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ARITHMETICO GEOMETRIC DECOMPOSITION OF SOME GRAPHS

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Abstract: A decomposition $(G_{ab}, G_{(a+d)br}, G_{(a+2d)br^2}, G_{(a+3d)br^3}, \ldots, G_{(a+(n-1)d)br^{n-1}})$ of G is said to be a Arithmetico Geometric Decomposition (ACOGD) or (a , d , b , r , n) – Decomposable if each $G_{(a+(i-1)d)br^{i-1}}$ is connected and $|E(G_{(a+(i-1)d)br^{i-1}})| = (a + (i-1)\,d)br^{i-1}$ for every i = 1, 2, …., n and a, d, b, r (>1) $\in$ N. Clearly, $q = \frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}$, for every $n \in$ N. In this paper, we seek to find Arithmetico Geometric Decomposition of some graphs.

Keywords: arithmetico geometric decomposition; arithmetico geometric path decomposition; arithmetico geometric star decomposition.

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1. INTRODUCTION

All basic terminologies from Graph Theory are used in the sense of Frank Harary[3]. Let

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$G = (V, E)$ be a simple connected graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots E(G_n)$, then $(G_1, G_2, \ldots, G_n)$, is said to be a Decomposition of G. The one point union $C_n^{(t)}$ ($n \geq 3$ and $t \geq 2$) of t - copies of cycle C_n is the graph obtained by taking v as a common vertex such that any t distinct cycles are edge disjoint and do not have any vertex in common except v. A caterpillar tree is a tree in which every vertex has distance at most 1 from a central path. The central path of a caterpillar tree is also called the spine of the tree and it is obtained by removing all endpoint vertices in the tree. In this paper, Caterpillar tree is represented by C$(l_1, l_2, \ldots, l_m)$, where l_i is the number of leaves attached to the node labeled with i on the central path, for $i = 1, 2, \ldots, m$. In a Caterpillar, A vertex with degree atleast 3 is called a junction. A vertex which is adjacent to k pendant vertices in a graph G is called a k-support.

2. ARITHMETICO GEOMETRIC DECOMPOSITION OF GRAPHS

Definition 2.1: A decomposition (G_{ab} , $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$) of G is said to be a Arithmetico Geometric Decomposition (ACOGD) or (a , d , b , r , n) – Decomposable if each $G_{(a+(i-1)d)br^{i-1}}$ is connected and $|E(G_{(a+(i-1)d)br^{i-1}})| = (a + (i-1)d)br^{i-1}$ for every $i = 1, 2, \ldots, n$ and a, d, b, r (> 1) $\in$ N. Clearly, $q = \frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}$, for every $n \in$ N.

Theorem 2.2: A Graph G admits ACOGD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, …, $G_{(a+(n-1)d)br^{n-1}}$) if and only if $q(G) = \frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}$, for every $n \in$ N .

Proof: Let $q(G) = \frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}$, for every $n \in$ N. Now consider H_1 is a subgraph induced by G – $G_{(a+(n-1)d)br^{n-1}}$ having $\frac{(a-(a+(n-1)d)r^n)b}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - (a + (n-1)d)br^{n-1}$ edges. Then $H_2 = H_1 - G_{(a+(n-2)d)br^{n-2}}$ having $\frac{(a-(a+(n-1)d)r^n)b}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - (a + (n-1)d)br^{n-1} - (a + (n-2)d)br^{n-2}$ edges. Proceeding like this, we get H_{n-1} having an edge ab. Thus $(G_{ab}, G_{(a+d)br}, G_{(a+2d)br^2}, G_{(a+3d)br^3}, \ldots, G_{(a+(n-1)d)br^{n-1}})$ is a ACOGD of G.

Conversely, Suppose G admits ACOGD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$). Then $q = ab + (a + d)br + (a + 2d)br^2 + (a + 3d)br^3 + \ldots\ldots + (a+(n-1)d)br^{n-1}$. Therefore, $q = \frac{(a-(a+(n-1)d)r^n)b}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}$, for every $n \in$N.

Theorem 2.3: The Wheel graph W_y is $(a, d, 2j, r, n)$ - Decomposable if and only if $y = \frac{1}{2}[\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]$ where $a, d, b, j, r, n \in$ N.

Proof: Let v and $v_1, v_2, v_3, \ldots, v_y$ be the central vertex and rim vertices of W_y. Assume that the Wheel graph W_y is $(a, d, 2j, r, n)$ - Decomposable. By theorem 2.2,

$q(G) = \frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}$, for every $n \in$ N . Since $q(W_y) = 2y$, we have

$y = \frac{1}{2}[\frac{b(a-(a+(n-1)d)r^n)}{1-r}] + \frac{dbr(1-r^{n-1})}{(1-r)^2}$.

Conversely, assume that $y = \frac{1}{2}[\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]$. If $n = 1$, then $G_{ab} = W_{\frac{ab}{2}}$. If $n > 1$, then $K_{1,y}$ is rooted at v. Also $K_{1,y} = K_{1,ab} \cup K_{1,(a+d)br} \cup K_{1,(a+2d)br^2} \cup \ldots \cup K_{1,(a+(n-2)d)br^{n-2}} \cup K_{1,\left[\frac{(a+nd)r^n}{1-r}+\frac{dr^n}{(1-r)^2}\right]\left[\frac{2-r}{2r}\right]-\frac{(2dr^{n-1}+a)}{1-r}+\frac{dr^n}{2(1-r)}-\frac{dr}{2(1-r)^2}}$.

Therefore $K_{1,y} - K_{1,\left[\frac{(a+nd)r^n}{1-r}+\frac{dr^n}{(1-r)^2}\right]\left[\frac{2-r}{2r}\right]-\frac{(2dr^{n-1}+a)}{1-r}+\frac{dr^n}{2(1-r)}-\frac{dr}{2(1-r)^2}}$ is decomposed into G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-2)d)br^{n-2}}$). Consider the cycle C: $v_1, v_2, v_3, \ldots, v_y$. Therefore, C $\cup$ $K_{1,\left[\frac{(a+nd)r^n}{1-r}+\frac{dr^n}{(1-r)^2}\right]\left[\frac{2-r}{2r}\right]-\frac{(2dr^{n-1}+a)}{1-r}+\frac{dr^n}{2(1-r)}-\frac{dr}{2(1-r)^2}} = G_{(a+(n-1)d)br^{n-1}}$. Hence W_y is $(a, d, 2j, r, n)$ – Decomposable.

Theorem 2.4: The Double wheel graph DW_y admits ACOGD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$) if and only if b is divisible by 4, where $a, d, r, n \in$ N.

Proof: Assume D W_y admits ACOGD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …,

$G_{(a+(n-1)d)br^{n-1}}$). By theorem 2.2, $q(\mathrm{D}W_y) = 4[\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]$. Then $b[\frac{(a-(a+(n-1)d)r^n)}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2}]$ is a multiple of 4. Therefore b is a multiple of 4. Thus b is divisible by 4. Conversely, assume that b is divisible by 4. Let u be a vertex such that $d(u) = \Delta$ in $\mathrm{D}W_y$. If $n = 1$, then $G_{ab} = \mathrm{D}W_{\frac{ab}{4}}$. If $n > 1$, then $S_{\frac{1}{2}[\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]}$ is a star rooted at u.

Let $G^* = S_{\frac{1}{2}[\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]} - S_{\frac{b(a-(a+(n-2)d)r^{n-1})}{1-r} + \frac{dbr(1-r^{n-2})}{(1-r)^2}}$

$= S_{\left[\frac{(a+nd)r^n}{1-r} + \frac{dr^n}{(1-r)^2}\right]\left[\frac{2-r}{2r}\right] - \frac{(2dr^{n-1}+a)}{1-r} + \frac{dr}{2(1-r)} - \frac{dr}{2(1-r)^2}}$.

Now, $S_{\left[\frac{(a+nd)r^n}{1-r} + \frac{dr^n}{(1-r)^2}\right]\left[\frac{2-r}{2r}\right] - \frac{(2dr^{n-1}+a)}{1-r} + \frac{dr^n}{2(1-r)} - \frac{dr}{2(1-r)^2}} \cup G_{\frac{1}{2}[\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]} = G_{(a+(n-1)d)br^{n-1}}$. Therefore, $\mathrm{D}W_y$ is decomposed into G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, ..., $G_{(a+(n-1)d)br^{n-1}}$).

Remark 2.5: The Double wheel graph $\mathrm{D}W_y$ is $(2i, 2j\text{-}1, b, 2k, 2t) -$ Decomposable if and only if $y = \frac{1}{4}[\frac{b(a-(a+(2t-1)d)r^{2t})}{1-r} + \frac{dbr(1-r^{2t-1})}{(1-r)^2}]$, where $t, b, i, j, k \in \mathrm{N}$.

3. Arithmetico Geometric Path Decomposition of Graphs

Definition 3.1: A decomposition (G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, ..., $G_{(a+(n-1)d)br^{n-1}}$) of G is said to be a Arithmetico Geometric Path Decomposition (ACOGPD) or (a, d, b, r, n) - Path Decomposable if

i) G admits ACOGD

ii) Each $G_{(a+(i-1)d)br^{i-1}}$ is a path for each i = 1, 2, ..., n and $a, d, b, r\ (>1) \in \mathrm{N}$.

Theorem 3.2: One point union of t - copies of cycles with order ab, $(a+d)br$, $(a+2d)br^2$, $(a+3d)br^3$, ..., $(a+(n-1)d)br^{n-1}$ admits ACOGPD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, ..., $G_{(a+(n-1)d)br^{n-1}}$), where $a, r, d, b \in \mathrm{N}$ and $n \neq 1$.

Proof: Let u be a vertex with degree Δ. Let $C_1, C_2, C_3, C_4, \ldots, C_t$ be the cycles with order

$ab, (a + d)br, (a + 2d)br^2, (a + 3d)br^3, \ldots, (a+(n-1)d)br^{n-1}$. Let $G_{ab} = (C_1 - e_1) \cup \{e_2\}$, where e_1 is an edge in C_1 which is incident to u and e_2 is an edge in C_2 which is incident to u. Also let $G_{(a+d)br} = (C_2 - e_2) \cup \{e_3\}$, where e_2 is an edge in C_2 which is incident to u and e_3 is an edge in C_3 which is incident to u. Proceeding like this, we get $G_{(a+(n-1)d)br^{n-1}} = (C_t - e_t) \cup \{e_1\}$, where e_t is an edge in C_t which is incident to u and e_1 is an edge in C_1 which is incident to u. Hence one point union of t - copies of cycles admits ACOGPD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$).

Remark 3.3: Let $C_1, C_2, C_3, C_4, \ldots, C_t$ be the cycles with $q = [\frac{b(a-(a+(n-1)d)r^n)}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2}]$. Then $C_1 \cup C_2 \cup C_3 \cup \ldots \cup C_t$ is a connected graph which admits ACOGPD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$), where $a, r, d, b \in$ N and $n \neq 1$.

4. ARITHMETICO GEOMETRIC STAR DECOMPOSITION OF GRAPHS.

Definition 4.1: A decomposition (G_{ab} , $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$) of G is said to be a Arithmetico Geometric Star Decomposition (ACOGSD) or **(a, d, b, r, n) -** Star Decomposable if

(i) G admits ACOGD

(ii) Each $G_{(a+(i-1)d)br^{i-1}}$ is a star for each i = 1, 2, ..., n and $a, d, b, r\ (>1) \in$ N.

Theorem 4.2: Let $C((1 + d)r - 1, 0, (1 + 2d)r^2 - 2, 0, (1 + 3d)r^3 - 2, 0, \ldots, 0, (1 + (n-1)d)r^{n-1} - 1)$ be a caterpillar with $a = b = 1$, r is even and d is odd. Then $C((1 + d)r - 1, 0, (1 + 2d)r^2 - 1, 0, (1 + 3d)r^3 - 1, 0, \ldots, 0, (1+(n-1)d)r^{n-1} - 1)$ admits ACOGSD(G_1, $G_{(1+d)r}$, $G_{(1+2d)r^2}$, $G_{(1+3d)r^3}$, ..., $G_{(1+(n-1)d)r^{n-1}}$) with the orgin or terminus of G_1 is incident with any one of the non supports in the spine path s^* of $C((1 + d)r - 1, 0, (1 + 2d)r^2 - 2, 0, (1 + 3d)r^3 - 2, 0, \ldots, 0, (1 + (n-1)d)r^{n-1} - 1)$ if and only if (i) There are n junction supports in $C((1 + d)r - 1, 0, (1 + 2d)r^2 - 2, 0,$

$(1+3d)r^3 - 2$, 0, …, 0, $(1+(n-1)d)r^{n-1} - 1$) (ii) There is only one junction support is odd.

Proof: Assume that $C((1+d)r - 1$, 0, $(1+2d)r^2 - 2$, 0, $(1+3d)r^3 - 2$, 0, …, 0, $(1+(n-1)d)r^{n-1} - 1$) admits ACOGSD(G_1, $G_{(1+d)r}$, $G_{(1+2d)r^2}$, $G_{(1+3d)r^3}$, …, $G_{(1+(n-1)d)r^{n-1}}$). Given $a = b = 1$, r is even and d is odd. Let $u_1, u_2, u_3,…, u_{2n-3}$ be the vertices of a spine path. Then $u_1, u_3, u_5,…, u_{2n-3}$ are the vertices in $C((1+d)r - 1$, 0, $(1+2d)r^2 - 2$, 0, $(1+3d)r^3 - 2$, 0, …, 0, $(1+(n-1)d)r^{n-1} - 1$) with degrees $(1+d)r$, $(1+2d)r^2$, $(1+3d)r^3$, …, $(1+(n-1)d)r^{n-1}$. Therefore $u_1, u_3, u_5, …, u_{2n-3}$ are distinct and supports. In caterpillar C, the degree of each non support is 2. Suppose G_1 is incident with one of u_i, i =2,4,6,…, $2n-4$. Then that u_i, i = 2, 4, 6, …, $2n-4$ must be a junction support and the degree is odd. It is clear that the spine path s^* having n - junction support. Since $a = b = 1$, r is even and d is odd, then each junction support of u_i, i =1, 3, 5, …, $2n-3$ have even degrees. But one of u_i, i = 2,4,6, …, $2n-4$ must have an odd degree 3. Conversely, the proof is obvious.

Remark 4.3: Let $C(ab - 1, 0, (a+d)r - 2, 0, (a+2d)r^2 - 2, 0, (a+3d)r^3 - 2, 0, …$, $0, (a+(n-1)d)r^{n-1} - 1)$ be a caterpillar with b is even and a (>1), b, r, $d \in$ N. Then $C^*(ab - 1, 0, (a+d)r - 2, 0, (a+2d)r^2 - 2, 0, (a+3d)r^3 - 2$, 0, …, 0, $(a+(n-1)d)r^{n-1} - 1$) admits ACOGSD (G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$) if and only if (i) There are n junction supports in $C(ab - 1$, 0, $(a+d)r - 2$, 0, $(a+2d)r^2 - 2$, 0, $(a+3d)r^3 - 2$, 0, …, 0, $(a+(n-1)d)r^{n-1} - 1$). (ii) All the vertices of junction support is even.

Theorem 4.4: Let ab =1. Then the caterpillar $C(l_1, l_2)$ with

$l_1 = rb\left(\frac{a+d-ar^2+dr^2-r^{2t}(a+d+2td)+r^{2t+2}(a-d+2td)}{(1-r^2)^2}\right)$,

$l_2 = br^2\left(\frac{a+2d-ar^2+dr^2-r^{2t}(a+2d+2td)+r^{2t+2}(a+2td)}{(1-r^2)^2}\right)$ admits ACOGSD (G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, … ,$G_{(a+(n-1)d)br^{n-1}}$) if and only if $n = 2t+1$, $t \in$ N.

Proof: Assume that the caterpillar $C(l_1, l_2)$ admits ACOGSD(G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$).

$$q(C(l_1,l_2)) = l_1 + l_2 + ab$$
$$= rb\left(\frac{a+d-ar^2+dr^2-r^{2t}(a+d+2td)+r^{2t+2}(a-d+2td)}{(1-r^2)^2}\right) +$$
$$r^2b\left(\frac{a+2d-ar^2+dr^2-r^{2t}(a+2d+2td)+r^{2t+2}(a+2td)}{(1-r^2)^2}\right) + ab$$
$$= \frac{b(a-[a+2td]r^{2t+1})}{1-r} + \frac{dbr(1-r^{2t})}{(1-r)^2}.$$

By theorem 2.2, we get $n = 2t + 1$.

Conversely, let w_1 and w_2 be the non pendant vertices of $C(l_1, l_2)$ and $u_1, u_2, u_3, \ldots, u_{l_1}$ be the edges incident to w_1 and $v_1, v_2, v_3, \ldots, v_{l_2}$ be the edges incident to w_2. Then w_1 and w_2 are junction supports. Consider U $=\{u_1, u_2, u_3, \ldots, u_{l_1}\}$ and V $=\{v_1, v_2, v_3, \ldots, v_{l_2}\}$ the path w_1- w_2 is decomposed into G_1. Since $a = 1$, $b = 1$, we have U edges incident to w_1 is decomposed into $(G_{(d+1)r}, G_{(3d+1)r^3}, \ldots, G_{((2t-1)d+1)r^{2t-1}})$ and V edges incident to w_2 is decomposed into $(G_{(2d+1)r^2}, G_{(4d+1)r^4}, \ldots, G_{(2td+1)r^{2t}})$. Therefore, caterpillar $C(l_1, l_2)$ is decomposed into $G_1, G_{(d+1)r}, G_{(2d+1)r^2}, G_{(3d+1)r^3}, \ldots, G_{((2t-1)d+1)r^{2t-1}}, G_{(2td+1)r^{2t}})$.

Remark 4.5:

(i) Let $ab = 2$. Then the caterpillar $C(l_1, 0, l_2)$ with

$l_1 = rb\left(\frac{a+d-ar^2+dr^2-r^{2t}(a+d+2td)+r^{2t+2}(a-d+2td)}{(1-r^2)^2}\right)$,

$l_2 = br^2\left(\frac{a+2d-ar^2+dr^2-r^{2t}(a+2d+2td)+r^{2t+2}(a+2td)}{(1-r^2)^2}\right)$ admits ACOGSD (G_{ab} , $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}, \ldots, G_{(a+(n-1)d)br^{n-1}}$) if and only if $n = 2t + 1$, $t \in$ N.

(ii) Let $ab = 1$. Then the caterpillar $C(l_1, 0, 0, l_2)$ with

$l_1 = rb\left(\frac{a+d-ar^2+dr^2-r^{2t}(a+d+2td)+r^{2t+2}(a-d+2td)}{(1-r^2)^2}\right) -1$,

$l_2 = br^2\left(\frac{a+2d-ar^2+dr^2-r^{2t}(a+2d+2td)+r^{2t+2}(a+2td)}{(1-r^2)^2}\right) - 1$ admits ACOGSD (G_{ab}, $G_{(a+d)br}$, $G_{(a+2d)br^2}$, $G_{(a+3d)br^3}, \ldots, G_{(a+(n-1)d)br^{n-1}}$) if and only if $n = 2t + 1$, $t \in$ N.

(iii) Let $ab = 2$. Then the caterpillar $C(l_1, 0,0,0, l_2)$ with

$l_1 = rb\left(\frac{a+d-ar^2+dr^2-r^{2t}(a+d+2td)+r^{2t+2}(a-d+2td)}{(1-r^2)^2}\right) -1$,

$l_2 = br^2\left(\frac{a+2d-ar^2+dr^2-r^{2t}(a+2d+2td)+r^{2t+2}(a+2td)}{(1-r^2)^2}\right) - 1$ admits ACOGSD (G_{ab}, $G_{(a+d)br}$,

$G_{(a+2d)br^2}$, $G_{(a+3d)br^3}$, …, $G_{(a+(n-1)d)br^{n-1}}$) if and only if $n = 2t + 1, t \in$ N.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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