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TIME DEPENDENT RETRIAL QUEUEING MODEL WITH ORBITAL SEARCH UNDER NON-PREEMPTIVE PRIORITY SERVICES

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Abstract: Transient solution of Single server Non-preemptive priority Retrial model with orbital search is studied using eigenvalues and eigenvectors. In this model, customers are arriving in Poisson process. Arrival rates of low priority and high priority customers are respectively λ_1 and λ_2 . The service times for low and high priority customers follow exponential distribution with parameters μ_1 and μ_2 respectively. Customer finding the system busy, on arrival, goes to the orbit and form a virtual queue. In this model of orbital search, when the server becomes free, he has two options. Either the server search for the customer in the orbit with probability p to provide service for a customer in the orbit or remains idle with the probability $1-p$. Whenever the server is free, customer from orbit try for service, under classical retrial policy with rate σ which follows Poisson process. In this paper, transient solution of average number of customers in the orbit and high priority queue, the probability of server being idle, the probabilities of server being busy with low and high priority customers for various values of $\lambda_1, \lambda_2, \mu_1, \mu_2, \sigma, p$ and t are estimated.

Keywords: retrial; orbit; classical retrial policy; orbital search; transient solution.

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1. INTRODUCTION

Retrial is a branch inherited from Queueing Theory. Customers finding the server busy, on arrival to the system, form a virtual queue with the intention to try for service after some random time. This virtual queue is termed as Orbit. Customers in the orbit retry independently of each other for service by following one of the retrial policy such as Classical retrial policy, Constant retrial policy and Linear retrial policy. Though Retrial Queueing theory was initiated in 1953, the momentum was gained only in the eighties. Falin, Templeton, Artalejo, Choi, Le Gall, Neuts, Stepanov and Yang were the major contributors to the Theory of Retrial Queue. Review of Literature [1], [2], [10], [12], [28] can be found from Surveys and Bibliographies. Falin et al. studied the Single server Retrial queue with priority services.

One of the main objective of Queueing theory is to minimize the idle time of the server. In Retrial Queueing Theory, when the server becomes free, he remains idle until either a low or a high priority customer arrives to the system or a customer from the orbit retry for the service. To minimize this idle time of the server, a new approach is advocated. The technique is that whenever the server becomes free, either he voluntarily search for the customer in the orbit to provide the service with probability p or remain idle with probability $1-p$. This process is called Orbital Search. Artalejo [3] introduced Orbital search and initiated the study in 2002. Dudin, Krishnamoorthy, Joshua and Deepak [8][9][16] extended the study of Orbital search to Retrial systems $M(X)/G/1$ with two types of search of customers, $BMAP/G/1$ and $M/G/1$ with non-persistent customers. In 2006, Chakravarthy et al [20], estimated performance evaluation of Multiserver retrial queue with Orbital search. Gopal Sekar et al [13] studied Single Server Retrial Queueing System with Orbital search under Erlang K Service in 2012.

In many of the applications, Transient solution to a Queueing Problem is considered more relevant than the steady state. Transient solutions to Queueing problem were initially explored by Clarke [5], Saaty [19] and Tacaks [27] in the middle of the Twentieth century. Nearly after three decades, the interest on study of transient solution was evinced by Parthasarathy [17], and Krishnakumar [15]. Many results on Transient solution to Various queueing models are published by Tarabia [25][26],

Kalidass[14], Sudhesh[23][24], Sheriff Ammar[21][22] and Rakesh Kumar[18].

Search of transient solution to Retrial Queueing problem is more difficult and much complicated than that of Queueing Problem. Due to the presence of differential equations, Analytical solution to Transient Retrial Queueing problem is hard to manage. The alternative is to look for Numerical solution. The transitive solution to M/M/1 Retrial queueing system is evolved using Eigen values and Eigenvectors. In this paper, Numerical solution to Transient M/M/1 Retrial Queueing model with non-preemptive priority service and orbital search is obtained using eigenvalues and eigenvectors.

2. MODEL DESCRIPTION

The model taken up for study is Single server Retrial Queueing system with orbital search under non-preemptive priority service. In this priority model, the arrival of low and high priority customers follow Poisson distribution with arrival rates respectively λ_1 and λ_2 . If the server is idle and a low priority customer arrives, then the server provides service to the low priority customer and the customer leaves the system after he gets the service. On the other hand, when a low priority customer, finds the server busy on his arrival, goes to the orbit with the objective to try for service independently after some time. The rate of retrial of customers from orbit follows Poisson Process with retrial rate σ .

The time taken by the server to provide service for low and high priority customers follow exponential distribution. The rates of service to low and high priority customers are respectively μ_1 and μ_2 . An arriving high priority customer to the server may find himself in one of the following situations

When high priority queue is empty

1. When the server is idle, he will get the service immediately and leaves the system.
2. When the server is busy with a low priority customer, as the model is Non-preemptive priority model, the high priority customer wait for the server to complete the service for the low priority customer and then get the service done and leaves the system.

3. When the server is busy with high priority customer, wait for the service to be completed and gets the service and leave the system.

When high priority queue is nonempty

4. Joins the high priority queue.

In this model of Orbital search, in order to minimize the idle time of the server, whenever the server becomes free, he has two options. Either he provides service for a customer in the orbit with chance p or remain idle with chance $1-p$. The time duration to call the low priority customer for service is considered as negligible.

2.1 RETRIAL POLICY

Customers from the Orbit try to get the service under Classical Retrial Policy. In this policy, Customers from the orbit try independently to get service from the server. The random time to get selected for the service is exponentially distributed with rate σ . In the interval $(t, t + \Delta t)$, the probability of retrial, when there are n customers in the orbit, is $n\sigma\Delta t + O(\Delta t)$.

2.2 RANDOM PROCESS

Let $N(t)$, $P(t)$ and $S(t)$ be the random variables representing, the number of customers in the orbit at time t , the number of high priority customers in the queue at time t and the state of the server at time t respectively.

The random process is described as $\langle N(t), P(t), S(t) \rangle$.

$S(t) = 0$ if server is idle at time t

$S(t) = 1$ if server is busy with low priority customer at time t

$S(t) = 2$ if server is busy with high priority customer at time t

The state spaces for M/M/1 retrial queueing with non-preemptive priority service and orbital search are $\{ (u, v, w) / u = 0, 1, 2, 3, \dots ; v = 0; w = 0, 1, 2 \} \cup$

$\{ (u, v, w) / u = 0, 1, 2, 3, \dots ; v = 1, 2, 3, \dots; w = 1, 2 \}$

Let A_{ij} be the matrix corresponding to the state of $N(t) = j$ from the state $N(t) = i$. The infinitesimal generator matrix is $Q = (A_{ij})$.

2.3 TRUNCATION

As the set of State spaces is infinite, the infinitesimal generator matrix will not be a matrix of finite order. Matrix of finite order is desired to facilitate the computation and it is obtained by restricting the number of customers in the high priority queue to s and the number of customers in the orbit needs to be truncated to M so that the probability that will be lost is negligible. In this model, open truncation is applied to overcome the difficulty.

Table 1: Transitions from the state when there are no customers in the orbit

From the state			Event	To the state		
N(t)	P(t)	S(t)		N(t)	P(t)	S(t)
0	0	0	$-(\lambda_1+\lambda_2)$	0	0	0
			λ_1	0	0	1
			λ_2	0	0	2
0	0	1	$-(\lambda_1+\lambda_2+\mu_1)$	0	0	1
			λ_1	1	0	1
			λ_2	0	1	1
			μ_1	0	0	0
0	0	2	$-(\lambda_1+\lambda_2+\mu_2)$	0	0	2
			λ_1	1	0	2
			λ_2	0	1	2
			μ_2	0	0	0
0	1,2,...,s-1	1	$-(\lambda_1+\lambda_2+\mu_1)$	0	i	1
			λ_1	1	i	1
			λ_2	0	i+1	1
			μ_1	0	i-1	2
0	1,2,...,s-1	2	$-(\lambda_1+\lambda_2+\mu_2)$	0	i	2
			λ_1	1	i	2
			λ_2	0	i+1	2
			μ_2	0	i-1	2
0	s	1	$-(\lambda_1+\lambda_2+\mu_1)$	0	s	1
			λ_1	1	s	1
			μ_1	0	s-1	2
0	s	2	$-(\lambda_1+\lambda_2+\mu_2)$	0	s	2
			λ_1	1	s	2
			μ_2	0	s-1	2

2.4 TRANSITIONS

The transitions can be broadly categorized into three cases depending on the state from which it is being transitioned. They are

- (a) Transitions from the state when there are no customers in the orbit
- (b) Transitions from the state when there are at least one customer in the orbit
- (c) Transitions from the state of Truncation level of M number of customers in the orbit

The three cases of transitions are shown in Tables 1, 2 and 3.

Table 2: Transitions from the state when there are at least one customer in the orbit

From the state			Event	To the state		
N(t)	P(t)	S(t)		N(t)	P(t)	S(t)
1,2,3,...	0	0	$-(\lambda_1+\lambda_2+i\sigma)$	i	0	0
			λ_1	i	0	1
			λ_2	i	0	2
			$i\sigma$	i-1	0	1
1,2,3,...	0	1	$-(\lambda_1+\lambda_2+\mu_1)$	i	0	1
			λ_1	i+1	0	1
			λ_2	i	1	1
			$(1-p)\mu_1$	i	0	0
1,2,3,...	0	2	$-(\lambda_1+\lambda_2+\mu_2)$	i	0	2
			λ_1	i+1	0	2
			λ_2	i	1	2
			$(1-p)\mu_2$	i	0	0
1,2,3,...	1,2,3,...,s-1	1	$-(\lambda_1+\lambda_2+\mu_1)$	i	j	1
			λ_1	i+1	j	1
			λ_2	i	j+1	1
			μ_1	i	j-1	2
1,2,3,...	1,2,3,...,s-1	2	$-(\lambda_1+\lambda_2+\mu_2)$	i	J	2
			λ_1	i+1	j	2
			λ_2	i	j+1	2
			μ_2	i	j-1	2
1,2,3,...	s	1	$-(\lambda_1 + \mu_1)$	I	s	1
			λ_1	i+1	s	1
			μ_1	I	s-1	2
1,2,3,...	s	2	$-(\lambda_1 + \mu_2)$	i	s	2
			λ_1	i+1	s	2
			μ_1	i	s-1	2

Table 3: Transitions from the state of Truncation level

From the state			Event	To the state		
N(t)	P(t)	S(t)		N(t)	P(t)	S(t)
M	0	0	$-(\lambda_1+\lambda_2)$	M	0	0
			λ_2	M	0	2
			$M\sigma$	M-1	0	1
M	0	1	$-(\lambda_1+\lambda_2+\mu_1)$	M	0	1
			λ_2	M	1	1
			$(1-p)\mu_1$	M	0	0
			$p\mu_1$	M-1	0	1
M	0	2	$-(\lambda_1+\lambda_2+\mu_2)$	M	0	2
			λ_2	M	1	2
			$(1-p)\mu_2$	M	0	0
			$p\mu_2$	M-1	0	1
M	1,2,3,...,s-1	1	$-(\lambda_1+\lambda_2+\mu_1)$	M	j	1
			λ_2	M	j+1	1
			μ_1	M	j-1	2
M	1,2,3,...,s-1	2	$-(\lambda_1+\lambda_2+\mu_2)$	M	J	2
			λ_2	M	j+1	2
			μ_2	M	j-1	2
M	s	1	$-(\lambda_1 + \mu_1)$	M	s	1
			μ_1	M	s-1	2
M	s	2	$-(\lambda_1 + \mu_2)$	M	s	2
			μ_2	M	s-1	2

2.5 GOVERNING DIFFERENTIAL EQUATIONS

Let $P_{ijk}(t)$ stands for the Probability at time t for $N(t) = i$, $P(t) = j$ and $S(t) = k$. Then from the transition tables Table 1,2,3, the Chapman – Kolmogorov Difference Differential equations for the Single server retrial queueing model with non-preemptive priority service with orbital search are obtained as shown in system of equations (1). The equations can be written in matrix form as

$$\frac{dX(t)}{dt} = X(t)Q \quad \dots (2)$$

where $X(t) = (X_0(t), X_1(t), X_2(t), \dots, X_M(t))$ with

$$X_i(t) = (P_{i00}(t), P_{i01}(t), P_{i02}(t), P_{i11}(t), P_{i12}(t), P_{i21}(t), P_{i22}(t), \dots, P_{is1}(t), P_{is2}(t))$$

for $i = 0,1,2,3,\dots,M$

2.6 CHAPMAN- KOLMOGOROV GOVERNING EQUATIONS

$$\begin{aligned}
 P_{000}'(t) &= -(\lambda_1 + \lambda_2)P_{000}(t) + \mu_1 P_{001}(t) + \mu_2 P_{002}(t) \\
 P_{001}'(t) &= -(\lambda_1 + \lambda_2 + \mu_1)P_{001}(t) + \lambda_1 P_{000}(t) + \sigma P_{100}(t) \\
 P_{002}'(t) &= -(\lambda_1 + \lambda_2 + \mu_2)P_{002}(t) + \lambda_2 P_{000}(t) + \mu_1 P_{011}(t) + \mu_2 P_{012}(t) \\
 P_{011}'(t) &= -(\lambda_1 + \lambda_2 + \mu_1)P_{011}(t) + \lambda_2 P_{001}(t) \\
 P_{0s-11}'(t) &= -(\lambda_1 + \lambda_2 + \mu_1)P_{0s-11}(t) + \lambda_2 P_{0s-21}(t) \\
 P_{0s1}'(t) &= -(\lambda_1 + \mu_1)P_{0s1}(t) + \lambda_2 P_{0s-11}(t) \\
 P_{012}'(t) &= -(\lambda_1 + \lambda_2 + \mu_2)P_{012}(t) + \lambda_2 P_{002}(t) + \mu_1 P_{021}(t) + \mu_2 P_{022}(t) \\
 P_{0s-12}'(t) &= -(\lambda_1 + \lambda_2 + \mu_2)P_{0s-12}(t) + \lambda_2 P_{0s-22}(t) + \mu_1 P_{0s1}(t) + \mu_2 P_{0s2}(t) \\
 P_{0s2}'(t) &= -(\lambda_1 + \mu_2)P_{0s2}(t) + \lambda_2 P_{0s-12}(t) \\
 P_{i00}'(t) &= -(\lambda_1 + \lambda_2 + i\sigma)P_{i00}(t) + (1-p)\mu_1 P_{i01}(t) + (1-p)\mu_2 P_{i02}(t) \\
 P_{i01}'(t) &= -(\lambda_1 + \lambda_2 + \mu_1)P_{i01}(t) + \lambda_1 P_{i-101}(t) + \lambda_1 P_{i00}(t) + (i+1)\sigma P_{i+100}(t) + p\mu_1 P_{i+101}(t) + p\mu_2 P_{i+102}(t) \\
 P_{i11}'(t) &= -(\lambda_1 + \lambda_2 + \mu_1)P_{i11}(t) + \lambda_1 P_{i-111}(t) + \lambda_2 P_{i01}(t) \\
 P_{is-11}'(t) &= -(\lambda_1 + \lambda_2 + \mu_1)P_{is-11}(t) + \lambda_1 P_{i-1s-11}(t) + \lambda_2 P_{is-21}(t) \\
 P_{is1}'(t) &= -(\lambda_1 + \mu_1)P_{is1}(t) + \lambda_1 P_{i-1s1}(t) + \lambda_2 P_{is-11}(t) \\
 P_{i02}'(t) &= -(\lambda_1 + \lambda_2 + \mu_2)P_{i02}(t) + \lambda_1 P_{i-102}(t) + \lambda_2 P_{i00}(t) + \mu_1 P_{i11}(t) + \mu_2 P_{i12}(t) \\
 P_{i12}'(t) &= -(\lambda_1 + \lambda_2 + \mu_2)P_{i12}(t) + \lambda_1 P_{i-112}(t) + \lambda_2 P_{i02}(t) + \mu_1 P_{i11}(t) + \mu_2 P_{i22}(t) \\
 P_{is-12}'(t) &= -(\lambda_1 + \lambda_2 + \mu_2)P_{is-12}(t) + \lambda_1 P_{i-1s-12}(t) + \lambda_2 P_{is-22}(t) + \mu_1 P_{is1}(t) + \mu_2 P_{is2}(t) \\
 P_{is2}'(t) &= -(\lambda_1 + \mu_2)P_{is2}(t) + \lambda_1 P_{i-1s2}(t) + \lambda_2 P_{is-12}(t) \\
 P_{M00}'(t) &= -\lambda_2 P_{M00}(t) + (1-p)\mu_1 P_{M01}(t) + (1-p)\mu_2 P_{M02}(t) \\
 P_{M01}'(t) &= -(\lambda_2 + \mu_1)P_{M01}(t) + \lambda_1 P_{M-101}(t) + \lambda_1 P_{M00}(t) \\
 P_{M11}'(t) &= -(\lambda_2 + \mu_1)P_{M11}(t) + \lambda_1 P_{M-111}(t) + \lambda_2 P_{M01}(t) \\
 P_{Ms-11}'(t) &= -(\lambda_2 + \mu_1)P_{Ms-11}(t) + \lambda_1 P_{M-1s-11}(t) + \lambda_2 P_{Ms-21}(t) \\
 P_{Ms1}'(t) &= -\mu_1 P_{Ms1}(t) + \lambda_1 P_{M-1s1}(t) + \lambda_2 P_{Ms-11}(t) \\
 P_{M02}'(t) &= -(\lambda_2 + \mu_2)P_{M02}(t) + \lambda_1 P_{M-102}(t) + \lambda_2 P_{M00}(t) + \mu_1 P_{M11}(t) + \mu_2 P_{M12}(t) \\
 P_{M12}'(t) &= -(\lambda_2 + \mu_2)P_{M12}(t) + \lambda_1 P_{M-112}(t) + \lambda_2 P_{M02}(t) + \mu_2 P_{M22}(t) \\
 P_{Ms-12}'(t) &= -(\lambda_2 + \mu_2)P_{Ms-12}(t) + \lambda_1 P_{M-1s-12}(t) + \lambda_2 P_{Ms-22}(t) + \mu_2 P_{Ms2}(t) \\
 P_{Ms2}'(t) &= -\mu_2 P_{Ms2}(t) + \lambda_1 P_{M-1s2}(t) + \lambda_2 P_{Ms-12}(t)
 \end{aligned}
 \tag{1}$$

2.7 TRUNCATED GENERATOR MATRIX

The truncated generator matrix is

$$Q = \begin{pmatrix} A_{00} & A_0 & 0 & \dots & 0 & 0 \\ A_{10} & A_{11} & A_0 & \dots & 0 & 0 \\ 0 & A_{21} & A_{22} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & A_{M-1M-1} & A_0 \\ 0 & 0 & 0 & \dots & A_{MM-1} & A_{MM} \end{pmatrix}$$

The matrices A_{ij} can be obtained from the system of Chapman – Kolmogorov equations.

Here $A_0 = A_{01} = A_{12} = \dots = A_{M-1,M}$

The solution of the system of equations (2) is

$$X(t) = X(0)e^{tQ} \text{ where } [X(0)] = (1, 0, 0, \dots, 0)$$

3. DESCRIPTION OF COMPUTATIONAL METHOD

To obtain the solution, the following sequence is used

1. Orbit size is truncated to a sufficiently large value of the number of customers in the orbit, say M , so that the loss probability is small.
2. Generator matrix Q is a square matrix of order $(2s+3)(M+1)$. The time dependent probabilities can be written as $X(t) = ((X_0(t), X_1(t), X_2(t), \dots, X_M(t)))$ where

$$X_i(t) = (P_{i00}(t), P_{i01}(t), P_{i02}(t), P_{i11}(t), P_{i12}(t), P_{i21}(t), P_{i22}(t), \dots, P_{is1}(t), P_{is2}(t)) \text{ for } i = 0, 1, 2, 3, \dots, M$$

3. Eigen values $d_1, d_2, \dots, d_{(2s+3)(M+1)}$ of tQ and the corresponding Eigenvectors

$$c_1, c_2, \dots, c_{(2s+3)(M+1)} \text{ are computed.}$$

4. Form the square matrix C with columns as of eigenvectors of tQ .
5. Form the Diagonal matrix D with $d_1, d_2, \dots, d_{(2s+3)(M+1)}$ as diagonal elements.
6. Diagonalize the matrix tQ using D and C and find e^{tQ}

7. The first column of e^{tQ} provides $X(t)$.

4. TIME DEPENDENT SYSTEM PERFORMANCE MEASURES

a. The probability mass function of Server state

Let $S(t)$ be the random variable which represents the server state at time t . In this model $S(t)$ takes the values $0, 1, 2$.

1) Probability that the server is ideal at time t is $P_0(t) = \sum_{i=0}^{\infty} P_{i00}(t)$

2) Probability that the server is busy with a low priority customer at time t is

$$P_1(t) = \sum_{i=0}^{\infty} \sum_{j=0}^s P_{ij1}(t)$$

3) Probability that the server is busy with a high priority customer at time t is

$$P_2(t) = \sum_{i=0}^{\infty} \sum_{j=0}^s P_{ij2}(t)$$

b. The probability mass function of number of customers in the orbit

Let $N(t)$ be the random variable which represents the number of low customers in the orbit at time t . $N(t)$ takes the values $0, 1, 2, 3, \dots$

1) Probability of no customers in the orbit at time $t = PL_0(t) = \sum_{j=0}^s \sum_{l=1}^2 P_{0jl}(t) + P_{000}(t)$

2) Probability of n customers in the orbit at time $t = PL_n(t) = \sum_{j=0}^s \sum_{l=1}^2 P_{njl}(t) + P_{n00}(t)$

c. The Probability mass function of number of high priority customers in front of the service station at time t

Let $P(t)$ be the random variable which represents the number of high priority customers in the queue at time t . In this model we assume that the capacity of high priority customers in the queue is finite and $P(t)$ takes the values $0, 1, 2, 3, \dots, s$

1) Probability of no customers in the high priority queue at time t is $PH_0(t) = \sum_{i=0}^{\infty} \sum_{l=0}^2 P_{i0l}(t)$

2) Probability of n customers in the high priority queue at time t is $PH_n(t) = \sum_{i=0}^{\infty} \sum_{l=1}^2 P_{inl}(t)$

d. The Mean Priority Queue Length at time t

$$MPQL(t) = \sum_{j=1}^s j \left[\sum_{i=0}^{\infty} \sum_{l=1}^2 P_{ijl}(t) \right] = \sum_{j=1}^s j PH_j(t)$$

e. The Mean Number of Customers in the Orbit at time t

$$MNCO(t) = \sum_{i=0}^{\infty} i \left[\sum_{j=0}^s \sum_{l=1}^2 P_{ijl}(t) + P_{i00}(t) \right] = \sum_{i=0}^{\infty} i PL_i(t)$$

f. The probability that the orbiting customer is blocked at time t

$$\text{Blocking Probability} = \sum_{i=1}^{\infty} \sum_{j=0}^s \sum_{l=1}^2 P_{ijl}(t)$$

g. The probability that an arriving customer (high or low) enters the service station

immediately at time t = $\sum_{i=0}^{\infty} P_{i00}(t)$.

5. TIME DEPENDENT NUMERICAL STUDY

The time dependent system performance measures of this model have been done and expressed in the form of tables which are shown below using the time dependent probability vector $X(t)$ for $\mu_1 = 10$, $\mu_2 = 20$, $\sigma = 10$, $s = 4$, $p = 0.3$ and various values λ_1, λ_2 .

Tables 4, 5 and 6 provide time dependent number of customers in the orbit at time t for different values of λ_1, λ_2 .

Observation

$$\text{As t increases, } PL_n(t) \rightarrow PL_n$$

Where PL_n is the Steady state probability that there are n customers in the orbit. These results coincides with Single server retrial queueing system with non-preemptive priority services.

- The sequence $\{PL_n(t)\} \rightarrow 0$ as $n \rightarrow \infty$ for all values of t .

Tables 7, 8 and 9 provide time dependent number of customers in the high priority queue in front of the service station at time t

Observation

- As the value of t increases the Transient Probabilities $PH_n(t) \rightarrow PH_n$ where PH_n is the Steady state probability that there are n customers in the high priority queue. These results coincides with Single server retrial queueing system with non-preemptive priority services.

Tables 10, 11 and 12 provide the time dependent probabilities that, the server is idle, busy with low priority customer, busy with high priority customer, MNCO and MPQL for different rate of arrivals times at time t

Observation

1. As the value of t increases and for various values of $\lambda_1, \lambda_2, \mu_1, \mu_2$ and σ

$$P_0(t) \rightarrow P_0, P_1(t) \rightarrow P_1, MNCO(t) \rightarrow MNCO, MPQL(t) \rightarrow MPQL \text{ where}$$

MNCO is the mean number of low priority customers in the Orbit and MPQL is the mean number of high priority customers in the steady state.

$$MNCO = \frac{\lambda_1(\lambda_1\beta_{1,2} + \lambda_2\beta_{2,2})}{2\left(1 - \frac{\lambda_2}{\mu_2}\right)\left(1 - \frac{\lambda_1}{\mu_1} - \frac{\lambda_2}{\mu_2}\right)} + \frac{\lambda_1\left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}\right)}{\sigma\left(1 - \frac{\lambda_1}{\mu_1} - \frac{\lambda_2}{\mu_2}\right)}$$

$$MPQL = \frac{\lambda_2(\lambda_1\beta_{1,2} + \lambda_2\beta_{2,2})}{2\left(1 - \frac{\lambda_2}{\mu_2}\right)} \text{ where } \beta_{1,2} = \frac{2}{\mu_1^2}, \beta_{2,2} = \frac{2}{\mu_2^2}$$

2. $P_0(t)$ decreases as arrival rates λ_1, λ_2 increases for all values of t
3. $P_1(t)$ increases as primary arrival rate λ_1 increases for all values t
4. $P_2(t)$ increases as arrival rate λ_2 increases for all values t
5. $MNCO(t)$ increases as arrival rate λ_1 increases for all values of t

6. $MPQL(t)$ increases as arrival rate λ_2 increases for all values of t

6. SPECIAL CASES

1. As $p \rightarrow 0$, the model coincides with the Transient Single server Retrial queueing model with non-preemptive priority service as discussed by Damodaran et al, [7].
2. As $p \rightarrow 0$ and $t \rightarrow \infty$, this model coincides with the Single server Retrial queueing model with non-preemptive priority service as discussed by Ayyappan et al, [4].

Table 4: Time dependent Probability distribution of number of customers in the orbit for $\lambda_1 = 1$ and $\lambda_2 = 4$.

t	PL ₀ (t)	PL ₁ (t)	PL ₂ (t)	PL ₃ (t)	PL ₄ (t)	PL ₅ (t)	PL ₆ (t)
1	0.947000	0.047473	0.004897	0.000558	0.000064	0.000007	0.000001
2	0.945290	0.048652	0.005292	0.000664	0.000088	0.000012	0.000002
3	0.945220	0.048696	0.005310	0.000670	0.000090	0.000012	0.000002
4	0.945217	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
5	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
6	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
7	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
8	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
9	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
10	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
11	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
12	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
13	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
14	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002
15	0.945216	0.048698	0.005311	0.000670	0.000090	0.000012	0.000002

TIME DEPENDENT RETRIAL QUEUEING MODEL

Table 5: Time dependent Probability distribution of number of customers in the orbit for $\lambda_1 = 3$ and $\lambda_2=4$.

t	PL₀(t)	PL₁(t)	PL₂(t)	PL₃(t)	PL₄(t)	PL₅(t)	PL₆(t)
1	0.747413	0.167677	0.056631	0.019253	0.006300	0.001951	0.000568
2	0.720169	0.173634	0.064791	0.025325	0.009937	0.003850	0.001460
3	0.714706	0.174298	0.066194	0.026578	0.010835	0.004418	0.001790
4	0.713311	0.174413	0.066520	0.026891	0.011075	0.004583	0.001894
5	0.712913	0.174438	0.066607	0.026979	0.011144	0.004632	0.001926
6	0.712791	0.174444	0.066632	0.027005	0.011165	0.004647	0.001937
7	0.712753	0.174446	0.066640	0.027013	0.011172	0.004652	0.001940
8	0.712741	0.174446	0.066643	0.027015	0.011174	0.004654	0.001941
9	0.712736	0.174446	0.066643	0.027016	0.011175	0.004654	0.001942
10	0.712735	0.174446	0.066644	0.027017	0.011175	0.004655	0.001942
11	0.712735	0.174446	0.066644	0.027017	0.011175	0.004655	0.001942
12	0.712734	0.174446	0.066644	0.027017	0.011175	0.004655	0.001942
13	0.712734	0.174446	0.066644	0.027017	0.011175	0.004655	0.001942
14	0.712734	0.174446	0.066644	0.027017	0.011175	0.004655	0.001942
15	0.712734	0.174446	0.066644	0.027017	0.011175	0.004655	0.001942

Table 6: Time dependent Probability distribution of number of customers in the orbit for $\lambda_1 = 6$ and $\lambda_2 = 4$.

t	PL ₀ (t)	PL ₁ (t)	PL ₂ (t)	PL ₃ (t)	PL ₄ (t)	PL ₅ (t)	PL ₆ (t)
1	0.402097	0.225934	0.151802	0.096774	0.057989	0.032619	0.017236
2	0.309378	0.194367	0.148665	0.111011	0.080241	0.056058	0.037851
3	0.272949	0.177257	0.141124	0.110780	0.085058	0.063799	0.046752
4	0.253684	0.167194	0.135556	0.108881	0.085979	0.066679	0.050797
5	0.241958	0.160734	0.131612	0.107050	0.085854	0.067831	0.052813
6	0.234210	0.156325	0.128767	0.105538	0.085447	0.068285	0.053889
7	0.228805	0.153183	0.126666	0.104333	0.084996	0.068437	0.054492
8	0.224889	0.150872	0.125081	0.103377	0.084575	0.068452	0.054841
9	0.221971	0.149129	0.123865	0.102617	0.084207	0.068405	0.055047
10	0.219747	0.147790	0.122917	0.102010	0.083893	0.068335	0.055169
11	0.218025	0.146745	0.122169	0.101521	0.083628	0.068257	0.055241
12	0.216670	0.145918	0.121572	0.101125	0.083406	0.068181	0.055282
13	0.215594	0.145258	0.121092	0.100802	0.083219	0.068109	0.055304
14	0.214730	0.144726	0.120703	0.100537	0.083063	0.068045	0.055315
15	0.214030	0.144294	0.120385	0.100319	0.082932	0.067988	0.055318

TIME DEPENDENT RETRIAL QUEUEING MODEL

Table 7: Time dependent Probability distribution of number of high priority customers for $\lambda_1 = 1$ and $\lambda_2 = 4$.

t	PH₀(t)	PH₁(t)	PH₂(t)	PH₃(t)	PH₄(t)
1	0.926711	0.054930	0.013789	0.003548	0.001023
2	0.925996	0.055339	0.013979	0.003627	0.001058
3	0.925970	0.055355	0.013986	0.003630	0.001059
4	0.925969	0.055355	0.013987	0.003630	0.001059
5	0.925969	0.055355	0.013987	0.003630	0.001059
6	0.925969	0.055355	0.013987	0.003630	0.001059
7	0.925969	0.055355	0.013987	0.003630	0.001059
8	0.925969	0.055355	0.013987	0.003630	0.001059
9	0.925969	0.055355	0.013987	0.003630	0.001059
10	0.925969	0.055355	0.013987	0.003630	0.001059
11	0.925969	0.055355	0.013987	0.003630	0.001059
12	0.925969	0.055355	0.013987	0.003630	0.001059
13	0.925969	0.055355	0.013987	0.003630	0.001059
14	0.925969	0.055355	0.013987	0.003630	0.001059
15	0.925969	0.055355	0.013987	0.003630	0.001059

Table 8: Time dependent Probability distribution of number of high priority customers for $\lambda_1 = 3$ and $\lambda_2 = 4$.

t	$PH_0(t)$	$PH_1(t)$	$PH_2(t)$	$PH_3(t)$	$PH_4(t)$
1	0.865493	0.097181	0.027276	0.007651	0.002398
2	0.859360	0.101042	0.028782	0.008200	0.002616
3	0.858196	0.101793	0.029061	0.008297	0.002653
4	0.857903	0.101984	0.029130	0.008321	0.002662
5	0.857820	0.102038	0.029150	0.008328	0.002665
6	0.857795	0.102054	0.029156	0.008330	0.002665
7	0.857787	0.102060	0.029158	0.008330	0.002666
8	0.857784	0.102061	0.029158	0.008331	0.002666
9	0.857783	0.102062	0.029159	0.008331	0.002666
10	0.857783	0.102062	0.029159	0.008331	0.002666
11	0.857783	0.102062	0.029159	0.008331	0.002666
12	0.857783	0.102062	0.029159	0.008331	0.002666
13	0.857783	0.102062	0.029159	0.008331	0.002666
14	0.857783	0.102062	0.029159	0.008331	0.002666
15	0.857783	0.102062	0.029159	0.008331	0.002666

TIME DEPENDENT RETRIAL QUEUEING MODEL

Table 9: Time dependent Probability distribution of number of high priority customers for $\lambda_1 = 6$ and $\lambda_2 = 4$.

t	PH₀(t)	PH₁(t)	PH₂(t)	PH₃(t)	PH₄(t)
1	0.795126	0.145844	0.042732	0.012335	0.003963
2	0.776915	0.157724	0.047034	0.013808	0.004518
3	0.769454	0.162700	0.048754	0.014372	0.004721
4	0.765357	0.165451	0.049689	0.014674	0.004828
5	0.762793	0.167180	0.050272	0.014860	0.004894
6	0.761062	0.168351	0.050664	0.014985	0.004938
7	0.759834	0.169182	0.050942	0.015074	0.004968
8	0.758932	0.169793	0.051146	0.015138	0.004991
9	0.758252	0.170254	0.051299	0.015187	0.005008
10	0.757730	0.170609	0.051417	0.015224	0.005021
11	0.757321	0.170886	0.051509	0.015253	0.005031
12	0.756998	0.171106	0.051581	0.015276	0.005039
13	0.756740	0.171281	0.051640	0.015294	0.005045
14	0.756531	0.171423	0.051686	0.015309	0.005050
15	0.756362	0.171538	0.051725	0.015321	0.005055

Table 10: Time dependent Probability distribution of number of high priority customers for $\lambda_1 = 1$ and $\lambda_2 = 4$.

t	P₀(t)	P₁(t)	P₂(t)	MNCO(t)	MPQL(t)
1	0.701702	0.098778	0.199520	0.059237	0.097242
2	0.700269	0.099950	0.199780	0.061652	0.098413
3	0.700215	0.099998	0.199788	0.061759	0.098454
4	0.700212	0.100000	0.199788	0.061764	0.098456
5	0.700212	0.100000	0.199788	0.061765	0.098456
6	0.700212	0.100000	0.199788	0.061765	0.098456
7	0.700212	0.100000	0.199788	0.061765	0.098456
8	0.700212	0.100000	0.199788	0.061765	0.098456
9	0.700212	0.100000	0.199788	0.061765	0.098456
10	0.700212	0.100000	0.199788	0.061765	0.098456
11	0.700212	0.100000	0.199788	0.061765	0.098456
12	0.700212	0.100000	0.199788	0.061765	0.098456
13	0.700212	0.100000	0.199788	0.061765	0.098456
14	0.700212	0.100000	0.199788	0.061765	0.098456
15	0.700212	0.100000	0.199788	0.061765	0.098456

TIME DEPENDENT RETRIAL QUEUEING MODEL

Table 11: Time dependent Probability distribution of number of high priority customers for $\lambda_1 = 3$ and $\lambda_2 = 4$.

t	$P_0(t)$	$P_1(t)$	$P_2(t)$	MNCO(t)	MPQL(t)
1	0.519403	0.282335	0.198262	0.378577	0.184279
2	0.504563	0.296144	0.199294	0.453230	0.193670
3	0.501604	0.298968	0.199427	0.471603	0.195419
4	0.500847	0.299697	0.199456	0.476792	0.195856
5	0.500630	0.299906	0.199464	0.478364	0.195980
6	0.500564	0.299970	0.199466	0.478862	0.196017
7	0.500543	0.299990	0.199467	0.479024	0.196029
8	0.500537	0.299997	0.199467	0.479078	0.196033
9	0.500534	0.299999	0.199467	0.479096	0.196034
10	0.500534	0.300000	0.199467	0.479103	0.196035
11	0.500533	0.300000	0.199467	0.479105	0.196035
12	0.500533	0.300000	0.199467	0.479105	0.196035
13	0.500533	0.300000	0.199467	0.479106	0.196035
14	0.500533	0.300000	0.199467	0.479106	0.196035
15	0.500533	0.300000	0.199467	0.479106	0.196035

Table 12: Time dependent Probability distribution of number of high priority customers for $\lambda_1 = 6$ and $\lambda_2 = 4$.

t	$P_0(t)$	$P_1(t)$	$P_2(t)$	MNCO(t)	MPQL(t)
1	0.308638	0.494814	0.196548	1.439031	0.284165
2	0.260944	0.540765	0.198292	2.177038	0.311289
3	0.240470	0.560870	0.198660	2.637822	0.322207
4	0.229036	0.572164	0.198800	2.956663	0.328164
5	0.221815	0.579317	0.198869	3.189648	0.331881
6	0.216912	0.584182	0.198907	3.365842	0.334386
7	0.213420	0.587650	0.198930	3.502296	0.336161
8	0.210849	0.590206	0.198945	3.609844	0.337463
9	0.208907	0.592137	0.198955	3.695760	0.338444
10	0.207412	0.593626	0.198962	3.765137	0.339198
11	0.206242	0.594790	0.198968	3.821649	0.339786
12	0.205315	0.595713	0.198971	3.868017	0.340252
13	0.204574	0.596452	0.198974	3.906294	0.340624
14	0.203974	0.597049	0.198976	3.938057	0.340924
15	0.203487	0.597535	0.198978	3.964533	0.341168

3. As $p \rightarrow 0$ and $\lambda_2 \rightarrow 0$ this model coincides with Transient single server retrial queueing model discussed Damodaran et al [6]
4. As $p \rightarrow 0$, $\lambda_2 \rightarrow 0$ and $t \rightarrow \infty$, this model coincides with Falin[11].
5. As $p \rightarrow 0$, $\lambda_2 \rightarrow 0$ and $\sigma \rightarrow \infty$, this model coincides with Transient behavior of single server queueing model discussed by Parthasarathy [17]

7. CONCLUSION

In this paper, with the intention of reducing the idle time of the server, Orbital search in the case of Single Server Retrial queueing system with non-preemptive priority is studied. Transient solution to the M/M/1 Retrial Queueing system is obtained using eigenvalues and eigenvectors. System performance measures and Probability distributions of customers and System performance measures are determined.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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