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# ANALYSIS OF TRI-CUM BISERIAL BULK QUEUE MODEL CONNECTED WITH A COMMON SERVER

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**Abstract:** The present paper demonstrates the tri-cum biserial bulk queue model linked with a common server with fixed batch size. The development of the model has been done in the steady-state condition. The arrival and servicing patterns of the customers are postulated to follow the Poisson law. Various queuing model performances have been assessed by using the probability generating function technique and other statistical tools. The broad parametric examination has been documented to show the adequacy of the current arrangement procedure.

**Keywords:** batch size; bulk arrival; mean queue length; moment generating function; Poisson law.

**2010 AMS Subject Classification:** 93A30, 05A15.

## 1. INTRODUCTION

Queuing theory is an assortment of mathematical models of several queuing systems. The formation of a queue is a natural phenomenon. We face this problem in our daily routine life

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everywhere. Several queuing models have been established so far, which enable the individual to take the precise choice in actual time circumstances. These models are viable while managing the practical issues underway businesses, banking parts, and business shopping centers, etc. Many investigations have been accomplished in the past, which dealt with the characteristics of queuing models.

A.K. Erlang developed the concept of queuing theory. Erlang [1] executed this hypothesis to analyze the impact of the fluctuating help request on the use of phones during the discussion. Suzuki [2] explored the queuing framework comprise of two queues in the arrangement. In the investigation, commonly autonomous irregular factors with the particular circulation work have been utilized to exhibit the administration time at all the administration counters. Maggu [3] explored the numerous waiting line parameters of the waiting line model with phase-type service. Sharma and Sharma [4] accomplished a specific capacity queuing model with a time-dependent analysis. They have expected that the bulk appearance rate relies upon the kind of administration accessible in the framework. Krishnamoorthy and Ushakumari [5] calculated several queue characteristics of the Markovian queuing model using Little’s method in the formation of various governing equations.

Singh et al. [6] studied the transient behavior of a queuing model in which servers were arranged in parallel in a bi-series way. Kumar et al. [7] investigated various queuing parameters of a complex queue network in which two subsystems connected in a biserial way further linked with a common server. Chen [8] built up the participation capacities to examine the consistent state conduct of lining frameworks having differing bunch sizes. Creator utilized a nonlinear programming system with the combination of Zadeh's augmentation rule to grow such capacity. Gupta et al. [9] explored a broad examination of a queuing model comprising of multi-server associated in a biserial way. Suhasini et al. [10] created a two-terminal couple queuing model to examine the queuing parameters. Uma and Manoj [11] played out an exhaustive investigation of single server bulk queuing model including three phases of heterogeneous assistance. A bulk waiting line framework with single help has been examined by Thangaraj and Rajendran [12]. Mittal and Gupta [13] developed a biseries bulk queuing model connected with a common server in a steady-state condition. Agrawal and Singh [14-17] performed detailed exploration to calculate the various queuing performance measures of some recently established tri-cum biserial based queuing models. Numerous applications can be observed in which the developed model can be efficiently implemented. For instant, in the gaming-club, three sections $Sr_a$, $Sr_b$, and $Sr_c$ exist. These sections comprise various games activities that can be played in a team only. The minimum players in a team are two and can go up to any higher number. The team enters in any of the sections and can randomly move from one section to another. It is also possible that after entering in only one section, they exit from the section $Sr_d$. Various combinations of the team's movements are possible. These types of gaming-clubs are widespread in metropolitan cities and malls. Therefore, such situations may arise when teams /Customers have to wait for a long time to get availed of the facility. This is really a very complex problem that can be effortlessly managed by the developed model.

## 2. MATHEMATICAL DESCRIPTION OF THE MODEL

In this queuing model, three servers are connected in parallel in tri cum biseries way, which are further linked with a common server in series. The queues associated with the servers $Sr_a$, $Sr_b$, $Sr_c$, and $Sr_d$ are $Q_a$, $Q_b$, $Q_c$, and $Q_d$, respectively. The customers entered the system with mean arrival rates $\lambda_a$, $\lambda_b$, and $\lambda_c$ arrive in batches of fixed sizes $B_a$, $B_b$, and $B_c$ follow the Poisson process and join the queues $Q_a$, $Q_b$, and $Q_c$, respectively. The customers $n_a$ coming at mean arrival rate $\lambda_a$ after completion of service at server $Sr_a$ can use the facility available at server $Sr_b$ or $Sr_c$ (both or either of two) with the probabilities $p_{ab}$ *and* $p_{ac}$ or directly can use the facility available at server $Sr_d$ with the probability $p_{ad}$ to such an extent that $p_{ab} + p_{ac} + p_{ad} = 1$. A similar criterion will apply to those customers who entered in servers $Sr_b$ and $Sr_c$. After availing the service at server $Sr_d$ the customer is permitted to exit the system. The pictorial representation of the considered problem is demonstrated in Figure 1.

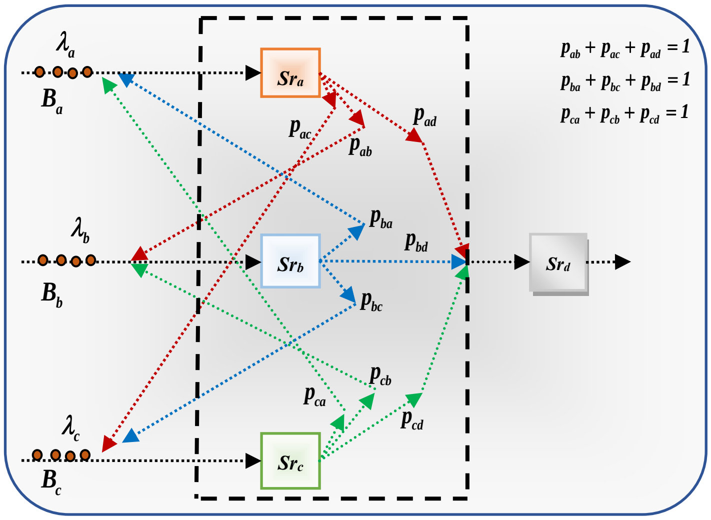


**FIGURE 1.** Queuing network

The following nomenclature has been used in the formulation and analysis of the model.

Probabilities: $p_{ab}$, $p_{ac}$, $p_{ad}$, $p_{ba}$, $p_{bc}$, $p_{bd}$, $p_{ca}$, $p_{cb}$, $p_{cd}$

Mean arrival rate: $\lambda_a$, $\lambda_b$, $\lambda_c$

Mean Servicing rates: $\mu_a$, $\mu_b$, $\mu_c$, $\mu_d$

Batch sizes: $B_a$, $B_b$, $B_c$

Number of Customers: $n_a$, $n_b$, $n_c$, $n_d$

Traffic intensities or utilization of server: $\rho_a$, $\rho_b$, $\rho_c$, $\rho_d$

Mean Queue length (average number of customers): L

Variance (Fluctuations in the queue): $V_{ar}$

Average waiting time for the customer: $E_{wt}$

## 3. SOLUTION METHODOLOGY

The steady-state governing differential-difference equation of the model can be written as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_d)P_{n_a,n_b,n_c,n_d} &= \lambda_a P_{n_a-B_a,n_b,n_c,n_d}+\lambda_b P_{n_a,n_b-B_b,n_c,n_d}+\lambda_c P_{n_a,n_b,n_c-B_c,n_d}\\ &+\mu_a p_{ab}P_{n_a+1,n_b-1,n_c,n_d}+\mu_a p_{ac}P_{n_a+1,n_b,n_c-1,n_d}\\ &+\mu_a p_{ad}P_{n_a+1,n_b,n_c,n_d-1}+\mu_b p_{ba}P_{n_a-1,n_b+1,n_c,n_d}\\ &+\mu_b p_{bc}P_{n_a,n_b+1,n_c-1,n_d}+\mu_b p_{bd}P_{n_a,n_b+1,n_c,n_d-1}\\ &+\mu_c p_{cb}P_{n_a,n_b-1,n_c+1,n_d}+\mu_c p_{ca}P_{n_a-1,n_b,n_c+1,n_d}\\ &+\mu_c p_{cd}P_{n_a,n_b,n_c+1,n_d-1}+\mu_d P_{n_a,n_b,n_c,n_d+1}\end{aligned} \tag{1}$$

To solve the governing equation, Generating function and partial generating functions are assumed as

$$f(z_1,z_2,z_3,z_4)=\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty}\sum_{n_c=0}^{\infty}\sum_{n_d=0}^{\infty} P_{n_a,n_b,n_c,n_d}\; z_1^{n_a} z_2^{n_b} z_3^{n_c} z_4^{n_d} \tag{2}$$

$$\text{such that } |z_1|\le 1,\ |z_2|\le 1,\ |z_3|\le 1,\ |z_4|\le 1$$

$$f_{n_b,n_c,n_d}(z_1)=\sum_{n_a=0}^{\infty} P_{n_a,n_b,n_c,n_d}\, z_1^{n_a}$$

$$f_{n_c,n_d}(z_1,z_2)=\sum_{n_b=0}^{\infty} f_{n_b,n_c,n_d}(z_1).z_2^{n_b}$$

$$f_{n_d}(z_1,z_2,z_3)=\sum_{n_c=0}^{\infty} f_{n_c,n_d}(z_1,z_2).z_3^{n_c}$$

$$f(z_1,z_2,z_3,z_4)=\sum_{n_d=0}^{\infty} f_{n_d}(z_1,z_2,z_3).z_4^{n_d}$$

On solving the governing equation with the help of the P.G.F. (probability generating function) technique, we can find the value of probability distribution function in a steady state.

$$f(z_1,z_2,z_3,z_4)=\frac{\psi}{\xi} \tag{3}$$

where

$$\begin{aligned}\psi &= \mu_a\left\{1-\frac{p_{ab}z_2}{z_1}-\frac{p_{ac}z_3}{z_1}-\frac{p_{ad}z_4}{z_1}\right\}f_a + \mu_b\left\{1-\frac{p_{ba}z_1}{z_2}-\frac{p_{bc}z_3}{z_2}-\frac{p_{bd}z_4}{z_2}\right\}f_b\\ &+\mu_c\left\{1-\frac{p_{ca}z_1}{z_3}-\frac{p_{cb}z_2}{z_3}-\frac{p_{cd}z_4}{z_3}\right\}f_c+\mu_d\left\{1-\frac{1}{z_4}\right\}f_d\end{aligned}$$

and

$$\xi = \lambda_a\left(1-z_1^{B_a}\right)+\lambda_b\left(1-z_2^{B_b}\right)+\lambda_c\left(1-z_3^{B_c}\right) \quad +\mu_a\left\{1-\frac{p_{ab}z_2}{z_1}-\frac{p_{ac}z_3}{z_1}-\frac{p_{ad}z_4}{z_1}\right\}$$
$$+\mu_b\left\{1-\frac{p_{ba}z_1}{z_2}-\frac{p_{bc}z_3}{z_2}-\frac{p_{bd}z_4}{z_2}\right\} \quad +\mu_c\left\{1-\frac{p_{ca}z_1}{z_3}-\frac{p_{cb}z_2}{z_3}-\frac{p_{cd}z_4}{z_3}\right\}+\mu_d\left\{1-\frac{1}{z_4}\right\}$$

Assuming $f\left(z_2,z_3,z_4\right)=f_a$ , $f\left(z_1,z_3,z_4\right)=f_b$, $f\left(z_1,z_2,z_4\right)=f_c$ , $f\left(z_1,z_2,z_3\right)=f_d$

Since $f(1,1,1,1)=1$, the total probability. Considering $z_1=1$ as $z_2\to 1, z_3\to 1, z_4\to 1$ $f(z_1,z_2,z_3,z_4)$ is of (0/0) indeterminate form. Therefore, using L- Hospital rule, we get

$$\mu_a f_a-\mu_b p_{ba} f_b-\mu_c p_{ca} f_c=-\lambda_a B_a+\mu_a-\mu_b p_{ba}-\mu_c p_{ca} \tag{4}$$

Again differentiating numerator and denominator of Eq. (3) separately w.r.t. $z_2$ by taking $z_2=1$ as $z_1\to 1,\ z_3\to 1,\ z_4\to 1$ we get

$$-\mu_a p_{ab} f_a+\mu_b f_b-\mu_c p_{cb} f_c=-\lambda_b B_b-\mu_a p_{ab}+\mu_b-\mu_c p_{cb} \tag{5}$$

Again differentiating numerator and denominator of Eq. (3) separately w.r.t. $z_3$ by taking $z_3=1$ as $z_1\to 1,\ z_2\to 1,\ z_4\to 1$ we get

$$-\mu_a p_{ac} f_a-\mu_b p_{bc} f_b+\mu_c f_c=-\lambda_c B_c-\mu_a p_{ac}-\mu_b p_{bc}+\mu_c \tag{6}$$

Again differentiating numerator and denominator of Eq. (3) separately w.r.t. $z_4$ by taking $z_4=1$ as $z_1\to 1,\ z_2\to 1,\ z_3\to 1$ we get

$$-\mu_a p_{ad} f_a-\mu_b p_{bd} f_b-\mu_c p_{cd} f_c+\mu_d f_d=-\mu_a p_{ad}-\mu_b p_{bd}-\mu_c p_{cd}+\mu_d \tag{7}$$

On solving equations (4)-(7), we get the following values of traffic intensities of servers

$$\rho_a=\left[\frac{\lambda_a B_a\left(1-p_{bc}p_{cb}\right)+\lambda_b B_b\left(p_{ba}+p_{bc}p_{ca}\right)+\lambda_c B_c\left\{p_{ca}\left(1-p_{bc}p_{cb}\right)+p_{cb}\left(p_{ba}+p_{bc}p_{ca}\right)\right\}}{\mu_a\left\{\left(1-p_{ac}p_{ca}\right)\left(1-p_{bc}p_{cb}\right)-\left(p_{ab}+p_{ac}p_{cb}\right)\left(p_{ba}+p_{bc}p_{ca}\right)\right\}}\right]$$

$$\rho_b=\left[\frac{\lambda_a B_a\left\{p_{ab}\left(1-p_{ca}p_{ac}\right)+p_{ac}\left(p_{cb}+p_{ca}p_{ab}\right)\right\}+\lambda_b B_b\left(1-p_{ca}p_{ac}\right)+\lambda_c B_c\left(p_{cb}+p_{ca}p_{ab}\right)}{\mu_b\left\{\left(1-p_{ba}p_{ab}\right)\left(1-p_{ca}p_{ac}\right)-\left(p_{bc}+p_{ba}p_{ac}\right)\left(p_{cb}+p_{ca}p_{ab}\right)\right\}}\right]$$

$$\rho_c=\left[\frac{\lambda_a B_a\left(p_{ac}+p_{ab}p_{bc}\right)+\lambda_c B_c\left(1-p_{ab}p_{ba}\right)\lambda_b B_b\left\{p_{bc}\left(1-p_{ab}p_{ba}\right)+p_{ba}\left(p_{ac}+p_{ab}p_{bc}\right)\right\}}{\mu_c\left\{\left(1-p_{cb}p_{bc}\right)\left(1-p_{ab}p_{ba}\right)-\left(p_{ca}+p_{cb}p_{ba}\right)\left(p_{ac}+p_{ab}p_{bc}\right)\right\}}\right]$$

$$\rho_d = \frac{p_{ad}}{\mu_d}\rho_a\mu_a + \frac{p_{bd}}{\mu_d}\rho_b\mu_b + \frac{p_{cd}}{\mu_d}\rho_c\mu_c$$

where $\rho_a = 1 - f_a,\ \rho_b = 1 - f_b,\ \rho_c = 1 - f_c,\ \rho_d = 1 - f_d$

The solution (Joint Probability) of the model in steady-state is written as

$$P_{n_a,n_b,n_c,n_d} = \rho_a^{n_a}\ \rho_b^{n_b}\ \rho_c^{n_c}\ \rho_d^{n_d}(1-\rho_a)(1-\rho_b)(1-\rho_c)(1-\rho_d)$$

The solution of this model exists if $\rho_a,\ \rho_b,\ \rho_c,\ \rho_d < 1$

## 4. PERFORMANCE MEASURES

**Mean queue length (average number of customers)** $L = \frac{\rho_a}{1-\rho_a} + \frac{\rho_b}{1-\rho_b} + \frac{\rho_c}{1-\rho_c} + \frac{\rho_d}{1-\rho_d}$

**Fluctuation (Variance) in queue length** $V_{ar} = \frac{\rho_a}{(1-\rho_a)^2} + \frac{\rho_b}{(1-\rho_b)^2} + \frac{\rho_c}{(1-\rho_c)^2} + \frac{\rho_d}{(1-\rho_d)^2}$

**Average waiting time for customer** $E_{wt} = \frac{L}{\lambda_{sum}}$ , *where* $\lambda_{sum} = \lambda_a + \lambda_b + \lambda_c$

## 5. VALIDATION STUDY

In this section, we consider some special cases by setting the value of an appropriate parameter to validate our result with existing models.

**Case I:**

If we assume $\lambda_c$=0 and $p_{ac} = p_{ad} = p_{bc} = p_{bd} = p_{ca} = p_{cb} = p_{cd}$ =0 then presently developed model reduced to the model studied by Mohammad et al. [18].

**Case II:**

By setting the values $\lambda_b = \lambda_c$=0, $p_{ac} = p_{ad} = p_{ba} = p_{bc} = p_{bd} = p_{ca} = p_{cb} = p_{cd}$ =0, $B_a = B_b = B_c = 1$ and $p_{ab}$=1 then this model reduces to the model obtained by Jackson [19].

**Case III:**

If we take $B_a = B_b = B_c = 1$ then the present model gives the same results as provided by Agrawal & Singh [14].

**Case IV:**

By considering $\lambda_c = 0$, $B_a = B_b = 1$ and $p_{ac} = p_{ad} = p_{bc} = p_{bd} = p_{ca} = p_{cb} = p_{cd} = 0$, In this case, the outcome of the model resembles with the model given by Maggu [20].

## 6. PARAMETRIC STUDY

The detailed description of the governing equation and solution methodology of the present model has been given in sections II and III. In section IV, various queuing performance measures have been given. In section V, some particular cases have been discussed by setting the value of an appropriate parameter to validate our result with existing models.

The details of various input parameters used to calculate the various queuing performance measures have been presented in Table 1.

**TABLE 1: Details of various input parameters**

| $p_{ab}$ | $p_{ac}$ | $p_{ad}$ | $p_{ba}$ | $p_{bc}$ | $p_{bd}$ | $p_{ca}$ | $p_{cb}$ | $p_{cd}$ |
|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.3 | 0.4 | 0.3 | 0.3 | 0.4 | 0.3 | 0.3 | 0.4 |

Table 2 displays the effect of mean arrival rate $\lambda_a$ and three different combinations of batch sizes of $B_a$, $B_b$ and $B_c$ on mean queue length, variance, and the average waiting time. It is clear from the results that as the mean arrival rate $\lambda_a$ increases from 2 to 4, the mean queue length, variance, and average waiting time also increases. This is on the expected line for the reason that as the number of clients at a specific server increases, the mean queue length, variance, and average waiting time also increase. It has been seen while computing the results that the traffic utilization of the servers are less than 1, which also fulfills the steady-state condition of the model. The same conclusion can be drawn from Tables 3 and 4, which display the effect of mean arrival rates ($\lambda_b$ and $\lambda_c$) and different batch size combinations of $B_a$, $B_b$ and $B_c$ on various queuing performance measures.

Table 5 exhibits the variation of mean queue length, variance, and the average waiting time with mean servicing rate ($\mu_a$) from 26 to 28 and three different combinations of batch sizes of $B_a$, $B_b$ and $B_c$. It is clear from the results that queue length, variance, and average waiting time decrease as the mean servicing rate ($\mu_a$) increases. It is valid for all intents and purposes and numerically likewise because when the servicing rate expands, the clients at different servers will be served quickly as the outcomes the queue length, variances, and waiting time diminishes. The same type of outcome can be seen in Tables 6-8.

**TABLE 2: Queue lengths, Variances and Average waiting time for various mean arrival rates of $\lambda_a$ (taking $\lambda_b = 3$, $\lambda_c = 4$, $\mu_a = 26$, $\mu_b = 27$, $\mu_c = 28$, $\mu_d = 30$)**

| $\lambda_a \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 2 | 6.846 | 18.702 | 0.761 | 8.008 | 24.671 | 0.890 | 9.685 | 35.826 | 1.076 |
| 2.2 | 7.450 | 21.523 | 0.810 | 8.460 | 27.004 | 0.920 | 10.273 | 39.587 | 1.117 |
| 2.4 | 8.139 | 25.012 | 0.866 | 8.951 | 29.658 | 0.952 | 10.918 | 43.929 | 1.162 |
| 2.6 | 8.935 | 29.422 | 0.931 | 9.487 | 32.699 | 0.988 | 11.630 | 48.982 | 1.211 |
| 2.8 | 9.869 | 35.149 | 1.007 | 10.074 | 36.210 | 1.028 | 12.418 | 54.912 | 1.267 |
| 3 | 10.989 | 42.851 | 1.099 | 10.720 | 40.299 | 1.072 | 13.299 | 61.944 | 1.330 |
| 3.2 | 12.368 | 53.697 | 1.213 | 11.437 | 45.107 | 1.121 | 14.289 | 70.376 | 1.401 |
| 3.4 | 14.131 | 69.972 | 1.359 | 12.237 | 50.826 | 1.177 | 15.412 | 80.624 | 1.482 |
| 3.6 | 16.508 | 96.757 | 1.557 | 13.139 | 57.719 | 1.240 | 16.699 | 93.274 | 1.575 |
| 3.8 | 19.997 | 147.689 | 1.852 | 14.163 | 66.157 | 1.311 | 18.192 | 109.181 | 1.684 |
| 4 | 25.942 | 272.366 | 2.358 | 15.342 | 76.677 | 1.395 | 19.950 | 129.636 | 1.814 |

**TABLE 3: Queue lengths, Variances and Average waiting time for various mean arrival rates of $\lambda_b$ (taking $\lambda_a = 2$, $\lambda_c = 4$, $\mu_a = 26$, $\mu_b = 27$, $\mu_c = 28$, $\mu_d = 30$)**

| $\lambda_b \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 2 | 5.301 | 12.481 | 0.663 | 5.267 | 12.314 | 0.658 | 7.336 | 22.627 | 0.917 |
| 2.2 | 5.571 | 13.476 | 0.679 | 5.695 | 13.942 | 0.695 | 7.735 | 24.659 | 0.943 |
| 2.4 | 5.858 | 14.578 | 0.697 | 6.173 | 15.881 | 0.735 | 8.165 | 26.945 | 0.972 |
| 2.6 | 6.165 | 15.802 | 0.717 | 6.708 | 18.219 | 0.780 | 8.630 | 29.529 | 1.004 |
| 2.8 | 6.493 | 17.169 | 0.738 | 7.314 | 21.086 | 0.831 | 9.135 | 32.467 | 1.038 |
| 3 | 6.846 | 18.702 | 0.761 | 8.008 | 24.671 | 0.890 | 9.685 | 35.826 | 1.076 |
| 3.2 | 7.227 | 20.432 | 0.786 | 8.815 | 29.264 | 0.958 | 10.286 | 39.694 | 1.118 |
| 3.4 | 7.639 | 22.395 | 0.813 | 9.769 | 35.329 | 1.039 | 10.948 | 44.181 | 1.165 |
| 3.6 | 8.086 | 24.638 | 0.842 | 10.924 | 43.662 | 1.138 | 11.680 | 49.428 | 1.217 |
| 3.8 | 8.575 | 27.221 | 0.875 | 12.367 | 55.736 | 1.262 | 12.495 | 55.624 | 1.275 |
| 4 | 9.111 | 30.219 | 0.911 | 14.247 | 74.575 | 1.425 | 13.409 | 63.021 | 1.341 |

**TABLE 4: Queue lengths, Variances and Average waiting time for various mean arrival rates of $\lambda_c$ (taking $\lambda_a = 2$, $\lambda_b = 3$, $\mu_a = 26$, $\mu_b = 27$, $\mu_c = 28$, $\mu_d = 30$)**

| $\lambda_c \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 2 | 4.119 | 8.446 | 0.588 | 4.772 | 10.828 | 0.682 | 4.073 | 8.252 | 0.582 |
| 2.2 | 4.324 | 9.077 | 0.600 | 5.009 | 11.655 | 0.696 | 4.393 | 9.260 | 0.610 |
| 2.4 | 4.539 | 9.766 | 0.613 | 5.261 | 12.564 | 0.711 | 4.744 | 10.431 | 0.641 |
| 2.6 | 4.768 | 10.522 | 0.627 | 5.529 | 13.565 | 0.727 | 5.129 | 11.803 | 0.675 |
| 2.8 | 5.010 | 11.353 | 0.642 | 5.813 | 14.672 | 0.745 | 5.556 | 13.429 | 0.712 |
| 3 | 5.267 | 12.271 | 0.658 | 6.117 | 15.899 | 0.765 | 6.033 | 15.379 | 0.754 |
| 3.2 | 5.542 | 13.287 | 0.676 | 6.442 | 17.267 | 0.786 | 6.569 | 17.754 | 0.801 |
| 3.4 | 5.834 | 14.418 | 0.695 | 6.790 | 18.797 | 0.808 | 7.179 | 20.698 | 0.855 |
| 3.6 | 6.147 | 15.681 | 0.715 | 7.165 | 20.517 | 0.833 | 7.881 | 24.427 | 0.916 |
| 3.8 | 6.484 | 17.100 | 0.737 | 7.569 | 22.461 | 0.860 | 8.703 | 29.283 | 0.989 |
| 4 | 6.846 | 18.702 | 0.761 | 8.008 | 24.671 | 0.890 | 9.685 | 35.826 | 1.076 |

**TABLE 5: Queue lengths, Variances and Average waiting time for various mean service rates of $\mu_a$ (taking $\lambda_a = 2$, $\lambda_b = 3$, $\lambda_c = 4$, $\mu_b = 27$, $\mu_c = 28$, $\mu_d = 30$)**

| $\mu_a \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 26 | 6.846 | 18.702 | 0.761 | 8.008 | 24.671 | 0.890 | 9.685 | 35.826 | 1.076 |
| 26.2 | 6.814 | 18.564 | 0.757 | 7.982 | 24.575 | 0.887 | 9.655 | 35.707 | 1.073 |
| 26.4 | 6.782 | 18.432 | 0.754 | 7.958 | 24.483 | 0.884 | 9.627 | 35.593 | 1.070 |
| 26.6 | 6.752 | 18.308 | 0.750 | 7.934 | 24.395 | 0.882 | 9.599 | 35.485 | 1.067 |
| 26.8 | 6.723 | 18.190 | 0.747 | 7.911 | 24.312 | 0.879 | 9.573 | 35.383 | 1.064 |
| 27 | 6.695 | 18.078 | 0.744 | 7.889 | 24.232 | 0.877 | 9.547 | 35.285 | 1.061 |
| 27.2 | 6.668 | 17.971 | 0.741 | 7.867 | 24.156 | 0.874 | 9.523 | 35.192 | 1.058 |
| 27.4 | 6.642 | 17.870 | 0.738 | 7.847 | 24.083 | 0.872 | 9.499 | 35.103 | 1.055 |
| 27.6 | 6.617 | 17.773 | 0.735 | 7.827 | 24.013 | 0.870 | 9.476 | 35.019 | 1.053 |
| 27.8 | 6.593 | 17.681 | 0.733 | 7.807 | 23.947 | 0.867 | 9.454 | 34.938 | 1.050 |
| 28 | 6.569 | 17.593 | 0.730 | 7.788 | 23.883 | 0.865 | 9.433 | 34.861 | 1.048 |

**TABLE 6: Queue lengths, Variances and Average waiting time for various mean service rates of $\mu_b$ (taking $\lambda_a = 2$, $\lambda_b = 3$, $\lambda_c = 4$, $\mu_a = 26$, $\mu_c = 28$, $\mu_d = 30$)**

| $\mu_b \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 26 | 6.998 | 19.327 | 0.778 | 8.351 | 26.776 | 0.928 | 9.890 | 36.808 | 1.099 |
| 26.2 | 6.965 | 19.188 | 0.774 | 8.275 | 26.288 | 0.919 | 9.845 | 36.587 | 1.094 |
| 26.4 | 6.934 | 19.057 | 0.770 | 8.203 | 25.836 | 0.911 | 9.803 | 36.379 | 1.089 |
| 26.6 | 6.903 | 18.933 | 0.767 | 8.134 | 25.419 | 0.904 | 9.762 | 36.184 | 1.085 |
| 26.8 | 6.874 | 18.814 | 0.764 | 8.069 | 25.032 | 0.897 | 9.722 | 36.000 | 1.080 |
| 27 | 6.846 | 18.702 | 0.761 | 8.008 | 24.671 | 0.890 | 9.685 | 35.826 | 1.076 |
| 27.2 | 6.819 | 18.596 | 0.758 | 7.949 | 24.336 | 0.883 | 9.649 | 35.663 | 1.072 |
| 27.4 | 6.793 | 18.494 | 0.755 | 7.893 | 24.023 | 0.877 | 9.614 | 35.508 | 1.068 |
| 27.6 | 6.768 | 18.398 | 0.752 | 7.840 | 23.730 | 0.871 | 9.581 | 35.361 | 1.065 |
| 27.8 | 6.744 | 18.306 | 0.749 | 7.789 | 23.457 | 0.865 | 9.549 | 35.223 | 1.061 |
| 28 | 6.721 | 18.218 | 0.747 | 7.741 | 23.200 | 0.860 | 9.518 | 35.091 | 1.058 |

**TABLE 7: Queue lengths, Variances and Average waiting time for various mean service rates of $\mu_c$ (taking $\lambda_a = 2$, $\lambda_b = 3$, $\lambda_c = 4$, $\mu_a = 26$, $\mu_b = 27$, $\mu_d = 30$)**

| $\mu_c \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 26 | 7.260 | 20.705 | 0.807 | 8.494 | 27.217 | 0.944 | 11.454 | 53.497 | 1.273 |
| 26.2 | 7.210 | 20.444 | 0.801 | 8.434 | 26.879 | 0.937 | 11.203 | 50.605 | 1.245 |
| 26.4 | 7.162 | 20.200 | 0.796 | 8.377 | 26.565 | 0.931 | 10.974 | 48.081 | 1.219 |
| 26.6 | 7.116 | 19.971 | 0.791 | 8.323 | 26.271 | 0.925 | 10.765 | 45.864 | 1.196 |
| 26.8 | 7.073 | 19.756 | 0.786 | 8.272 | 25.997 | 0.919 | 10.572 | 43.907 | 1.175 |
| 27 | 7.031 | 19.554 | 0.781 | 8.223 | 25.740 | 0.914 | 10.395 | 42.169 | 1.155 |
| 27.2 | 6.991 | 19.363 | 0.777 | 8.176 | 25.499 | 0.908 | 10.232 | 40.619 | 1.137 |
| 27.4 | 6.952 | 19.184 | 0.772 | 8.131 | 25.273 | 0.903 | 10.080 | 39.229 | 1.120 |
| 27.6 | 6.916 | 19.014 | 0.768 | 8.088 | 25.061 | 0.899 | 9.939 | 37.979 | 1.104 |
| 27.8 | 6.880 | 18.854 | 0.764 | 8.047 | 24.860 | 0.894 | 9.807 | 36.850 | 1.090 |
| 28 | 6.846 | 18.702 | 0.761 | 8.008 | 24.671 | 0.890 | 9.685 | 35.826 | 1.076 |

**TABLE 8: Queue lengths, Variances and Average waiting time for various mean service rates of $\mu_d$ (taking $\lambda_a = 2$, $\lambda_b = 3$, $\lambda_c = 4$, $\mu_a = 26$, $\mu_b = 27$, $\mu_c = 28$)**

| $\mu_d \downarrow$ | $B_a = 3, B_b = 2, B_c = 2$ | | | $B_a = 2, B_b = 3, B_c = 2$ | | | $B_a = 2, B_b = 2, B_c = 3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ | L | $V_{ar}$ | $E_{wt}$ |
| 26 | 8.180 | 27.147 | 0.909 | 9.874 | 38.734 | 1.097 | 12.435 | 61.264 | 1.382 |
| 26.2 | 8.072 | 26.334 | 0.897 | 9.713 | 37.241 | 1.079 | 12.173 | 58.190 | 1.353 |
| 26.4 | 7.971 | 25.593 | 0.886 | 9.563 | 35.906 | 1.063 | 11.935 | 55.514 | 1.326 |
| 26.6 | 7.877 | 24.915 | 0.875 | 9.424 | 34.706 | 1.047 | 11.717 | 53.170 | 1.302 |
| 26.8 | 7.788 | 24.294 | 0.865 | 9.295 | 33.624 | 1.033 | 11.518 | 51.104 | 1.280 |
| 27 | 7.704 | 23.723 | 0.856 | 9.174 | 32.644 | 1.019 | 11.335 | 49.274 | 1.259 |
| 27.2 | 7.624 | 23.196 | 0.847 | 9.062 | 31.753 | 1.007 | 11.165 | 47.644 | 1.241 |
| 27.4 | 7.549 | 22.710 | 0.839 | 8.956 | 30.942 | 0.995 | 11.009 | 46.186 | 1.223 |
| 27.6 | 7.478 | 22.259 | 0.831 | 8.856 | 30.199 | 0.984 | 10.863 | 44.876 | 1.207 |
| 27.8 | 7.411 | 21.841 | 0.823 | 8.763 | 29.519 | 0.974 | 10.728 | 43.695 | 1.192 |
| 28 | 7.346 | 21.452 | 0.816 | 8.674 | 28.894 | 0.964 | 10.601 | 42.625 | 1.178 |

## 7. Conclusion

In the present study, a complex queuing model has been established with the help of moment generating function and other statistical tools to find the various queuing performance measures such as length of queues, fluctuation in queues, and average waiting time for customers. The legitimacy of the present queuing model is checked by thinking about particular cases. A broad parametric examination has been acquainted with show the suitability of the current solution methodology. This parametric examination can be helpful in different viable applications, for example, shopping complexes, sports centers, businesses, and so forth.

## Conflict of Interests

The authors declare that there is no conflict of interests.

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