ON NUMERICAL ANALYSIS AND SIMULATION OF LINEAR ELASTICITY OF TAPES USED IN BIOMECHANICS

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Abstract. Deformation of materials occurs when materials are subjected to various external forces. In biomechanics, these deformations vary as different materials have different elastic parameters. In this paper, effects of elastic parameters of elastic tapes on the displacement parameters are investigated. PDEs modeling elasticity are solved and the simulations of different materials are done. An appropriate discretization scheme and solution method are used to solve the system of PDEs modeling elasticity. We used finite element method to discretize and a solution method with linear complexity. The Validation of error estimates is done. This paper highlights the different elastic properties; the tensile stresses at different levels of elongation, the maximum elongation, and the stiffness of the Kinesio tapes, Athletic tapes, and Surgical tapes with different values of the elastic parameters.

Keywords: elastic tape; material properties; PDEs modeling elasticity.

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1. INTRODUCTION

Forces acting on living things can cause motion, be a healthy stimulus for growth and development, or overload tissues and causing injury. The studies of the interactions between

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these forces and their effects on the function and motion of our bodies promoted quality of life. Movements are a set of complex and coordinated mechanical interactions which are happened between bones, muscles, ligaments and joints. The injury to, or lesion in one of these parts may cause the disability of movement[1].

Elastic materials (elastic tube, elastic tapes, elastic band) have been used in rehabilitative medicine to correct and support the movements, to prevent the injuries due to movements, to help a rapid recover from overuse or to increase the strength of dynamic muscular in the health fitness industry. Elastic tapes are materials applied on the skin in order to maintain a stable position of bones and muscles during movements[1].

The term biomechanics is the branch of physics which studies the relationship between forces, matters and motions. So, biomechanics can be defined as study of the movement of living things using the science of mechanics [2].

As mechanics provides the tools necessary to analyse the strength of structures, to predict and measure the movements of machines, biomechanics provides conceptual and mathematical tools that are necessary for understanding how to improve or make movements safer[3]. The term biomechanics was adopted by the international community of scientists to describe the science that studies the mechanical aspects of living organisms[2].

2. PDEs Modeling the Elasticity

The PDEs modeling elasticity are

$$(2.1) \qquad -\nabla.\sigma = f, in \ \Omega$$

$$(2.2) \qquad with, \sigma = \lambda trace(\varepsilon)I + 2\mu\varepsilon,$$

$$(2.3) \qquad \varepsilon = \frac{1}{2}(\nabla u + (\nabla u)^T)$$

where σ is the stress tensor, f is the body force per unit volume, μ and λ are Lamé's elasticity parameters for the material in Ω domain , I is the identity tensor, ε is the symmetric strain-rate tensor, and u is the displacement vector field.

By combining(2.2) and (2.3)and using the definition of divergence operator $div(u) = trace(grad(u))$ and the property of the trace operator $trace(A^T) = trace(A)$, we get the new expression of the stress.

$$(2.4) \qquad\qquad \sigma = \lambda(\nabla \cdot u)I + \mu(\nabla u + (\nabla u)^T)$$

3. ELASTIC PARAMETERS

An elastic parameter also known as modulus of elasticity is a quantity that measures an object or substance's resistance to being deformed elastically when a stress is applied to it[4].

Bulk modulus (K): Bulk modulus is a proportion of volumetric stress related to the volumetric strain. A material that is difficult to compress has a huge bulk modulus, $K = -\frac{V dp}{dV}$, with dp the change in pressure, dV the change in volume, and V the initial volume.

Young's modulus (E): It is ratio between stress applied to the material and the strain. It characterizes the behavior of the material when stretched in one direction[5]. $E = \frac{\sigma}{\varepsilon}$, where the stress σ equals to the forces applied to the material divided by the area of contact $\sigma = \frac{F}{A}$ and strain ε equals to the change in length divided by the original length $\varepsilon = \frac{l - l_o}{l_o} = \frac{\Delta l}{l_o}$.

Lamé modulus (λ): It relates stresses and strains in perpendicular directions. It is closely related to the incompressibility. $\lambda = \rho(V_p^2 - 2V_s^2)$, where, V_p is compressional velocity, V_s is shear velocity, and ρ is density of elastic material.

Shear modulus (μ): It is a measure of the ability of a material to resist transverse deformations. It is the ratio between the shear stress and the shear strain. $\mu = \frac{F/A}{\Delta x/l} = \frac{Fl}{A\Delta x}$, with F the force which acts on the material, A the area on which the force acts, Δx the transverse displacement, and l the initial length.

Poisson's ratio (ν): The negative ratio between transverse strain and longitudinal strain in the direction of stretching force[5]. It is a unitless because it is strain divided by another strain of the same units. $\nu = -\frac{\varepsilon t}{\varepsilon l}$, where εt is ratio between the change in transverse and the original transverse length, and εl is ratio between the change in longitude and the original longitude length[6].

Mass modulus (M): Also known as the longitudinal modulus, it is defined as the ratio of axial stress to axial strain in an uniaxial strain state $\sigma_{zz}=M\varepsilon_{zz}$, where all the other strains ε_{**} are null. $M=\rho V_P^2$, with V_P, the velocity of a P-wave.

Given two of these parameters, others can be derived. The formulas can be found in[7].

4. LINEAR ELASTICITY

The elasticity is the capacity of a material to return to its shape and size when the load on it is removed. The elasticity is linear when the force necessary to stretch or compress a material is proportional to how much it is stretched or compressed. In other words, the elasticity is linear when the Hooke's law is satisfied. The Hooke's law is expressed by $\sigma = E \cdot \varepsilon$, where σ is the stress, E is the Young's modulus, and ε is the strain.

In this paper, we are interested in the elastic tapes which we considered to be linear elastic materials. The linear elastic materials are elastic (attain their original shape and size when the stresses are removed) and their loading and unloading paths are the same[8, 9]. See the figures 1a and 1b.
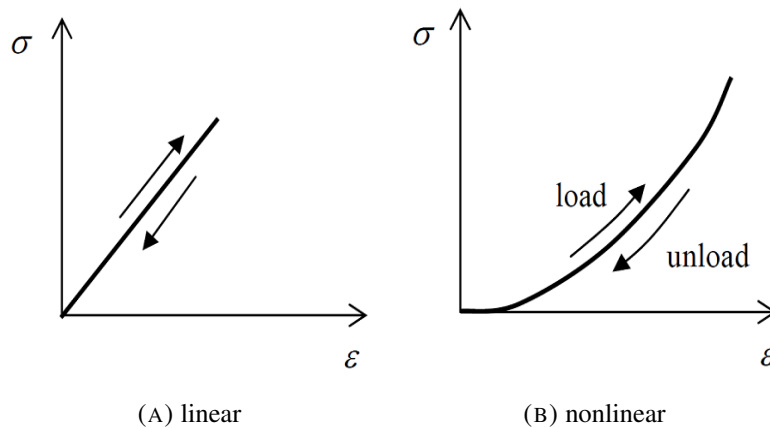

(A) linear   (B) nonlinear

FIGURE 1. Different stress-strain relationships:(a) linear elastic (isotropic and homogeneous materials):(b)non-linear elastic[9]

The generalized Hooke's law is the statement that the component of stress at a given point inside a linear elastic material is linear homogeneous functions of the strain components at the point. Mathematically, this implies that $\sigma^{ij} = D^{ijkl}\varepsilon_{kl}$, where the quantity D^{ijkl} is the tensor of elastic constants and it characterizes the elastic properties of the material[10].

The relation between the stress and strain tensors in a Cartesian frame, is $\varepsilon^{11} = \frac{\sigma^{11}}{E}$ when only one stress σ^{11} is present. Applying σ^{22} which will be perpendicular, there will be an additional term $-\frac{v}{E}\sigma^{22}$ to ε^{11}. With σ^{33}, there is a similar contribution of $-\frac{v}{E}\sigma^{33}$[9].

5. METHODOLOGY

5.1. Displacement.

The displacement is a difference vector between the positions of materials at two instances of time. The different particles of a material can move differently between the same two instances of time. In general, the displacement varies spatially and temporally. Displacement is what we can observe and measure while forces, traction and stress tensors are introduced to explain the displacement[11].

Displacement can also be defined as movement of individual points on a system due to the external forces. There is a displacement when the entire body is moving; translation of points of a body or rotation of lines. The displacement occurs also when related to the deformation; change in length (contraction or elongation) or distortion; change of angle between lines[12]. In this paper, we were interested in this second type of displacement.

5.2. Von Mises stress.

In general, a material under stress does not change the shape or size permanently. It returns to its original shape when the load is removed. But every material has its maximum stress that represents the upper limit of the load above which the material will be permanently deformed. This stress is called yield stress (σ_Y).

The stress on material can be divided into two components; the hydrostatic stress (σ_H) that acts to change the volume of the material only and the deviatoric stress (σ_{dev}) that acts to change the shape only. That is, in matrix form

$$
\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_H & 0 & 0 \\ 0 & \sigma_H & 0 \\ 0 & 0 & \sigma_H \end{pmatrix} + \begin{pmatrix} \sigma_{xx} - \sigma_H & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - \sigma_H & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - \sigma_H \end{pmatrix}
$$

or

$$\sigma(u) = \sigma_H + \sigma_{dev}$$

with

$$\sigma_H = \frac{1}{3} tr(\sigma(u)) I$$

The Von Mises stress is a special measure of stress that serves as an approach to combine all stress components into one value. The Von Mises stress is compared with the yield strength of a material to predict if that material will yield. Mathematically, the Von Mises stress, is expressed as [13]

(5.1) $$\sigma_M = \sqrt{\frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\tau_{xy} + \tau_{yz} + \tau_{xz})]}$$

where σ_x, σ_y, and σ_z are the normal stresses and τ_{xy}, τ_{yz}, and τ_{xz} are the shear stresses.

The concept of Von Mises stress arises from the distortion energy failure theory. The distortion energy is the energy required to change the shape of a material. During pure distortion, the shape of the material changes, but not the volume. Distortion energy failure theory is a comparison between two kinds of energies; distortion energy in the actual case and distortion energy at the time of failure. So, the failure occurs when the distortion energy in actual case is more than the distortion energy in a simple tension case at the time of failure [14].

Distortion energy required for the deformation of a shape's material is given by the expression

(5.2) $$U_d = \frac{1+v}{3E} \sigma_M^2$$

Distortion energy at the time of failure is defined as:

(5.3) $$U_{d,fail} = \frac{1+v}{3E} \sigma_Y^2$$

The condition of failure will be $U_d \geq U_{d,fail}$ that is a material will yield if the Von Mises stress of any part of it is equal or exceed the yield strength of the material. So, the failure condition can be simplified as $\sigma_M \geq \sigma_Y$.

In this paper, the Von Mises stresses of surgical tape, athletic tape, and kinesio tape are computed and compared with their yield strength to determine the stress and at which level of elongation the tapes will be deformed in irreversible way.

5.3. Finite element method.

The finite element method (FEM) is a numerical method for approximate the solution of various engineering problems. The problems are described by partial differential equations or can be formulated as functional minimization.

The FEM is based on subdividing the domain of interest (a complex object) into a collection of small areas or volumes which are simple to analyse. Then, the solution is formulated for each element not for the entire domain. When the solutions of all elements are found, they are combined to obtain a solution for the whole domain [15].

The process of subdividing is called discretization. The small volumes or areas thus obtained are called finite elements. The finite elements can have various forms (see figure 2), in two dimensions they can be quadrilateral or triangular, and in three-dimensions, hexahedral, tetrahedral, etc. The finite elements are connected each other by nodes, boundary lines and surfaces[16, 17]. An object subdivided into finite elements (see figure 3) is called mesh.
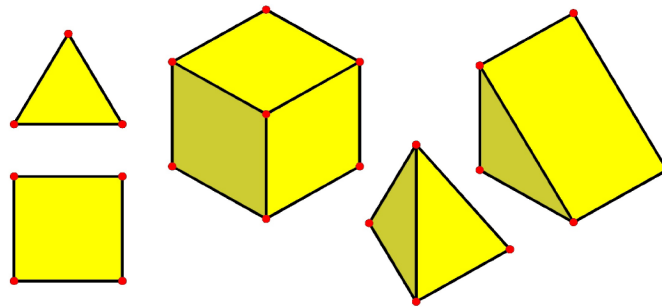
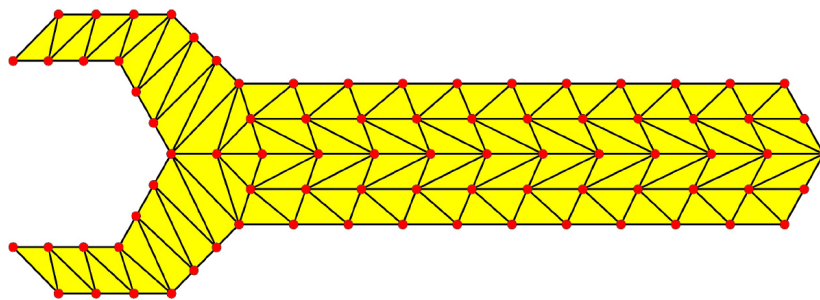FIGURE 2. Common forms of finite elements[18].

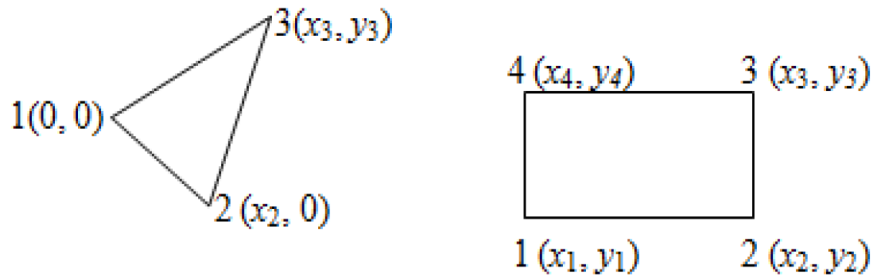FIGURE 3. Mesh of an object subdivided into finite elements [18].

FIGURE 4. Triangular and rectangular elements[19].

Finite element equation is obtained by incorporating governing equation into element equation. This last is obtained from the shape function and geometry equation. The number of equations is equal to the number of nodes. Consider two degree of freedom for each node (figure 4), linear shape function for triangular and rectangular elements consists of 2 unknowns that represent 2 degree of freedom. For the triangular, there are 3 equations whereas rectangular there are 4 equations. In summary, we can say that the number of equations for an element represents total number of nodes while the number of unknowns in each element equation represents degree of freedom of a node.

The steps of solving a problem with the FEM are: The problem is transformed into a variational problem, the variational problem is discretized (divided into simple elements), the discrete variational problem leads to a set of linear equations and then the equations are solved[6, 20].

The FEM has many advantages such that to model easily and perfectly irregular shaped bodies, to manipulate general loading and boundary conditions, to model bodies composed of composite and multiphase materials, to improve the accuracy of approximate solution by varying the size and type of element, to handle a variety of nonlinear effects including material behavior, large deformation, etc[18].

The FEM is a general and efficient numerical method for solving PDEs. The starting point for solving a PDE with the FEM is to express the PDE in variational form. In following paragraphs we formulate the variational form of the equations (2.1), (2.2), and (2.3).

5.3.1. *Variational formulation.*

To formulate a variational form of a PDE we, firstly, multiply the PDE by a function v, called a test function, secondly, integrate the result of the multiplication over the domain, and finally, perform integration by parts. The unknown function u to be approximated is called trial function. The trial and test functions are so-called function spaces.

In our case, the variational formulation of the PDEs modeling elasticity is obtained by multiply (2.1) by the test function $v \in \hat{V}$, where $\hat{V}$ is a vector-valued test function space, and integrating over the domain Ω, we get[21]:

$$-\int_{\Omega} (\nabla \cdot \sigma) \cdot v \, dx = \int_{\Omega} f \cdot v \, dx$$

By product rule of the gradient, we have $\nabla(\sigma \cdot v) = (\nabla \cdot \sigma)v + \sigma(\nabla \cdot v)$

$$(\nabla \cdot \sigma)v = \nabla(\sigma \cdot v) - \sigma(\nabla \cdot v)$$

Now, we integrate by parts over the domain Ω

$$-\int_{\Omega} (\nabla \cdot \sigma) \cdot v \, dx = -\int_{\Omega} \nabla(\sigma \cdot v) \, dx + \int_{\Omega} \sigma(\nabla \cdot v) \, dx$$

We know that if a is a constant vector then $\nabla(\sigma \cdot v \cdot a) = \nabla(\sigma \cdot v) \cdot a$

so, we can write

$$\int_{\Omega} \nabla(\sigma \cdot v \cdot a) \, dx = \int_{\Omega} \nabla(\sigma \cdot v) \, dx \, a = \int_{\partial\Omega} (\sigma \cdot v \cdot a)n \, ds = \int_{\partial\Omega} \sigma \cdot v \, n \, ds \, a$$

where n is the unit normal to $\partial\Omega$.

Since a was arbitrary, we have

$$\int_{\Omega} \nabla(\sigma \cdot v) \, dx = \int_{\partial\Omega} \sigma \cdot v n \, ds$$

Thus

$$-\int_{\Omega} (\nabla \cdot \sigma) \cdot v \, dx = \int_{\Omega} \sigma \cdot (\nabla v) \, dx - \int_{\partial\Omega} (\sigma \cdot n) \cdot v \, ds$$

The quantity $\sigma \cdot n$ is known as the traction or stress vector at the boundary and is often prescribed as a boundary condition. We here assume that it is prescribed on a part $\partial\Omega_T$ of the

boundary as $\sigma \cdot n = T$. We thus obtain[21]:

$$\int_\Omega \sigma \cdot \nabla v dx = \int_\Omega f \cdot v dx + \int_{\partial \Omega_T} T \cdot v ds$$

By inserting the expression(2.4)for σ gives the variational form with u as unknown.

In summary, the variational formulation of our equations can be given as [21]: find $u \in V$ such that

$$(5.4) \qquad\qquad a(u,v) = L(v) \quad \forall v \in \hat{V}$$

where

$$(5.5) \qquad\qquad a(u,v) = \int_\Omega \sigma(u) \cdot \nabla v dx$$

$$(5.6) \qquad\qquad \sigma(u) = \lambda(\nabla \cdot u)I + \mu(\nabla u + (\nabla u)^T)$$

$$(5.7) \qquad\qquad L(v) = \int_\Omega f \cdot v dx + \int_{\partial \Omega_T} T \cdot v ds$$

by replacing ∇v by the symmetric gradient $\varepsilon(v)$ gives rise to the slightly different variational form

$$(5.8) \qquad\qquad a(u,v) = \int_\Omega \sigma(u) \cdot \varepsilon(v) dx$$

where $\varepsilon(v)$ is the symmetric part of ∇v:

$$\varepsilon(v) = \frac{1}{2}(\nabla v + (\nabla v)^T)$$

Thus the equations(5.6), (5.7),(5.8) are variational form of the equations governing the small deformation. These equations are discretized using FEM and so, compute the displacement and the stress of the tapes.

5.4. Stiffness.

Stiffness is the property of a material which shows its ability to resist deformations. The deformations can be bending, stretching or twisting, when the forces are applied on it. The stiffness of a material shows how much it deflects under a given forces. The stiffness depends on the Young's modulus, on the shape, on the size of material and on how the material is loaded[22].

Stiffness is an important aspect of materials and structures which determines how much a material is deformed under load and how it transfers part of the load to other materials or structures in contact with it.

Stiffness is a physical quantity that represents the magnitude of force required to cause a unit deformation of a material. More is the value of stiffness more is the rigidity of the material, that is if it requires more force to shift a unit of material A than B, we would say that A is stiffer than B[23].

The stiffness is expressed by $k = \frac{F}{\delta}$, with F the applied force, and $\delta = \frac{FL}{AE}$, where L is the length of the object, E the Young's modulus, and A the area. Finally, the expression of the stiffness is

$$(5.9) \qquad\qquad k = \frac{AE}{L}$$

5.5. Discretization error and convergence rate.

Since we solved the PDE approximately, we need to know the error of approximation. In FEM, there are three main sources of error; discretization errors that arise from the creation of the mesh, formulation errors due to incorrect model of the real world, and numerical errors from the solution of the FEM equations[24].

The error is defined as the difference between the exact solution and the approximated solution. Let us call $u^{ex}(x)$ the exact solution and $u(x)$ the corresponding approximate solution in FEM. The error, $e(x)$ is defined as $e(x) = u^{ex}(x) - u(x)$.

The error we are interested in here is from the discretization because it can be used to prove the accuracy of the method used for solving the equations and to show how approximated solution is closer to the exact solution.

5.5.1. *Discretization error.*

Discretization error results from transforming the physical system into a finite element model. The discretization error is the direct consequence of mesh quality. The discretization error is what is meant typically when referring to error in the computed solution.

There are a number of approaches available for estimating discretization error[25, 26]. Here, we measure the discretization error by L_2 norm.

The L_2 norm is expressed by

$$(5.10) \qquad \|e\|_{L_2} = \sqrt{\int_0^l (u^{ex}(x) - u(x))^2 dx}$$

The discretization error can be computed through the comparison between the stress, stain, or displacement from exact solution and those approximated. In this paper, we compute the displacement discretization error.

5.5.2. *Convergence rate.*

After computing the error, an important question for any numerical method is its convergence rate; how fast the error approach zero when the number of finite elements or the order polynomial of interpolation are increased. For FEM, the error $e(x) = u^{ex}(x) - u(x)$ is bounded by the size of the mesh h to some power r; that is, $\|e(x)\| \leqslant Ch^r$ for some positive constant C and $r > 0$ called the convergence rate of the method used to approximate the solution [21].

Mathematically, the convergence is expressed by $u(x) \to u^{ex}(x)$ as $h \to 0$. That is the solution approximated by the FEM should converge (as the number of finite elements is increased) to the analytical (exact) solution of the equations that govern the response of the mathematical model [27].

6. NUMERICAL EXPERIMENT

Tapes are considered to be the 3D objects of 200 mm in length, 30 mm in width, and 0.01 mm in thickness(see the figure 5).

6.1. Displacement.

For the displacement, we discretize the model of the tapes. The finite elements of the mesh are triangular. Two elastic parameters; Lamé modulus (λ) and shear modulus(μ) are necessary.
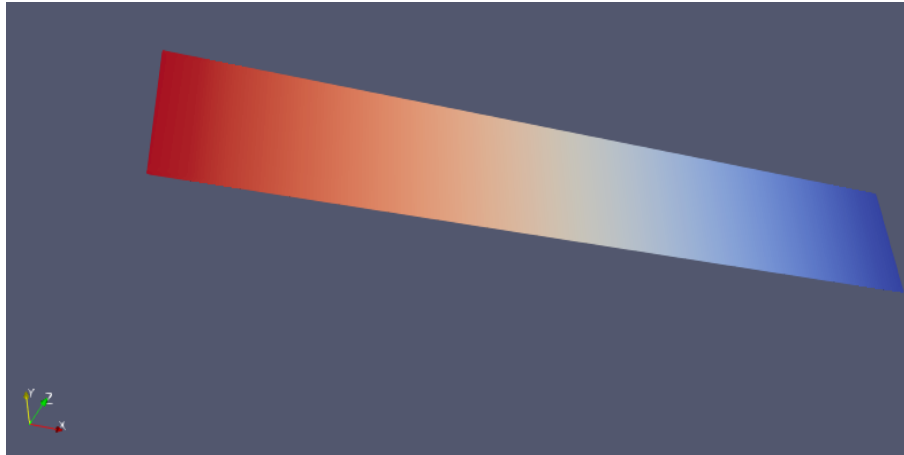
FIGURE 5. simulated image of tapes used in numerical experiment

The values of these parameters are in table 1, and we make variations to see the effects of these on the displacement.

After the discretization, we apply to the mesh the force of 5N to cause the deformations and get the different displacements of the tapes. The displacement will be computed to all nodes and the results of some taken as references will be shown.

	Tapes		
Elastic parameters	Kinesio	Surgical	Athletic
λ(Gpa)	3.19	1.82	29.4
μ(Gpa)	1.37	0.93	11.43

TABLE 1. values of elastic parameters

6.2. Tensile and Von Mises stress.

The tensile stress is computed at the different level of elongation that is at 15%, 25%, 50%, 75%, and 100% to determine how the tapes are stressed at those levels; deformed or permanently deformed at those levels of elongation and so, make a classification of kinesio tape, surgical tape, and athletic tape in different technique used in taping. The percent of the elongation will be computed by using the expression; Elongation (%)=$(D(mm)$ x 100%$)/L(mm)$, with D the displacement and L the initial length of the tapes. The different levels of stress have been

chosen due to the different technique used in taping finding in the literature; lymphatic, fascial, tendon, and corrective[28], in which different values of tension are necessary.

The Von Mises stress is computed using the equation(5.1) and compared with the yield stress at different level of elongation for each tape, to determine at which elongation the tapes lose their elasticity; maximun elongation.

6.3. Stiffness.

The stiffness is computed using the expression (5.9). The stiffness is an indicator of the tendency for an element to return to its original form after being subjected to a force.

6.4. Discretization error and convergence rate.

We compute the L_2 norm of the discretisation error using the mesh hp-refinement adaptation strategies. Hp-refinement is a combination of two refinements h and p. h-refinement is the modification of the size of the finite elements by changing the mesh connectivity. The simple strategy for this type of refinement is subdividing cells, while more complex procedures insert or remove nodes or cells[29].

The strategy used here is the subdivision of cells. The advantage of this procedure is that the overall mesh topology remains the same with the new cells taking the place of the ancient cells in the connectivity arrangement [29]. In our case, we start by 4x4 finite elements and continue subdividing till 256x256 finite elements.

P-refinement in FEM achieves increased resolution by increasing the order of accuracy of the polynomial in each finite element. For the order of the polynomial, we use three different degrees $P_1 = 1$, $P_2 = 2$, and $P_3 = 3$.

Normally, the error decreases when the size of the finite elements decreases or when the number of the finite elements increases and when the order polynomial of interpolation increases.

For computing the convergence rate, we define the element size to be $h = 1/n$, where n is the number of cells in one direction. So, we compute the errors (E_i for $i = 1 \cdots 6$) for finer and finer meshes. Assuming $E_i = Ch_i^r$ for unknown constants C and r, we can compare two consecutive

experiments, $E_{i-1} = Ch_{i-1}^r$ and $E_i = Ch_i^r$, and solve for

(6.1)
$$r = \frac{ln(E_i/E_{i-1})}{ln(h_i/h_{i-1})}$$

The r values should approach the expected convergence rate as i increases; for us the value of r is of order h^{P+1} since we used the L_2 norm in measuring the error.

7. RESULTS AND DISCUSSIONS

7.1. Introduction.

In this part, the results of the simulations are presented, followed by discussions at each step. We firstly, present the results of computed displacement with the values of the elastic parameters; shear modulus (μ) and Lamé modulus (λ) in table 1. Secondly, we present the displacement after making variations on the said elastic parameters to investigate the effects of elastic parameters on displacement. The force applied on the tapes is of F=5N.

We also present the results of the Von Mises stress and the tensile stress (at different level of elongation), and the stiffness of the kinesio, surgical, and athletic tapes.

7.2. Displacement.

The figure 6 represents the displacements of the kinesio, surgical, and athletic tapes with the values of elastic parameters (μ and λ) in table 1. It is observed that the athletic tape has the lowest displacement and surgical tape has the highest displacement.
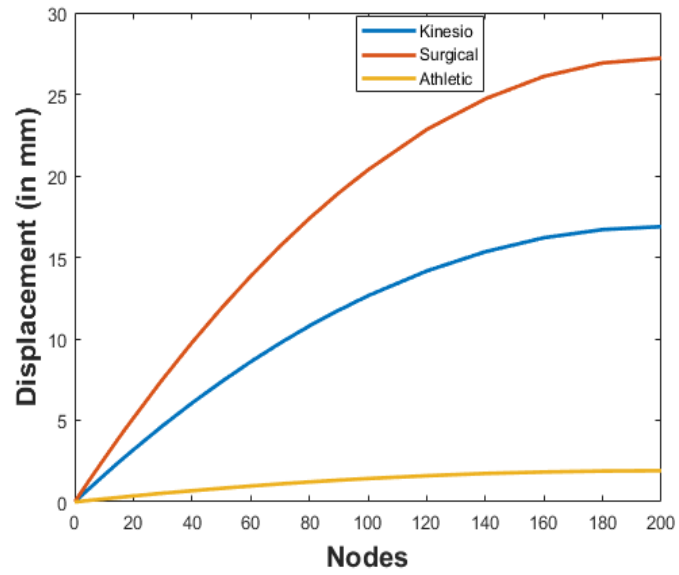
FIGURE 6. Displacements of the Kinesio, Surgical, and Athletic tapes

The figures 7, 8, and 9 show respectively the displacements of the kinesio, surgical, and athletic tapes by varying the shear modulus $\mu(=2.5, 4.5, 6.5, 8.5, 10.5)$ while the values of Lamé modulus (λ) are those in table 1, and force applied F=5N. It is observed that an increase in shear modulus leads to decrease in the displacement and this for all the studied tapes. Even in this case, athletic tape has the lowest displacement and surgical tape has the highest displacement, if we take the equal values of the shear modulus(μ).
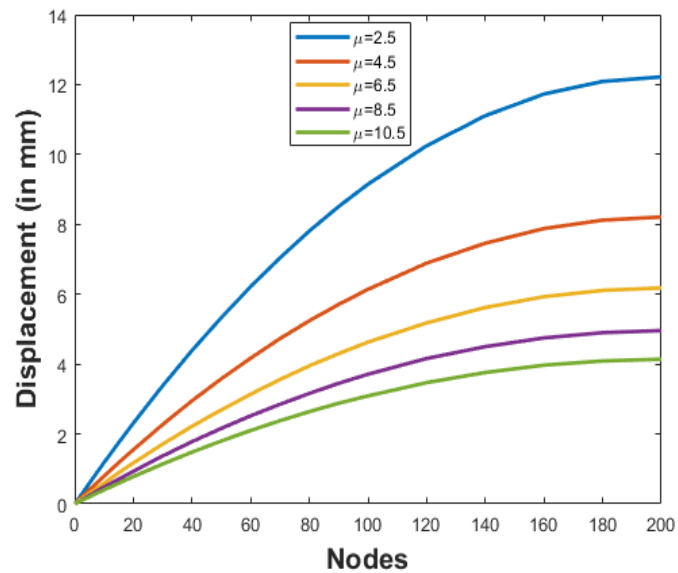
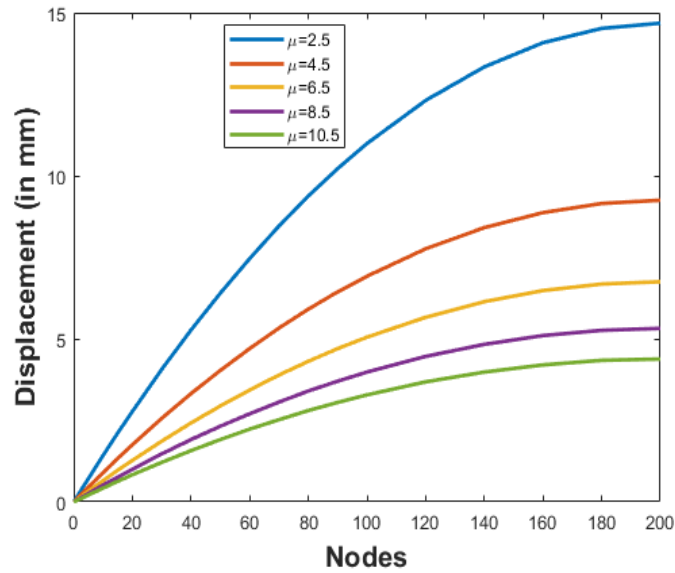FIGURE 7. Displacements of the kinesio tape for different values of shear modulus(μ)

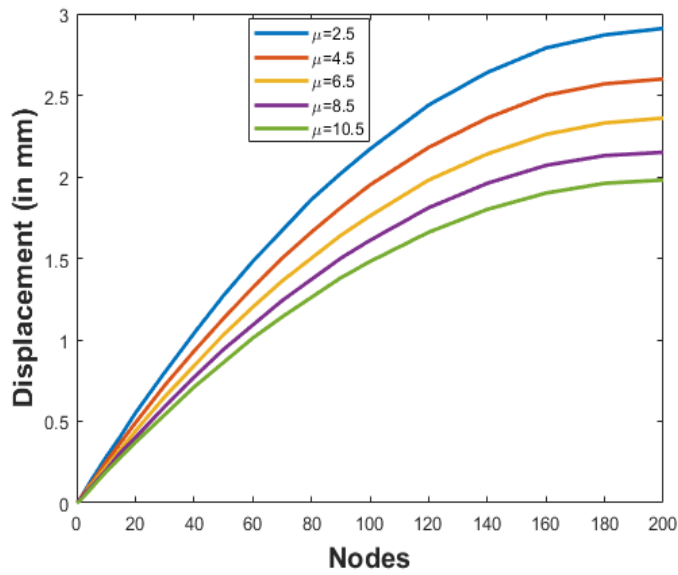FIGURE 8. Displacements of the surgical tape for different values of shear modulus(μ)

FIGURE 9. Displacements of the athletic tape for different values of shear modulus(μ)

In the figures 10, 11, and 12 are presented respectively the displacements of the kinesio, surgical, and athletic tapes with variations of the values of the Lamé modulus , $\lambda (= 5.5, 9.5, 13.5, 17.5, 21.5, 25.5)$ while the values of shear modulus (μ) are those in table 1, and force applied F=5N. It is observed that an increase in Lamé modulus leads to decrease in the displacement and this is the same for the three studied tapes. Consider the same values of the Lamé modulus (λ), the athletic tape has the lowest displacement and surgical tape has the highest.
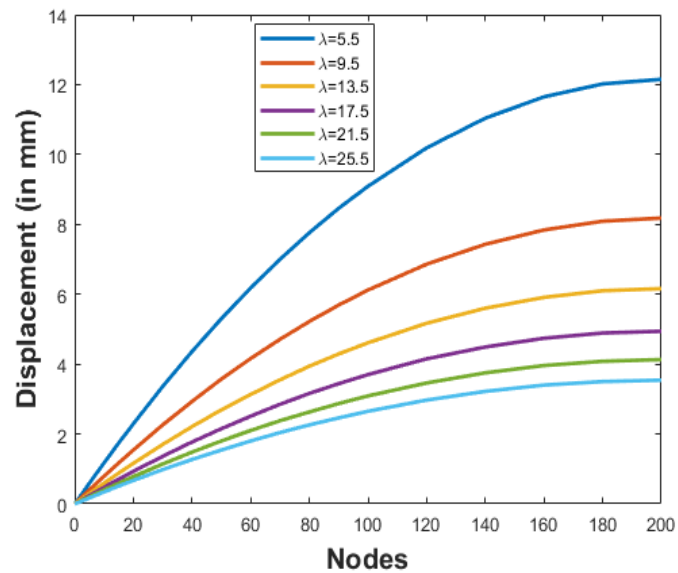

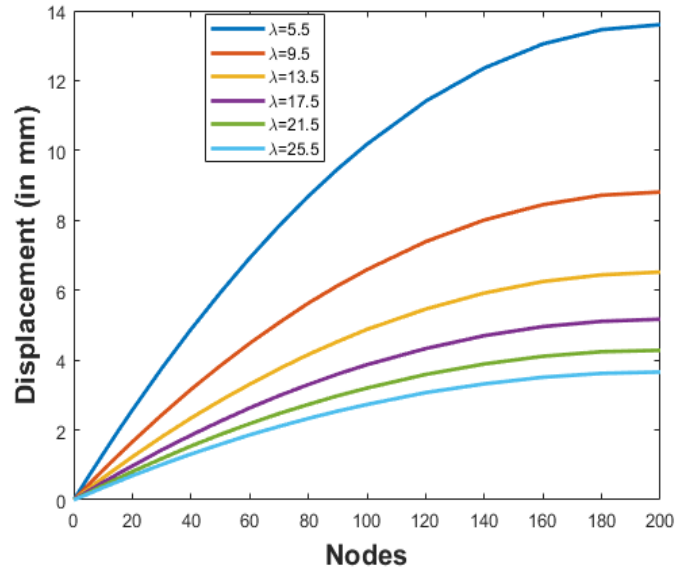FIGURE 10. Displacements of the kinesio tape for different values of Lamé modulus(λ)

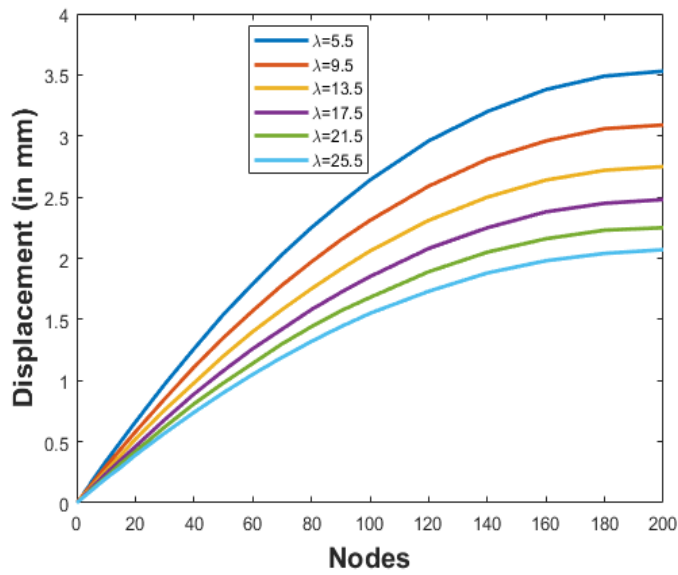FIGURE 11. Displacements of the surgical tape for different values of Lamé modulus(λ)

FIGURE 12. Displacements of the athletic tape for different values of Lamé modulus (λ)

7.3. Tensile and Von Mises stress.

In the table 2, are collected the force, and the stress of the kinesio, surgical, and athletic tapes at 15%, 25%, 50%, 75%, and 100% of elongation. The original length is of 200 mm. We applied the different values of force on them and observe the force that causes the different levels of elongation.

It is observed that athletic tape needs more force to change the length than others and the surgical tape needs less. For the stress, it is observed that, at all levels, the athletic tape is the most stressed while the surgical tape is the least.

Elong.(%)	L(mm)	Force(N)			Stress(Mpa)		
		Kin	Sur	Ath	Kin	Sur	Ath
0	200	0	0	0	0	0	0
15	230	8.9	5.5	78.5	29.6	18.3	261.6
25	250	14.8	9.2	130	49.3	30.6	433.3
50	300	30	18.4	261	100	61.3	870
75	350	44.4	27.5	391	148	91.6	1303
100	400	59	36.7	522	196	122	1740

TABLE 2. Stress at different levels of elongation

The tables 3, 4, and 5 presented the values of the Von Mises stress. It has observed that the Von Mises stress is not homogeneous on all the surface of the tapes that is why we presented in 3, 4, and 5, the minimum and maximum found. The athletic tape has a highest Von Mises stress and the surgical tape has the lowest in maximum in all different levels of elongation. In minimum, kinesio and surgical tapes have the same Von Mises stress at elongation of 15%. The minimum Von Mises stress at 15 and 25% of elongation are equal for the surgical tape.

	Von Mises stress(Mpa)	
Elong.(%)	Min.	Max.
15	0.01	1.76
25	0.02	2.93
50	0.06	5.95
75	0.08	8.81
100	0.11	11.71

TABLE 3. Von Mises stress of Kinesio tape

	Von Mises stress(Mpa)	
Elong.(%)	Min.	Max.
15	0.01	1.09
25	0.01	1.82
50	0.03	3.65
75	0.05	5.45
100	0.07	7.28

TABLE 4. Von Mises stress of Surgical tape

	Von Mises stress(Mpa)	
Elong.(%)	Min.	Max.
15	0.16	15.58
25	0.26	25.8
50	0.53	51.81
75	0.8	77.65
100	1.07	103.63

TABLE 5. Von Mises stress of Athletic tape

7.4. Stiffness.

For the stiffness, as shown in table 6, the athletic tape is stiffer than others while the surgical is less.

	Tapes		
	Kinesio	**Surgical**	**Athletic**
Stiffness(N/m)	5.55×10^3	3.75×10^3	4.665×10^4

TABLE 6. Stiffness of tapes

7.5. Discretization error and convergence rate.

In the figures 13, 14, and 15, are presented the discretization errors and the convergence rates found using respectively 1, 2, and 3 as values of the degrees of the piecewise polynomial.
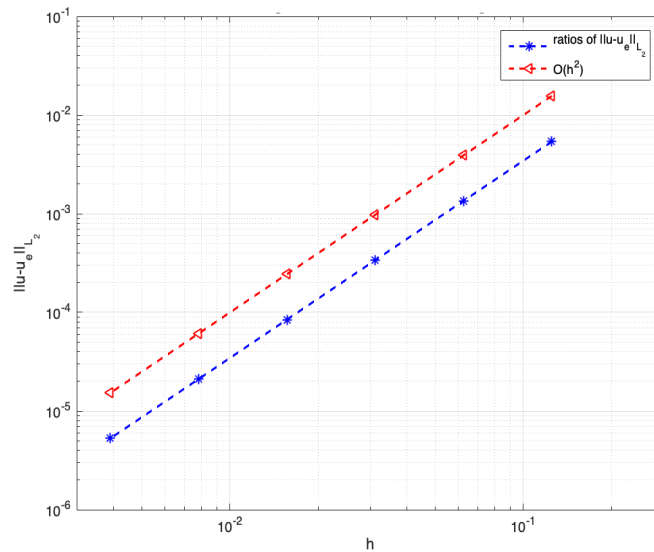

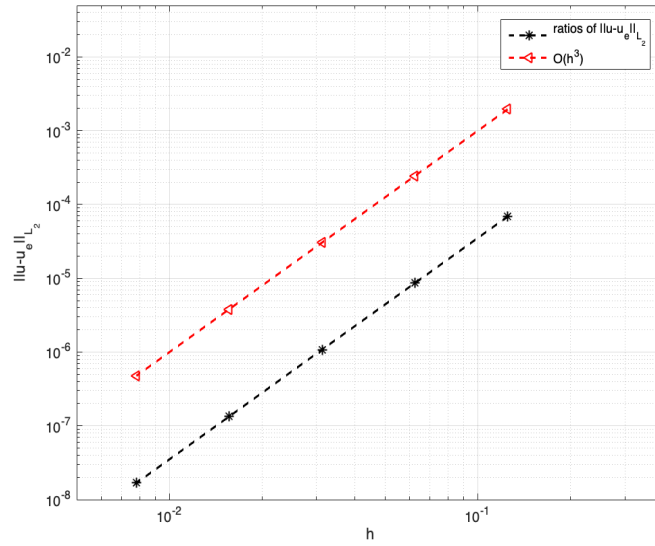FIGURE 13. Error and convergence rate for P=1
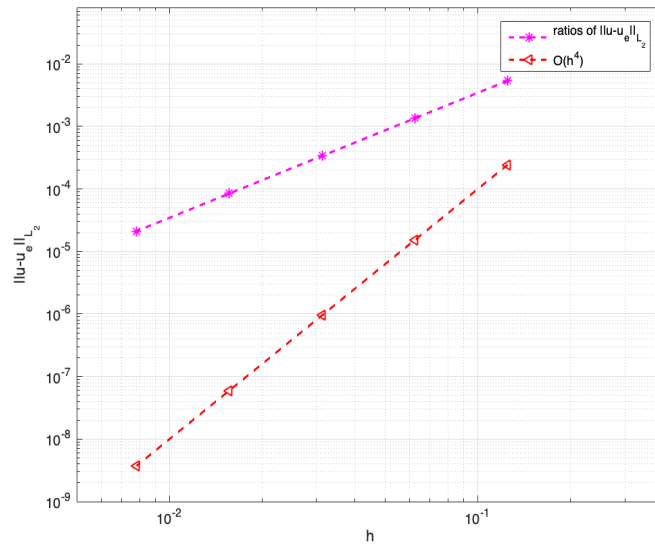
FIGURE 14. Error and convergence rate for P=2

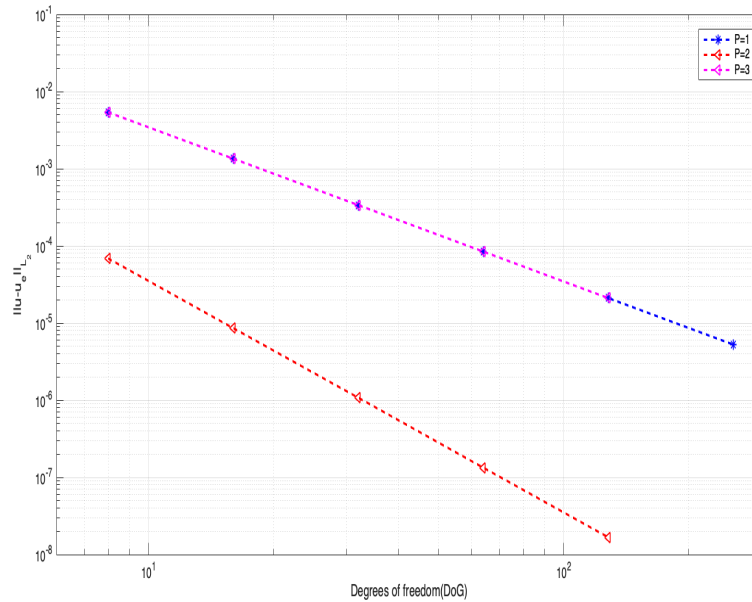FIGURE 15. Error and convergence rate for P=3

FIGURE 16. Rates of convergence for different P

It is observed that, for all the values of P,the errors decrease with the size of the meshes(h), and is the expected results since the estimated solution approach the exact solution when the number of the mesh increases. For $P = 3$, the error does not decrease quickly as for $P = 1$,and $P = 2$.

For the convergence rate, the order of the convergence values is of order h^{P+1}. This is the expected rate since we used the L_2 norm in measuring the error.

The figure 16 presents the rates of convergence for different values of the polynomial function (P). It is observed that, increase both the degree of freedom and degree of the polynomial function , leads to improved the accuracy.

8. CONCLUSION

In this paper, were investigated the effects of two elastic parameters; Lamé modulus (λ) and shear modulus(μ) on the displacement of Kinesio tapes, Athletic tapes, and Surgical tapes. It has observed that a tape with a high Lamé modulus or shear modulus, has the lowest displacement and this is the same when varying the value of these two parameters. This paper highlights also the tensile stresses at different levels of elongation, the maximum elongation, and the stiffness of the Kinesio tapes, Athletic tapes, and Surgical tapes with different values of the elastic

parameters. In treatments, the techniques are different from the tensions applied For example, in the muscle technique, tapes are applied by maximal tissues extension without tension, while in lymphatic technique 0% to 15% tension is necessary. In fascial technique, the tension should be between 25% and 50%, in ligament and tendon techniques it should be between 50% and 75%. The highest tension of tape is necessary in corrective technique where it may reach 50 to 100%. So, for all tapes studied, they can be used in these four techniques since their yield stresses are greater than their Von Mises stresses in all levels of elongation. The results are very helpful for the users of the elastic tapes by avoiding the confusions, of the elastic tape to use and the increase in their effectiveness.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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