) becomes,

$$H_1(z) = \frac{\eta z\left(\frac{\mu q H_1(z) - \mu_v \mathrm{P}_{0,0}(1-z)}{g(z)}\right) + rzH_2(z) - \lambda z(1-z)\mathrm{P}_{3,0}}{\mathrm{f(z)}}, \tag{13}$$

where

$$f(z) = \lambda z(1-z) + \alpha z + \mu(z-p),$$

and

$$\mathrm{g(z)} = (\eta + \mu_v)z - \mu_v + \lambda z(1-z).$$

By rewriting the equation (13) we get,

$$H_1(z) = \frac{1}{F(z)}\left\{\frac{-\eta z\mu_v \mathrm{P}_{0,0}(1-\mathrm{z})}{g(z)} + rz\frac{\alpha H_1(1)}{\lambda(1-z)+r} - \lambda z(1-z)\mathrm{P}_{3,0}\right\}, \tag{14}$$

where

$$F(z) = f(z) - \frac{\eta z\mu q}{g(z)} \ .$$

Theorem 1

The relationship connecting $\mathrm{P}_{0,0}$, $H_1(1)$and $\mathrm{P}_{3,0}$ is

$$\frac{-\eta\mu_v \mathrm{P}_{0,0}(1-\mathrm{z}^*)}{\mathrm{g}(\mathrm{z}^*)} + \frac{r\alpha H_1(1)}{r+\lambda(1-z^*)} = \lambda(1-z^*)\mathrm{P}_{3,0} \tag{15}$$

where z* is the unique root of

$$F(z) = f(z) - \frac{\eta z\mu q}{g(z)} \tag{16}$$

in $|z| < 1$.

Proof

By substituting $z = 0$ and $z = 1$ in the equation (16) we get,

$$F(0) = -\mu p < 0,$$

$$F(1) = \alpha > 0.$$

Also

$$F'(z) = \alpha + \lambda - 2\lambda z + \mu - \frac{\eta\mu q}{[(\eta + \mu_v)z - \mu_v + \lambda z(1-z)]} + \frac{\eta z\mu q(\eta + \mu + \lambda - 2\lambda z)}{[(\eta + \mu_v)z - \mu_v + \lambda z(1-z)]^2},$$

and

$$F''(z) = -2\lambda + \frac{2\mu q\eta(\eta + \mu_v + \lambda - 3\lambda z)}{[(\eta + \mu_v)z - \mu_v + \lambda z(1-z)]^2} + \frac{2\eta z\mu q(\eta + \mu + \lambda - 2\lambda z)^2}{[(\eta + \mu_v)z - \mu_v + \lambda z(1-z)]^3} < 0.$$

Therefore $F'(z)$ is a monotonous decreasing function in $|z| < 1$.

Now

$$F'(0) = \alpha + \lambda + \mu + \frac{\eta\mu q}{\mu_v} > 0.$$

$$F'(1)\eta = (\alpha + \mu)\eta - \lambda(\eta + \mu q) + \mu q\mu_v.$$

When $\lambda \leq \frac{(\lambda+\mu)\eta+\mu q\mu_v}{\eta+\mu q}$, $F'(1) \geq 0$, by $F'(0) > 0$ and $F''(z) < 0$, we have that $F(z)$ is a monotonous increasing function in $|z| < 1$. From $F(0) < 0, F(1) > 0$, we obtain that F(z) has a unique root in $|z| < 1$.

When $\lambda > \frac{(\lambda+\mu)\eta+\mu q\mu_v}{\eta+\mu q}, F'(1) \geq 0$, by $F'(0) > 0$ and $F''(z) < 0,$ we note that there exists $k \in (0,1)$ so that $F'(k) = 0$, then we have that $F(z)$ is a monotonous increasing function in $(0, k)$ and a monotonous decreasing function in $(k, 1)$. From $F(0) < 0$ and $F(1) > 1$ we have that $F(z)$ has a unique root in $|z| < 1$.

In summary, $F(z)$ has a unique root in $|z| < 1$. Due to the occurrence of the disaster, the system in consideration is always stable. Therefore, the power series $H_1(z)$ in (14) is converges in the unit circle |z| < 1, i.e., $H_1(z)$ must be finite for all |z| < 1. Let $z = z*$ be the unique root of $F(z)$. In the equation (14), as the denominator vanishes as $z \to z*$, the numerator must vanish for the root as well. Then, substituting the equation (11) into the numerator, (15) follows.

Theorem 2

The relationship connecting $H_1(z)$ and $P_{0,0}$ is

$$\mu q H_1(z^{**}) = \mu_v(1 - z^{**})P_{0,0} \tag{17}$$

where z^{**} is the unique root of

$$g(z)=(\eta+\mu_v)z-\mu_v+\lambda z(1-z)$$

in$|z|<1$.

Proof

Similar to the proof of Theorem 1.

Evaluation of $\mathbf{P_{0,0}}$

From normality condition,

$$\mathrm{P}_{3,0}+H_0(1)+H_1(1)+H_2(1)=1.$$

Using the equations (15), (10), (11) the above equation can be written as,

$$\frac{-\eta\mu_v \mathrm{P}_{0,0}}{\lambda g(z^*)}+H_1(1)\left[\frac{r\alpha}{\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)}+\frac{\mu q}{\eta}+1+\frac{\alpha}{r}\right]=1.$$

On further simplification we get,

$$\mathrm{P}_{0,0}=\frac{(H_1(1)f_1-1)\lambda g(z^*)}{\eta\mu_v} \tag{18}$$

where

$$f_1=\frac{r^2\alpha\eta+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)}{\lambda\eta r(1-z^*)\big(r+\lambda(1-z^*)\big)}.$$

Evaluation of $\boldsymbol{H_1(1)}$:

Replacing the equation (18) in normality condition we get,

$$H_1(1)=\frac{\lambda\eta r(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}{r^2\eta\lambda\alpha g(z^*)+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}. \tag{19}$$

Evaluation of $\boldsymbol{H_2(z)}$:

Substituting the equation (19) in the equation (11) we get,

$$H_2(z)$$
$$=\frac{\alpha\eta r\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)\big(\lambda g(z^*)+\eta\mu_v\big)}{(\lambda(1-z)+r)\{r^2\eta\lambda\alpha g(z^*)+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)\big(\lambda g(z^*)+\eta\mu_v\big)\}} \tag{20}$$

Evaluation of $\boldsymbol{P_{3,0}}$

Substituting the equation (17) in the equation (15) we get,

$$\mathrm{P}_{3,0}=\frac{\eta r^{2}\alpha\{\eta\mu_{v}+\lambda g(z^{*})-\eta^{2}\mu_{v}\}}{r^{2}\eta\lambda\alpha g(z^{*})+(\mu qr+\alpha\eta+\eta r)\lambda(1-z^{*})(r+\lambda(1-z^{*}))(\lambda g(z^{*})+\eta\mu_{v})}. \qquad (21)$$

Evaluation of $\boldsymbol{H_1(z)}$

Substituting the equation (18), (19) in the equation (14) we get,

$$H_1(z)=\frac{1}{F(z)}\left\{\frac{(1-H_1(1)f_1)\lambda g(z^{*})(1-z)}{g(z)}+\frac{rz\alpha H_1(1)}{\lambda(1-z)+r}\right.$$

$$\left.-\lambda z(1-z)\left(\eta-\eta H_1(1)f_1+\frac{r\alpha H_1(1)}{\lambda(1-z^{*})(r+\lambda(1-z^{*}))}\right)\right\}$$

$$H_1(z)=f_2\left\{\frac{\lambda\eta r(1-z^{*})\big(r+\lambda(1-z^{*})\big)(\lambda g(z^{*})+\eta\mu_{v})}{r^{2}\eta\lambda\alpha g(z^{*})+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^{*})\big(r+\lambda(1-z^{*})\big)(\lambda g(z^{*})+\eta\mu_{v})}\right\}+f_3 \qquad (22)$$

where

$$f_2=\frac{1}{F(z)}\left\{\frac{rz\alpha}{r+\lambda(1-z)}+\lambda\eta z(1-z)f_1-\frac{z(1-z)r\alpha}{(1-z^{*})}-\frac{zf_1\lambda g(z^{*})(1-z)}{g(z)}\right\},$$

and

$$f_3=\frac{\lambda(1-z)}{g(z)}\{zg(z^{*})-\eta z\, g(z)\}.$$

Evaluation of $\boldsymbol{H_0(z)}$

Similarly substituting the equation (18) and (22) in the equation (10) we get

$$H_0(z)=\frac{1}{g(z)}\left\{\frac{\lambda\eta r(1-z^{*})\big(r+\lambda(1-z^{*})\big)(\lambda g(z^{*})+\eta\mu_{v})}{r^{2}\eta\lambda\alpha g(z^{*})+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^{*})\big(r+\lambda(1-z^{*})\big)(\lambda g(z^{*})+\eta\mu_{v})}f_4+f_5\right\}, \qquad (23)$$

where

$$f_4=\frac{1}{\eta}(\eta\mu qf_2-\lambda(1-z)g(z^{*})f_1),$$

and $$f_5=\frac{1}{\eta}\big(\eta\mu qf_3+\lambda(1-z)g(z^{*})\big).$$

Theorem 3

The probability generating function of the number of customers in the system is

$$H(z)=P_{3,0}+H_0(z)+H_1(z)+H_2(z)$$

where $H_2(z), P_{3,0}, H_1(z)$ and $H_0(z)$ are given by the equations (20), (21), (22) and (23) respectively.

4. Performance Measures

4.1 The probability of the server in different states

Let the probability of the server in the busy state, repair state and vacation state be P_b, P_r and P_v respectively. Then

$$P_b = H_1(1) = \frac{\lambda\eta r(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}{r^2\eta\lambda\alpha g(z^*)+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}$$

$$P_r = H_2(1) = \frac{\alpha}{r}H_1(1)$$

$$P_v = H_0(1) = \frac{\mu q}{r}H_1(1)$$

and

$$P_{3,0} = Probability\ that\ the\ server\ is\ idle$$

$$= \frac{\eta r^2\alpha\{\eta\mu_v+\lambda g(z^*)-\eta^2\mu_v\}}{r^2\eta\lambda\alpha g(z^*)+(\mu qr+\alpha\eta+\eta r)\lambda(1-z^*)(r+\lambda(1-z^*))(\lambda g(z^*)+\eta\mu_v)}.$$

4.2 The mean number of customers in the system

$$E(number\ of\ customers\ in\ the\ system) = H_0{}'(1)+H_1{}'(1)+H_2{}'(1).$$

4.3 Rate Arguments

Disaster Rate:

Disaster is allowed to take place during the busy state. Therefore, the rate at which disaster occurs is given by

$$\text{Disaster Rate} = \alpha H_1(1)$$

$$= \frac{\alpha\lambda\eta r(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}{r^2\eta\lambda\alpha g(z^*)+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}$$

Busy Period Termination Rate:

Next, we determine the rate at which busy terminates. The busy period termination rate is given by

$$Busy\ period\ termination\ rate = \mu\,\mathrm{P}_{1,1} = \frac{1}{p}(\lambda \mathrm{P}_{3,0}-\eta \mathrm{P}_{0,0}-r\mathrm{P}_{2,0})$$

where

$$\mathrm{P}_{2,0}=H_2(0)=\frac{\alpha\eta r\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)(\lambda g(z^*)+\eta\mu_v)}{(\lambda+r)\{r^2\eta\lambda\alpha g(z^*)+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)\big(\lambda g(z^*)+\eta\mu_v\big)\}},$$

and $\mathrm{P}_{0,0,}\mathrm{P}_{3,0}$ are given by the equation (18), (21) respectively.

Rate of initiations of busy period

$$Rate\ of\ initiations\ of\ busy\ period = Busy\ period\ termination\ rate + \text{Disaster Rate}.$$

Special case:

As $\mu_v=0,$ the above equations (20), (21), (22) and (23) becomes

$$H_2(z)=\frac{\alpha\eta r\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)}{(\lambda(1-z)+r)\{r^2\eta\alpha+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big)\}},$$

$$\mathrm{P}_{3,0}=\frac{\eta r^2\alpha}{r^2\eta\alpha+(\mu qr+\alpha\eta+\eta r)\lambda(1-z^*)(r+\lambda(1-z^*))},$$

$$H_1(z)=\frac{\lambda\eta\alpha r^2 z\big\{(1-z^*)\big(r+\lambda(1-z^*)\big)-\lambda(1-z)(r+\lambda(1-z))\big\}}{f(z)(r+\lambda(1-z))(\alpha\eta r^2+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big))},$$

and

$$H_0(z)=\frac{\lambda\eta\alpha r^2 z\big\{(1-z^*)\big(r+\lambda(1-z^*)\big)-\lambda(1-z)(r+\lambda(1-z))\big\}}{f(z)(\eta+\lambda(1-z))(r+\lambda(1-z))(\alpha\eta r^2+(\mu qr+\eta r+\alpha\eta)\lambda(1-z^*)\big(r+\lambda(1-z^*)\big))}$$

The above values are seen to coincide with the equations obtained by Ye et al [7].

5. CONCLUSION AND FUTURE SCOPE

Single server queue with system disaster and repair under Bernoulli working vacation schedule is considered. Using balance equations the probability generating function of the number of customers in the system and performance measures are derived explicitly. This model can also be extended by allowing disaster to occur in working vacation state.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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