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SOME OUTCOMES ON b-METRIC SPACE

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Abstract: In this article we generate a common fixed point theorem in b-metric space by employing the conditions CLR-property and weakly compatible mappings. Further our result is also substantiated by a suitable example.

Keywords: common fixed point; b-metric space; weakly compatible mappings and CLR-Property.

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1. INTRODUCTION

Fixed point theory plays vital role in mathematics and it has main areas of research in analysis. In the present scenario several fixed point theorems have been evolved on different platforms. Junck[8] proved many results by introducing the concept of compatible mappings in metric space. Thereafter many authors [5], [6] and [7] proved common fixed point theorems using weaker conditions. In this process b-metic space turned out as one of the generalizations of

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metric space. The concept of $b-$metric space was introduced by Czerwik [2]. In the recent past some more common fixed point theorems like [2],[3] and [4] came into existence in b-metric space using several conditions. J.R. Roshan, N. Shobkolaei, S. Sedghi and M. Abbas [1] proved a common fixed point theorem using compatible and continuous mappings in b-metric space. In this paper we extend their result by using some weaker conditions such as weakly compatible mappings and CLR-property.

2. PRELIMINARIES

2.1 Definition [2]:

A function $d : X \times X \to R^+$ where X is a nonempty set and $k \geq 1$ is a b-metric if and only if for each $\alpha, \beta, \gamma \in X$, the following conditions are satisfied

(b1) $d(\alpha, \beta) = 0 \Leftrightarrow \alpha = \beta$

(b2) $d(\alpha, \beta) = d(\beta, \alpha)$

(b3) $d(\alpha, \gamma) \leq k[d(\alpha, \beta) + d(\beta, \gamma)]$.

Then the pair (X, d) is called a $b-$metric space.

2.2 Definition [1]:

Suppose (X, d) is a $b-$metric space. Then the sequence $\{\alpha_j\}$ in X is said to be

(i) convergent if and only if there exists $\alpha \in X$ such that $d(\alpha_j, \alpha) \to 0$ *as* $j \to \infty$.

(ii)Cauchy $\Leftrightarrow d(\alpha_j, \alpha_l) \to 0$, *as* $j, l \to \infty$.

2.3 Definition:

Mappings G and H defined on $b-$metric space (X, d) then the pair (G, H) is called weakly commuting on X if $d(GH\alpha, HG\alpha) \leq d(G\alpha, H\alpha)\ \forall\ \alpha \in X$.

2.4 Definition [5]:

In a $b-$ metric space (X,d) we define mappings G and H are compatible if $d(GH\alpha_k, HG\alpha_k) = 0$ as $k \to \infty$ whenever $\{\alpha_k\}$ is a sequence in X such that $G\alpha_k = H\alpha_k = \mu$ for some $\mu \in X$.

2.5 Definition:

Suppose the mappings G and H of a $b-$metric space (X,d) in which if $G\mu = H\mu$ for some $\mu \in X$ such that $GH\mu = HG\mu$ holds then G and H are known as weakly compatible mappings.

2.6 Definition:

Suppose the mappings G and H of a $b-$metric space (X,d) in which if there exists a sequence $\{\alpha_k\}$ in X for some $\mu \in X$ such that $\lim_{n\to\infty} G\alpha_k = \lim_{n\to\infty} H\alpha_k = H\mu$ then G and H are known as common limit in the range of H property and it is denoted by $CLR_H -$ Property.

Now we discuss an example on $CLR_H -$Property.

Example:

Take $X = [0,1]$ is a b - metric space with $d(\alpha,\beta) = |\alpha - \beta|^2$, *where* $k = 2$.

Define $G(\alpha) = \begin{cases} \dfrac{1-2\alpha}{3}, if \ 0 < \alpha \le \dfrac{1}{5} \\ 2+3\alpha \quad \text{if} \ \ \alpha > \dfrac{1}{5} \end{cases}$ and $H(\alpha) = \begin{cases} \dfrac{1+\alpha}{6} =, if\, 0 < \alpha \le \dfrac{1}{5} \\ 2\alpha+3, \text{if} \quad \alpha > \dfrac{1}{5} \end{cases}$

Take a sequence $\{\alpha_j\}$ *as* $\alpha_j = \frac{1}{5} - \frac{1}{j}$ for $j \ge 0$.

$$\text{Now } G\alpha_k = G\left(\frac{1}{5}-\frac{1}{k}\right) = \left(\frac{1-2\left(\frac{1}{5}-\frac{1}{j}\right)}{3}\right) = \left(\frac{\frac{3}{5}-\frac{2}{j}}{3}\right) = \left(\frac{1}{5}-\frac{2}{3j}\right) = \frac{1}{5} \ as \ \ j \to \infty$$

$$\text{and } \ H\alpha_k = H\left(\frac{1}{5}-\frac{1}{j}\right) = \frac{1+\left(\frac{1}{5}-\frac{1}{j}\right)}{6} = \frac{\left(\frac{6}{5}-\frac{1}{k}\right)}{6} = \left(\frac{1}{5}-\frac{1}{6k}\right) = \frac{1}{5} \text{ as } j \to \infty.$$

Hence, as $j \to \infty$, *we get* $G\alpha_j = H\alpha_j = \frac{1}{5}. = \mathrm{H}\left(\frac{1}{5}\right)$ where $\frac{1}{5} \in$ X.

Thus the pair (G,H) satisfies CLR_H – property.

The following theorem was proved in [1].

2.7 Theorem:

Suppose that the four self maps f,g,S and T on a complete $b-$metric space (X,d) satisfying the following conditions:

(C1) $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$

(C2) $d(f\alpha, g\beta) \le \frac{q}{k^4} \max\left\{d(S\alpha, T\beta), d(f\alpha, S\alpha), d(g\beta, T\beta), \frac{1}{2}\left(d(S\alpha, g\beta) + d(f\alpha, T\beta)\right)\right\}$

holds for every $\alpha, \beta \in X$ with $q \in (0,1)$.

(C3) T and S are continuous

(C4) the pair of maps (f,S) and (g,T) are compatible mappings.

Then the four maps f,g,S and T have a unique common fixed point.

Now we prove the existence of the above Theorem under some modified conditions.

For this we need to recall the following lemma which is useful in the proof of our main result.

2.8 Lemma:[3] Assume that (X,d) is a $b-$metric space with $k \ge 1$ and suppose that the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ are b-convergent to α and β respectively. Then we have

$$\frac{1}{k^2} d(\alpha, \beta) \le \liminf_{n\to\infty} d(\alpha_n, \beta_n) \le \limsup_{n\to\infty} d(\alpha_n, \beta_n) \le k^2 d(\alpha, \beta).$$

3. MAIN RESULT

3.1. Theorem:

Suppose that the self maps f, g, S and T on a complete $b-$metric space (X,d) satisfying the following conditions:

(C1) $f(X)\subseteq T(X)$ and $g(X)\subseteq S(X)$

(C2) $d(f\alpha, g\beta)\leq \frac{q}{k^4}\max\left\{d(S\alpha,T\beta), d(f\alpha,S\alpha), d(g\beta,T\beta), \frac{1}{2}\left(d(S\alpha,g\beta)+d(f\alpha,T\beta)\right)\right\}$,

holds for every $\alpha,\beta\in X$ with $q\in(0,1)$.

(C3) The pair (f,S) satisfies CLR_S property or the pair (g,T) satisfies CLR_T property

(C4) the pair of mappings (f,S) and (g,T) are weakly compatible.

Then the four maps f, g, S and T have a unique common unique fixed point.

PROOF:

Let $\alpha_0\in X$ then by the condition(C1) there exists $\alpha_1\in X$ such that $f\alpha_0=T\alpha_1$. For this point α_1 we can select a point $\alpha_2\in X$ such that $g\alpha_1=S\alpha_2$ and so on. Continuing this process it is possible to construct a sequence $\{\beta_n\}$ such that $\beta_{2j}=f\alpha_{2j}=T\alpha_{2j+1}$ and $\beta_{2j+1}=g\alpha_{2j+1}=S\alpha_{2j+2}$ for all $j\geq 0$. Now we show that $\{\beta_j\}$ is a cauchy sequence.

Consider $d(\beta_{2j},\beta_{2j+1})=d(f\alpha_{2j},g\alpha_{2j+1})$

$$\leq\frac{q}{k^4}\max\{d(S\alpha_{2j},T\alpha_{2j+1}),d(f\alpha_{2j},S\alpha_{2j}),d(g\alpha_{2j+1},T\alpha_{2j+1}),$$
$$\frac{1}{2}(d(S\alpha_{2j},g\alpha_{2j+1})+d(f\alpha_{2j},T\alpha_{2j+1}))\}$$

$$=\frac{q}{k^4}\max\{d(\beta_{2j-1},\beta_{2j}),d(\beta_{2j},\beta_{2j-1}),d(\beta_{2j+1},\beta_{2j}),$$
$$\frac{1}{2}(d(\beta_{2j-1},\beta_{2j+1})+d(\beta_{2j},\beta_{2j}))\}.$$

$$=\frac{q}{k^4}\max\{d(\beta_{2j-1},\beta_{2j}),d(\beta_{2j},\beta_{2j+1}),\frac{d(\beta_{2j-1},\beta_{2j+1})}{2}\}$$

$$\leq\frac{q}{k^4}\max\{d(\beta_{2j-1},\beta_{2j}),d(\beta_{2j},\beta_{2j+1}),\frac{k}{2}(d(\beta_{2j-1},\beta_{2j})+d(\beta_{2j},\beta_{2j+1}))\}.$$

If $d(\beta_{2j},\beta_{2j+1})>d(\beta_{2j-1},\beta_{2j})$ for some j, then the above inequality gives

$$d(\beta_{2j},\beta_{2j+1})\le\frac{q}{k^3}d(\beta_{2j},\beta_{2j+1})$$

a contradiction.

Hence $d(\beta_{2j},\beta_{2j+1})\le d(\beta_{2j-1},\beta_{2j})$ for all $j\in N$.

Now the above inequality gives

$$d(\beta_{2j},\beta_{2j+1})\le\frac{q}{k^3}d(\beta_{2j-1},\beta_{2j}).\text{-------------------}(1)$$

Similarly

$$d(\beta_{2j-1},\beta_{2j})\le\frac{q}{k^3}d(\beta_{2j-2},\beta_{2j-1}).\text{------------}(2)$$

From (1) and (2) we have

$$d(\beta_j,\beta_{j-1})\le\lambda d(\beta_{j-1},\beta_{j-2}),\quad \text{where } \lambda=\frac{q}{k^3}<1 \text{ and } j\ge 2.$$

Hence for all $j\ge 2$,we obtain

$$d(\beta_j,\beta_{j-1})\le\lambda d(\beta_{j-1},\beta_{j-2})\le\dots\dots\dots\le\lambda^{j-1}d(\beta_1,\beta_0)\text{ .----------}(3)$$

So for all $j>l$, we have

$$d(\beta_j,\beta_l)\le kd(\beta_l,\beta_{l+1})+k^2d(\beta_{l+1},\beta_{l+2})+\dots\dots\dots+k^{j-l-1}d(\beta_{j-1},\beta_j).$$

Now from (3), we have

$$d(\beta_j,\beta_l)\le(k\lambda^l+k^2\lambda^{l+1}+\dots\dots\dots\dots+k^{j-l-1}\lambda^{j-1})d(\beta_1,\beta_0)$$

$$\le k\lambda^l(1+k\lambda+k^2\lambda^2+\dots\dots\dots\dots)d(\beta_1,\beta_0)$$

$$\le\frac{k\lambda^l}{1-k\lambda}d(\beta_1,\beta_0).$$

Taking limits as $l,j\to\infty$,we have $d(\beta_j,\beta_l)\to 0$ as $k\lambda$ is less than one.

Therefore $\{\beta_j\}$ is a Cauchy sequence in X and by completeness of X, it converges to some point μ in X such that $\lim_{j\to\infty}f\alpha_{2j}=\lim_{j\to\infty}T\alpha_{2j+1}=\lim_{j\to\infty}g\alpha_{2j+1}=\lim_{j\to\infty}S\alpha_{2j+2}=\mu$.

Case(I): The pair (f,S) satisfies CLR_S-Property.

Suppose the pair (f,S) satisfies CLR_S Property , then there exists a sequence $\{\alpha_j\} \in$ X such that

$f\alpha_j = S\alpha_j = S\mu$ as $j \to \infty$ for some $\mu \in X$.--------------------(3')

Since $f \subseteq T(X)$ there exists a sequence $\{\beta_j\}$ in X such that $f\alpha_j = T\beta_j$ as $j \to \infty$.

This gives $\lim_{j\to\infty} T\beta_j = S\mu$.-----------------(4)

From (3') and (4) we get

$$\lim_{j\to\infty} f\alpha_j = \lim_{j\to\infty} S\alpha_j = \lim_{j\to\infty} T\beta_j = S\mu \quad \textit{for some } \mu \in X.--------(5)$$

Now we prove that $g\beta_j = S\mu$ as $j \to \infty$.

Putting $\alpha = \alpha_j$ and $\beta = \beta_j$ in (C2), we get

$$d(f\alpha_j, g\beta_j) \le \frac{q}{k^4} \max\left\{ d(S\alpha_j, T\beta_j), d(f\alpha_j, S\alpha_j), d(g\beta_j, T\beta_j), \frac{1}{2}\left(d(S\alpha_j, g\beta_j) + d(f\alpha_j, T\beta_j)\right) \right\}$$

taking both sides the upper limit as $j \to \infty$ and using the Lemma (2.7), which gives

$$d(S\mu, g\beta_j) \le \lim_{j\to\infty} Sup\, d(f\alpha_j, g\beta_j)$$

$$\frac{q}{k^4}\max\left\{ \begin{array}{l} \lim\limits_{j\to\infty} Sup\, d(S\alpha_j, T\beta_j), \lim\limits_{j\to\infty} Sup\, d(f\alpha_j, S\alpha_j), \lim\limits_{j\to\infty} Sup\, d(g\beta_j, T\beta_j), \\ \frac{1}{2}\left(\lim\limits_{j\to\infty} Sup\, d(S\alpha_j, g\beta_j) + \lim\limits_{j\to\infty} Sup\, d(f\alpha_j, T\beta_j) \right) \end{array} \right\}$$

$$\le \frac{q}{k^4} \max\left\{ 0, 0, k^2 d(g\beta_j, S\mu), \frac{k^2}{2}\left(d(S\mu, g\beta_j) + 0\right) \right\}$$

$$= \frac{q}{k^4} k^2 d(S\mu, g\beta_j)$$

$$= \frac{q}{k^2} d(S\mu, g\beta_j)$$

$$\frac{d(S\mu, g\beta_j)}{k^2} \le \frac{q}{k^2} d(S\mu, g\beta_j)$$

$$d(S\mu, g\beta_j) \le q d(S\mu, g\beta_j).$$

As $0<q<1$,so $S\mu = g\beta_j$

Now this gives $f\alpha_j = S\alpha_j = T\beta_j = g\beta_j = S\mu$ for some $\mu \in X$.---------------(6)

Since the pair of mapping (f,S) is weakly compatible with $f\alpha_j = S\alpha_j$ such that $fS\alpha_j = Sf\alpha_j$ then $f\mu = S\mu$.

Now we show that $f\mu = \mu$.

Putting $\alpha = \mu$ and $\beta = \beta_{2j+1}$ in (C2)

$$d(f\mu, g\beta_{2j+1}) \le \frac{q}{k^4} \max\left\{ d(S\mu, T\beta_{2j+1})\beta, d(f\mu, S\mu), d(g\beta_{2j+1}, T\beta_{2j+1}), \frac{1}{2}\left(d(S\mu, g\beta_{2j+1}) + d(f\mu, T\beta_{2j+1})\right) \right\}$$

taking both sides the upper limit as $j \to \infty$ and using the Lemma (2.7), which gives

$$\frac{d(f\mu,\mu)}{k^2} \le \lim_{j\to\infty} Sup\, d(f\mu, g\beta_{2j+1}) \le \frac{q}{k^4} \max\left\{ k^2 d(f\mu,\mu), 0, 0, \frac{k^2}{2}\left(d(f\mu,\mu) + d(f\mu,\mu)\right) \right\}$$

$$\frac{d(f\mu,\mu)}{k^2} \le \frac{q}{k^4} \max\left\{ k^2 d(f\mu,\mu), 0, 0, k^2 d(f\mu,\mu) \right\}$$

$$\frac{d(f\mu,\mu)}{k^2} \le \frac{q}{k^4} k^2 d(f\mu,\mu)$$

$$d(f\mu,\mu) \le q d(f\mu,\mu).$$

As $0<q<1$,so $f\mu = \mu$.

Which implies $S\mu = f\mu = \mu$.--------------(7)

Again since the pair (g,T) is weakly compatible with $g\beta_j = T\beta_j$ such that $gT\beta_j = Tg\beta_j$ then $g\mu = T\mu$.

Now we show that $g\mu = \mu$.

Putting $\alpha = \mu$ and $\beta = \mu$ in (C2)

$$d(f\mu, g\mu) \le \frac{q}{k^4} \max\left\{ d(S\mu, T\mu)\beta, d(f\mu, S\mu), d(g\mu, T\mu), \frac{1}{2}\left(d(S\mu, g\mu) + d(f\mu, T\mu)\right) \right\}$$

taking both sides the upper limit as $j \to \infty$ and using the Lemma (2.7), which gives

$$\frac{d(\mu, g\mu)}{k^2} \le \lim_{j\to\infty} Sup\, d(f\mu, g\mu) \le \frac{q}{k^4} \max\left\{ k^2 d(\mu, g\mu), 0, 0, \frac{k^2}{2}\left(d(\mu, g\mu) + d(\mu, g\mu)\right) \right\}$$

$$\frac{d(\mu, g\mu)}{k^2} \le \frac{q}{k^4} \max\left\{ k^2 d(\mu, g\mu), k^2 d(\mu, g\mu) \right\}$$

$$\frac{d(\mu, g\mu)}{k^2} \le \frac{q}{k^4} k^2 d(\mu, g\mu)$$

$$d(\mu, g\mu) \le q d(\mu, g\mu).$$

$As\ 0 < q < 1,\ \ g\mu = \mu.$

This implies $g\mu = T\mu = \mu.$--------------(8)

From (7) and (8) we obtain

$f\mu = S\mu = g\mu = T\mu = \mu.$

Hence the four maps f, g, S and T have a common fixed point.

Similarly we can prove the result when the pair (g, T) satisfies CLR_T-Property.

Uniqueness:

Assume that $\eta\ (\eta \ne \mu)$ is another common fixed point of four mappings f,S,g and T.

Put $\alpha = \eta$ and $\beta = \mu$ in (C2)

$$d(f\eta, g\mu) \le \frac{q}{k^4} \max\left\{ d(S\eta, T\mu), d(f\eta, S\eta), d(g\mu, T\mu), \frac{1}{2}\left(d(S\eta, g\mu) + d(f\eta, T\mu)\right) \right\}$$

$$\le \frac{q}{k^4} \max\left\{ k^2 d(\eta, \mu), k^2 d(\eta, \eta), k^2 d(\mu, \mu), \frac{k^2}{2}\left(d(\eta, \mu) + d(\eta, \mu)\right) \right\}$$

$$= \frac{q}{k^4} \max\left\{ k^2 d(\eta, \mu), 0, 0, k^2 d(\eta, \mu) \right\}$$

$$\frac{d(\eta,\mu)}{k^2}=\frac{q}{k^4}k^2 d(\eta,\mu)$$

$$d(\eta,\mu)\le qd(\eta,\mu).$$

As $0<q<1$, $\eta=\mu$.

Hence the four maps f,g,S and T have a common unique fixed point.

Now we give an illustration to support our result.

3.2. Example:

Suppose $X=[-4,1]$ is a $b-$metric space with $d(\alpha,\beta)=|\alpha-\beta|^2$ where $\alpha,\beta\in X$.

Define the four self maps f,g,S and T as follows

$$f(\alpha)=\begin{cases}\frac{\alpha+3}{5}, & \alpha\in(-1\,,\,0.25)\\ 0.25, & \alpha\in[-4,1]\cup[0.25\,,\,1)\end{cases};$$

$$g(\alpha)=\begin{cases}\frac{\alpha-2}{3}, & \alpha\in(-1\,,\,0.25)\\ 0.25, & \alpha\in[-4,-1]\cup[0.25,1]\end{cases};$$

$$S(\alpha)=\begin{cases}1\,,\ \alpha\in(-1\,,\,0.25)\\ 0.25\,,\alpha\in[-4,-1]\cup\{0.25\}\\ 1-3\alpha\,,\ \alpha\in(0.25,1]\end{cases};$$

and

$$T(\alpha)=\begin{cases}1\,,\ \alpha\in(-1\,,\,0.25)\\ 0.25\,,\alpha\in[-4,-1]\cup\{0.25\}\\ 1-\alpha\,,\ \alpha\in(0.25,\,1]\end{cases}$$

Then

this gives $f(X)=(0.4,\,0.625)$, $g(X)=(-0.58,\,-0.66)\cup\{0.25\}$, $S(X)=[-2,0.5)\cup\{1\}$ and $T(X)=(0,\,1)$.

Clearly the condition (C1) is satisfied.

Take a sequence as $\alpha_j=\frac{1}{4}+\frac{1}{j}$ for $j\ge 0$.

Now $\lim\limits_{j\to\infty} f\alpha_j = \lim\limits_{j\to\infty} f\left(\frac{1}{4}+\frac{1}{k}\right) = \lim\limits_{j\to\infty} 0.25 = 0.25$

and $\lim\limits_{j\to\infty} S\alpha_j = \lim\limits_{j\to\infty} S\left(\frac{1}{4}+\frac{1}{k}\right) = \lim\limits_{j\to\infty}\left[1-3\left(\frac{1}{4}+\frac{1}{j}\right)\right] = 0.25.$

This gives $\lim\limits_{j\to\infty} f\alpha_j = \lim\limits_{j\to\infty} S\alpha_j = 0.25 = S\alpha$ *for all* $\alpha \in (0.25, 1)$.

$\Rightarrow \quad \lim\limits_{j\to\infty} f\alpha_j = \lim\limits_{j\to\infty} S\alpha_j = 0.25 = S\alpha$ *for some* $\alpha \in X$.

Similarly $\lim\limits_{j\to\infty} g\alpha_j = \lim\limits_{j\to\infty} g\left(\frac{1}{4}+\frac{1}{k}\right) = \lim\limits_{j\to\infty} 0.25 = 0.25$

and $\lim\limits_{j\to\infty} T\alpha_j = \lim\limits_{j\to\infty} T\left(\frac{1}{4}+\frac{1}{k}\right) = 0.25.$

This gives $\lim\limits_{j\to\infty} g\alpha_j = \lim\limits_{j\to\infty} T\alpha_j = 0.25 = T\alpha$ *for some* $\alpha \in X$.

Therefore the pairs (f,S) and (g,T) are satisfying CLR_s and CLR_T properties respectively.

Also $f(0.25) = S(0.25) = 0.25$ and $fS(0.25) = Sf(0.25)$ as $j \to \infty$

and $g(0.25) = T(0.25) = 0.25$ and $gT(0.25) = Tg(0.25)$ as $j \to \infty$.

Therefore the pairs of mappings (f,S) and (g,T) are weakly compatible.

But $fS\alpha_j = fS\left(\frac{1}{4}+\frac{1}{j}\right) = f\left(1-3\left(\frac{1}{4}+\frac{1}{j}\right)\right) = f\left(1-\frac{3}{4}-\frac{1}{j}\right) = f\left(\frac{1}{4}-\frac{1}{j}\right) = \frac{13}{20}$ as $k \to \infty$

and $Sf\alpha_j = Sf\left(\frac{1}{4}+\frac{1}{j}\right) = S(0.25) = S(0.25) = 0.25 = \frac{1}{4}$ as $j \to \infty$

so that $\lim\limits_{j\to\infty} d\left(fS\alpha_j, Sf\alpha_j\right) = d\left(\frac{13}{20},\frac{1}{4}\right) = \left|\frac{13}{20}-\frac{1}{4}\right|^2 = \frac{4}{25} \neq 0.$

Similarly, $\lim\limits_{j\to\infty} d\left(gT\alpha_j, Tg\alpha_j\right) = \frac{2}{3} \neq 0$,

showing that the pairs (f,S) and (g,T) are not compatible mappings.

We now establish that the mappings f,g,S and T satisfy the condition**(C2)** .

Case (i):

If $\alpha,\beta\in(-1,0.25)$, we define $d(\alpha,\beta)=|\alpha-\beta|^2$, where k=2

Putting $\alpha=-0.5$ and $\beta=0.1$, then the inequality (C2) gives

$$d(f(-0.5),g(0.1))\leq\frac{q}{k^4}\max\left\{\begin{matrix}d(S(-0.5),T(0.1)),d(f(-0.5),S(-0.5)),d(g(0.1),T(0.1)),\\ \frac{1}{2}(d(S(-0.5),g(0.1))+d(f(-0.5),T(0.1)))\end{matrix}\right\}$$

$$\frac{1}{k^2}d(0.5,-0.63))\leq\frac{q}{k^4}\max\left\{k^2d(1,1)),k^2d(0.5,1),k^2d(-0.63,1),\frac{k^2}{2}(d(1,-0.63)+d(0.5,1))\right\}$$

$$\frac{1.27}{2^2}\leq\frac{q}{2^4}\max\{0,1,10.6,5.8\},$$

$$0.3175\leq\frac{q}{2^4}\times10.6$$

$$0.3175\leq0.6625q\quad\Rightarrow q=0.47\quad where\ q\in(0,1).$$

Hence the condition (C2) is satisfied.

Case (ii):

If $\alpha,\beta\in[-4,-1]\cup\{0.25\}$, we define $d(\alpha,\beta)=|\alpha-\beta|^2$, where $k=2$

Putting $\alpha=-2$ and $\beta=0.25$ then the inequality (C2) gives

$$d(f(-2),g(0.25))\leq\frac{q}{k^4}\max\left\{\begin{matrix}d(S(-2),T(0.25)),d(f(-2),S(-2)),d(g(0.25),T(0.25)),\\ \frac{1}{2}(d(S(-2),g(0.25))+d(f(-2),T(0.25)))\end{matrix}\right\}$$

$$\frac{1}{k^2}d(0.25,0.25))\leq\frac{q}{k^4}\max\left\{k^2d(0.25,0.25)),k^2d(0.25,0.25),k^2d(0.25,0.25),\frac{k^2}{2}(d(0.25,0.25)+d(0.25,0.25))\right\}.$$

It can be observed that q $\in(0,1)$ satisfies the above inequality.

Hence the condition (C2) is satisfied.

Case (iii):

If $\alpha,\beta \in (0.25,1]$, we define $d(\alpha,\beta)=|\alpha-\beta|^2$, where $k=2$.

Putting $\alpha=0.4$ and $\beta=0.8$, then the inequality (C2) gives

$$d\left(f(0.4),g(0.8)\right)\le \frac{q}{k^4}\max\left\{\begin{array}{l} d\left(S(0.4),T(0.8)\right), d\left(f(0.4),S(0.4)\right), d\left(g(0.8),T(0.8)\right), \\ \frac{1}{2}\left(d\left(S(0.4),g(0.8)\right)+d\left(f(0.4),T(0.8)\right)\right)\end{array}\right\}$$

$$\frac{1}{k^2}d\left(0.25,0.25\right)\le \frac{q}{k^4}\max\left\{k^2 d\left(-0.2,0.2\right), k^2 d\left(0.25,-0.2\right), k^2 d\left(0.25,0.2\right), \frac{k^2}{2}\left(d\left(-0.2,0.25\right)+d\left(0.25,0.2\right)\right)\right\}$$

$$\frac{1}{2^2}\times 0\le \frac{q}{2^4}\max\{0.64,0.8,0.01,0.4\}.$$

It can be obsrved that q $\in(0,1)$ satisfies the above inequality.

Hence the condition (C2) is satisfied.

It is arrived that 0.25 is the unique common fixed point of the four self maps f,g, S and T.

CONCLUSION

In this paper we generated a result on b-metric space using weaker conditions weakly compatible mappings and CLR-property resulting the generalisation of Theorem (2.7). Further our result is also justified by discussing a valid example.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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