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MATHEMATICAL MODEL ANALYSIS OF CRIME DYNAMICS INCORPORATING MEDIA COVERAGE AND POLICE FORCE

MESKEREM ABEBAW MEBRATIE, MOHAMMED YIHA DAWED*

Department of Mathematics, Hawassa University, Hawassa, Ethiopia

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Abstract: In this paper, a mathematical model is proposed to recognize the dynamics of crime. Unlike some of other previous model, we have taken into account the impact of media coverage, police force and moral/religious activity on crime. Some fundamental properties of the model including existence and positivity as well as boundedness of the solutions of the model are investigated. The model exhibits two equilibria: the crime-free and the persistent equilibrium points. We sufficiently analyze asymptotic behavior of the solutions which depends on the basic reproduction number. Numerical simulation is carried out using *Ode45* of Matlab, sensitivity analysis of the basic reproduction number is also constructed.

Keywords: mathematical model; crime free; crime persistence; basic reproduction number; sensitivity analysis.

2010 AMS Subject Classification: 93A30, 49Q12.

1. INTRODUCTION

Several authors have presented empirical as well as practical study on the generation and prevention of crime. Crime is an illegal act for which someone can be punished by the government. The possible contributions of mathematical modeling of crime has been nicely reviewed in [17]. Iglesias et al., [38] presented an economic analysis of crime based on the idea that a

*Corresponding author

E-mail address: mohammedyiha2017@gmail.com

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crime results from a trade off between exported benefit and the risk of punishment. In particular, the trade off between crime and punishment has been studied. H. Zhao et al.,[19] proposed a mathematical model to study the interplay between criminality and poverty, and explored the possibility of crime control via government interventions. Vito GF et al.,[36] proposed the prevention of juvenile delinquency is an essential part of any crime prevention program in the society. Modeling of delinquent behavior using an infectious disease approach has been done for fanatical and violent ideology [4, 7, 30] and the transmission of violent crime and burglary in the United Kingdom[5]. Becker et al.,[15], has suggested that people drift towards criminality when the benefits of crime are more than potential punishment. The presence of police increases the chances of punishment and hence the expected cost of committing crime is less. For effectual prevention of crime, the increment in the police force should be made with the upsurge in crime rates. It is evident from the literature that the increased level of police has a substantial effect to reduce crime in society[34]. Hence, making additional recruitment in the police force in accordance to the level of crime in the society is a rational strategy for crime control. Many of the social problems are assumed to be contiguous like an epidemic[21, 23]. The media coverage is obviously not the most important factor responsible for fighting the transmission of the infectious disease, but it is a very important issue which has to be considered seriously[1]. Voluminous literature is available on epidemic models comprising the transmission and control of infectious diseases[18, 29]. Some of epidemiological studies have also focused on the effect of immigration of infectives on the dynamics of disease[13, 14]. Using the same approach, a mathematical model to assess the effect of police force on the prevailing of crime in the society is proposed by A.K.Misra[2]. It is considered that criminals in the society increase when people living in that region get involved in criminal activities due to contact with criminals. In this research [2] importance of media coverage, non criminal individuals, susceptible individuals those are not aware about crime, susceptible individuals those are aware about crime moral/religious situation in the society are not considered. Here, we try to fill this gap in our research. Crime free and persistent equilibrium points are also computed. In this paper, we seek to understand the effects of media coverage, informal learning and moral/religious activity in

the transmission of crime. The rest of the paper is organized as follows: the model is formulated in Section 2, existence and stability analysis of the endemic equilibrium is done in Section 3, numerical simulations and discussion are carried out in Section 4. The paper ends with a conclusion in Section 5.

2. MATHEMATICAL MODEL DESCRIPTION

In this thesis we extended the A.K.Misra model SCR [2] to the form $S_u S_a CRQ$ where the total population N is divided into unaware susceptible class (S_u), aware susceptible class (S_a), class of criminal (C), class of non-criminal (Q) and class of prisoner (R). The model assumes that unaware susceptible class (S_u) increases by a constant recruitment rate $\sigma\Lambda$, decreases by a rates $\beta S_u C$ and ηS_u due to interaction with criminals and family advice awareness creation, respectively. The aware susceptible class (S_a) increases by a constant recruitment rate $(1 - \sigma)\lambda$ which is the remaining fraction rate, by rate ηS_u and by rate $(1 - \theta)vR$ which is the jail leaving fraction rate, and it decreases by rate τS_a due to moral or religious case. The criminal class (C) increases by rate $\beta S_u C$ and decreases by efficacy ρ of media coverage, and by jail rate γCP using legal authority or police force. The prisoner class (R) increases by imprisonment rate γCP and decrease by jail leaving rate (vR) where $\frac{1}{v}$ is prisoner period in jail. The non-criminal class Q increases by rates θvR and τS_a where θ is a proportion. The natural death rate μ is the same for compartments S_u, S_a, C, R and Q . The police force (P) is incorporated explicitly in the model. It increases with rate ϕC which is proportional to the criminal C and decreases by rate $\phi_0(P - P_0)$ due to retirement or natural death.

In addition to the above, we also consider the following assumptions:

- i. An individual can be criminal only through contacts with criminal individuals.
- ii. The non-criminal individuals do not prone to any criminal activities.
- iii. The police force do not commit any crime.

The flow chart of the mathematical model is explained as follows:

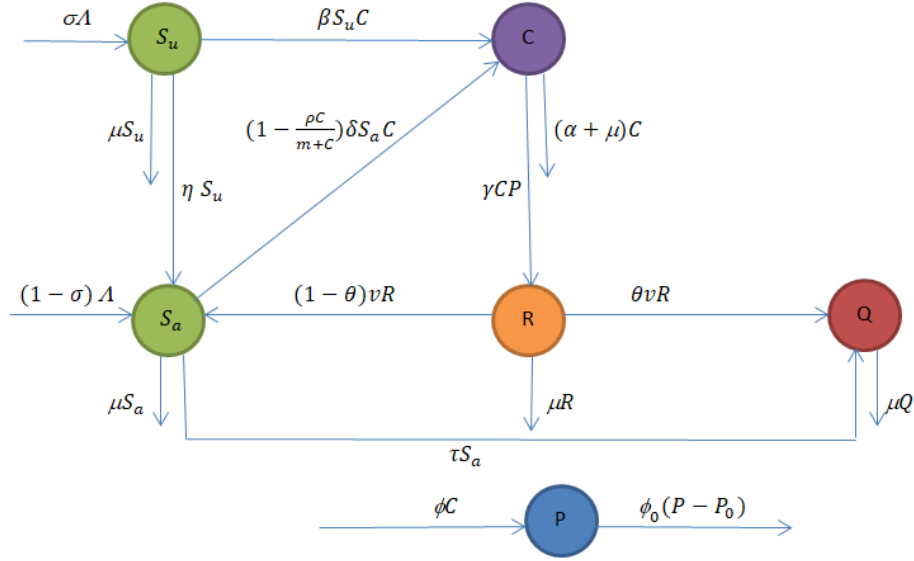


FIGURE 1. Flow chart of mathematical model

Considering the assumptions, the dynamics of the crime is described by using the following system of differential equations(see table 1 for the description of the involved paramers):

$$(1) \quad \frac{dS_u}{dt} = \sigma\Lambda - \beta S_u C - \eta S_u - \mu S_u,$$

$$(2) \quad \frac{dS_a}{dt} = (1 - \sigma)\Lambda + \eta S_u + (1 - \theta)vR - \tau S_a - \left(1 - \frac{\rho C}{m + C}\right) \delta S_a C - \mu S_a,$$

$$(3) \quad \frac{dC}{dt} = \beta S_u C + \left(1 - \frac{\rho C}{m + C}\right) \delta S_a C - \gamma C P - (\alpha + \mu)C,$$

$$(4) \quad \frac{dR}{dt} = \gamma C P - (v + \mu)R,$$

$$(5) \quad \frac{dQ}{dt} = \tau S_a + \theta v R - \mu Q,$$

$$(6) \quad \frac{dP}{dt} = \phi C - \phi_0(P - P_0),$$

where $S_u(0) > 0, S_a(0) > 0, C(0) \geq 0, R(0) \geq 0, Q(0) \geq 0, P(0) > 0$, and $0 < \theta < 1, 0 < \sigma < 1$, and $\frac{\rho \delta C}{m + C}$ is the reduced rate of contact with criminals due to media coverage.

| Var/Par | Description of variables and parameters |
|------------------|--|
| $S_u(t)$ | Susceptible individuals those are unaware about crime. |
| $S_a(t)$ | Susceptible individuals those are aware about crime. |
| $C(t)$ | Active criminals who are currently involved in various criminal activities. |
| $R(t)$ | Prisoners who are essentially criminals residing in the jails |
| $Q(t)$ | Non-criminals who are not currently involved in any criminal activities. |
| $P(t)$ | Police force used to protect (deter) the level of crime in the society. |
| Λ | Constant recruitment rate of susceptible individuals. |
| σ, θ | Proportions. |
| η | Rate of crime unaware susceptible become crime aware susceptible because of informal learning. |
| m | Media coverage. |
| ρ | Measures efficacy of media coverage. |
| β | Transmission rate (crime unaware susceptible to criminal). |
| μ | Natural mortality rate. |
| γ | Incarceration rate. |
| α | The crime related death rate. |
| ν | Jail leaving rate. |
| δ | Transfer rate from crime aware susceptible to criminal. |
| τ | Rate of crime aware susceptible becomes non-criminal due to moral/religious activity. |
| ϕ | Recruitment rate of individuals of police force . |
| ϕ_0 | Mortality or retirement rate. |
| P_0 | Baseline police force population size. |

TABLE 1. Description of the model variables and parameters.

2.1. Existence, positivity and boundedness of solution of the model

Well-Posedness: The mathematical model system is well-posed. Infact, by Picard's (Or Cauchy-Lipschitz) theorem its solution exists, is unique and continually depends on the initial data. Since we are dealing with population, we should ensure positivity and boundedness of solutions.

Theorem 1. *The solution of the system (1) – (6) is positive.*

Proof. From the the first equation of the model system we have,

$$\begin{aligned}\frac{dS_u}{dt} &= \sigma\Lambda - \beta S_u C - \eta S_u - \mu S_u, \\ &\geq -\beta S_u C - \eta S_u - \mu S_u, \quad \text{since } \sigma \text{ \& } \Lambda \text{ are positive} \\ &= -(\beta C + \eta + \mu)S_u. \\ \Rightarrow \frac{dS_u}{S_u} &\geq -(\beta C(t) + \eta + \mu)dt.\end{aligned}$$

After integration,

$$(7) \quad S_u(t) = S_u(0)e^{-\int(\beta C + \eta + \mu)dt}$$

which is positive for all time $t \geq 0$. □

The positivity of the remaining state variables can be proved in the same way. Let N be the total population, $N(t) = S_u + S_a + C + R + Q$. Consider the set

$$\Omega = \left\{ (S_u, S_a, C, R, Q, P) \in \mathbb{R}_+^6 : 0 \leq N \leq \frac{\Lambda}{\mu}, 0 \leq P \leq P_0 \right\}$$

The theorem below establishes the boundedness of the solution.

Theorem 2. *All solutions $(S_u(t), S_a(t), C(t), R(t), Q(t), P(t))$ of the model system (1)-(6) are bounded in the region Ω .*

Proof. Here, one can show that if $N(t) = S_u + S_a + C + R + Q$, then it follows that $N(t) \leq \max \left\{ N(0), \frac{\Lambda}{\mu} \right\}$. On the other hand $\frac{dP}{dt} = \phi C - \phi_0(P - P_0)$ implies that $P(t) \leq \max \{ P_0, P(0) \}$. Hence, the solution of the model system is bounded in the region Ω . □

2.2. Equilibrium points

i) Crime-Free Equilibrium Point (E_{cfe}): Crime-free equilibrium point (E_{cfe}) is steady state solution, where there is no crime in the society, i.e., $C = 0$.

Thus, the crime free equilibrium point of the model is obtained to be

$$E_{cfe} = \left(\frac{\sigma A}{k_1}, \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2}, 0, 0, \frac{A\tau(\varepsilon_1 k_1 + \sigma \eta)}{\mu k_1 k_2}, P_0 \right)$$

where $k_1 = \eta + \mu$, $k_2 = \tau + \mu$ and $\varepsilon_1 = 1 - \sigma$.

Basic Reproduction Number (R_0): The basic reproduction number (R_0), which is important for the qualitative analysis of the model, is obtained by using the next generation matrix [37]. Rewriting the system of the model starting with the criminal compartments for both population gives as

$$\begin{aligned} \frac{dC}{dt} &= \beta S_u C + \left(1 - \frac{\rho C}{m+C}\right) \delta S_a C - \gamma C P - (\alpha + \mu) C, \\ \frac{dR}{dt} &= \gamma C P - (v + \mu) R, \\ \frac{dS_u}{dt} &= \sigma A - \beta S_u C - (\eta + \mu) S_u, \\ \frac{dS_a}{dt} &= (1 - \sigma) A + \eta S_u + (1 - \theta) v R - (\tau + \mu) S_a - \left(1 - \frac{\rho C}{m+C}\right) \delta S_a C, \\ \frac{dQ}{dt} &= \delta S_a + \theta v R - \mu Q, \\ \frac{dP}{dt} &= \phi C - \phi_0 (P - P_0). \end{aligned} \tag{8}$$

For the model under consideration, using notation $X = (C, R)$ we have the vector functions

$$F(X) = \begin{pmatrix} \beta S_u C + \left(1 - \frac{\rho C}{m+C}\right) \delta S_a C \\ 0 \end{pmatrix}, \tag{9}$$

,

$$V(X) = \begin{pmatrix} \gamma C P + (\alpha + \mu) C \\ (v + \mu) R - \gamma C P \end{pmatrix}.$$

The Jacobian matrix at the crime-free equilibrium point $J_F(E_{cfe})$, $J_V(E_{cfe})$:

$$J_F(E_{cfe}) = \begin{pmatrix} \beta S_u^0 + \delta S_a^0 & 0 \\ 0 & 0 \end{pmatrix} \tag{10}$$

where $S_u^0 = \frac{\sigma A}{k_1}$, $S_a^0 = \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2}$,

$$J_V(E_{cfe}) = \begin{pmatrix} \gamma P^0 + \alpha + \mu & 0 \\ -\gamma P^0 & \nu + \mu \end{pmatrix}$$

Now, the inverse of the matrix $J_V(E_{cfe})$ is found as

$$J_V(E_{cfe})^{-1} = \begin{pmatrix} \frac{1}{\gamma P^0 + \alpha + \mu} & 0 \\ \frac{\gamma P^0}{(\nu + \mu)(\gamma P^0 + \alpha + \mu)} & \frac{1}{\nu + \mu} \end{pmatrix}$$

Therefore, the generation matrix is

$$J_F(E_{cef})J_V(E_{cfe})^{-1} = \begin{pmatrix} \frac{\beta S_u^0 + \delta S_a^0}{\gamma P^0 + \alpha + \mu} & 0 \\ 0 & 0 \end{pmatrix}$$

Hence, the reproduction number, R_0 , is found to be

$$(11) \quad R_0 = \frac{\Lambda \sigma \beta (\tau + \mu) + \Lambda \delta ((1 - \sigma)(\eta + \mu) + \sigma \eta)}{(\tau + \mu)(\eta + \mu)(\gamma P^0 + \alpha + \mu)}.$$

This represents the average number of secondary criminal cases generated by a single criminal during his or her entire life as criminal when introduced into a completely susceptible population.

ii) Crime-Persistent Equilibrium Point (E_{cpe}): Crime persistent equilibrium point, E_{cpe} , is steady state solution where the crime persists in the population. If $E_{cpe}(S_u^*, S_a^*, C^*, R^*, Q^*, P^*)$ is crime-persistent equilibrium point, it satisfies the algebraic equations

$$\begin{aligned} \sigma A - \beta S_u^* C^* - \eta S_u^* - \mu S_u^* &= 0, (1 - \sigma)A + \eta S_u^* + (1 - \theta)\nu R^* - \tau S_a^* - \left(1 - \frac{\rho C^*}{m + C^*}\right) \delta S_a^* C^* - \mu S_a^* \\ &= 0, \beta S_u^* C^* + \left(1 - \frac{\rho C^*}{m + C^*}\right) \delta S_a^* C^* - \gamma C^* P^* - (\alpha + \mu)C^* = 0, \gamma C^* P^* - (1 - \theta)\nu R^* - \theta \nu R^* - \mu R^* \\ &= 0, \tau S_a^* + \theta \nu R^* - \mu Q^* = 0, \phi C^* - \phi_0(P^* - P_0) = 0. \end{aligned}$$

After simplifications we obtain E_{cpe} in terms of C^* as

$$\begin{aligned} S_u^* &= \frac{\sigma A}{\beta C^* + \eta + \mu}, S_a^* = \frac{(1 - \sigma)A\phi_0(\nu + \mu)(\beta C^* + \eta + \mu) + \sigma \eta A + (1 - \theta)\nu \gamma C^*(\phi C^* + \phi_0 P_0)(\beta C^* + \eta + \mu)}{\phi_0(\nu + \mu)\left(\left(1 - \frac{\rho C^*}{m + C^*}\right)\delta C^* + \eta + \mu\right)(\beta C^* + \eta + \mu)}, \\ R^* &= \frac{\gamma C^*(\phi C^* + \phi_0 P_0)}{\phi_0(\nu + \mu)}, Q^* = \frac{\tau[(1 - \sigma)A\phi_0(\nu + \mu)(\beta C^* + \eta + \mu) + \sigma \eta A + (1 - \theta)\nu \gamma C^*(\phi C^* + \phi_0 P_0)(\beta C^* + \eta + \mu)]}{\mu \phi_0(\nu + \mu)\left(\left(1 - \frac{\rho C^*}{m + C^*}\right)\delta C^* + \eta + \mu\right)(\beta C^* + \eta + \mu)} \\ &+ \frac{\theta \nu \gamma C^*(\phi C^* + \phi_0 P_0)\left(\left(1 - \frac{\rho C^*}{m + C^*}\right)\delta C^* + \tau + \mu\right)(\beta C^* + \eta + \mu)}{\mu \phi_0(\nu + \mu)\left(\left(1 - \frac{\rho C^*}{m + C^*}\right)\delta C^* + \eta + \mu\right)(\beta C^* + \eta + \mu)}, \text{ and } P^* = \frac{\phi C^* + \phi_0 P_0}{\phi_0}, \end{aligned}$$

where C^* is positive solution of

$$(12) \quad k_4 C^{*5} + k_5 C^{*4} + k_6 C^{*3} + k_7 C^{*2} + k_8 C^* + k_9 = 0,$$

$$k_4 = -\delta\beta d - \rho\beta d,$$

$$k_5 = \delta\beta e - \delta\beta dm - \delta\rho k_1 d - \delta\beta\rho e + \delta k_1 d - g\beta\delta u + g\beta\delta\rho u,$$

$$k_6 = \delta\beta(b+f) + \delta\beta em - \delta\rho k_1 e + \delta k_1 dm + \delta k_1 e - \delta\beta\rho(b+f) - g\beta\delta um \\ - g\beta n - g u k_1 \delta + g u k_1 \delta\rho - h\beta\delta u + h\beta\delta\rho u,$$

$$k_7 = au\delta - au\delta\rho + \delta\beta m(b+f) - \delta\rho k_1(b+f) + c\delta\beta + \delta k_1 em - \delta\rho k_1 e + \delta k_1(b+f) \\ - c\delta\beta\rho - g\beta nm - g\beta\delta u k_1 m - g n k_1 - h\beta\delta um - h\beta n - h\delta u k_1 + h\delta\rho u k_1,$$

$$k_8 = au\delta m + an + c\delta\beta m - c\delta\rho k_1 + \delta k_1 m(b+f) + \delta k_1 c - g n k_1 m - h\beta nm - hu\delta k_1 m - h n k_1,$$

$$k_9 = anm + \delta cm - h n k_1 m = \phi_0 k_3 m [A\sigma\beta k_2 + A\delta(\varepsilon_1 k_1 + \sigma\eta) - k_1 k_2(\gamma P_0 + \alpha + \mu)],$$

$$k_9 = k_1 k_2 \phi_0 k_3 m (\gamma P_0 + \alpha + \mu) \left(\frac{A\sigma\beta k_2 + A\delta(\varepsilon_1 k_1 + \sigma\eta)}{k_1 k_2 (\gamma P_0 + \alpha + \mu)} - 1 \right) \\ = k_1 k_2 \phi_0 k_3 m (\gamma P_0 + \alpha + \mu) (R_0 - 1).$$

Here, $a = \beta\sigma\Lambda$, $b = \varepsilon_1\Lambda\phi_0 k_3\beta$, $c = \varepsilon_1\Lambda\phi_0 k_3 k_1 + \eta\sigma\Lambda\phi_0 k_3$, $d = \varepsilon_2 v \gamma \phi \beta$, $e = \varepsilon_2 v \gamma (\phi k_1 + \phi_0 P_0 \beta)$, $f = \varepsilon_2 v \gamma \phi_0 P_0 k_1$, $g = \frac{\gamma\phi}{\phi_0}$, $h = \gamma P_0 + \alpha + \mu$, $u = \phi_0 k_3$, $n = \phi_0 k_3 k_2$.

The equation (12) has at least one positive solution, by Lemma 2.2 [39].

3. STABILITY ANALYSIS

Theorem 3. *The equilibrium solution E_{cfe} of the model system of equation (1)-(6) is locally asymptotically stable if $R_0 < 1$.*

Proof. The Jacobian matrix J at the state variables is given by

$$\begin{pmatrix} -\beta C - \eta - \mu & 0 & -\beta S_u & 0 & 0 & 0 \\ \eta & -\tau - \delta C + \frac{\rho C}{m+C} \delta C - \mu & -\delta S_a + w & (1-\theta)v & 0 & 0 \\ \beta C & \delta C - \frac{\rho C}{m+C} \delta C & \beta S_u + \delta S_a - w - \gamma P - \alpha - \mu & 0 & 0 & -\gamma C \\ 0 & 0 & \gamma P & -v - \mu & 0 & \gamma C \\ 0 & \tau & 0 & \theta v & -\mu & 0 \\ 0 & 0 & \phi & 0 & 0 & -\phi_0 \end{pmatrix}$$

where $w = \frac{\rho m \delta S_a C}{(m+c)^2} - \frac{\rho C \delta S_a}{m+C}$. Hence,

$$J(E_{cfe}) = \begin{pmatrix} -k_1 & 0 & \frac{-\beta \sigma A}{k_1} & 0 & 0 & 0 \\ \eta & -k_2 & -\frac{\delta A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2} & (1-\theta)v & 0 & 0 \\ 0 & 0 & \frac{\beta \sigma A}{k_1} + \frac{\delta A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2} - \gamma P_0 - (\alpha + \mu) & 0 & 0 & 0 \\ 0 & 0 & \gamma P_0 & -k_3 & 0 & 0 \\ 0 & \tau & 0 & \theta v & -\mu & 0 \\ 0 & 0 & \phi & 0 & 0 & -\phi_0 \end{pmatrix}.$$

After simplification the eigenvalues of the matrix, $J(E_{cfe})$, is found as

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} = - \begin{pmatrix} \eta + \mu \\ \tau + \mu \\ (\eta + \mu)(\tau + \mu)(\gamma P_0 + \alpha + \mu)(1 - R_0) \\ v + \mu \\ \mu \\ \phi_0 \end{pmatrix}.$$

All the eigenvalues have negative real part if $R_0 < 1$. Hence, the theorem. \square

Global stability of the crime free equilibrium point (E_{cfe}): Based on Iggider [22] we write our system (1)-(6) in the form

$$(13) \quad \begin{aligned} \frac{dY_n}{dt} &= A(Y_n - Y_{E_{cfe},n}) + A_1 Y_i, \\ \frac{dY_i}{dt} &= A_2 Y_i, \end{aligned}$$

where $Y_n = (S_u, S_a, Q, P)$, $Y_i = (C, R)$. The components of Y_n denote the non criminal individuals and the components of Y_i denote the criminal individuals, and $Y_{E_{cfe},n}$ is vector at crime free equilibrium point E_{cfe} of the same vector length as Y_n . According to Iggider [22] the crime-free equilibrium point is globally asymptotically stable if the following conditions hold:

- (i) A should be a matrix with real negative eigenvalues.
- (ii) A_2 should be a Metzler matrix.

Theorem 4. *The crime free-equilibrium point, E_{cfe} , is globally asymptotically stable if $R_0 < 1$.*

Proof. Referring to system (1)-(6) we define

$$Y_n = (S_u, S_a, Q, P)^T, Y_i = (C, R)^T, E_{cfe} = \left(\frac{\sigma A}{k_1}, \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2}, 0, 0, \frac{A\tau(\varepsilon_1 k_1 + \sigma \eta)}{\mu k_1 k_2}, P_0 \right)^T$$

$$Y_{E_0, n} = \left(\frac{\sigma A}{k_1}, \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2}, \frac{A\tau(k_1 \varepsilon_1 + \sigma \eta)}{\mu k_1 k_2}, P_0 \right)^T$$

Now,

$$Y_n - Y_{E_0, n} = \begin{pmatrix} S_u \\ S_a \\ Q \\ P \end{pmatrix} - \begin{pmatrix} \frac{\sigma A}{k_1} \\ \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2} \\ \frac{A\tau(k_1 \varepsilon_1 + \sigma \eta)}{\mu k_1 k_2} \\ P_0 \end{pmatrix} = \begin{pmatrix} S_u - \frac{\sigma A}{k_1} \\ S_a - \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2} \\ Q - \frac{A\tau(k_1 \varepsilon_1 + \sigma \eta)}{\mu k_1 k_2} \\ P - P_0 \end{pmatrix}.$$

Equation (13) together with the model system (1)-(6) is written to the form:

$$\begin{pmatrix} \sigma A - \beta S_u C - (\eta + \mu) S_u \\ (1 - \sigma) A + \eta S_u + (1 - \theta) v R - \tau S_a - \left(1 - \frac{\rho C}{m + C}\right) \delta S_a C - \mu S_a \\ \tau S_a + \theta v R - \mu Q \\ \phi C - \phi_0 (P - P_0) \end{pmatrix} = A \begin{pmatrix} S_u - \frac{\sigma A}{k_1} \\ S_a - \frac{A(\varepsilon_1 k_1 + \sigma \eta)}{k_1 k_2} \\ Q - \frac{A\tau(k_1 \varepsilon_1 + \sigma \eta)}{\mu k_1 k_2} \\ P - P_0 \end{pmatrix} + A_1 \begin{pmatrix} C \\ R \end{pmatrix},$$

$$\begin{pmatrix} \beta S_u C + \left(1 - \frac{\rho C}{m + C}\right) \delta S_a C - \gamma C P - (\alpha + \mu) C \\ \gamma C P - (v + \mu) R \end{pmatrix} = A_2 \begin{pmatrix} C \\ R \end{pmatrix},$$

where the matrices A , A_1 and A_2 make the above equations meaningful.

Using the non-criminal elements of the Jacobian matrix of system (1)-(6) and the representation

in eq'n (13) we calculate

(14)

$$A = \begin{pmatrix} -(\eta + \mu) & 0 & 0 & 0 \\ \eta & -(\tau + \mu) & 0 & 0 \\ 0 & \tau & -\mu & 0 \\ 0 & 0 & 0 & -\phi_0 \end{pmatrix}, A_1 = \begin{pmatrix} -\beta S_u & 0 \\ \frac{\rho C}{(m+C)^2} \delta S_a C - (1 - \frac{\rho C}{m+C}) \delta S_a & (1 - \theta)v \\ 0 & \theta v \\ \phi & 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} \beta S_u - \frac{\rho m}{(m+C)^2} \delta S_a C + (1 - \frac{\rho C}{m+C}) \delta S_a - \gamma P - (\alpha + \mu) & 0 \\ \gamma P & -(v + \mu) \end{pmatrix}$$

Hence, the sufficient conditions are satisfied. Therefore, the crime free equilibrium point E_{cfe} is globally asymptotically stable if $R_0 < 1$. \square

Local stability of crime persistent equilibrium point (E_{cpe}):

Theorem 5. *If $R_0 > 1$, then the persistent equilibrium point, E_{cpf} , is locally asymptotically stable.*

Proof. The stability of the persistent equilibrium point is determined based on the signs of the eigenvalues of the Jacobian matrix which is computed at the crime persistent equilibrium.

The Jacobian matrix of the model system at E_{cpe} is given by:

$$J(E_{cpe}) = \begin{pmatrix} -a_{11} & 0 & -b_{11} & 0 & 0 & 0 \\ \eta & -d_{11} & -e_{11} & (1 - \theta)v & 0 & 0 \\ f_{11} & g_{11} & -h_{11} & 0 & 0 & -i_{11} \\ 0 & 0 & j_{11} & -k_{11} & 0 & i_{11} \\ 0 & \tau & 0 & \theta v & -\mu & 0 \\ 0 & 0 & \phi & 0 & 0 & -\phi_0 \end{pmatrix}$$

Here, $a_{11} = \beta C^* + \eta + \mu$, $b_{11} = \beta S_u^*$, $d_{11} = \left(1 - \frac{\rho C^*}{m+C^*}\right) \delta C^* + \tau + \mu$,

$e_{11} = -\left(\frac{\rho m C^*}{(m+C^*)^2} - \left(1 - \frac{\rho C^*}{m+C^*}\right)\right) \delta S_a^*$, $f_{11} = \beta C^*$, $g_{11} = \left(1 - \frac{\rho C^*}{m+C^*}\right) \delta C^*$,

$h_{11} = -\beta S_u^* + \left(\frac{\rho m C^*}{(m+C^*)^2} - \left(1 - \frac{\rho C^*}{m+C^*}\right)\right) \delta S_a^* + \gamma P^* + \alpha + \mu$, $i_{11} = \gamma C^*$, $j_{11} = \gamma P^*$.

After simplification the characteristic polynomial is obtained as

$$(15) \quad P(\lambda) = (\mu + \lambda) \left(a_5 \lambda^5 + a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 \right)$$

where

$$a_5 = 1, a_4 = a_{11} + d_{11} + k_3 + \mu + \phi_0, a_3 = a_{11}d_{11} + a_{11}k_3 + a_{11}\mu + a_{11}\phi_0 + d_{11}k_3 + \mu\phi_0 + d_{11}\mu + d_{11}\phi_0 + a_{11}d_{11}k_3 + a_{11}d_{11}\mu + a_{11}d_{11}\phi_0 + k_3\mu + k_3\phi_0 + i_{11}\phi + e_{11}g_{11} + b_{11}f_{11},$$

$$a_2 = a_{11}k_3\mu + a_{11}k_3\phi_0 + a_{11}\mu\phi_0 + d_{11}k_3\mu + d_{11}k_3\phi_0 + d_{11}\mu\phi_0 + k_3\mu\phi_0 + a_{11}i_{11}\phi + d_{11}i_{11}\phi + k_3i_{11}\phi + a_{11}e_{11}g_{11} + k_3e_{11}g_{11} + \phi_0e_{11}g_{11} - (1 - \theta)v_{g_{11}}j_{11} + b_{11}\eta_{g_{11}} + b_{11}f_{11}d_{11} + b_{11}f_{11}k_3 + b_{11}f_{11}\phi_0,$$

$$a_1 = a_{11}d_{11}k_3\mu + a_{11}d_{11}k_3\phi_0 + a_{11}d_{11}\mu\phi_0 + a_{11}k_3\mu\phi_0 + d_{11}k_3\mu\phi_0 + a_{11}d_{11}i_{11}\phi + a_{11}k_3i_{11}\phi + d_{11}k_3i_{11}\phi + a_{11}k_3e_{11}g_{11} + a_{11}e_{11}g_{11}\phi_0 + k_3e_{11}g_{11}\phi_0 - a_{11}(1 - \theta)v_{g_{11}}j_{11} - \phi_0(1 - \theta)v_{g_{11}}j_{11} - (1 - \theta)v_{g_{11}}i_{11}\phi + b_{11}\eta_{g_{11}}k_3 + b_{11}\eta_{g_{11}}\phi_0 + b_{11}f_{11}d_{11}k_3 + b_{11}f_{11}d_{11}\phi_0 + b_{11}f_{11}k_3\phi_0,$$

$$a_0 = a_{11}d_{11}k_3\mu\phi_0 + a_{11}d_{11}k_3i_{11}\phi + a_{11}k_3e_{11}g_{11}\phi_0 - a_{11}\phi_0(1 - \theta)v_{g_{11}}j_{11} - a_{11}(1 - \theta)v_{g_{11}}i_{11}\phi + b_{11}\eta_{g_{11}}k_3\phi_0 + b_{11}f_{11}d_{11}k_3\phi_0.$$

One of the eigenvalues of $J(E_{cpe})$ is $\lambda_1 = -\mu$ and the remaining five roots of equation(15) are analyzed by Routh-Hurwitz criteria. The coefficients $a_0, a_1, a_2, a_3, a_4, a_5$ of the characteristic polynomial equation are real positive. Thus, the necessary condition for stability of the equilibrium point is satisfied. From the sufficient condition for stability of the system, the Hurwitz array for the characteristic polynomial is presented as follows:

$$\begin{array}{c|ccc} \lambda^5 & a_5 & a_3 & a_1 \\ \lambda^4 & a_4 & a_2 & a_0 \\ \lambda^3 & b_1 & b_2 & b_3 \\ \lambda^2 & c_1 & c_2 & c_3 \\ \lambda^1 & d_1 & d_2 & d_3 \\ \lambda^0 & e_1 & e_2 & e_3 \end{array}$$

where a_0, a_1, a_2, a_3, a_4 and a_5 are characteristic polynomial coefficients and the elements in the rest of the array are computed using the following ways.

$$b_1 = \frac{-1}{a_4} \begin{vmatrix} a_5 & a_3 \\ a_4 & a_2 \end{vmatrix} = \frac{a_3a_4 - a_2}{a_4} > 0, \quad \text{since } a_5 = 1,$$

$$\begin{aligned}
b_2 &= \frac{-1}{a_4} \begin{vmatrix} a_5 & a_1 \\ a_4 & a_0 \end{vmatrix} = \frac{a_1 a_4 - a_0}{a_4}, b_3 = \frac{-1}{a_4} \begin{vmatrix} a_5 & 0 \\ a_4 & 0 \end{vmatrix} = 0, \\
c_1 &= \frac{-1}{b_1} \begin{vmatrix} a_4 & a_2 \\ b_1 & b_2 \end{vmatrix} = \frac{a_2 b_1 - a_4 b_2}{b_1} > 0, c_2 = \frac{-1}{b_1} \begin{vmatrix} a_4 & a_0 \\ b_1 & b_3 \end{vmatrix} = a_0, c_3 = \frac{-1}{b_1} \begin{vmatrix} a_4 & 0 \\ b_1 & 0 \end{vmatrix} = 0, \\
d_1 &= \frac{-1}{c_1} \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} = \frac{b_2 c_1 - b_1 c_2}{c_1} > 0, d_2 = \frac{-1}{c_1} \begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix} = 0, d_3 = \frac{-1}{c_1} \begin{vmatrix} b_1 & 0 \\ 0 & 0 \end{vmatrix} = 0, \\
e_1 &= \frac{-1}{d_1} \begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix} = a_0 > 0, e_2 = \frac{-1}{d_1} \begin{vmatrix} c_1 & c_3 \\ d_1 & d_2 \end{vmatrix} = \frac{b_2 c_1 - b_1 c_2}{c_1}, e_3 = \frac{-1}{d_1} \begin{vmatrix} c_1 & 0 \\ d_1 & 0 \end{vmatrix} = 0.
\end{aligned}$$

The coefficients of the characteristic polynomial equation $a_0, a_1, a_2, a_3, a_4, a_5$ are real positive. More over the first column of the Routh-Hurwitz array have the same positive sign. Hence, by Routh-Hurwitz criteria all eigenvalues of the characteristics polynomial equation are negative. Therefore the crime persistent equilibrium point is locally asymptotically stable if $R_0 > 1$. \square

Bifurcation: For all values of R_0 , the E_{cfe} always exists and is never destroyed. When $R_0 < 1$ the crime free equilibrium point is stable and there is no other equilibrium point. As soon as $R_0 > 1$, the E_{cfe} becomes unstable and a new equilibrium point, the crime persistent equilibrium point, E_{cpe} , is created and is expected to be stable. Moreover for $R_0 = 1$, $E_{cfe} = E_{cpe}$. Hence a transcritical bifurcation occurs in the model at the bifurcation point $R_0 = 1$.

4. NUMERICAL SIMULATION AND DISCUSSION

In this section, we present numerical simulations to validate our analytical findings and stability results obtained in the previous sections. Numerical experiments are carried out to monitor the dynamics of the system (1)-(6). The main focus of the simulation is to investigate the response of model parameters for the spread of crime. The simulation is carried out for different values of the parameters. The full list of parameter values used in the simulation are given in the following table.

| Parameter(symbol) | Value | Source |
|-------------------|----------|------------|
| Λ | 30 | Assumption |
| η | 0.02 | Assumption |
| ϕ_0 | 0.005 | [2] |
| P_0 | 700 | [2] |
| δ | 0.000003 | Assumption |
| γ | 0.00001 | Assumption |
| α | 0.00005 | [2] |
| ν | 0.00004 | Assumption |
| θ | 0.05 | Assumption |
| m | 0.00001 | Assumption |
| ϕ | 0.000032 | [2] |
| β | 0.000007 | Assumption |
| μ | 0.00052 | [2] |
| σ | 0.002 | varies |
| ρ | 0.6 | varies |
| τ | 0.008 | Assumption |

TABLE 2. The set of parameter values.

Additionally we also assumed the initial conditions as $S_u(0) = 3000, S_a(0) = 4000, C(0) = 2000, R(0) = 350, P(0) = 700, Q(0) = 1000$. For the above set of parameter values, the equilibrium values are obtained as: $S_u^* = 2.6525, S_a^* = 3393.7, C^* = 300, R^* = 376.03, Q^* = 5221.4, P^* = 701.92$. The real part of the eigenvalues of the Jacobian matrix corresponding to the equilibrium E_{cpe} for the model system (1)-(6) are $-0.0182, -0.0087, -0.0023, -0.0002$ and -0.00052 . It is apparent that all the six eigenvalues of the Jacobian matrix have negative real part. Hence, the persistent equilibrium, E_{cpe} , is locally asymptotically stable.

The population for different class with time to compare the distribution of the population when $R_0 < 1$ and $R_0 > 1$ is shown in figure 2. In figure 2a, the basic reproduction number, $R_0 = 0.0104 < 1$. The figure tells us that the crime aware susceptible, non-criminals and the

police force increase, while criminal and prisoner population decrease. In figure 2b the basic reproduction number, $R_0 = 1.3025 > 1$. The figure tells us that criminal population increases, crime aware susceptible population and prisoner population decrease. The variation in criminal population $C(t)$ and prisoner population $R(t)$ with respect to time for different values of ϕ is shown in Fig.3. It is clear that from figure 3a and 3b as the values of additional recruitment rate of police force ϕ increases the criminal population decreases while the prisoner increases as additional recruitment rate of police force ϕ increases. Also, the variation in criminal population $C(t)$ and police force $P(t)$ with respect to time for different values of incarceration rate γ is shown in Fig.4. It is apparent from this figure that as the value of γ increases the criminal population as well as police force decrease. It is because the incarceration of criminals with higher rate will reduce the number of criminals in the society and hence the police force. From figures 5a and 5b we observe aware susceptible population $S_a(t)$ decreases while the non-criminal population $C(t)$ increase as τ increases and from figures 6a and 6b we observe as media coverage increases aware susceptible population $S_a(t)$ increases while criminal population $C(t)$ decreases. The figure 7 describes R_0 decreases as the baseline police force P_0 increases. In this it is observed P_0 attains its critical value P_c which is required for the eradication of crime. From here it is concluded that for the establishment of crime free society, the baseline police force P_0 must be greater than the critical value P_c .

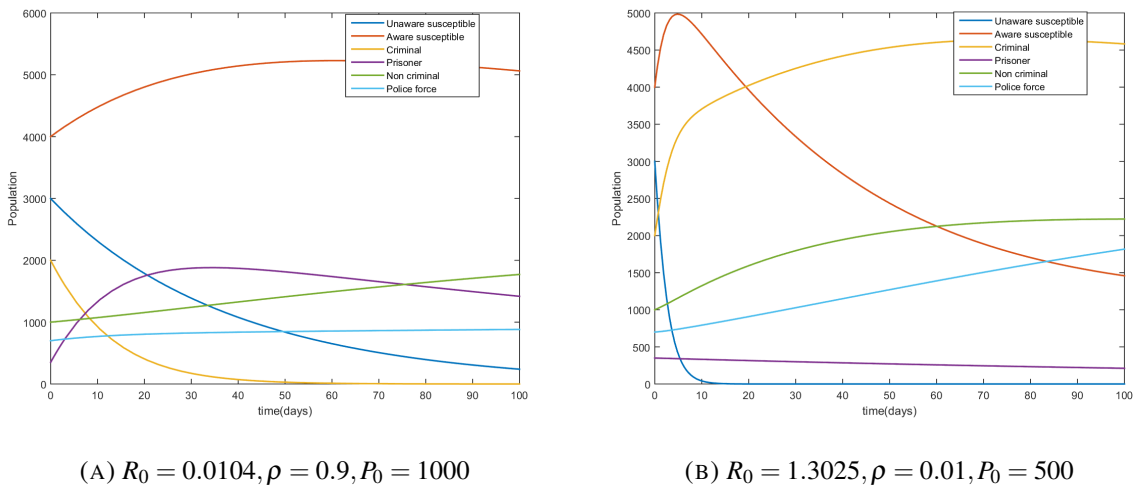


FIGURE 2. Time series plot of state variables

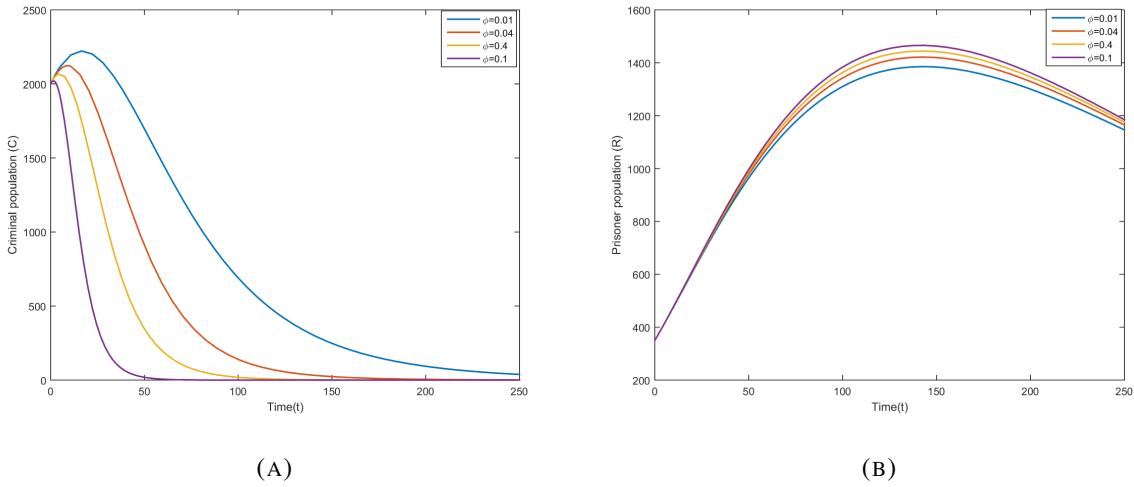


FIGURE 3. Variations of criminal population (C) and prisoner population (R) w.r.t. time t for different values of additional recruitment rate of police force ϕ .

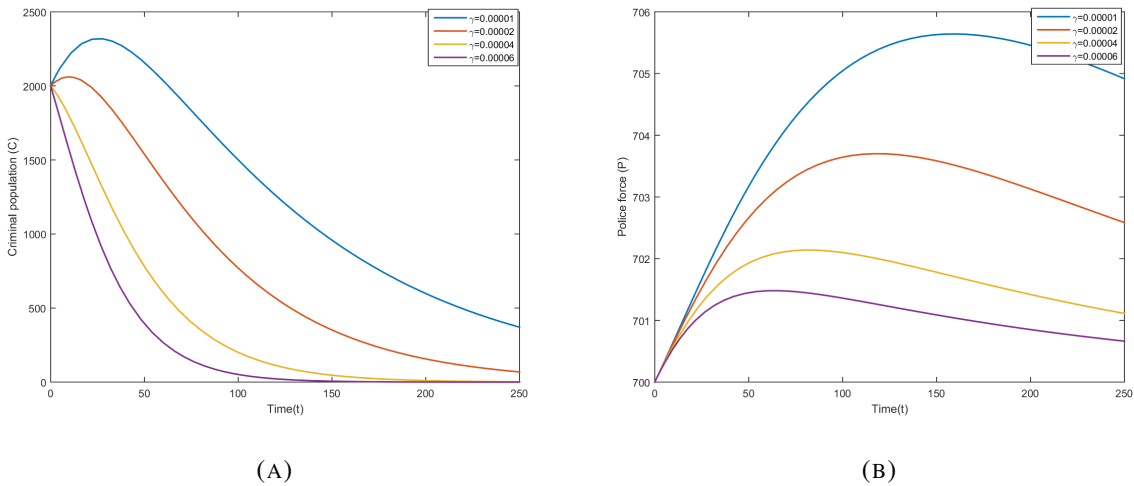


FIGURE 4. Variations of criminal population(C) and police force (P) w.r.t. time t for different values of γ .

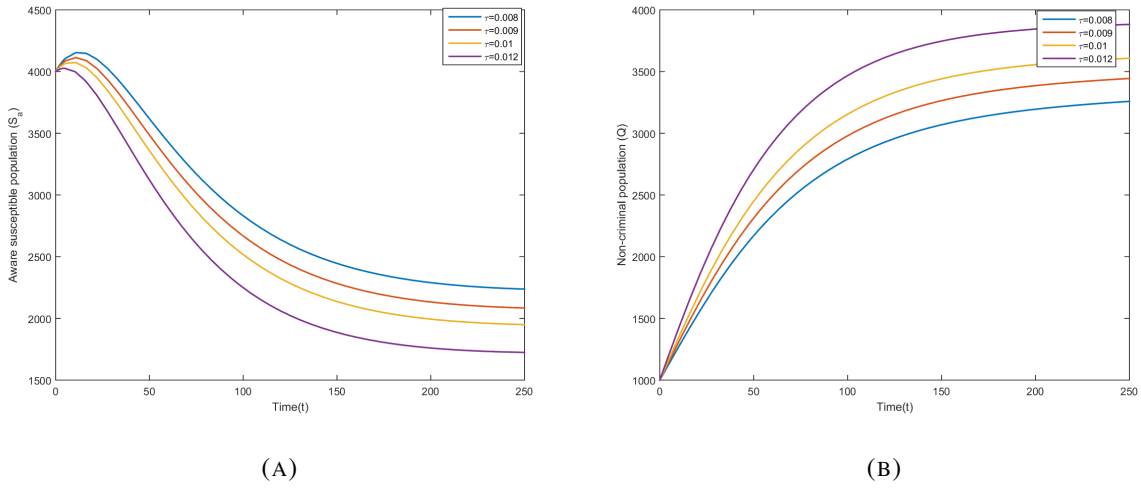


FIGURE 5. Aware susceptible(S_a) and non-criminal population(Q) w.r.t. time t for different values of τ .

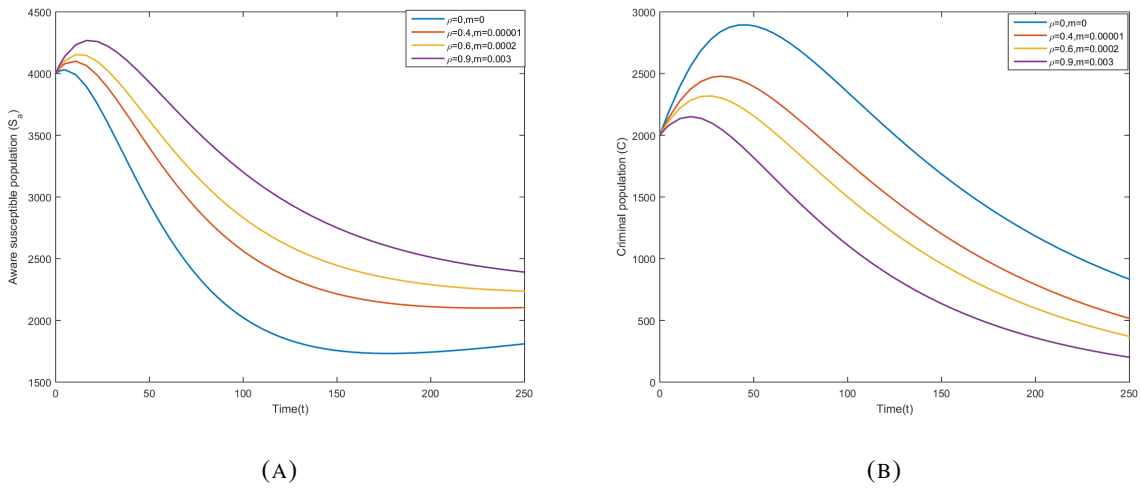


FIGURE 6. Variations of aware susceptible population (S_a) and criminal population (C) w.r.t. time t for different values of ρ & m .

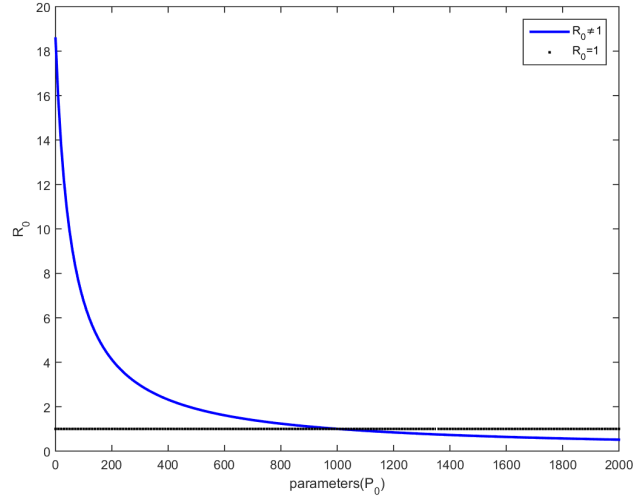


FIGURE 7. Variation of threshold R_0 w.r.t. baseline police force P_0 .

Numerical simulation of sensitivity analysis: In this part we observe the influence of parameters on the reproduction number R_0 . The absolute change of R_0 with respect to P_0, τ and γ is given by $\frac{\partial R_0}{\partial P_0} < 0, \frac{\partial R_0}{\partial \tau} < 0$ and $\frac{\partial R_0}{\partial \gamma} < 0$, respectively. This implies R_0 decreases as P_0, τ and γ increase.

The normalized forward sensitivity index M_I of a variable R_0 that depends on a parameter S , as [32] is defined as

$$(16) \quad M_I = \frac{\partial R_0}{\partial S} \times \frac{S}{R_0}$$

Since we have explicit formula for reproduction number R_0 in equation(11) it follows that $M_I > 0$ for the parameters: $\Lambda, \sigma, \beta, \delta$ and $M_I < 0$ for the parameters: $\tau, \mu, \eta, \gamma, P_0$ and α . Increasing awareness on media coverage, incarceration rate and base line police force are best strategies to reduce R_0 .

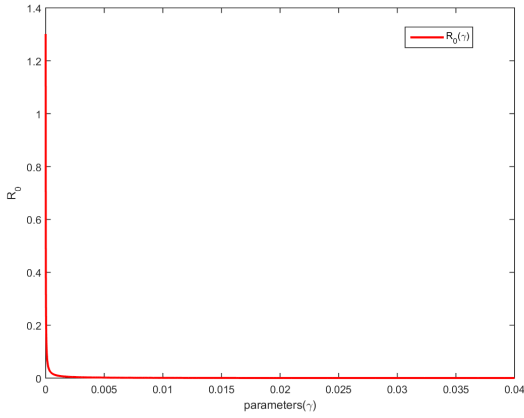
| Parameter | Sign | Sensitivity indices |
|---------------------|------|---------------------|
| $M_{\Lambda}^{R_0}$ | + | 1 |
| $M_{\sigma}^{R_0}$ | + | 0.001883 |
| $M_{\beta}^{R_0}$ | + | 0.001934 |
| $M_{\tau}^{R_0}$ | - | 0.937151 |
| $M_{\mu}^{R_0}$ | - | 0.016708 |
| $M_{\delta}^{R_0}$ | + | 0.998066 |
| $M_{\eta}^{R_0}$ | - | 0.218837 |
| $M_{\gamma}^{R_0}$ | - | 0.924703 |
| $M_{\alpha}^{R_0}$ | - | 0.006605 |
| $M_{P_0}^{R_0}$ | - | 0.924703 |

TABLE 3. Sensitivity indices of R_0 evaluated at the parameter values.

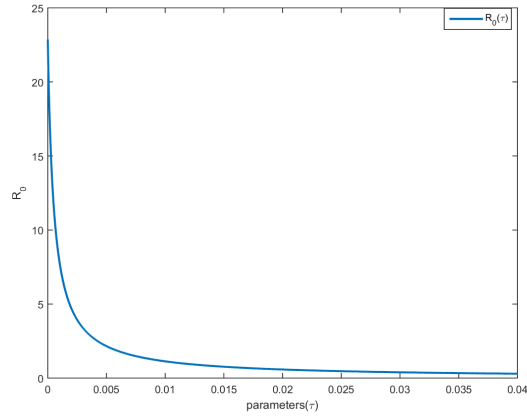
We have numerically computed the relative sensitivity of R_0 with respect to the above parameters (using the parameters values in Table 2) and have displayed the results in Table 3.

From table 3, the negative signs indicate parameters are inversely proportional to R_0 while the positive sign parameters are directly proportional to R_0 . Model parameters whose sensitivity index values are near -1 or 1 suggest that a change in their magnitude has a significant impact on either increasing or decreasing the size of R_0 , respectively. We noted also that R_0 is sensitive to the parameters γ , P_0 and τ . An increase(decrease) in γ , P_0 and τ by 10% will decrease (increase) R_0 by 9.247% and 9.371%, respectively. Further, an increase (decrease) δ by 10% will increase (decrease) R_0 by 9.98%. A similar change in β , α , μ , η , σ has its own effect on R_0 .

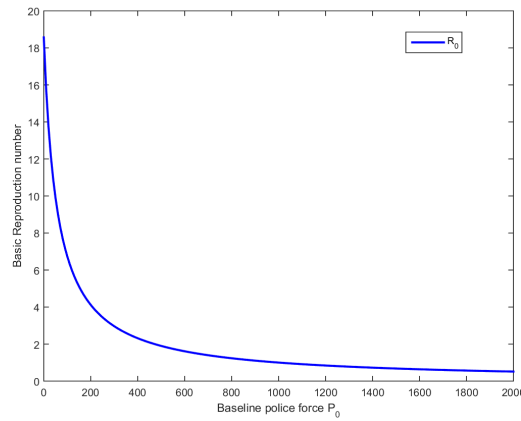
Fig8 *a, b, c* show that the crime persistent cease to exist as γ , τ and P_0 increase.



(A) R_0 Vs. γ



(B) R_0 Vs. τ



(C) R_0 Vs. P_0

FIGURE 8. Variation of R_0 for different parameters γ , τ and P_0 .

5. CONCLUSION

Crime affects everyone in one way or the other. It is observed that criminal behavior is contagious like epidemics and spreads through peer influence. Thus, the epidemic modeling approach can readily be applied to model the dynamics of crime in the society and its control. In view of this, unlike some of other previous model, we have taken into account the impact of media coverage, police force on crime and moral/religious activity and we also included additional compartments. We have considered that criminals in the society increase due to interaction of criminals with people having a tendency to commit a crime. The police force discourage

crime in the society by thesaurussing criminals. Crucial mathematical features which include; Wel-posedness, positivity of the solution, existence and stability criteria of the crime-free(E_{cfe}), the persistent equilibria(E_{cpe}) have been derived in terms of the basic reproduction number, R_0 . Stability of the disease free and endemic equilibrium is studied. The persistent equilibrium point is determined and shown to be locally asymptotically stable when the threshold parameter value, R_0 , is greater than unity. The results of the crime free equilibrium showed that the model is both locally and globally stable when $R_0 < 1$, thus reducing R_0 to less than unity reduces the crime spread. The numerical analysis shows that in the presence of media coverage, police force and moral/religious the crime dies out faster while lack of these reporting the presence of the crime and preventive measures greatly increases the number of criminal people in the population which is not encouraging for the eradication of the crime.

RECOMMENDATION

The model that we adjusted has not carried out optimal control and cost effectiveness of different crime intervention strategies, which can be investigated in future to find out which strategy is the top in the control of the crime.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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