MATHEMATICAL MODEL FOR ESTIMATING UNCONFIRMED CASES OF COVID-19 IN ETHIOPIA, AND TARGETING SENSITIVE PARAMETERS

TESFAYE TADESSE EGA*, MOHAMMED YIHA DAWED, BELAY KASSA GEBREMESKEL, TADELE

TESFA TEGEGN

Department of Mathematics, Hawassa University, P.O. Box 05, Hawassa, Ethiopia

Abstract. The novel coronavirus (COVID-19) pandemic was originated from the Wuhan city of China at the end of December, 2019. The virus has spread to 216 countries and territories around the globe. The outbreak of this sever acute respiratory syndrome (Covid-19) killed $757,727$ with, $21,092,096$ cases as of August $14,2020$. In this paper, we propose an SPI_uI_cR deterministic mathematical model which contain protected class to incorporate people who follow the guidelines of WHO to keep themselves and others from Covid-19 infection in Ethiopia. Unconfirmed cases of new incidences were estimated, and sensitive parameters which exasperate the transmission of the virus were identified. According to the sensitivity result, the transmission rate(β_1), the rate of applying protective measures(η), the rate of being reluctant (ω) and the rate of Covid-19 test (ψ) are found to be the most sensitive parameters. For estimated and fitted values of parameters R_e work out to be 1.5, which indicate the disease will be epidemic in the population. We found that the current protective measures are not enough to reduce the transmission of Covid-19 in Ethiopia. The numerical result of our model shows that new incidence of unconfirmed cases are higher than confirmed cases, and this escalate the transmission of the virus in the community. Increasing both the rate of transfer of individuals from susceptible class to protected class and the rate of Covid-19 test play a significant role in capturing unconfirmed cases and reduce the secondary infection R_e less than unity.

Keywords: Covid-19; sensitive parameters; protective measures; effective reproduction number.

2010 AMS Subject Classification: 92B05.

---

*Corresponding author

E-mail address: testify1314@gmail.com

1. INTRODUCTION

New corona virus(COVID-19) is an infectious disease caused by the novel coronavirus(2019-nCoV), which resultes in severe acute respiratory syndrome [1]. The virus was emerged from Wuhan City of China at the end of December 2019, and it has spred rapidly around the globe [2]. In January 30, 2020, WHO has declared a global health emergency, and later in March 11, 2020 COVID-19 was labled as a global pandemic disease[3]. The pandemic has spread throughout the world with less than three months and affected more than 193 countries and territories [4]. Covid-19 becomes more sever in people with low immunity, old age, and medical problems like cardiovascular disease, diabetes, chronic respiratory disease, and cancer [5].

People who are infected often have symptoms of illness after an incubation period of approximately 5.2 days. The common symptoms of Covid-19 are fever, cough, cold, headache,breathing problem and fatigue [6]. The period from the onset of COVID-19 symptoms to death ranges from 6 to 41 days , in average 14 days, but it dependens on the age and status of the immune system [1]. There is a high probability of transmitting the virus without any of the symptoms listed, this situation complicates the process of controling the spread of the virus. The disease mainly spreads through close contact with infected person through respiratory droplets [7]. As the virus is a novel one, there are still unclear matters about the spread and severe causes of illness [8].

While the scientific research is underway for effective vaccine, protective measures are recommended by WHO. Governments and policy makers are taking and designing protective measures in all countries in which Covid-19 has appeared. Among these mandated travel restrictions, lockdowns, canceling or postpone many sports and events, festivals, exhibitions and tournaments, massive closure of schools and universities [9, 10]. Medical professionals are also recommending individuals to take protective measures,which includes frequent hand wash or sanitizing, maintaining social distancing, wearing mask, avoid crowded places, staying home and cleaning exposed surfaces with disinfectants [11].

On 14 February 2020 the first case of Covid-19 was reported from Africa, and it has taken one month to reach Africa after it's emergence in China [12]. The first confirmed case of Covid-19 was reported from Egypt and later from other three African countries: South Africa, Morocco, and Algeria. Most of the identified imported cases have arrived from Europe and the United States rather than China. After some time Covid-19 has spread to other African Countries which have poor healthcare systems [13, 14]. As of Augest 14, 2020 more than 874, 036 cases of COVID-19 have been confirmed and 18, 498 deaths have been reported from different African countries [14]. Due to this pandemic , many Africa nations are currently experiencing partial lockdown while, others are experiencing a total lockdown [15]. Generally the COVID-19 has not only led to increased mortality, but has also affected over all social, political and economic activities of Africa, specially in developing countries like Ethiopia [16].

In Ethiopia the first case of Covid-19 was reported in 13 March ,2020 and the victim was a Japanese citizen who had arrived to Addis Ababa in 4, March from Burkina Faso. Two days later,three additional cases of the virus were reported in relation to close contact with the Japanese's man [17]. Following this, the goverment of Ethiopia tried

to take different protective measures, but new cases of Covid-19 has started to increase from time to time [18]. As of March 23, 2020(after 33 days) there were 94 conformed case with 5 deaths. Two months later on May 23, 2020 (after 70 days) the confirmed cases have increased to 494 with no addtional death [19]. Within 37 days interval ,the confirmed cases of COVID-19 has increased four times compared to the previous 33 days report. From May 18, 2020 to May 23, 2020 around 113 cases were reported .At the time of writing this manuscript, August 14, 2020 the total confirmed cases and deaths have reached to 27, 242 and 492 respectively [20].

People awareness on the transmission of the disease and on how to apply protective measures is very important, as the virus have no any medication and vaccination yet. Currently, it is believed that preventive measures have no any other substitution to fight the spread of the disease [21, 22]. Addressing the people with the right information and assessing the status of the virus in relation to expected measures are very important. Lack of awareness comes due to the absence, inaccessibility or inaccuracy of information. In addition to that cultural taboos, myths and fear, interfere to take preventative measures or visiting medical institutions [22].

Even-though majority of the people of Ethiopia who are living in town seem to be aware of Covid-19, there are still a significant number of people in different parts of the country who have wrong believe about the realness and spread of the virus. While some people believe that the virus stay abroad, others believe taking alcoholic drinks, eating garlic, lemon, and the likes, could cure or prevent the virus [23]. On the other hand, people who were very dedicate to take all preventive measures during the first cases of Covid-19, become relectunt as the transmission increases. This situation has continued still now, on the time of more than 100 confirmed cases are reported per day[18].

Studying epidemics using Mathematical model is useful to control and understand the dynamics of any infectious disease. It is also a significant and powerful tool that can be used in analyzing the spread and control of infectious diseases, such as COVID-19 [24, 25]. There are a number of mathematical models which have been developed recently to study the spread of Covid-19 [26, 27, 28, 29, 30], but the case of people attitude in using protective measures and the severity of infection in unconfirmed cases has not been addressed to the level.

Therefore, the main goal of this paper is to formulate and analyze a mathematical model of COVID-19. By this, we are able to assess the status and effect of people awareness to apply strict protective measures in combating the transmission of the virus. The total population is classified in to five compartments including protected and unprotected groups.

2. MATERIAL AND METHODS

2.1. Model Formulation.

In this section we formulate SPI_uI_cR (Susceptible→ Protected→unconfirmed-Infectious→ confirmed-Infectious → Recovered) model of Covid-19 for the case of Ethiopia. **Susceptible** class includes individuals who have no enough awareness about the virus and those who are reluctant to follow protective measures. Susceptible have no

disease, but they are likely to be infected in case of contact with unconfirmed-Infectious or confirmed-Infectious individuals. The **Protected** class contains those individuals who strictly apply all protective measures according to the the guidelines of WHO. Measures include regularly and thoroughly cleaning hands with an alcohol-based hand rub or wash them with soap and water, maintain social distancing, avoid going to crowded places, avoid touching eyes, nose and mouth, wearing mask and Keep up to date on the latest information from trusted sources. **Unconfirmed-Infectious** class includes those who have already contracted the virus but they have not been confirmed through Covid-19 test. The **confirmed-Infectious** class contains those individuals who have been tested and they are identified as positive for Covid-19 infection. **Recovered** class includes those who recovered from Covid-19 through immunity. Infectious individuals recover from both unconfirmed-Infectious and confirmed-Infectious classes through immunity.

Populations are recruited at a rate of ϕ with a proportion θ and $(1-\theta)$ to the susceptible and Protective classes respectively. The susceptible class is reduced by a rate of η to the Protective group and by the infections λ_1 and λ_2 from both unconfirmed and confirmed infectious groups. When any individual hesitate, forget or become reluctant to take any of the protective measure, will be transferred or return back to the susceptible class at a rate of ω. Unconfirmed-Infectious individuals recover at a rate of δ and confirmed-Infectious recover at a rate of ε. After losing immunity (waning), the unconfirmed-Infectious join susceptible class, while the confirmed-Infectious join the protected class. This is due to awareness in the process of recovery from confirmed cases.

- Our model is formulated based on the following further assumptions.

    i The population is mixed homogeneously.

    ii There is more movement in unconfirmed-Infectious class than confirmed-Infectious class due to quarantine. It is also assumed that more number of asymptomatic individuals are found in unconfirmed-Infectious class.

    iii Individuals who start showing Covid-19 symptoms may have high probability to be confirmed than those individuals who are asymptomatic.

    iv Susceptible individuals who contracted Covid-19 virus becomes infectious right away.

    v Individuals in protected class are not infected by covid-19, since they follow all the rules and guidelines strictly.

2.2. Model Compartment. Using the above assumptions, definition of variables and parameters, the model flow diagram which depicts the dynamics of Covid-19 is shown in Fig.1.

2.3. Model Parameter and Description. The parameters of the model and their description is given in Table 1.

2.4. Model Equation. Based on the assumptions and interrelation between the variables and parameters in Fig.1. Covid-19 transmission dynamics can be described by using system of ordinary differential equations of the
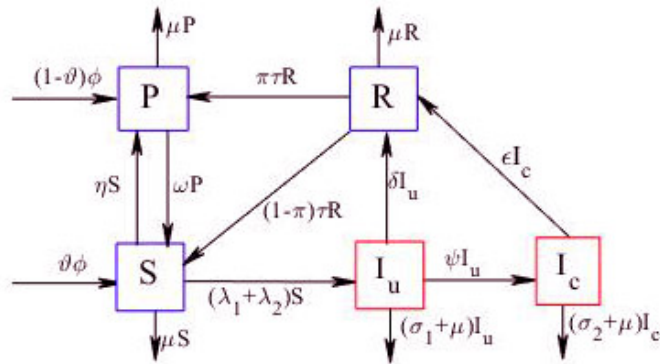
FIGURE 1. Model Compartment

TABLE 1. Parameters of the model and their description

Parameter	Description
ϕ	Rate of new recruitment.
θ	Proportion.
β_1, β_2	Force of infection due to I_u, and I_c respectively.
ω	The rate of being reluctant or careless to use protective measures.
η	The rate of awareness and strictly applying protective measures.
δ	The rate of recovery from unconfirmed infection of Covid-19.
μ	The natural death rate of human population.
ψ	The rate of confirmed cases of Covid-19.
ε	The rate of recovery from confirmed infection of Covid-19.
π	Proportion.
τ	Lose of immunity (Waning).
σ_1	Disease induced death rate of unconfirmed cases.
σ_2	Disease induced death rate of confirmed cases.

form,

(1)
$$\frac{dS}{dt} = \theta\phi + \omega P + (1 - \pi)\tau R - (\eta + \mu + \lambda_1 + \lambda_2)S,$$
$$\frac{dP}{dt} = (1 - \theta)\phi + \eta S + \pi\tau R - (\mu + \omega)P,$$
$$\frac{dI_u}{dt} = (\lambda_1 + \lambda_2)S - (\mu + \delta + \psi + \sigma_1)I_u,$$
$$\frac{dI_c}{dt} = \psi I_u - (\varepsilon + \sigma_2 + \mu)I_c,$$
$$\frac{dR}{dt} = \delta I_u + \varepsilon I_c - (\mu + \tau)R.$$

where $\lambda_1 = \beta_1 \frac{I_u}{N}$ and $\lambda_2 = \beta_2 \frac{I_c}{N}$.

3. BASIC PROPERTIES OF THE MODEL

3.1. Invariant Region.
We assume that all state variables and parameters of the model are non-negative for $\forall t \geqslant 0$, We analyze the model system in a suitable feasible region where all state variables are positive. This region will be obtained under the following theorem;

Theorem:1 All forward solutions in $\mathbb{R}_+^5$ are feasible $\forall t \geqslant 0$ if they enter the invariant region Φ, We have

$$(2) \qquad \Phi = \{(S, P, I_u, I_c, R) \in \mathbb{R}_+^5 : S + P + I_u + I_c + R \leqslant N\}$$

3.2. Positivity of the solution.
The model system (1) to be epidemiological meaningful and well posed, we need to prove that all state variables are non-negative, $\forall t \geqslant 0$.

Theorem:2 Let the initial values of the system $\{S(0) > 0, P(0) > 0, I_u(0) > 0, I_c(0) > 0, R(0) > 0)\} \in \Phi$, Then, the solution set $\{S(t), P(t), I_u(t), I_c(t), R(t)$ of the model system (1) is positive, $\forall t > 0$.

4. MODEL ANALYSIS

In this section we investigate the existence and stability of equilibrium points and reproduction number.

4.1. Disease Free Equilibrium Point.
In the absence of Covid-19 the compartments I_u and I_c become zero, This leads R to becomes automatically zero. Then, the disease free equilibrium point is give by

$$(3) \qquad \varepsilon_0 = (S, P, I_u, I_c, R) = \left(\frac{\phi(\theta\mu + \omega)}{\mu(\mu + \omega + \eta)}, \frac{\phi(\mu(1-\theta) + \eta)}{\mu(\mu + \omega + \eta)}, 0, 0, 0 \right)$$

4.2. Reproduction Number.
Definition 1. The effective reproduction number, R_e, is defined as the measure of average number of infections caused by a single infectious individual introduced in a community in which intervention strategies (in our case is strict protective measures) are administered [31, 32].

We use next generation operator method proposed by [33]. Let $f_i(x)$ be the rate of appearance of new infection in compartment i, $v_i^-(x)$ be the rate of transfer of individuals out of compartment i and $v_i^+(x)$ be the rate of transfer of individuals into compartment i by all other means, and it is assumed that each function is continuously differentiable at least twice in each variable. The disease transmission model of system (1) consists of non-negative initial conditions together with the following system of equations:

$$(4) \qquad \dot{x} = \mathscr{F}_i(x) = f_i(x) - v_i(x), \qquad i = 1, 2.$$

where $v_i = v_i^- - v_i^+$, We consider expressions in which the infection is in progression. These are I_u and I_c, and hence,

(5)
$$\frac{dI_u}{dt} = (\lambda_1 + \lambda_2)S - (\mu + \delta + \psi + \sigma_1)I_u$$
$$\frac{dI_c}{dt} = \psi I_u - (\varepsilon + \sigma_2 + \mu)I_c$$

where $f - v$ is given by

(6)
$$\frac{d}{dt}\begin{bmatrix} I_u \\ I_c \end{bmatrix} = f - v = \begin{bmatrix} \beta_1 \frac{I_u S}{N} + \beta_2 \frac{I_c S}{N} \\ 0 \end{bmatrix} - \begin{bmatrix} (\mu + \delta + \psi + \sigma_1)I_u \\ (\mu + \sigma_2 + \varepsilon)I_c - \psi I_u \end{bmatrix}$$

Here, f and v are the rates of new infections and transfer among compartments respectively. The corresponding Jacobian matrices of the disease-free equilibrium of the above system(6) are as follows;

$$F = \begin{bmatrix} \beta_1 \frac{S}{N} & \beta_2 \frac{S}{N} \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} (\mu + \delta + \psi + \sigma_1) & 0 \\ -\psi & (\mu + \sigma_2 + \varepsilon) \end{bmatrix}$$

Then, the next generation matrix is given by,

$$FV^- = \begin{bmatrix} \frac{K}{\mu + \delta + \psi + \sigma_1}\left(\beta_1 + \frac{\beta_2 \psi}{(\sigma_2 + \mu + \varepsilon)}\right) & \frac{\beta_2 K}{(\mu + \sigma_2 + \varepsilon)} \\ 0 & 0 \end{bmatrix}$$

where $K = \frac{S}{N}$, and the largest eigenvalues of the matrix is given by:

$$\rho(FV^-) = R_e = \frac{K}{(\mu + \delta + \psi + \sigma_1)}\left(\beta_1 + \frac{\beta_2 \psi}{(\sigma_2 + \mu + \varepsilon)}\right)$$

Hence, the effective reproduction number of system (1) is given by,

(7)
$$R_e = \frac{(\omega + \mu\theta)(\psi\beta_2 + \beta_1(\varepsilon + \mu + \sigma_2))}{(\eta + \mu + \omega)(\delta + \mu + \psi + \sigma_1)(\varepsilon + \mu + \sigma_2)}$$

4.3. Stability of the Equilibria.

4.3.1. Local Stability of the Disease Free Equilibrium Point (DFE). Theorem 1: If $R_e < 1$, then (a) the disease-free equilibrium ε_0 of system (1) is locally asymptotically stable; (b) the disease-free equilibrium ε_0 of system (1) is globally asymptotically stable in the region Φ.

Proof (a): The Jacobian Matrix of system 1 is given by

$$J_{\varepsilon_0} = \begin{bmatrix} -(\mu + \eta) & \omega & -\beta_1 K & -\beta_2 K & (1 - \pi)\tau \\ \eta & -(\mu + \omega) & 0 & 0 & \pi\tau \\ 0 & 0 & \beta_1 K - (\mu + \delta + \psi + \sigma_1) & \beta_2 K & 0 \\ 0 & 0 & \psi & -(\mu + \sigma_2 + \varepsilon) & 0 \\ 0 & 0 & \delta & \varepsilon & -(\mu + \tau) \end{bmatrix}$$

When m_i, $i = 1, 2, ..., 5$ represent the eigenvalues of the Jacobian matrix, they are given as,

$$m_1 = -K_5,$$

$$m_2 = -\frac{1}{2}K_4 - \frac{1}{2}K_3 + \frac{1}{2}\beta_1 K + \frac{1}{2}\sqrt{(\beta_1 K - K_3 + K_4)^2 + 4\psi\beta_2 K},$$

(8)

$$m_3 = -\frac{1}{2}K_4 - \frac{1}{2}K_3 + \frac{1}{2}\beta_1 K - \frac{1}{2}\sqrt{(\beta_1 K - K_3 + K_4)^2 + 4\psi\beta_2 K},$$

$$m_4 = -\frac{1}{2}K_2 - \frac{1}{2}K_1 + \frac{1}{2}\sqrt{(K_2 - K_1)^2 + 4\eta\omega},$$

$$m_5 = -\frac{1}{2}K_2 - \frac{1}{2}K_1 - \frac{1}{2}\sqrt{(K_2 - K_1)^2 + 4\eta\omega},$$

Where

$$K_1 = \mu + \eta, \quad K_2 = \mu + \omega, \quad K_3 = \mu + \delta + \psi + \sigma_1, \quad K_4 = \mu + \sigma_2 + \varepsilon, \quad K_5 = \mu + \tau$$

For the system to be stable all eigenvalues must be negative. Since all parameters are positive, obviously m_1 and m_5 are negative. Showing the negativity of m_3 and m_4 is trivial, but for m_2 to be negative the condition $K_3 + K_4 > \beta_1 K + \sqrt{(\beta_1 K - K_3 + K_4)^2 + 4\psi\beta_2 K}$ must be satisfied. Through some computation we arrive at $K\left(\frac{\psi\beta_2 + \beta_1 K_4}{K_3 K_4}\right) < 1$, and substituting the expressions for K_3 and K_4 we get

(9)
$$R_e = \frac{K}{(\mu + \delta + \psi + \sigma_1)}\left(\beta_1 + \frac{\beta_2\psi}{(\sigma_2 + \mu + \varepsilon)}\right) < 1.$$

Therefore, for the DFE to be stable the condition $R_e < 1$, must be satisfied.

4.3.2. *Global Stability of the DFE.* Proof (b): Using the method proposed by [34] system (1) can be written as

(10)
$$\begin{cases} \frac{dM}{dt} = F(M,N) \\ \frac{dN}{dt} = G(M,N), \qquad G(N,0) = 0 \end{cases}$$

Where $M \in \mathbb{R}^3$ denotes the number of uninfected individuals and $N \in \mathbb{R}^2$ denotes the number of infected individuals including latent, infectious, etc. $T_0 = (M^0, 0)$ as a disease free equilibrium of this system.

The conditions (H_1) and (H_2) must be met to guarantee the global asymptotic stability of the disease free state.

$H_1: \frac{dM}{dt} = F(M^0, 0), M^0$ is globally asymptotically stable(g.a.s).

$H_2: G(M,N) = AN - \hat{G}(M,N)$, where $\hat{G}(M,N) \geqslant 0$ for $(M,N) \in \Omega$

and $A = D_N G(M^0, 0)$ is an Metzler matrix and Ω is the region where the model makes biological sense.

From our model we have

$$F(M,N) = \begin{bmatrix} \theta\phi + \omega P + (1 - \pi)\tau R - (\lambda_1 + \lambda_2 + \eta + \mu)S \\ (1 - \theta)\phi + \eta S + \pi\tau R - (\mu + \omega)P \\ \varepsilon I_c + \delta I_u - (\tau + \mu)R \end{bmatrix},$$

$$G(M,N) = \begin{bmatrix} (\lambda_1 + \lambda_2)S - (\mu + \delta + \psi + \sigma_1)I_u \\ \psi I_u - (\varepsilon + \sigma_2 + \mu)I_c \end{bmatrix},$$

Where $M^0 = (S^0, P^0, 0, 0, 0)$ and $\Omega = \mathbb{R}_+^5$ Therefore:

$$\frac{dN}{dt} = F(M,0) = \begin{bmatrix} \theta\phi + \omega P - (\eta + \mu)S \\ (1 - \theta)\phi + \eta S - (\mu + \omega)P \\ 0 \end{bmatrix},$$

Which shows that $M^0 = (S^0, P^0, 0, 0, 0)$ is globally asymptotically stable (g.a.s). By this the condition H_1 is shown.

To show the second condition H_2 consider,

$$\hat{G}(M,N) = \begin{bmatrix} (\beta_1 I_u + \beta_2 I_c)(1 - \frac{S}{N}) \\ 0 \end{bmatrix}, A = \begin{bmatrix} \beta_1 - (\mu + \psi) & \beta_2 \\ \psi & -(\mu + \sigma + \varepsilon) \end{bmatrix}$$ Since $0 \leqslant S \leqslant N$, this indicate $\hat{G}(M,N) \geqslant 0$

Now by considering the right hand side of H_2,

$$AN - \hat{G}(M,N) =$$

$$\begin{bmatrix} \beta_1 - (mu + \delta + \psi + \sigma_1) & \beta_2 \\ \psi & -(\mu + \sigma_2 + \varepsilon) \end{bmatrix} \begin{bmatrix} I_u \\ I_c \end{bmatrix} - \begin{bmatrix} (\beta_1 I_u + \beta_2 I_c)(1 - \frac{S}{N}) \\ 0 \end{bmatrix}$$

$$= G(M,N) = \begin{bmatrix} (\lambda_1 + \lambda_2)S - (\mu + \delta + \sigma_2 + \psi)I_u \\ \psi I_u - (\mu + \sigma_2 + \varepsilon)I_c \end{bmatrix}$$

So the condition H_2 is also satisfied. Thus, $T_0 = (M^0, 0)$ is globally asymptotically stable(g.a.s). Therefore we have proved theorem 1.

4.3.3. *Global Stability of the Endemic Equilibrium Point(EEP).* In this section we investigate the conditions under which the endemic equilibrium points are stable or unstable. We prove whether the solution starting sufficiently close to the equilibrium remains close to the equilibrium and approaches the equilibrium as $t \to \infty$, or if there are solutions starting arbitrary close to the equilibrium which do not approach it respectively. As postulated in the study by Korobeinikov (2004, 2007), we assert that the local stability of the DFE advocates for local stability of the Endemic Equilibrium for the reverse condition. We thus find the global stability of Endemic equilibrium using a Korobeinikov approach as described by [31].

The endemic equilibrium point $E^*(S^*, P^*, I_u^*, I_c^*, R^*)$ is obtained by setting each of the equations of system(1) equal to zero.

We then have system(11) which exists for $R_e > 1$.

$$\theta\phi + \omega P + (1 - \pi)\tau R - (\eta + \mu + \lambda_1 + \lambda_2)S = 0$$

$$(1 - \theta)\phi + \eta S + \pi\tau R - (\mu + \omega)P = 0$$

(11)
$$(\lambda_1 + \lambda_2)S - (\mu + \delta + \psi + \sigma_1)I_u = 0$$

$$\psi I_u - (\varepsilon + \sigma_2 + \mu)I_c = 0$$

$$\varepsilon I_u + \delta I_c - (\mu + \tau)R = 0$$

The existence of *EEP* can be shown using the method described by [35, 36]. The conditions $S \neq 0$ or $P \neq 0$ or $I_u \neq 0$ or $I_c \neq 0$ or $R \neq 0$, that is $S > 0$ or $P > 0$ or $I_u > 0$ or $I_c > 0$ or $R > 0$ must be satisfied. Then adding system(11) we have $\phi - \sigma_1 I_u - \sigma_2 I_c - \mu N$, since $S + P + I_u + I_c + R = N$, It follows that $\phi = \sigma_1 I_u + \sigma_2 I_c + \mu N$; Now, since $\phi > 0, \sigma_1 > 0, \sigma_2 > 0$ we can discern that $\mu N > 0$, $\sigma_1 I_u > 0, \sigma_2 I_c > 0$, which implies that that $S > 0$, $P > 0, I_u > 0, I_c > 0, R > 0$. This prove that the endemic equilibrium point of system(1) exists.

We formulate a suitable Lyapunove function for Covid-19 model as given in the form below,

(12)
$$V = \sum a_i(y_i - y_i^* ln y_i)$$

where a_i is defined as a properly selected positive constant, y_i is defined as the population of the ith compartment, and y_i^* is the equilibrium point. We will then have

$$V = W_1(S - S^* ln S) + W_2(P - P^* ln P)$$

$$+ W_3(I_u - I_u^* ln I_u) + W_4(I_c - I_c^* ln I_c) + W_5(R - R^* ln R)$$

The constants W_i are non negative in Φ such that $W_i > 0$ for $i = 1, 2, 3, ..., 5$. The Lyapunove function V together with it's constants $W_1, W_2, ..., W_5$ are chosen in such a way that V is continuous and differentiable is a space. We then compute the time derivative of V from which we get:

$$\frac{dV}{dt} = W_1\left(1 - \frac{S^*}{S}\right)\frac{dS}{dt} + W_2\left(1 - \frac{P^*}{P}\right)\frac{dP}{dt}$$

$$+ W_3\left(1 - \frac{I_u^*}{I_u}\right)\frac{dI_u}{dt} + W_4\left(1 - \frac{I_c^*}{I_c}\right)\frac{dI_c}{dt} + W_5\left(1 - \frac{R^*}{R}\right)\frac{dR}{dt}$$

Now using the Covid-19 model of system (1) we have,

$$\frac{dV}{dt} = W_1\left(1 - \frac{S^*}{S}\right)[\theta\phi + \omega P + (1 - \pi)\tau R - (\eta + \mu + \lambda_1 + \lambda_2)S]$$

$$+ W_2\left(1 - \frac{P^*}{P}\right)[(1 - \theta)\phi + \eta S + \pi\tau R - (\mu + \omega)P]$$

$$+ W_3\left(1 - \frac{I_u^*}{I_u}\right)[(\lambda_1 + \lambda_2)S - (\mu + \delta + \psi + \sigma_1)I_u]$$

$$+ W_4\left(1 - \frac{I_c^*}{I_c}\right)[\psi I_u - (\varepsilon + \sigma_2 + \mu)I_c]$$

$$+W_5 \left(1 - \frac{R^*}{R}\right)[\varepsilon I_u + \delta I_c - (\mu + \tau)R]$$

Substituting the endemic equilibrium points after simplification we obtain,

$$\frac{dV}{dt} = -W_1 \left(1 - \frac{S^*}{S}\right)^2 - W_2 \left(1 - \frac{P^*}{P}\right)^2$$

$$-W_3 \left(1 - \frac{I_u^*}{I_u}\right)^2 - W_4 \left(1 - \frac{I_c^*}{I_c}\right)^2 - W_5 \left(1 - \frac{R^*}{R}\right)^2 + F(S, P, I_u, I_c, R)$$

where the function $F(S, P, I_u, I_c, R)$ is non positive. Now following the procedure in [35, 36], we have $F(S, P, I_u, I_c, R) \leqslant 0$ for all S, P, I_u, I_c, R, then $\frac{dV}{dt} \leqslant 0$ for all S, P, I_u, I_c, R and it is zero when $S = S^*, P = P^*, I_u = I_u^*, I_c = I_c^*, R = R^*$, Hence the largest compact invariant set in S, P, I_u, I_c, R such that $\frac{dV}{dt} = 0$ is the sigleton E^* which is endemic equilibrium point of the model system (1). Using Lasalles's invariant principle postulated by [37] we assert that E^* is globally asymptotically stable in the interior of the region of S, P, I_u, I_c, R and leads to the following theorem. **Theorem 3** If $R_e > 1$, then the Covid-19 disease model of system (1) has a unique endemic equilibrium point E^*, which is globally asymptotically stable in S, P, I_u, I_c, R.

5. NUMERICAL SIMULATION AND SENSITIVITY ANALYSIS

5.1. Estimation of Parameters. The parameter values in Table 2 are estimated using the data found from Ethiopian Ministry of Health(MoE) from May 1, 2020 to July 31, 2020 [38]. We used the reported new cases of Covid-19(confirmed cases) to fit our model and estimate most of the parameter values. Other parameter values were assumed and referred from literature.

According to [39, 40] the population and life expectancy of Ethiopia were estimated to be 114, 963, 588 and 66.71 respectively in the year 2020. Using these figures we have assumed the recruitment rate to be $\mu \times N_0$.

The time of recovery from Covid-19 ranges between 2 weeks to 6 weeks, but we assume that the recovery rate for the confirmed cases and the unconfirmed case of Covid-19 is different. Confirmed people may recover more faster than the unconfirmed cases due to taking different targeted intakes which can increase the immune system. Further, the confirmed people are staying in a confined place and given more care to create emotional stability. Therefore, we take 2 weeks for the confirmed people to recover while we take 4 weeks in average for the unconfirmed people to recover. Using these assumptions the value of $\delta = 0.015$ day^{-1} and $\varepsilon = 0.1152$day^{-1}.

The values of some parameters were bounded and others were estimated using the Least Square optimization method. Figure 2 shows the reported data and simulation of our model. It can be observed that the incidence of the disease obtained from the model fits the plot of the reported new-case data. This increases the accuracy of our prediction and further investigation. In our model taking strict protective measure were taken as a control strategy. According to this, the effective reproduction number, R_e is found to be 1.5. When $\eta = \omega = 0$ the basic reproduction number is $R_0 = 2.7$.

TABLE 2. Parameter Values and their Description

| Parameter	Description	Value	Source
θ	Proportion	0.6039	Assumed
ϕ	Rate of recruitment	4721.5	Assumed
ω	Reluctant rate	0.0017	Fitted
μ	Natural death rate	4.11×10^{-5}	Assumed
π	Proportion	0.3	Assumed
β_1	Infection rate of Unconfirmed case	0.2584	Fitted
β_2	Infection rate of Confirmed case	0.0108	Fitted
η	Rate of applying protective measure	0.0033	Fitted
τ	Waning	0.0050	[26]
ψ	Rate of confirmed case	0.0216	Fitted
δ	Rate of recovery of Unconfirmed case	0.015	Fitted
σ_1	Death rate of unconfirmed cases	0.0209	Fitted
σ_2	Death rate of confirmed case	0.0062	Fitted
ε	Rate of recovery of Confirmed case	0.1152	Fitted
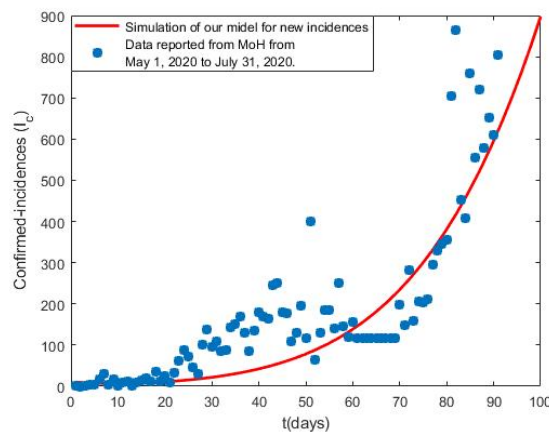
FIGURE 2. Model fit to the data

5.2. Sensitivity Analysis. Sensitivity analysis is commonly used to discover parameters that have a high impact on R_e and should be targeted by intervention strategies. The normalized forward sensitivity index is the ratio of relative change of a variable to the relative change in parameter [25]. If the variable is a differentiable function of the parameter then the sensitivity index is defined as follows:

TABLE 3. Sensitivity Indices of R_e

No	Parameter	Sensitivity Indices
1	β_1	0.9926
2	η	−0.6546
3	ω	0.6484
4	ψ	−0.368
5	σ_1	−0.3632
6	δ	−0.2607
7	θ	0.0144
8	β_2	0.0074
9	ε	−0.007
10	μ	0.0055
11	σ_2	−0.0004

Definition 2: The normalized forward sensitivity index of variable g that depends on parameter b is defined as:

$$\Gamma_b^g = \frac{\partial g}{\partial b} \times \frac{b}{g}$$

By using the same procedure, the sensitivity indices of R_e are computed with respect to all parameters embedded to R_e.

As can be seen from the sensitivity index β_1, η, ω, ψ, σ_1 and δ are the most sensitive parameters of R_e respectively. Since η has inverse co-relation with R_e increasing the rate of applying strict protective measures will reduce R_e very fast. Very small change of η from 0.0033 to 0.0053 reduces R_e to 0.7 which is less than unity. On the other hand reducing reluctant rate ω is another important measure, that can be taken to reduce the effective reproduction number. Contact rate β_1 can be reduce using social distancing and wearing mask. Increasing Covid-19 test is also another important parameter to be increased so that R_e should be reduced. The parameter ψ is directly proportional to R_e, that means increasing covid-19 test is very important in capturing the unconfirmed cases and reducing the effective reproduction number.

5.3. Numerical Simulation. We used ode Matlab solver to show the numerical result of the model. The effect of sensitive parameters are observed in the numerical result of the model. The confirmed case incidence was extended to estimate the unconfirmed cases of Covid-19 in Ethiopia. For the numerical simulation the initial conditions $S(0) = 7 \times 10^7, P(0) = 4 \times 10^7, I_u(0) = 26, I_c(0) = 2, R(0) = 1$, were used.

As can be seen from Figure 3 the unconfirmed cases of Covid-19 in Ethiopia is very much higher than confirmed cases. This is due to low testing rate in comparison with the number of exposed people. Figure 4 shows
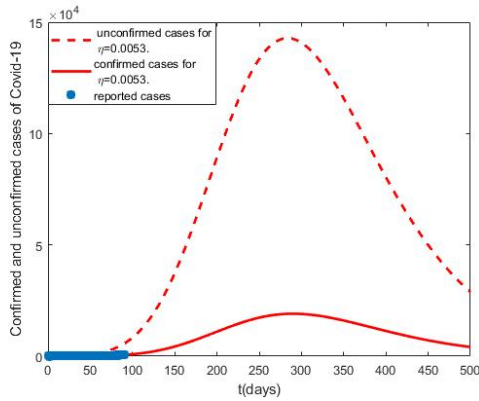
FIGURE 3. Confirmed and Unconfirmed cases of Covid-19 with $\eta = 0.0033$
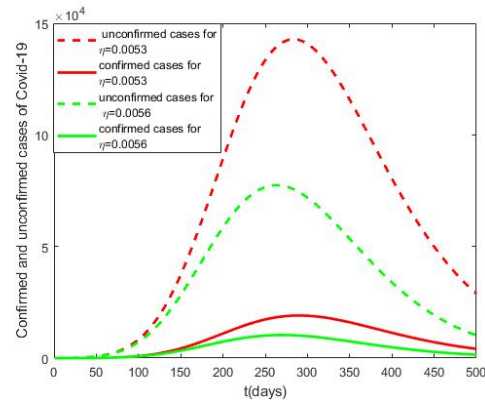

FIGURE 4. Comparison between Confirmed and Unconfirmed cases of Covid-19 with $\eta = 0.0053$ and $\eta = 0.0056$

the sensitivity of η. Increasing the value of η from 0.0053 to 0.0056 significantly reduce the number of new incidences. Increasing the rate of Covid-19 test gives more opportunity to capture unconfirmed cases. Figure 5 shows the combined intervention, that is raising the value of $\eta = 0.0053$ to $\eta = 0.0056$ and ψ(confirmation rate) from 0.0216 to 0.027 gives better control strategy to reduce the transmission of the virus from invading the population. The comparison between combined intervention and new incidences with the current situation is shown in figure 6. Based on the current rate of Covid-19 test, there will be time that the daily new case will reach to more than 19,000 confirmed cases and 145,000 unconfirmed cases in Ethiopia. The transmission of the virus will peak at the end of January, 2021 with more than 164,000 daily new cases.

6. DISCUSSION AND CONCLUSION

Covid-19 has become a global public health burden since December 2020. After the first case of the virus reported in Ethiopia on March, 2020 the incidence has been increasing from time to time. In the first days majority of the reported cases were from Addis Ababa and surrounding areas. As time goes (in two months time) the virus has spread in many parts of the country. Especially, beginning from the mid of May, 2020 the number of average confirmed cases have increased rapidly. We believe that unless serious control measures are not applied, the pandemic seem to invade the population in a short period of time.

We found that the current control measures which are taken by the concerned bodies to control the spread of the virus, is not enough. This can be seen from the result of R_e which is 1.5. We found increasing protective measures (η) using the guidelines provided by WHO is one of the important measure which should be taken by the people and enforced by the government. On the other hand, reducing the flow of people from protected class to susceptible class (ω) should be reduced and increase the reverse movement (η). We believe that medias play
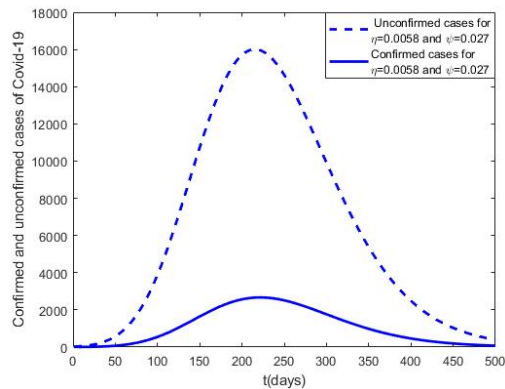
FIGURE 5. Confirmed and Unconfirmed cases of Covid-19 with $\eta = 0.0058$ and $\psi = 0.027$.
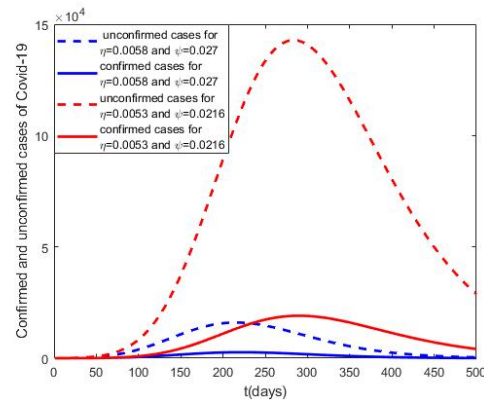

FIGURE 6. Comparison between Confirmed and Unconfirmed cases of Covid-19 with different values of η and ψ

an important role to increase the culture of peoples attitude towards applying control measures. The severity of the disease and wrong perception about its transmission must be described from top to bottom. One can observe that individuals are highly dependent on the the daily information they are getting from medias to apply protective measures. As η and ψ are sensitive parameters and inversely proportional to R_e, increasing both the protective measures and Covid-19 test is the best measures that can be taken to fight the spread of the virus.

We believe that protective measures must be applied continuously and strictly. Inconsistent attitude towards applying protective measures will cost the life of individuals and the people around them. Covid-19 virus is highly transmitted virus, therefore, all individuals are responsible to one another. Authorities and concerned bodies should focus on instigating people to use their maximum effort to increase their interest of using strict protective measures. Legal measures should be taken on people who involve themselves and others in dangerous activities.

From the numerical solution of our model the unconfirmed cases of Covid-19 in Ethiopia is more than the confirmed cases. In six months time, while the daily confirmed cases reaches more than $19,000$, the unconfirmed cases will reach more than $145,000$. That means we will be having a total of more than $164,000$ daily cases at around the peak. Increasing the other sensitive parameter ψ helps to capture unconfirmed cases, and hence reduce the spread of the virus rapidly. The combination of intervention, that is increasing η and ψ will eradicate the spread of the virus very fast by reducing R_e less than unity.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] H.A. Rothan, S.N. Byrareddy, The epidemiology and pathogenesis of coronavirus disease (COVID-19) outbreak, J. Autoimmunity. 109 (2020), 102433.

[2] R. Singh, R. Adhikari, Age-structured impact of social distancing on the COVID-19 epidemic in India, ArXiv:2003.12055 [Cond-Mat, q-Bio]. (2020).

[3] J.A. Siordia, Epidemiology and clinical features of COVID-19: A review of current literature, J. Clinic. Virol. 127 (2020), 104357.

[4] A. Radulescu, K. Cavanagh, Management strategies in a SEIR model of COVID 19 community spread, ArXiv:2003.11150 [Physics, q-Bio]. (2020).

[5] M.K. Arti. Modeling and Predictions for COVID 19 Spread in India. ResearchGate. (2020). https://doi.org/10.13140/RG.2.2.11427.81444.

[6] S.A. Lauer, K.H. Grantz, Q. Bi, F.K. Jones, Q. Zheng, H.R. Meredith, A.S. Azman, N.G. Reich, J. Lessler, The Incubation Period of Coronavirus Disease 2019 (COVID-19) From Publicly Reported Confirmed Cases: Estimation and Application, Ann. Intern. Med. 172 (2020), 577–582.

[7] World Health Organization. Modes of transmission of virus causing COVID-19: implications for IPC precaution recommendations: scientific brief, 27 March 2020 (No. $WHO/2019 - nCoV/Sci_Brief/Transmission_modes/2020.1$). (2020).

[8] Y. Yang, F. Peng, R. Wang, K. Guan, T. Jiang, G. Xu, J. Sun, C. Chang, The deadly coronaviruses: The 2003 SARS pandemic and the 2020 novel coronavirus epidemic in China, J. Autoimmunity. 109 (2020), 102434.

[9] J.A.L. Ludvigsen, J.W. Hayton, Toward COVID-19 secure events: considerations for organizing the safe resumption of major sporting events, Manag. Sport Leis. (2020) 1–11. https://doi.org/10.1080/23750472.2020.1782252.

[10] P. Khrapov, A. Loginova, Mathematical modelling of the dynamics of the Coronavirus COVID-19 epidemic development in China. Int. J. Open Inform. Technol. 8 (4) (2020), 13-16.

[11] S.K. Chaturvedi, Covid-19, Coronavirus and Mental Health Rehabilitation at Times of Crisis, J. Psychosoc. Rehabil. Ment. Health. 7 (2020) 1–2.

[12] M. Gilbert, G. Pullano, F. Pinotti, et al. Preparedness and vulnerability of African countries against importations of COVID-19: a modelling study, Lancet. 395 (2020), 871–877.

[13] P. Ozili, COVID-19 in Africa: socio-economic impact, policy response and opportunities, Int. J. Sociol. Soc. Policy. (2020). https://doi.org/10.1108/IJSSP-05-2020-0171.

[14] K. Zandvoort, C. Jarvis, C. Pearson, et. al. Response strategies for COVID-19 epidemics in African settings: a mathematical modelling study. medRxiv 2020.04.27.20081711. doi: https://doi.org/10.1101/2020.04.27.20081711.

[15] O.M. Ibrahim, D.D. Ekundayo, COVID-19 Pandemic in Nigeria: Misconception among Individuals, Impact on Animal and the Role of Mathematical Epidemiologists. Preprints 2020, 2020040492. doi: 10.20944/preprints202004.0492.v1.

[16] N. Davies, S. Sweeney, S. Torres-Rueda, et al. The impact of Coronavirus disease 2019 (COVID-19) on health systems and household resources in Africa and South Asia. medRxiv 2020.05.06.20092734; doi: https://doi.org/10.1101/2020.05.06.20092734.

[17] https://www.afro.who.int/news/first-case-covid-19-confirmed-ethiopia: Retrieved on April 13, 2020.

[18] K. Baye, COVID-19 prevention measures in Ethiopia: Current realities and prospects. ESSP Working Paper 141. International Food Policy Research Institute (IFPRI), Washington, 2020.

[19] G.T. Abate, A. Brauw, K. Hirvonen, Food and nutrition security in Addis Ababa, Ethiopia during COVID-19 pandemic: June 2020 report. ESSP Working Paper 145. International Food Policy Research Institute (IFPRI). Washington, 2020.

[20] https://www.worldometers.info/coronavirus/country/ethiopia/: Retrieved on July 31, 2020.

[21] Z. Shigute, A.D. Mebratie, G. Alemu, A. Bedi, Containing the spread of COVID-19 in Ethiopia, J. Glob. Health. 10 (2020), 010369.

[22] World Health Organization. Rational use of personal protective equipment (PPE) for coronavirus disease (COVID-19): interim guidance, 19 March 2020 (No. $WHO/2019 - nCoV/IPCPPE_{u}se/2020.2$). (2020).

[23] D. Bekele, T. Tolossa, R. Tsegaye, W. Teshome. The knowledge and practice towards COVID-19 pandemic prevention among residents of Ethiopia: An Online Cross-Sectional Study. bioRxiv 2020.06.01.127381; doi: https://doi.org/10.1101/2020.06.01.127381.

[24] N.C. Grassly, C. Fraser, Mathematical models of infectious disease transmission. Nat. Rev. Microbiol. 6 (6) (2008), 477-487.

[25] N.B. Dimitrov, L.A. Meyers, Mathematical Approaches to Infectious Disease Prediction and Control, in: J.J. Hasenbein, P. Gray, H.J. Greenberg (Eds.), Risk and Optimization in an Uncertain World, INFORMS, 2010: pp. 1–25. https://doi.org/10.1287/educ.1100.0075.

[26] S.M. Kassa, J.B.H. Njagarah, Y.A. Terefe, Analysis of the mitigation strategies for COVID-19: From mathematical modelling perspective, Chaos Solitons Fractals. 138 (2020), 109968.

[27] Z. Liu, P. Magal, O. Seydi, G. Webb, Understanding Unreported Cases in the COVID-19 Epidemic Outbreak in Wuhan, China, and the Importance of Major Public Health Interventions, Biology. 9 (2020), 50.

[28] B. Ivorra, M.R. Ferrández, M. Vela-Pérez, A.M. Ramos, Mathematical modeling of the spread of the coronavirus disease 2019 (COVID-19) considering its particular characteristics. The case of China. Commun. Nonlinear Sci. Numer. Simul. 88 (2020), 105303.

[29] Z. Cakir, H.B. Savas, A Mathematical Modelling Approach in the Spread of the Novel 2019 Coronavirus SARS-CoV-2 (COVID-19) Pandemic. Electron. J. Gen. Med. 17 (4) (2020), em205.

[30] Q. Lin, S. Zhao, D. Gao, Y. Lou, S. Yang, S.S. Musa, M.H. Wang, Y. Cai, W. Wang, L. Yang, D. He, A conceptual model for the coronavirus disease 2019 (COVID-19) outbreak in Wuhan, China with individual reaction and governmental action, Int. J. Infect. Dis. 93 (2020), 211–216.

[31] D. Okuonghae, A. Korobeinikov, Dynamics of tuberculosis: the effect of direct observation therapy strategy (DOTS) in Nigeria. Math. Model. Nat. Phen. 2 (1) (2007), 113-128.

[32] D. Okuonghae, V. Aihie, Case detection and direct observation therapy strategy (DOTS) in Nigeria: its effect on TB dynamics. J. Biol. Syst. 16 (01) (2008), 1-31.

[33] P. Driessche, J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Math. Biosci. 180 (1-2) (2002), 29-48.

[34] C. Castillo-Chavez, Z. Feng, and W. Huang, On the computation of R0 and its role on global stability, in Mathematical approaches for emerging and reemerging infectious diseases: models, methods and theory, C. Castillo-Chavez, S. Blower, P. van den Driessche, D. Kirschner, and A.-A. Yakubu, eds., Springer, Berlin Heidelberg New York, 2002, pp. 229–250.

[35] L.N. Massawe, E.S. Massawe, O.D. Makinde, Temporal model for dengue disease with treatment. Adv. Infect. Dis. 5 (01) (2015), 54208.

[36] J. Tumwiine, J. Mugisha, L.S. Luboobi, A mathematical model for the dynamics of malaria in a human host and mosquito vector with temporary immunity. Appl. Math. Comput. 189 (2) (2007), 1953–1965

[37] J.P. LaSalle, The Stability of Dynamical Systems, SIAM. Philadelphia. (1976).

[38] http://www.moh.gov.et/ejcc/am/COVID19. Retrieved from May 1, 2020 to July 31, 2020.

[39] https://www.worldometers.info/world-population/ethiopia-population. Retrieved on June 21, 2020.

[40] https://www.macrotrends.net/countries/ETH/ethiopia/life-expectancy. Retrieved on June 21, 2020.