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MATHEMATICAL STUDY AIMING AT ADOPTING AN EFFECTIVE STRATEGY TO COEXIST WITH CORONAVIRUS PANDEMIC

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Abstract. In this paper, we propose a discrete mathematical model that describes the evolution of the "covid-19" virus in a human population and the efforts made to control it. Our objective is to develop a simple, logical and an optimal strategy to reduce the negative impact of this infectious disease on countries. This objective is achieved through maximizing the number of people applying the preventive measures recommended by WHO against the pandemic in order to reduce the infection as much as possible. The tools of optimal control theory were used in this study, in particular Pontryagin's maximum principle.

Keywords: optimal control; SARS-COV-2; mathematical model; discrete model; COVID-19.

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1. INTRODUCTION

According to WHO, Covid-19, which appeared in the fourth quarter of 2019 in China, belongs to the coronavirus family which has a large number of viruses that can cause mild diseases

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in humans such as cold but also more severe or even fatal diseases. This virus is close to the one that caused the SARS epidemic (Severe Acute Respiratory Syndrome in 2002-2003). It seems less violent than the one that caused SARS but could be more contagious. It may therefore be possible that patients may be infected with this virus without yet showing symptoms (fever, persistent cough, shortness of breath, respiratory difficulties) while being contagious.

Coronaviruses are a family of various viruses that can infect both humans and animals (see [5]). Their name means "crown virus" which comes from the fact that they all have a crown-shaped appearance when viewed under a microscope. Coronaviruses were first identified in humans in the 1960s. They cause emerging diseases, that is, new infections caused by changes or mutations in the virus. Human coronaviruses mainly cause respiratory infections. They can also be accompanied by digestive disorders such as gastroenteritis. The incubation time, duration between exposure to the virus and the manifestation of the first symptoms, is between 5 and 14 days.

This coronavirus or "Sars-CoV-2" has many similarities to that of SARS (animal origin, genetics identification to 80%, causing pulmonary infections) but also there are notable differences according to scientists in terms of its contagion. Sars-CoV-2 is contagious from the onset of symptoms or sometimes in the absence of symptoms, while SARS becomes infectious few days after the first symptoms. There are also mild and asymptomatic forms of Sars-CoV-2 while Sars only caused severe forms.

According to the World Health Organization Weekly Operational Update on COVID-19 in August 2020 (see [14]), 24 257 989 people were infected with coronavirus worldwide, with more than 827 246 dead. It is a pandemic that is advancing at high speed in Asia, America and lately in Europe.

Coronavirus is an RNA virus with a high mutation rate like that of the flu or the HIV virus. Coronaviruses exist in many animal species and circulate fairly easily from one species to another which can lead to death. Coronaviruses are transmitted from human to human during close contact (touching or shaking hands, for example) and by air, coughing or sneezing. Touching an object or a surface with the virus on it, and then touching your mouth, nose, or eyes before

washing your hands can also spread this virus. More rarely the contamination can be transmitted by facial contact.

Most commonly, it causes mild to moderate respiratory illnesses like the common cold with symptoms such as:

- ⇒ **headache**
- ⇒ **cough**
- ⇒ **sore throat**
- ⇒ **fever**
- ⇒ **a general feeling of discomfort**

More seriously it can cause respiratory diseases of the lower respiratory system such as pneumonia or bronchitis, particularly in people with cardiopulmonary disease and those whose immune system is weakened and the elderly.

There is currently no vaccine available to protect against coronavirus. However, scientists at Pasteur Institute are multiplying initiatives to find a vaccine against this virus. They have managed to isolate and culture strains of this new virus and they are working on developing a vaccine within a year and a half, said researchers from the National Reference Center for Respiratory Viruses in a press release dated January 30, 2020. Researchers at the Pasteur Institute now have the virus that causes this infection. This viral isolation opens the way to new diagnostic, therapeutic and prophylactic approaches.

To reduce the risk of coronavirus infection, the WHO recommends the following preventive measures:

- ⇒ **CE washing hands regularly with soap and water for at least 20 seconds (not to mention the handy hydro-alcoholic solutions when you cannot wash your hands)**
- ⇒ **avoiding touching the eyes, nose or mouth when hands are not washed**
- ⇒ **avoiding contact with sick people**
- ⇒ **wearing a mask**

2. BACKGROUND

Since the onset of the coronavirus pandemic, researchers around the world have focused on studying the virus and finding effective strategies to combat it. For example in [5], the authors tried to develop a multiregional model in which they introduced controls representing four different strategies. The first represents the effort of awareness campaigns in region j that aims at introducing health measures to protect individuals from being infected by the virus (covering all cuts, shoes and long sleeve shirts when handling animals, washing hands thoroughly on a regular basis, taking shower and cleaning up work place and home...), the second represents security campaigns and health measures to prevent the movement of individuals from region j to region r , the third represents the effort to encourage the exposed individuals to join quarantine centers in the same region and the fourth is the effort to dispose of the infected animals.

In the article Optimal Control of the COVID-19 Pandemic with Non-pharmaceutical Interventions [1], the authors modeled the evolution of the pandemic by introducing a vaccinated population by ordinary differential equations and they introduced an optimal control aiming to minimize deaths.

In the article A Mathematical Modeling with Optimal Control Strategy of Transmission of COVID-19 Pandemic Virus [7], the authors carried out a continuous model describing the evolution of the disease within the country by introducing three optimal controls, the first of which can be interpreted as the proportion to be subjected to awareness and prevention, the second can be interpreted as quarantine and health surveillance and the third can be interpreted as the proportion to be subjected to diagnosis and monitoring.

In this paper, our work is a little different since we will try to indirectly minimize the number of infected people by preventing their contact with susceptible people and directly the number of people who do not apply preventive measures against the coronavirus. The aim is to find an optimal strategy that will reduce the impact of the coronavirus and at the same time open up all the productive sectors of the country, including tourism.

The mathematical processing employed calls upon the tools of the theory of optimal control and more precisely the Pontryagin's maximum principle [20–30].

3. MODEL FORMULATION

The model that we propose below makes it possible to describe the evolution of the pandemic by distinguishing two populations likely to be infected (S1) and (S2), as well as two hospitalized populations (H1) and (H2).

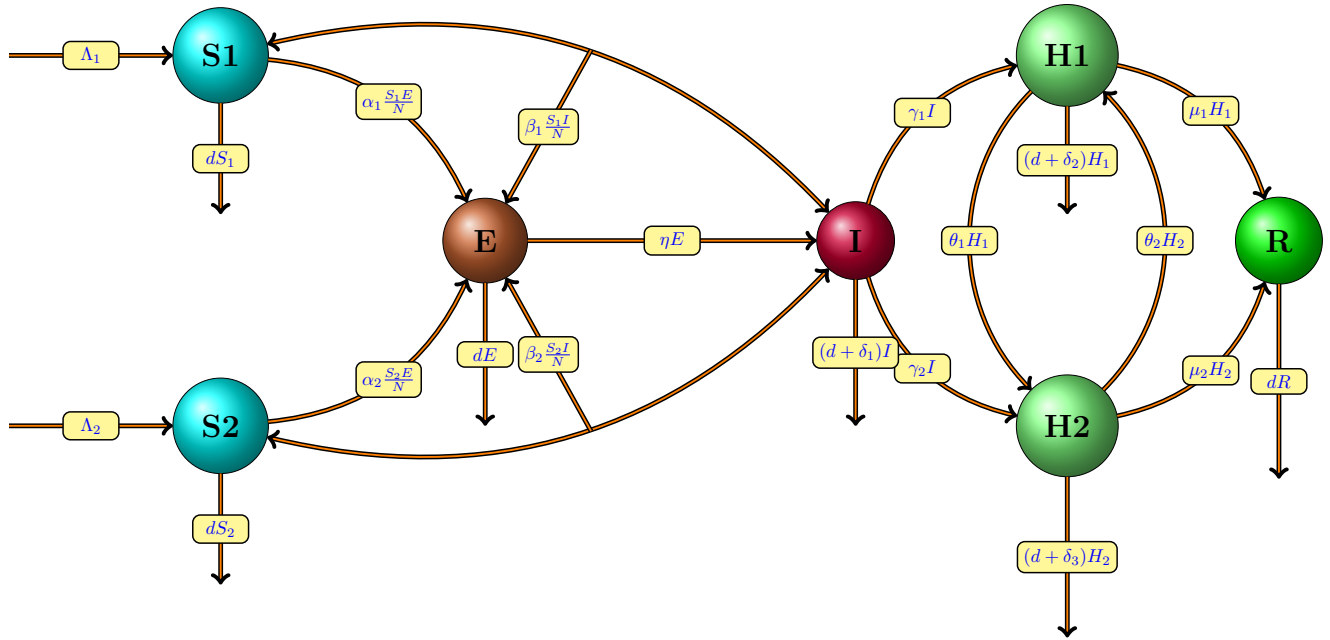


FIGURE 1. Diagram of the evolution of the Corona pandemic

The different compartments of our model are described below:

Compartment S₁: represents the number of people likely to be infected by the coronavirus and who do not apply preventive measures. It is increased by adding a number Λ_1 each day (the time unit used in this study) and decreases by contact with infected people without symptoms and others with symptoms respectively with reports $\alpha_1 \frac{S_1 E}{N}$ and $\beta_1 \frac{S_1 I}{N}$ and also by a natural mortality rate d .

Compartment S₂: represents the number of people who may be infected by coronavirus and who take preventive measures. It is increased by adding a number Λ_2 each day (the time unit used in this study) and decreases by contact with infected people without symptoms and others with symptoms respectively with reports $\alpha_2 \frac{S_2 E}{N}$ and $\beta_2 \frac{S_2 I}{N}$ and also by a natural mortality rate d .

Compartment E: represents the number of infected people not showing symptoms. It increases by the contact with people who apply preventive measures and those who do not apply them respectively by the ratios $\alpha_1 \frac{S_1 E}{N}$ and $\alpha_2 \frac{S_2 E}{N}$ and also by a contact between these people and infected cases showing symptoms with reports $\beta_1 \frac{S_1 I}{N}$ and $\beta_2 \frac{S_2 I}{N}$, and decreases by natural mortality d . Logically speaking, these contacts occur before the infected cases are declared "infected" (i.e. before the screening test) and after they will be isolated and hospitalized.

Compartment I: represents the number of infected people showing symptoms. It increases by the rate η of infected people without symptoms and decreases by the rates of hospitalized people γ_1 and γ_2 and by the mortality rate $d + \delta_1$.

Compartment H₁: represents the number of hospitalized people requiring normal medical care (their state of health does not require intensive care). It increases by the rate γ_1 of infected persons showing symptoms and decreases by the rate θ_1 of recovered persons as well as by a mortality rate of $d + \delta_2$.

Compartment H₂: represents the number of hospitalized people requiring intense medical care. It increases by the rate γ_2 of infected persons showing symptoms and decreases by the rate θ_2 of recovered persons as well as with a mortality rate of $d + \delta_3$.

Compartment R: represents the number of the recovered people. It increases by the rates θ_1 and θ_2 of people hospitalized and decreases by the rate of natural mortality d .

The previous diagram is mathematically translated by the following nonlinear system:

$$(1) \quad \begin{cases} S_{1,k+1} = \Lambda_1 - \alpha_1 \frac{S_{1,k} E_k}{N_k} - \beta_1 \frac{S_{1,k} I_k}{N_k} + (1-d) S_{1,k} \\ S_{2,k+1} = \Lambda_2 - \alpha_2 \frac{S_{2,k} E_k}{N_k} - \beta_2 \frac{S_{2,k} I_k}{N_k} + (1-d) S_{2,k} \\ E_{k+1} = \alpha_1 \frac{S_{1,k} E_k}{N_k} + \alpha_2 \frac{S_{2,k} E_k}{N_k} + \beta_1 \frac{S_{1,k} I_k}{N_k} + \beta_2 \frac{S_{2,k} I_k}{N_k} + (1-d-\eta) E_k \\ I_{k+1} = \eta E_k + (1-\gamma_1-\gamma_2-d-\delta_1) I_k \\ H_{1,k+1} = \gamma_1 I_k + (1-\theta_1-\mu_1-d-\delta_2) H_{1,k} + \theta_2 H_{2,k} \\ H_{2,k+1} = \gamma_2 I_k + (1-\theta_2-\mu_2-d-\delta_3) H_{2,k} + \theta_1 H_{1,k} \\ R_{k+1} = \mu_1 H_{1,k} + \mu_2 H_{2,k} + (1-d) R_k \end{cases}$$

Where $S_{1,0} \geq 0, S_{2,0} \geq 0, E_0 \geq 0, I_0 \geq 0, H_{1,0} \geq 0, H_{2,0} \geq 0$, and $R_0 \geq 0$. The total companies size at time k is denoted by N_k with $N_k = S_{1,k} + S_{2,k} + E_k + I_k + H_{1,k} + H_{2,k} + R_k$, and it is supposed to be constant.

4. THE OPTIMAL CONTROL PROBLEM

In this paragraph, we aim to introduce two controls that enable us minimizing the number of people who do not apply preventive measures (S1) to protect them from the coronavirus pandemic and decrease the contact of healthy people with infected people. The first control u actually represents an increase in the number of people applying preventive measures against the coronavirus (S2) and a decrease in the number of people who do not apply these preventive measures (S1) thanks to awareness campaigns on media or through direct contact with people on the street. As for the second control v , it represents a rate of increase in the compartments (S1) and (S2) of people likely to be infected and a rate of the decrease in the compartment (E) and (I) of infected persons by applying the measures aiming at reducing social contact. So, the controlled mathematical system is given by the following system of difference equations:

$$(2) \quad \left\{ \begin{array}{l} S_{1,k+1} = \Lambda_1 - (1 - v_k) \left(\alpha_1 \frac{S_{1,k}E_k}{N_k} + \beta_1 \frac{S_{1,k}I_k}{N_k} \right) + (1 - d - u_k) S_{1,k} \\ S_{2,k+1} = \Lambda_2 - (1 - v_k) \left(\alpha_2 \frac{S_{2,k}E_k}{N_k} + \beta_2 \frac{S_{2,k}I_k}{N_k} \right) + u_k S_{1,k} + (1 - d) S_{2,k} \\ E_{k+1} = (1 - v_k) \left(\alpha_1 \frac{S_{1,k}E_k}{N_k} + \beta_1 \frac{S_{1,k}I_k}{N_k} + \alpha_2 \frac{S_{2,k}E_k}{N_k} + \beta_2 \frac{S_{2,k}I_k}{N_k} \right) + (1 - d - \eta) E_k \\ I_{k+1} = \eta E_k + (1 - \gamma_1 - \gamma_2 - d - \delta_1) I_k \\ H_{1,k+1} = \gamma_1 I_k + (1 - \theta_1 - \mu_1 - d - \delta_2) H_{1,k} + \theta_2 H_{2,k} \\ H_{2,k+1} = \gamma_2 I_k + (1 - \theta_2 - \mu_2 - d - \delta_3) H_{2,k} + \theta_1 H_{1,k} \\ R_{k+1} = \mu_1 H_{1,k} + \mu_2 H_{2,k} + (1 - d) R_k \end{array} \right.$$

Where $S_{1,0} \geq 0, S_{2,0} \geq 0, E_0 \geq 0, I_0 \geq 0, H_{1,0} \geq 0, H_{2,0} \geq 0$, and $R_0 \geq 0$.

The optimal control problem to minimize the objective functional is given by

$$(3) \quad J(u, v) = S_{1,T} + E_T + I_T + \sum_{k=0}^{T-1} \left(S_{1,k} + E_k + I_k + \frac{A_k u_k^2}{2} - \frac{B_k v_k^2}{2} \right).$$

Where the parameters $A_k > 0$ and $B_k > 0$ are selected to weigh, respectively, the relative importance of the cost of awareness campaigns and preventive measures to reduce social contact.

The aim is to find an optimal control, (u^*, v^*) such that

$$J(u^*, v^*) = \min_{(u,v) \in U_{ad}} J(u, v).$$

where U_{ad} is the set of admissible controls defined by

$$U_{ad} = \{X = (X_0, X_1, \dots, X_{T-1}) / 0 \leq X_{\min} \leq X_k \leq X_{\max} \leq 1; \text{ for } X = u, v, k = 0, 1, \dots, T-1.\}$$

The sufficient condition of the existence of an optimal control (u^*, v^*) for problem (2) and (3) comes from the following theorem.

Theorem 1. *There exists the optimal control (u^*, v^*) such that*

$$J(u^*, v^*) = \min_{(u,v) \in U_{ad}} J(u, v),$$

subject to the control system (2) with initial conditions.

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps, $S_1 = (S_{1,0}, S_{1,1}, S_{1,2}, \dots, S_{1,T})$, $S_2 = (S_{2,0}, S_{2,1}, S_{2,2}, \dots, S_{2,T})$, $E = (E_0, E_1, E_2, \dots, E_T)$, $I = (I_0, I_1, I_2, \dots, I_T)$, $H_1 = (H_{1,0}, H_{1,1}, H_{1,2}, \dots, H_{1,T})$, $H_2 = (H_{2,0}, H_{2,1}, H_{2,2}, \dots, H_{2,T})$, $R = (R_0, R_1, R_2, \dots, R_T)$ are uniformly bounded for all (u, v) in the control set U_{ad} thus $J(u, v)$ is bounded for all $(u, v) \in U_{ad}$. Since $J(u, v)$ is bounded, $\inf_{(u,v) \in U_{ad}} J(u, v)$ is finite, and there exists a sequence $(u^j, v^j) \in U_{ad}$ such that $\lim_{j \rightarrow +\infty} (u^j, v^j) = \inf_{(u,v) \in U_{ad}} J(u, v)$ and corresponding sequences of states $S_1^j, S_2^j, E^j, I^j, H_1^j, H_2^j$, and R^j .

Since there are a finite number of uniformly bounded sequences, there exist (u^*, v^*) and $S_1^*, S_2^*, E^*, I^*, H_1^*, H_2^*$, and $R^* \in \mathbb{R}^{T-1}$ such that, on a subsequence, $\lim_{j \rightarrow +\infty} (u^j, v^j) = (u^*, v^*)$, $\lim_{j \rightarrow +\infty} S_1^j = S_1^*$, $\lim_{j \rightarrow +\infty} S_2^j = S_2^*$, $\lim_{j \rightarrow +\infty} E^j = E^*$, $\lim_{j \rightarrow +\infty} I^j = I^*$, $\lim_{j \rightarrow +\infty} H_1^j = H_1^*$, $\lim_{j \rightarrow +\infty} H_2^j = H_2^*$ and $\lim_{j \rightarrow +\infty} R^j = R^*$.

Finally, due to the finite dimensional structure of system (2) and the objective function $J(u, v)$, (u^*, v^*) is an optimal control with corresponding states S_1^* , S_2^* , E^* , I^* , H_1^* , H_2^* and R^* . Therefore, $\inf_{(u,v) \in U_{ad}} J(u, v)$ is achieved. \square

5. CHARACTERISATION OF THE OPTIMAL CONTROL

We apply the discrete version of Pontryagin's maximum principle [20–30].

The key idea is introducing the adjoint function to attach the system of difference equations to the objective functional resulting in the formation of a function called the Hamiltonian.

This principle converts the problem of finding the control to optimize the objective functional subject to the state of difference equation with initial condition into finding the control to optimize Hamiltonian pointwise (with respect to the control).

Now, we have the Hamiltonian \widehat{H}_k at time step k , defined by

$$\widehat{H}_k = S_{1,k} + E_k + I_k + \frac{A_k u_k^2}{2} - \frac{B_k v_k^2}{2} + \sum_{i=1}^7 \lambda_{i,k+1} f_{i,k+1}$$

where $f_{i,k+1}$ is the right side of the system of difference equation (2) of the i^{th} state variable at time step $k + 1$.

Theorem 2. *Given an optimal control $(u^*, v^*) \in U_{ad}$ and solutions $S_{1,k}^*, S_{2,k}^*, E_k^*, I_k^*, H_{1,k}^*, H_{2,k}^*$ and R_k^* of corresponding state system there exist adjoint functions $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and λ_6 satisfying the following equations:*

$$\begin{aligned} \lambda_{1,k} &= \frac{\partial \widehat{H}_k}{\partial S_{1,k}} = 1 + (\lambda_{3,k+1} - \lambda_{1,k+1}) \left(\alpha_1 \frac{E_k}{N_k} + \beta_1 \frac{I_k}{N_k} \right) (1 - v_k) + (\lambda_{2,k+1} - \lambda_{1,k+1}) u_k + \lambda_{1,k+1} (1 - d) \\ \lambda_{2,k} &= \frac{\partial \widehat{H}_k}{\partial S_{2,k}} = (\lambda_{3,k+1} - \lambda_{2,k+1}) \left(\alpha_2 \frac{E_k}{N_k} + \beta_2 \frac{I_k}{N_k} \right) (1 - v_k) + \lambda_{2,k+1} (1 - d) \\ \lambda_{3,k} &= \frac{\partial \widehat{H}_k}{\partial E_k} = 1 + (\lambda_{3,k+1} - \lambda_{1,k+1}) \alpha_1 \frac{S_{1,k}}{N_k} (1 - v_k) + (\lambda_{3,k+1} - \lambda_{2,k+1}) \alpha_2 \frac{S_{2,k}}{N_k} (1 - v_k) + \lambda_{3,k+1} (1 - d - \eta) \\ &\quad + \eta \lambda_{4,k+1} \\ \lambda_{4,k} &= \frac{\partial \widehat{H}_k}{\partial I_k} = 1 + (\lambda_{3,k+1} - \lambda_{1,k+1}) \beta_1 \frac{S_{1,k}}{N_k} (1 - v_k) + (\lambda_{3,k+1} - \lambda_{2,k+1}) \beta_2 \frac{S_{2,k}}{N_k} (1 - v_k) + \gamma_1 (\lambda_{5,k+1} - \lambda_{4,k+1}) \\ &\quad + \gamma_2 (\lambda_{6,k+1} - \lambda_{4,k+1}) + \lambda_{4,k+1} (1 - d - \delta_1) \\ \lambda_{5,k} &= \frac{\partial \widehat{H}_k}{\partial H_{1,k}} = \mu_1 (\lambda_{7,k+1} - \lambda_{5,k+1}) + \theta_1 (\lambda_{6,k+1} - \lambda_{5,k+1}) + \lambda_{5,k+1} (1 - d - \delta_2) \\ \lambda_{6,k} &= \frac{\partial \widehat{H}_k}{\partial H_{2,k}} = \mu_2 (\lambda_{7,k+1} - \lambda_{6,k+1}) + \theta_2 (\lambda_{5,k+1} - \lambda_{6,k+1}) + \lambda_{6,k+1} (1 - d - \delta_3) \\ \lambda_{7,k} &= \frac{\partial \widehat{H}_k}{\partial R_k} = \lambda_{7,k+1} (1 - d) \end{aligned}$$

with the transversality conditions at time T :

$$\lambda_1(T) = 1;$$

$$\lambda_2(T) = 0;$$

$$\lambda_3(T) = 1;$$

$$\lambda_4(T) = 1;$$

$$\lambda_5(T) = 0;$$

$$\lambda_6(T) = 0;$$

$$\lambda_7(T) = 0;$$

Furthermore, for $k = 0, 1, \dots, T - 1$, we obtain the optimal control (u^*, v^*) as

$$u_k^* = \min \left\{ \max \left\{ \frac{S_{1,k} (\lambda_{1,k+1} - \lambda_{2,k+1})}{A_k}, u_{\min} \right\}, u_{\max} \right\}$$

$$v_k^* = \min \left\{ \max \left\{ \frac{\left(\alpha_1 \frac{S_{1,k} E_k}{N_k} + \beta_1 \frac{S_{1,k} I_k}{N_k} \right) (\lambda_{3,k+1} - \lambda_{1,k+1}) + \left(\alpha_2 \frac{S_{2,k} E_k}{N_k} + \beta_2 \frac{S_{1,k} I_k}{N_k} \right) (\lambda_{3,k+1} - \lambda_{2,k+1})}{B_k}, v_{\min} \right\}, v_{\max} \right\}$$

Proof. The Hamiltonian \widehat{H}_k at time step k is given by

$$\begin{aligned} \widehat{H}_k = & S_{1,k} + E_k + I_k + \frac{A_k u_k^2}{2} - \frac{B_k v_k^2}{2} \\ & + \lambda_{1,k+1} \left[\Lambda_1 - (1 - v_k) \left(\alpha_1 \frac{S_{1,k} E_k}{N_k} + \beta_1 \frac{S_{1,k} I_k}{N_k} \right) + (1 - d - u_k) S_{1,k} \right] \\ & + \lambda_{2,k+1} \left[\Lambda_2 - (1 - v_k) \left(\alpha_2 \frac{S_{2,k} E_k}{N_k} + \beta_2 \frac{S_{2,k} I_k}{N_k} \right) + u_k S_{1,k} + (1 - d) S_{2,k} \right] \\ & + \lambda_{3,k+1} \left[(1 - v_k) \left(\alpha_1 \frac{S_{1,k} E_k}{N_k} + \beta_1 \frac{S_{1,k} I_k}{N_k} + \alpha_2 \frac{S_{2,k} E_k}{N_k} + \beta_2 \frac{S_{2,k} I_k}{N_k} \right) + (1 - d - \eta) E_k \right] \\ & + \lambda_{4,k+1} [\eta E_k + (1 - \gamma_1 - \gamma_2 - d - \delta_1) I_k] \\ & + \lambda_{5,k+1} [\gamma_1 I_k + (1 - \theta_1 - \mu_1 - d - \delta_2) H_{1,k} + \theta_2 H_{2,k}] \\ & + \lambda_{6,k+1} [\gamma_2 I_k + (1 - \theta_2 - \mu_2 - d - \delta_3) H_{2,k} + \theta_1 H_{1,k}] \\ & + \lambda_{7,k+1} [\mu_1 H_{1,k} + \mu_2 H_{2,k} + (1 - d) R_k] \end{aligned}$$

For $k = 0, 1, \dots, T - 1$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's maximum principle, in discrete time, given in [20–30]. Such that

$$\lambda_{1,k} = \frac{\partial \hat{H}_k}{\partial S_{1,k}}, \lambda_1(T)$$

$$\lambda_{2,k} = \frac{\partial \hat{H}_k}{\partial S_{2,k}}, \lambda_2(T)$$

$$\lambda_{3,k} = \frac{\partial \hat{H}_k}{\partial E_k}, \lambda_3(T)$$

$$\lambda_{4,k} = \frac{\partial \hat{H}_k}{\partial I_k}, \lambda_4(T)$$

$$\lambda_{5,k} = \frac{\partial \hat{H}_k}{\partial H_{1,k}}, \lambda_5(T)$$

$$\lambda_{6,k} = \frac{\partial \hat{H}_k}{\partial H_{2,k}}, \lambda_6(T)$$

$$\lambda_{7,k} = \frac{\partial \hat{H}_k}{\partial R_k}, \lambda_7(T)$$

For $k = 0, 1, \dots, T - 1$, the optimal control (u^*, v^*) can be solved from the optimality condition

$$\frac{\partial \hat{H}_k}{\partial u_k} = A_k u_k + S_{1,k} (\lambda_{2,k+1} - \lambda_{1,k+1}) = 0.$$

$$\frac{\partial \hat{H}_k}{\partial v_k} = -B_k v_k + \left(\alpha_1 \frac{S_{1,k} E_k}{N_k} + \beta_1 \frac{S_{1,k} I_k}{N_k} \right) (\lambda_{1,k+1} - \lambda_{3,k+1}) + \left(\alpha_2 \frac{S_{2,k} E_k}{N_k} + \beta_2 \frac{S_{1,k} I_k}{N_k} \right) (\lambda_{2,k+1} - \lambda_{3,k+1}) = 0.$$

So, we have

$$u_k = \frac{S_{1,k} (\lambda_{1,k+1} - \lambda_{2,k+1})}{A_k}.$$

$$v_k = \frac{\left(\alpha_1 \frac{S_{1,k} E_k}{N_k} + \beta_1 \frac{S_{1,k} I_k}{N_k} \right) (\lambda_{1,k+1} - \lambda_{3,k+1}) + \left(\alpha_2 \frac{S_{2,k} E_k}{N_k} + \beta_2 \frac{S_{1,k} I_k}{N_k} \right) (\lambda_{2,k+1} - \lambda_{3,k+1})}{B_k}.$$

By the bounds in U_{ad} of the control, it is easy to obtain (u_k^*, v_k^*) in the form of (10). \square

6. NUMERICAL SIMULATION AND PRESENTATION OF RESULTS

We carried out a numerical simulation to test the reliability of our mathematical model with and without control. The results will be presented in the following.

We start by presenting the results with the different controls by adopting several strategies that will be explained in details :

	Values
$(S_{1,0}, S_{2,0}, E_0, I_0, H_{1,0}, H_{2,0}, R_0, \Lambda_1, \Lambda_2)$	(1000,800,400,200,120,80,40,100,80)
$(\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \mu_1, \mu_2, \eta, \delta_1, \delta_2, \delta_3, d)$	(0.4,0.3,0.5,0.5,0.002,0.001,0.002,0.001,0.05,0.0001,0.0001,0.0001,0.065)

TABLE 1. Parameters and initial conditions for the model with and without control.

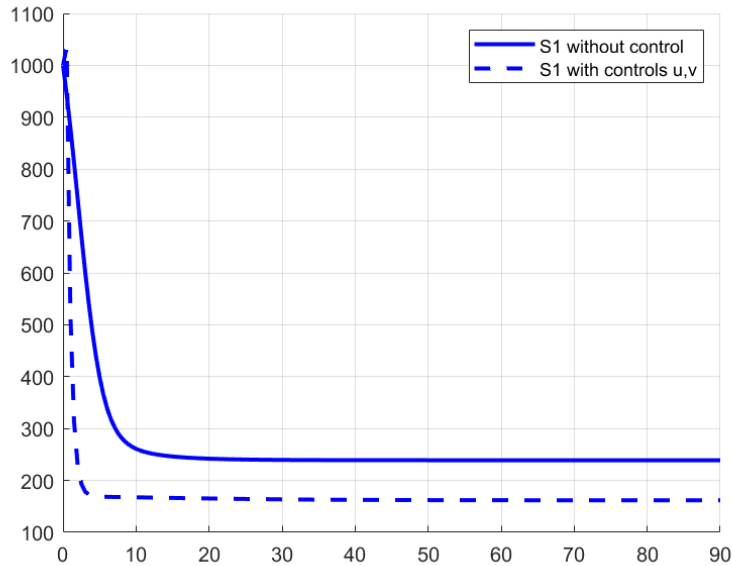


FIGURE 2. Model results of people likely to be infected and not applying preventive measures

We notice that after having applied the controls, the number of people likely to be infected by the coronavirus has decreased. This outcome was expected due to the implementation of awareness campaigns as well as reducing social contact which make people follow certain preventive measures against the disease and consequently cause a decrease in (S1).

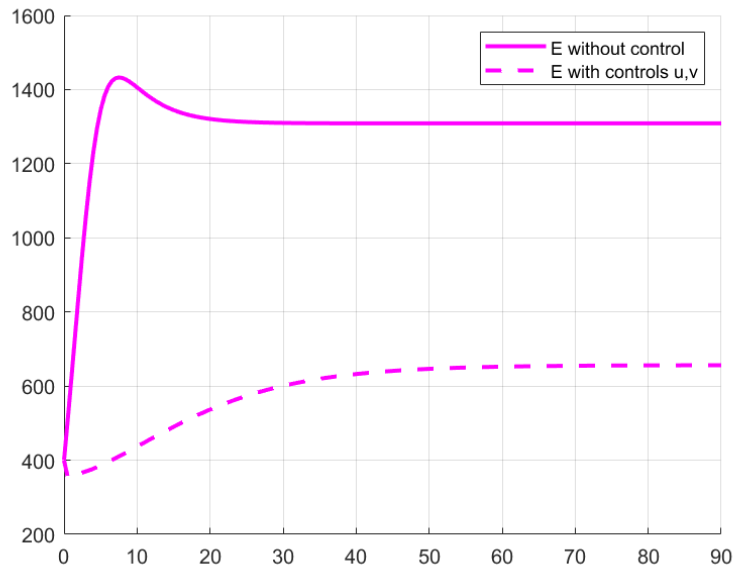


FIGURE 3. Model results of infected people without symptoms

The number of infected people without symptoms (E) decreased dramatically after the application of the control v . This result was expected since the application of the measure of reduction of social contact makes it possible to reduce the number of infected.

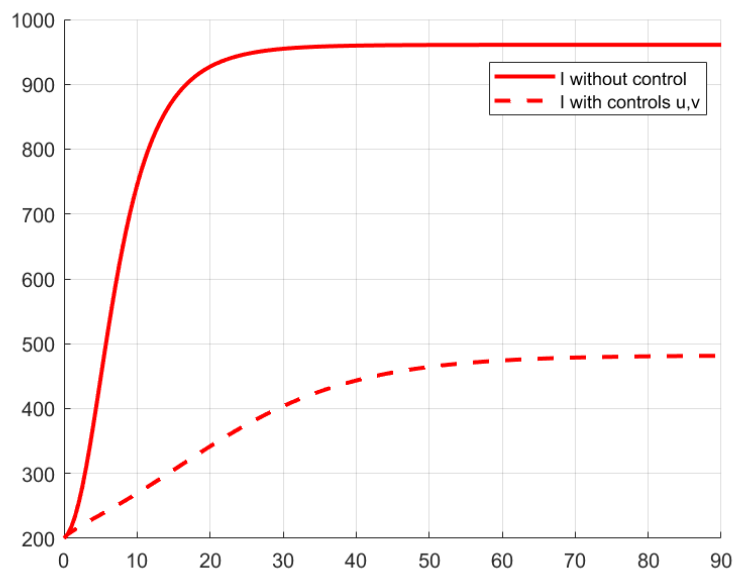


FIGURE 4. Model results of infected people with symptoms

The decrease that took place in the number of people during the incubation period of the disease (E), resulted in a remarkable decrease in the number (I) of infected people showing symptoms which is a desirable result.

7. CONCLUSION

The model we formulated to describe the spread of the pandemic (see the system 2) uses logical relationships between the different compartments established from the understanding of spread of the disease by consulting a few WHO reports and checking their reliability using a database containing 500 case records in USA. We conclude by saying that it is possible to achieve the defined objective (which is the effective fight against the pandemic coronavirus) by using two controls. The first represents the awareness campaigns aimed at maximizing the number of people applying the preventive measures against the coronavirus and reduce the number of people who do not apply these measures. The second represents decreasing social contact in public places which aims to reduce the number of infected people due to social contact.

In summary, we say confidently that a global strategy to fight against the pandemic coronavirus must encompass these two controls, that is to say adopt a strategy based on:

- Awareness campaigns aimed at getting people to apply the preventive measures against the coronavirus mentioned in the introduction to this paper.
- Decreasing in social contact in public places to prevent contamination as much as possible

We are therefore making efforts to strengthen the health system, raise awareness and reduce contact in places at risk instead of closing the country. So, in this article, we offer the possibility to see the results of our effective strategy.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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