Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 1, 735-768
https://doi.org/10.28919/jmcs/5083
ISSN: 1927-5307

MEASURE OF SLOPE ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS UNDER INTRA-CLASS CORRELATED STRUCTURE OF ERRORS USING CENTRAL COMPOSITE DESIGNS

B. SULOCHANA*, B. RE. VICTOR BABU

Department of Statistics, Acharya Nagarjuna University, Guntur 522510, Andhra Pradesh, India

Abstract: In the design of experiments for estimating the slope of the response surface, slope rotatability is a desirable property. In this paper, measure of slope rotatability for second order response surface designs using central composite designs under intra-class correlated structure of errors is suggested and illustrated with examples.

Keywords: response surface design; slope-rotatability; intra-class correlated structure of errors; central composite designs; weak slope rotatability region.

2010 AMS Subject Classification: 62K05

1. INTRODUCTION

Response surface methodology is a collection of mathematical and statistical techniques useful for analyzing problems where several independent variables influence a dependent variable. The independent variables are often called the input or explanatory variables and the dependent variable is often the response variable. An important step in development of response

---

*Corresponding author
E-mail address: sulochana.statistics@gmail.com
Received October 3, 2020

surface designs was the introduction of rotatable designs by Box and Hunter (1957). Das and Narasimham (1962) constructed rotatable designs using balanced incomplete block designs (BIBD). The study of rotatable designs mainly emphasized on the estimation of absolute response. Estimation of response at two different points in the factor space will often be of great importance. If differences at two points close together, estimation of local slope (rate of change) of the response is of interest. Hader and Park (1978) extended the notion of rotatability to cover the slope for the case of second order models. In view of slope rotatability of response surface methodology, a good estimation of derivatives of the response function is more important than estimation of mean response. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in animal etc. (cf. Park 1987). Victorbabu and Narasimham (1991) studied second order slope rotatable designs (SOSRD) using BIBD. Victorbabu (2007) suggested a review on SOSRD. To access the degree of slope rotatability Park and Kim (1992) introduced a measure for second order response surface designs. Park et.al (1993) introduced measure of rotatability for second order response surface designs. Surekha and Victorbabu (2011) studied measure of slope rotatability for second order response surface designs using central composite designs (CCD).

Many authors have studied rotatable designs and slope rotatable designs assuming errors to be uncorrelated and homoscedastic. However, it is not uncommon to come across practical situations when the errors are correlated, violating the usual assumptions. Panda and Das (1994) introduced robust first order rotatable designs. Das (1997, 1999, 2003a) introduced and studied robust second order rotatable designs. Das (2003b) introduced slope rotatability with correlated errors and gave conditions for the different variance-covariance error structures. Das and Park (2006) studied robust slope-rotatable designs over all directions. Das and Park (2007) introduced measure of robust rotatability for second order response surface designs. To access the degree of slope rotatability for correlated errors a new measure for second order response surface designs was introduced by Das and Park (2009). Rajyalakshmi (2014) studied some contributions to second order response surface designs under different correlated structure of errors. Rajyalakshmi and Victorbabu (2014a, 14b, 15) studied SOSRD under intra-class structure of errors using CCD, symmetrical unequal block arrangements with two unequal block sizes and BIBD respectively.

In this paper, following the works of Park and Kim (1992), Das (2003b, 2014), Das and Park (2009), Surekha and Victorbabu (2011), Rajyalakshmi (2014), Rajyalakshmi and Victorbabu (2014a), the measure of slope-rotatability for second order response surface designs under intra-class correlated structure of errors using CCD for $\rho(0 \le \rho \le 0.9)$ and for $2 \le k \le 17$ (k number of factors) is suggested.

2. PRELIMINARIES

2. Second order response surface designs with correlated structure of errors (cf. Das (2003b, 2014))

The second order surface model $D=(x_{\mu i})$ is

$$y_{\mu}=b_0+\sum_{i=1}^{k} b_i x_i+\sum_{i=1}^{k} b_{ii} x_{\mu i}^2+\sum\sum_{i<j=1}^{k} b_{ij} x_{\mu i} x_{\mu j}+e_{\mu};1\le \mu \le N \tag{2.1}$$

where x_{ui} denotes the level of the $i^{th}(i=1,2,...,k)$ factor in the $\mu^{th}(\mu=1,2,...,N)$ run of the experiment, $e_{\mu}^{'}s$ are correlated errors. Here b_0, b_i, b_{ii}, b_{ij} are the parameters of the model and y_{μ} is the observed response at the μ^{th} design point.

2.1. Conditions for slope-rotatability for second order response surface designs with correlated errors (cf. Das (2003b, 2014), Das and Park (2009))

Following Das (2003b, 2014), Das and Park (2009), the necessary and sufficient conditions for slope-rotatability for second order model with correlated errors are as follows.

The estimated response at x_i is given by

$$\hat{y}_{\mu} = \hat{b}_0 + \sum_{i=1}^{k} \hat{b}_i x_i + \sum_{i=1}^{k} \hat{b}_{ii} x_i^2 + \sum\sum_{i<j=1}^{k} \hat{b}_{ij} x_i x_j \tag{2.2}$$

For the second order model as in (2.2), we have

$$\frac{\partial \hat{y}_{\mu}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii} x_i + \sum_{j=1, j\neq i}^{k} \hat{b}_{ij} x_j \tag{2.3}$$

The variance of $\frac{\partial \hat{y}_{\mu}}{\partial x_i}$ is given by

$$V\left(\frac{\partial \hat{y}_{\mu}}{\partial x_i}\right) = V(\hat{b}_i) + 4x_i^2 V(\hat{b}_{ii}) + 4x_i Cov(\hat{b}_i, \hat{b}_{ii})$$

$$+ \sum_{j=1, i\neq j}^{k} x_j^2 V(\hat{b}_{ij}) + \sum_{j=1, s=1} \sum_{j\neq s\neq i}^{k} x_j x_s Cov(\hat{b}_{ij}, \hat{b}_{is})$$

$$+ 2 \sum_{j=1, j\neq i}^{k} Cov(\hat{b}_i, \hat{b}_{ij}) + 4 \sum_{j=1, j\neq i}^{k} x_i x_j Cov(\hat{b}_{ii}, \hat{b}_{ij})$$

$$V\left(\frac{\partial \hat{y}_{\mu}}{\partial x_i}\right) = \vartheta^{i.i} + 4x_i^2 \vartheta^{ii.ii} + 4x_i \vartheta^{i.ii} + \sum_{j=1, i\neq j}^{k} x_j^2 \vartheta^{ij.ij} + \sum_{j=1, s=1} \sum_{j\neq s\neq i}^{k} x_j x_s \vartheta^{ij.is}$$

$$+ 2 \sum_{j=1, j\neq i}^{k} \vartheta^{i.ij} + 4 \sum_{j=1, j\neq i}^{k} x_i x_j \vartheta^{ii.ij}. \tag{2.4}$$

The variance of estimated first order derivative with respect to each independent variable x_i as in (2.4) will be a function of $s^2 = \sum_{i=1}^{k} x_{\mu i}^2$ if and only if,

1) $\vartheta^{i.ii} = 0;\ 1 \le j \le k,\ \vartheta^{i.ij} = 0;\ 1 \le j,\ j \le k,\ i \neq j$

2) $\vartheta^{ij.ij'} = 0;\ 1 \le i \neq j \neq j' \le k$
$\vartheta^{ij.ij} = 0;\ 1 \le i,\ i \le k,\ i \neq k$

3) $\vartheta^{i.i} = \text{constant};\ 1 \le i \le k$

4) $\vartheta^{ii.ii} = \text{constant}; 1 \le i \le k$

5) $\vartheta^{ij.ij} = \text{constant}; 1 \le i < j \le k$, and

6) $\vartheta^{ii.ii} = \frac{1}{4}\vartheta^{ij.ij}; 1 \le i < j \le k$ (2.5)

The following are the equivalent conditions of (1) to (5) in (2.5) for slope rotatability in second order correlated errors model (2.1)

1)* (i) $\vartheta_{0.j} = \vartheta_{0.jl} = 0;\ 1 \le j < l \le k;$

(ii) $\vartheta_{i.j} = 0;\ 1 \le i, j \le k, i \ne j;$

(iii) a) $\vartheta_{ii.j} = 0;\ 1 \le i, j \le k;$

b) $\vartheta_{i.jl} = 0;\ 1 \le i, j < l \le k;$

c) $\vartheta_{ii.jl} = 0;\ 1 \le i, j < l \le k,\ (j,l) \ne (i,j)$

d) $\vartheta_{ij.lt} = 0;\ 1 \le i, l < j, t \le k,\ (i,j) \ne (l,t)$

2)* (i) $\vartheta_{0.jj} = \text{constant} = a_1$, say; $1 \le \text{i} \le \text{k}$

(ii) $\vartheta_{i.i} = \text{constant} = \frac{1}{g}$, say; $1 \le \text{i} \le \text{k}$

(iii) $\vartheta_{ii.ii} = \text{constant} = \eta\left(\frac{2}{f} + e\right)$, say; $1 \le i \le k$

3)* (i) $\vartheta_{ii.jj} = \text{constant} = e$, say; $1 \le i, j \le k,\ i \ne j$

(ii) $\vartheta_{ij.ij} = \text{constant} = \frac{1}{f}$, say; $1 \le i < j \le k$ (2.6)

where a_1, g, f, e, η are constants.

The variances and covariances of the estimated parameters of the model (2.1) for the slope-rotatability are as follows:

$$V\left(\hat{b}_0\right)=\vartheta^{0.0}=\frac{\eta\left(\frac{2}{f}+e\right)+(k-1)e}{B};\ 1\le i\le k;$$

$$V\left(\hat{b}_i\right)=\vartheta^{i.i}=g;\ 1\le i\le k;$$

$$V\left(\hat{b}_{ij}\right)=\vartheta^{ij.ij}=f;\ 1\le i<j\le k;$$

$$V\left(\hat{b}_{ii}\right)=\vartheta^{ii.ii}=\frac{\vartheta_{00}\{\eta(\frac{2}{f}+e)+(k-2)a_1^2\}}{B\{\eta(\frac{2}{f}+e)-e\}};\ 1\le i\le k;$$

$$Cov\left(\hat{b}_0,\hat{b}_{ii}\right)=\vartheta^{00.ii}=\frac{-a_1}{B};\ 1\le i\le k;$$

$$Cov\left(\hat{b}_{ii},\hat{b}_{ij}\right)=\vartheta^{ii.ij}=\frac{a_1^2-e\vartheta_{00}}{B\{\eta(\frac{2}{f}+e)-e\}};\ 1\le i\ne j\le k; \qquad (2.7)$$

where $B=\left[\vartheta_{00}\{\eta(\frac{2}{f}+e)+(k-1)e\}-ka_1^2\right]$ and the other covariances are zero.

An inspection of the $V(\hat{b}_0)$ shows that a necessary and sufficient condition for the existence of a non-singular second order designs $B>0$.

4)* $$B=\left[\vartheta_{00}\{\eta(\frac{2}{f}+e)+(k-1)e\}-ka_1^2\right]>0. \qquad (2.8)$$

For the second order slope rotatability with correlated errors,

$$V(\hat{b}_{ii})=\frac{1}{4}V(\hat{b}_{ij})\text{ i.e., }\vartheta^{ii.ii}=\frac{1}{4}\vartheta^{ij.ij}. \qquad (2.9)$$

On simplification of (2.9) using (2.7), we get,

5)* $$\eta\left(\frac{2}{f}+e\right)\left[4\vartheta_{00}-f\vartheta_{00}\eta\left(\frac{2}{f}+e\right)-f\vartheta_{00}g(k-1)+fka_1^2+\vartheta_{00}gf\right]$$
$$+\vartheta_{00}g\{4(k-2)+(k-1)fg\}-a_1^2\{4(k-1)+kfg\}=0. \qquad (2.10)$$

From (2.4), using slope rotatability conditions as in (2.6) and (2.7), we derive

$$V\left(\frac{\partial \hat{y}_\mu}{\partial x_i}\right) = g + 4x_i^2\left(\frac{f}{4}\right) + \sum_{j=1, i\neq j}^{k} x_j^2 f$$
$$= g + f\sum_{i=1}^{k} x_i^2$$
$$= g + fs^2 \tag{2.11}$$

where $s^2 = \sum_{i=1}^{k} x_i^2$ and g, f are as in (2.7).

(cf. Das (2003b, 2014), Das and Park (2009))

2.2. Intra-class correlated structure of errors (cf. Das (1997, 2003b, 2014))

Intra-class structure is the simplest variance-covariance structure which arises when errors of any two observations have the same correlation and each has the same variance. It is also known as uniform correlation structure. This can happen easily in a situation when all the observations studied are from the same batch or from the same run in a furnace.

Let ρ is the correlation between errors of any two observations, each having the same variance σ^2. Then intra-class variance covariance structure of errors given by the class:

$$W_0 = \left\{W_{N\times N(\rho)} = D(e) = \sigma^2\left[(1-\rho)I_N + \rho E_{N\times N}\right] : \sigma > 0, -(N\text{-}1)^{-1} < \rho < 1\right\}.$$

Here I_N denotes an identity matrix of order N and $E_{N\times N}$ is a $N\times N$ matrix of all elements1.

First observe that,

$$W_{N\times N}^{-1}(\rho) = \sigma^2\left[(\delta_0 - \gamma_0)I_N + \gamma_0 E_{N\times N}\right]$$

where $\delta_0 = \dfrac{1+(N\text{-}1)\rho}{(1\text{-}\rho)\{1+(N\text{-}1)\rho\}}$, $\gamma_0 = \dfrac{\rho}{(1\text{-}\rho)\{1\text{-}(N\text{-}1)\rho\}}$ and $\rho > (N\text{-}1)^{-1}$.

(cf. Das (1997, 2003b and 2014))

2.3. Conditions of slope rotatability for second order response surface designs under intra-class correlated structure of errors (cf. Das (2003b, 2014))

Following (2.6), the necessary and sufficient conditions for the second order slope rotatability under the intra-class structure after some simplifications turn out to be

I $\quad \sum_{\mu=1}^{N} \prod_{\mu=1}^{k} x_{\mu i}^{\alpha_i} = 0$; for any α_i odd and $\sum_{i=1}^{N} \alpha_i \leq 4$.

II (i) $\quad \sum_{\mu=1}^{N} x_{\mu i}^{2} = \text{constant} = \text{N}\gamma_2;\ 1 \leq i \leq k$; and,

(ii) $\quad \sum_{\mu=1}^{N} x_{\mu i}^{4} = \text{constant} = \text{cN}\gamma_4;\ 1 \leq i \leq k$,

III $\quad \sum_{\mu=1}^{N} x_{\mu i}^{2} x_{\mu j}^{2} = \text{constant} = \text{N}\gamma_4;\ 1 \leq i, j \leq k,\ i \neq k,$ (2.12)

The parameters of the second order slope rotatable design parameters under intra-class structure are as follows

$$a_1 = \frac{N\gamma_2}{\sigma^2\{1+(N-1)\rho\}},\quad e = \frac{\{1+(N-1)\rho\}N\gamma_4 - N\gamma_2^2}{\sigma^2(1-\rho)\{1-(N-1)\rho\}},\quad \frac{1}{g} = \frac{N\gamma_2}{\sigma^2(1-\rho)},\quad \frac{1}{f} = \frac{N\gamma_4}{\sigma^2(1-\rho)}$$

$$\vartheta_{00} = \frac{N}{\sigma^2\{1+(N-1)\rho\}},\ \eta\left(\frac{2}{f}+e\right) = \frac{\eta\{1+(N-1)\rho\}3N\gamma_4 - \rho N^2\gamma_2^2}{\sigma^2(1-\rho)\{1+(N-1)\rho\}}. \qquad (2.13)$$

where $c = 3\eta,\ \gamma_2,\ \gamma_4$ and η are constants.

Note that if $\rho = 0,$ (i.e., when errors are uncorrelated and homoscedastic) the conditions (2.12) and (2.13) becomes

I*: $\quad \sum_{\mu=1}^{N} x_{\mu i_1}^{\alpha_1} x_{\mu i_2}^{\alpha_2} x_{\mu i_3}^{\alpha_3} x_{\mu i_4}^{\alpha_4} = 0$; for any α_i odd and $\sum_{i=1}^{N} \alpha_i \leq 4$

II*: (i) $\quad \sum_{\mu=1}^{N} x_{\mu i}^{2} = \text{constant} = N\gamma_2;\ 1 \leq i \leq k$; and

(ii) $\sum_{\mu=1}^{N} x_{\mu i}^4 = \text{constant} = cN\gamma_4;\ 1 \le i \le k$

III*: $\sum_{\mu=1}^{N} x_{\mu i}^2 x_{\mu j}^2 = \text{constant} = N\gamma_4;\ 1 \le i, j \le k,\ i \ne k.$ (2.14)

Note that (I), (II) and (III) as in (2.14) are second order slope rotatable conditions when errors are uncorrelated and homoscedastic.

Using (2.13), the expression

$\vartheta_{00}\left[\{\eta\left(\frac{2}{f}+e\right)+(k-1)e\}-ka_1^2\right]$ simplifies to

$$\frac{N}{\sigma^2\{1+(N-1)\rho\}}\left[\{c+(k-1)\}N\gamma_4\{1+(N-1)\rho\}-\{\eta+(k-1)\}\rho N^2\gamma_2^2-kN\gamma_2^2\right].$$

The non-singularity condition (2.8) for the intra-class structure leads to

$$\left[\{c+(k-1)\}N\gamma_4\{1+(N-1)\rho\}-\{\eta+(k-1)\}\rho N^2\gamma_2^2-kN\gamma_2^2\right]>0$$

(2.15)

where $c = 3\eta$.

Using (2.13), the condition (5)* in (2.9) rises to

$$\frac{\eta\{1+(N-1)\rho\}3N\gamma_4-\rho N^2\gamma_2^2}{(1-\rho)}\left[\begin{array}{l}4N-\dfrac{\eta\{1+(N-1)\rho\}3N\gamma_4-\rho N^2\gamma_2^2}{\gamma_4\{1+(N-1)\rho\}}+k\dfrac{N\gamma_2^2(1-\rho)}{\gamma_4\{1+(N-1)\rho)} \\ -(k-2)\dfrac{\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2}{\gamma_4\{1+(N-1)\rho\}}\end{array}\right]$$
$$+\frac{N[\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2]}{(1-\rho)}\left[4(k-2)+(k-1)\frac{\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2}{N\gamma_4\{1+(N-1)\rho\}}\right]$$
$$-N^2\gamma_2^2\left[\frac{4(k-1)+\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2}{N\gamma_4\{1+(N-1)\rho}\right]=0.$$

(2.16)

(cf. Das (2003b))

for $\rho = 0,$ (i.e., when errors are uncorrelated and homoscedastic) (2.16) reduces to

$$\gamma_4\left[k(5-c)-(c-3)^2\right]+\gamma_2^2\left[k(c-5)+4\right]=0 \tag{2.17}$$

The above equation (2.17) is equal to slope rotatability for second order response surface designs with errors are uncorrelated and homoscedastic. (cf. Victorbabu and Narasimham (1991))

2.4. Slope rotatability for second order response surface designs under intra-class correlated structure of errors using CCD (cf. Rajyalakshmi (2014), Rajyalakshmi and victorbabu (2014a))

Following the works of Hader and Park (1978), Victorbabu and Narasimham (1991), Das (2003b, 2014), Rajyalakshmi (2014), Rajyalakshmi and Victorbabu (2014a), the method construction of slope rotatability for second order response surface designs under intra-class correlated structure of errors using CCD is given below. Let $\rho\left(-\frac{1}{N-1}<\rho<1\right)$ be correlation between errors of any two observations, each having the same variance $\sigma^2.$

Central composite designs are obtained by adding suitable factorial combinations $(\pm 1,\ \pm 1,...,\pm 1)$ to those obtained from $2^{t(k)}$ fractional (or a suitable fractional replicate of 2^k in which no interaction less than five factors is confounded). The $2k$ additional fractional combinations in CCD are $(\pm\alpha,\ 0,...,0),(0,\pm\alpha,\ 0,...,0),...,(0,0,...\pm\alpha)$ and n_0 central points $(0,\ 0,...,0)$ if necessary. The total number of factorial combinations in the design can be written as $N=F+T.$ Here F is total number of fractional points. i.e., $F=2^{t(k)}$ and $T=2k+n_0.$

Result (2.1): The design points $(\pm 1,\ \pm 1,\ ...,\ \pm 1)F\cup(\pm\alpha,\ ...,\ 0)2^1\cup n_0$ generated from the SOSRD under intra-class correlated structure of errors using CCD in design points $N=F+T,$ the simple symmetry conditions are true. Where α^2 is positive real root of the fourth degree polynomial equation,

$$\begin{aligned}&\left[(8k-4N)\{1+(N-1)\rho\right]\{1+(N-1)\rho\}\alpha^{8}\\&+\left[8Fk\{1+(N-1)\rho\}\right]\{1+(N-1)\rho\}\alpha^{6}\\&+\left[\left(2F(4-k)N+2F^{2}k+16F(1-k)\right)\{1+(N-1)\rho\right]\{1+(N-1)\rho\}\alpha^{4}\\&+\left[\left(16F^{2}(1-k)\right)\{1+(N-1)\rho\}\right]\{1+(N-1)\rho\}\alpha^{2}\\&+\left[\left(4F^{2}(k-1)N+4F^{3}(1-k)\right)\{1+(N-1)\rho\}\right]\{1+(N-1)\rho\}=0\end{aligned}$$

Note: Values of SOSRD under intra-class correlated structure of errors using CCD can be obtained by solving the above equation.

3. MAIN RESULTS

3.1. Measure of second order slope rotatability for correlated structure of errors (cf. Das and Park (2009))

Following Das and Park (2009), equations (2.5), (2.6) and (2.7) give necessary and sufficient conditions for a measure for any general second order response surface designs with correlated errors. Further we have

ϑ^{ii} eual for all i,

$\vartheta^{ii.ii}$ eual for all i,

$\vartheta^{ij.ij}$ eual for all i, j, where $i \neq j$

$$\vartheta^{i.ii}=\vartheta^{i.ij}=\vartheta^{ij.ij}=\vartheta^{ij.il}=0 \text{ for all } i\neq j\neq l, \text{ and for all } \rho \qquad (3.1)$$

Das and Park (2009) proposed that, if the conditions in (2.5) together (2.6), (2.7) and (3.1) are met, $M_k(D)$ is the proposed measure of slope rotatability for second order response surface designs for any general correlated error structure.

$$M_k(D)=\frac{1}{1+Q_k(D)}$$

$$\text{where } Q_k(D)=\frac{1}{2(k-1)\sigma^4}\left\{(k+2)(k+4)\sum_{i=1}^{k}\left[(\vartheta^{i.i}-\bar{\vartheta})+\frac{a_i-\bar{a}}{k+2}\right]^2\right.$$

$$+\frac{4}{k(k+2)}\sum_{i=1}^{k}(a_i-\bar{a})^2+2\sum_{i=1}^{k}\left[\left(4\vartheta^{ii.ii}-\frac{a_i}{k}\right)^2+\sum_{\text{i}=1;\ \text{j}\neq\text{i}}^{k}\left(\vartheta^{ij.ij}-\frac{a_i}{k}\right)^2\right]$$

$$+4(k+4)\left(4(\vartheta^{i.ii})^2+\sum_{j=1;\,j\neq i}^{k}(\vartheta^{i.ij})^2\right)$$

$$+4\sum_{i=1}^{k}\left(4\sum_{j=1;\,j\neq i}^{k}(\vartheta^{ij.ij})^2\right)+\sum_{j<l;\,j,l\neq i}\sum^{k}(\vartheta^{ij.il})^2\Bigg\} \qquad (3.2)$$

here $\bar{\vartheta}=\frac{1}{k}\sum_{i=1}^{k}\vartheta^{i.i}$, $a_i=4\vartheta^{ii}+\sum_{j=1;\,j\neq i}^{k}(\vartheta^{ij.ij})^2$ $(1\leq i\leq k)$ and $\bar{a}=\frac{1}{k}\sum_{i=1}^{k}a_i$.

It can be easily shown that $Q_k(D)$ in equation (3.2) becomes zero for all values ρ, if and only if the conditions in equations (3.1) hold.

Further, it is simplified to

$$Q_k(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2. \qquad (3.3)$$

Note that $0\leq M_k(D)\leq 1$, and it can be easily shown that $M_k(D)$ is one if and only if the design is slope rotatable with any correlated error structure for all values of ρ, and $M_k(D)$ approaches to zero as the design 'D' deviates from the slope-rotatability under specified correlated error structure.

In this paper, the degree of slope rotatability for second order response surface designs under intra-class correlated structure of errors using central composite designs for $\rho\left(0\leq\rho\leq 0.9\right)$ and for $2\leq k\leq 17$ (k number of factors) is suggested.

4. Measure of Slope Rotatability for Second Order Response Surface Designs Under Intra-Class Correlated Structure of Errors Using Central Composite Designs

Following Park and Kim (1992), Das and Park (2009), Surekha and Victorbabu (2011), the proposed measure of slope-rotatability for second order response surface designs under intra-class correlated structure of errors using CCD is given below.

This well-known type of design consists of $2^{t(k)}$ factorial points $(\pm 1, \pm 1, \ldots, \pm 1)$, 2k axial points of the form $(\pm\alpha, 0, \ldots, 0)$ and a centre point $(0, 0, \ldots, 0)$ may be replicated n_0 times if necessary. The total number of factorial combinations in the design can be written as $N = F + T$. Here F is total number of fractional points. i.e., $F = 2^{t(k)}$ and $T = 2k + n_0$.

The design points, $(\pm 1, \pm 1, \ldots, \pm 1)F \cup (\pm\alpha, \ldots, 0)2^1 \cup n_0$ will give slope rotatability for second order rotatable designs under intra-class correlated structure of errors using CCD equations in (3.1) are true. Further, from equations in (3.1), we have,

(I) $\sum x_{\mu i}^2 = F + 2\alpha^2 = N\gamma_2$

(II) $\sum x_{\mu i}^4 = F + 2\alpha^4 = cN\gamma_4$

(III) $\sum x_{\mu i}^2 x_{\mu j}^2 = F = N\gamma_4$ (4.1)

Measure of slope rotatability of second order response surface designs under intra-class correlated structure of errors using CCD can be obtained by

$$M_k(D) = \frac{1}{1 + Q_k(D)}$$

$$Q_k(D) = \frac{1}{\sigma^4}\left[4V(b_{ii}) - V(b_{ij})\right]^2$$
$$= \frac{1}{\sigma^4}\left[4\vartheta^{ii.ii} - \vartheta^{ij.ij}\right]^2$$
$$= \frac{1}{\sigma^4}\left[4G - \frac{(1-\rho)\sigma^2}{F}\right]^2 \tag{4.2}$$

where $G = V(b_{ii}) = \vartheta^{ii.ii} = (1-\rho)\sigma^2 \left[\dfrac{(k-1)FT - 4(k-1)F\alpha^2 + 2\left[N - 2(k-1)\right]\alpha^4}{2\alpha^4[kFT - 4kF\alpha^2 + 2[N-2k]\alpha^4]} \right]$

By substituting (2.13) and (4.1) in $V(b_{ii})$ of (2.7) we get above G value.

If $M_k(D)$ is one if and only if the design 'D' is slope rotatable under intra-class correlated structure of errors using CCD for all values of ρ, and $M_k(D)$ approaches to zero as the design 'D' deviates from the slope-rotatability under intra-class correlated structure of errors using CCD.

Example: We illustrate the measure of slope-rotatability for second order response surface designs under intra-class correlated structure of errors with the help of CCD for k=3 factors.

The design points, $(\pm1, \pm1, \pm1)2^3 \cup (\pm\alpha, ..., 0)2^1 \cup n_0$ will give slope rotatability for second order response surface designs under intra-class correlated structure of errors in $N=15$ design points for 3 factors. From equations in (4.1), we have

(I) $\sum x_{\mu i}^2 = 8 + 2\alpha^2 = N\gamma_2$

(II) $\sum x_{\mu i}^4 = 8 + 2\alpha^4 = cN\gamma_4$

(III) $\sum x_{\mu i}^2 x_{\mu j}^2 = 8 = N\gamma_4$ (4.3)

From (I), (II) and (III) of (4.3), we get $\gamma_2 = \dfrac{8+2\alpha^2}{15}$, $\gamma_4 = \dfrac{8}{15}$ and $c = \dfrac{8+2\alpha^4}{8}$. Substituting γ_2, γ_4 and c in (3.5) and on simplification, we get the following biquadratic equation in α^2.

$$\begin{aligned} &\left[24(1+14\rho) - 60(1+14\rho)\right](1+14\rho)\alpha^8 + 192(1+14\rho)^2\alpha^6 \\ &+ \left[240(1+14\rho) + 384(1+14\rho) - 256(1+14\rho)\right](1+14\rho)\alpha^4 + 2048(1+14\rho)^2\alpha^2 \\ &+ \left[7680(1+14\rho) - 4096(1+14\rho)\right](1+14\rho) = 0 \end{aligned} \tag{4.4}$$

Equation (4.4) has only one positive real root for all values of $\rho(-\dfrac{1}{N-1} \le \rho \le 0.9)$, $\alpha^2 = 5.9166$. This can be alternatively written directly from result (2.1). Solving (4.4), we get $\alpha = 2.4324$. From (4.2) we get $Q_k(D) = 0$, $M_k(D) = 1$ for all values of

$\rho(-\frac{1}{N-1} \leq \rho \leq 0.9)$.

Suppose if we take $\alpha = 1.9$ instead of taking $\alpha = 2.4324$ for 3 factors we get $Q_k(D) = 0.0905,$ then $M_k(D) = 0.9169$ (taking $\rho = 0.1$) . Here $M_k(D)$ deviates from slope rotatability for second order response surface designs under intra-class correlated structure of errors.

4.1.Weak slope rotatability region for correlated errors (cf. Das and Park (2009)

Following Das and Park (2009), we also find weak slope rotatability region (WSRR) for second order response surface designs under intra-class correlated structure of errors using CCD.

$$M_k(D) \geq \text{v}$$

$M_k(D)$ involves the correlation parameter $\rho \in W$ and as such, $M_k(D) \geq \text{v}$ for all ρ is too strong to be met. On the other hand, for a given v, we can find range of values of ρ for which $M_k(D) \geq \text{v}$. Das and Park (2009) call this range as the weak slope rotatability region $(\text{WSRR}(\text{R}_{D(\text{v})}(\rho))$ of the design 'D'. Naturally, the desirability of using 'D' will rest on the wide nature of $(\text{WSRR}(\text{R}_{D(\text{v})}(\rho))$ along with its strength v. Generally, we would require 'v' to be very high say, around 0.95 (cf. Das and Park (2009)).

Table 1 and 2, gives the values of $M_k(D)$ and weak slope rotatability region for second order slope rotatable designs under intra-class correlated structure of errors using CCD for $\rho(0 \leq \rho \leq 0.9)$ and $2 \leq k \leq 17$ (k number of factors) respectively.

Table 1: Values of $M_k(D)$'s for second order slope rotatable designs under intra-class correlated structure of errors using CCD for $\rho (0 \le \rho \le 0.9)$ and $2 \le k \le 17$ (k number of factors)

$k = 2, N = 9, \alpha^* = 2.0903$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2462	0.2873	0.3378	0.3999	0.4756	0.5664	0.6711	0.7839	0.8909	0.9708
1.3	0.3566	0.4062	0.4641	0.5307	0.6003	0.6892	0.7760	0.8603	0.9327	0.9822
1.6	0.6935	0.7363	0.7795	0.8219	0.8627	0.9005	0.9339	0.9617	0.9826	0.9956
1.9	0.9765	0.9809	0.9848	0.9883	0.9914	0.994	0.9961	0.9978	0.999	0.9998
2.0903	1	1	1	1	1	1	1	1	1	1
2.2	0.9968	0.9974	0.9979	0.9984	0.9989	0.9992	0.9995	0.9997	0.9998	0.9999
2.5	0.979	0.9829	0.9865	0.9896	0.9923	0.9947	0.9966	0.9981	0.9991	0.9998
2.8	0.9655	0.9718	0.9776	0.9828	0.9873	0.9911	0.9943	0.9968	0.9986	0.9996
3.1	0.9571	0.9649	0.9721	0.9785	0.9841	0.9885	0.9929	0.9959	0.9982	0.9996
3.4	0.9519	0.9607	0.9687	0.9759	0.9822	0.9875	0.9919	0.9955	0.9979	0.9995
3.7	0.9487	0.9581	0.9666	0.9742	0.9809	0.9867	0.9914	0.9952	0.9978	0.9995
4	0.9466	0.9563	0.9652	0.9731	0.9801	0.9861	0.9911	0.9948	0.9976	0.9994
4.6	0.9442	0.9533	0.9635	0.9718	0.9791	0.9854	0.9906	0.9947	0.9976	0.9994
4.9	0.9435	0.9537	0.9631	0.9715	0.9789	0.9852	0.9905	0.9946	0.9976	0.9994
5.2	0.9429	0.9533	0.9627	0.9712	0.9787	0.9851	0.9904	0.9945	0.9976	0.9994

Note: Here α^* indicates that the values of slope rotatability for second order response surface designs under intra-class correlated structure of errors using CCD.

$k = 3, N = 15, \alpha^* = 2.4324$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.3282	0.3763	0.4329	0.4993	0.5758	0.6615	0.7533	0.8445	0.9243	0.9799
1.3	0.6764	0.7207	0.7655	0.8101	0.8531	0.8932	0.9289	0.9587	0.9812	0.9952
1.6	0.7509	0.7882	0.8249	0.8602	0.8933	0.9234	0.9496	0.9710	0.9869	0.9967
1.9	0.8994	0.9169	0.9332	0.9481	0.9613	0.9728	0.9824	0.9900	0.9955	0.9988
2.2	0.9916	0.9932	0.9946	0.9959	0.9969	0.9979	0.9986	0.9992	0.9997	0.9999
2.4324	1	1	1	1	1	1	1	1	1	1
2.5	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9957	0.9965	0.9973	0.9979	0.9984	0.9989	0.9993	0.9996	0.9998	0.9999
3.1	0.9922	0.9937	0.9950	0.9962	0.9972	0.9980	0.9987	0.9992	0.9997	0.9999
3.4	0.9898	0.9917	0.9935	0.9949	0.9963	0.9974	0.9984	0.9990	0.9996	0.9999
3.7	0.9873	0.9897	0.9918	0.9937	0.9954	0.9968	0.9979	0.9988	0.9995	0.9999
4	0.9866	0.9891	0.9914	0.9934	0.9951	0.9966	0.9978	0.9988	0.9995	0.9999
4.3	0.9865	0.9890	0.9913	0.9933	0.9951	0.9965	0.9978	0.9988	0.9995	0.9999
4.6	0.9861	0.9887	0.9910	0.9931	0.9949	0.9965	0.9977	0.9987	0.9994	0.9999
4.9	0.9857	0.9884	0.9908	0.9929	0.9948	0.9964	0.9977	0.9987	0.9994	0.9999
5.2	0.9855	0.9882	0.9907	0.9928	0.9947	0.9963	0.9977	0.9987	0.9994	0.9999

$k = 4, N = 25, \alpha^* = 2.7988$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.3054	0.3518	0.4072	0.4729	0.5498	0.6375	0.7331	0.8301	0.9166	0.9777
1.3	0.7569	0.7935	0.8295	0.8640	0.8963	0.9257	0.9511	0.9719	0.9873	0.9967
1.6	0.8960	0.9141	0.9309	0.9462	0.9599	0.9718	0.9817	0.9896	0.9954	0.9988
1.9	0.9092	0.9251	0.9399	0.9533	0.9653	0.9757	0.9843	0.9911	0.9960	0.9990
2.2	0.9655	0.9719	0.9776	0.9828	0.9873	0.9911	0.9943	0.9967	0.9985	0.9996
2.5	0.9966	0.9973	0.9978	0.9983	0.998	0.9992	0.9995	0.9996	0.9998	0.9999
2.7988	1	1	1	1	1	1	1	1	1	1
2.8	1	1	1	1	1	1	1	1	1	1
3.1	0.9993	0.9994	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999
3.4	0.9984	0.9987	0.9989	0.9992	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999
3.7	0.9978	0.9982	0.9986	0.9989	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999
4	0.9973	0.9978	0.9983	0.9987	0.9990	0.9993	0.9996	0.9998	0.9999	0.9999
4.3	0.9970	0.9976	0.9981	0.9985	0.9989	0.9992	0.9995	0.9997	0.9999	0.9999
4.6	0.9968	0.9974	0.9979	0.9984	0.9988	0.9992	0.9995	0.9997	0.9999	0.9999
4.9	0.9996	0.9973	0.9978	0.9984	0.9988	0.9992	0.9995	0.9997	0.9999	0.9999
5.2	0.9965	0.9972	0.9977	0.9983	0.9987	0.9991	0.9994	0.9997	0.9999	0.9999

$k = 5, N = 27, \alpha^* = 2.8722$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2876	0.3326	0.3868	0.4517	0.5286	0.6176	0.7162	0.8177	0.9098	0.9758
1.3	0.7695	0.8048	0.8392	0.8720	0.9027	0.9304	0.9543	0.9737	0.9882	0.9970
1.6	0.9376	0.9489	0.9592	0.9685	0.9766	0.9837	0.9895	0.9941	0.9973	0.9993
1.9	0.9655	0.9719	0.9776	0.9828	0.9873	0.9912	0.9943	0.9968	0.9986	0.9996
2.2	0.9703	0.9758	0.9808	0.9852	0.9891	0.9924	0.9951	0.9973	0.9987	0.9997
2.5	0.9933	0.9946	0.9957	0.9967	0.9976	0.9983	0.9989	0.9994	0.9999	0.9999
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8722	1	1	1	1	1	1	1	1	1	1
3.1	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9986	0.9988	0.9991	0.9993	0.9995	0.9996	0.9998	0.9998	0.9999	0.9999
3.7	0.9978	0.9983	0.9986	0.9989	0.9992	0.9995	0.9997	0.9998	0.9999	0.9999
4	0.9974	0.9979	0.9983	0.9987	0.9990	0.9993	0.9996	0.9998	0.9999	0.9999
4.3	0.9970	0.9976	0.9981	0.9985	0.9989	0.9993	0.9995	0.9997	0.9998	0.9999
4.6	0.9968	0.9974	0.9979	0.9984	0.9988	0.9992	0.9995	0.9997	0.9999	0.9999
4.9	0.9966	0.9973	0.9978	0.9984	0.9988	0.9992	0.9995	0.9997	0.9999	0.9999
5.2	0.9965	0.9971	0.9978	0.9983	0.9988	0.9991	0.9994	0.9997	0.9999	0.9999

$k = 6, N = 45, \alpha^* = 3.265$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2681	0.3114	0.3640	0.4278	0.5044	0.5944	0.6960	0.8027	0.9016	0.9734
1.3	0.7522	0.7893	0.8258	0.8610	0.8939	0.9239	0.9499	0.9712	0.9869	0.9967
1.6	0.9392	0.9502	0.9603	0.9693	0.9773	0.9841	0.9898	0.9942	0.9974	0.9994
1.9	0.9785	0.9825	0.9862	0.9894	0.9922	0.9946	0.9965	0.9980	0.9991	0.9998
2.2	0.9828	0.9860	0.9889	0.9915	0.9937	0.9956	0.9972	0.9984	0.9993	0.9998
2.5	0.9856	0.9883	0.9907	0.9928	0.9947	0.9963	0.9976	0.9987	0.9994	0.9999
2.8	0.9973	0.9978	0.9982	0.9986	0.9990	0.9993	0.9996	0.9998	0.9999	0.9999
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.265	1	1	1	1	1	1	1	1	1	1
3.4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9996	0.9997	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9993	0.9995	0.9996	0.9999	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999
4.9	0.9992	0.9993	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999
5.2	0.9992	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999

$k = 7, N = 79, \alpha^* = 3.7178$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2554	0.2975	0.3489	0.4117	0.4879	0.5784	0.6819	0.7921	0.8955	0.9717
1.3	0.7381	0.7767	0.8149	0.8519	0.8867	0.9185	0.9462	0.9690	0.9860	0.9965
1.6	0.9362	0.9477	0.9582	0.9676	0.9760	0.9832	0.9892	0.9939	0.9972	0.9993
1.9	0.9810	0.9846	0.9877	0.9906	0.9931	0.9952	0.9969	0.9983	0.9992	0.9998
2.2	0.9903	0.9921	0.9937	0.9952	0.9964	0.9976	0.9984	0.9991	0.9996	0.9999
2.5	0.9893	0.9913	0.9931	0.9947	0.9961	0.9973	0.9982	0.9990	0.9996	0.9999
2.8	0.9935	0.9947	0.9958	0.9968	0.9976	0.9983	0.9989	0.9994	0.9997	0.9999
3.1	0.9989	0.9991	0.9993	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999
3.4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1	1	1
3.7178	1	1	1	1	1	1	1	1	1	1
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$k = 8, N = 81, \alpha^* = 3.768$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2482	0.2896	0.3403	0.4026	0.4784	0.5691	0.6736	0.7858	0.8919	0.9706
1.3	0.7325	0.7717	0.8106	0.8482	0.8838	0.9163	0.9448	0.9682	0.9856	0.9964
1.6	0.9363	0.9477	0.9582	0.9677	0.9761	0.9832	0.9892	0.9939	0.9972	0.9993
1.9	0.9828	0.9861	0.9881	0.9915	0.9937	0.9956	0.9972	0.9984	0.9993	0.9998
2.2	0.9935	0.9947	0.9958	0.9968	0.9976	0.9984	0.9989	0.9994	0.9997	0.9999
2.5	0.9940	0.9951	0.9960	0.9968	0.9976	0.9982	0.9988	0.9995	0.9998	0.9999
2.8	0.9942	0.9953	0.9963	0.9979	0.9985	0.9991	0.9995	0.9998	0.9999	0.9999
3.1	0.9981	0.9985	0.9988	0.9991	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999
3.4	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
3.768	1	1	1	1	1	1	1	1	1	1
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.99999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$k = 9, N = 147, \alpha^* = 4.3$										
α \ ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2412	0.2818	0.3318	0.3935	0.4689	0.5598	0.6652	0.7793	0.8882	0.9695
1.3	0.7229	0.7631	0.8030	0.8419	0.8788	0.9126	0.9422	0.9667	0.9849	0.9962
1.6	0.9325	0.9446	0.9557	0.9658	0.9746	0.9822	0.9886	0.9935	0.9971	0.9992
1.9	0.9818	0.9851	0.9882	0.9909	0.9934	0.9954	0.9970	0.9983	0.9993	0.9998
2.2	0.9937	0.9948	0.9959	0.9969	0.9978	0.9984	0.9989	0.9994	0.9997	0.9999
2.5	0.9966	0.9973	0.9978	0.9983	0.9988	0.9991	0.9995	0.9997	0.9998	0.9999
2.8	0.9962	0.9969	0.9976	0.9982	0.9986	0.9990	0.9994	0.9997	0.9998	0.9999
3.1	0.9968	0.9974	0.9979	0.9984	0.9988	0.9992	0.9995	0.9997	0.9998	0.9999
3.4	0.9993	0.9994	0.9995	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
3.7	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	1	1	1	1	1	1	1	1	1	1
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$k = 10, N = 149, \alpha^* = 4.3372$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2369	0.2770	0.3266	0.3878	0.4630	0.5538	0.6598	0.7752	0.8858	0.9687
1.3	0.7187	0.7593	0.7997	0.8391	0.8765	0.9109	0.9411	0.9659	0.9845	0.9961
1.6	0.9317	0.9439	0.9552	0.9653	0.9742	0.9820	0.9884	0.9934	0.9971	0.9993
1.9	0.9820	0.9853	0.9884	0.9911	0.9934	0.9954	0.9971	0.9984	0.9993	0.9998
2.2	0.9942	0.9953	0.9963	0.9972	0.9979	0.9985	0.9991	0.9995	0.9998	0.9999
2.5	0.9975	0.9979	0.9984	0.9988	0.9991	0.9994	0.9996	0.9998	0.9999	0.9999
2.8	0.9980	0.9984	0.9987	0.9990	0.9992	0.9995	0.9997	0.9998	0.9999	0.9999
3.1	0.9974	0.9979	0.9983	0.9987	0.9991	0.9993	0.9996	0.9998	0.9999	0.9999
3.4	0.9989	0.9991	0.9993	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999
3.7	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1	1	1
4.3372	1	1	1	1	1	1	1	1	1	1
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$k = 11, N = 151, \alpha^* = 4.3774$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2334	0.2732	0.3223	0.3832	0.4582	0.5491	0.6555	0.7718	0.8838	0.9681
1.3	0.7152	0.7561	0.7969	0.8367	0.8745	0.9094	0.9401	0.9654	0.9843	0.9960
1.6	0.9309	0.9432	0.9547	0.9649	0.9739	0.9818	0.9883	0.9934	0.9970	0.9993
1.9	0.9820	0.9854	0.9884	0.9911	0.9935	0.9954	0.9971	0.9984	0.9993	0.9998
2.2	0.9944	0.9955	0.9964	0.9973	0.9979	0.9986	0.9991	0.9995	0.9998	0.9999
2.5	0.9979	0.9983	0.9986	0.9989	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999
2.8	0.9988	0.9989	0.9992	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999
3.1	0.9985	0.9988	0.9991	0.9993	0.9995	0.9996	0.9998	0.9999	0.9999	0.9999
3.4	0.9986	0.9989	0.9991	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999
3.7	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1
4.3774	1	1	1	1	1	1	1	1	1	1
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$k = 12, N = 281, \alpha^* = 5.0152$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2298	0.2692	0.3179	0.3784	0.4532	0.5441	0.6509	0.7682	0.8817	0.9675
1.3	0.7096	0.7510	0.7924	0.8330	0.8716	0.9072	0.9385	0.9644	0.9839	0.9959
1.6	0.9284	0.9412	0.9529	0.9635	0.9729	0.9811	0.9878	0.9931	0.9969	0.9992
1.9	0.9810	0.9846	0.9878	0.9906	0.9931	0.9952	0.9969	0.9982	0.9992	0.9998
2.2	0.9940	0.9952	0.9962	0.9971	0.9978	0.9984	0.9990	0.9995	0.9998	0.9999
2.5	0.9977	0.9982	0.9985	0.9989	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999
2.8	0.9989	0.9991	0.9993	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999
3.1	0.9990	0.9992	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999
3.4	0.9986	0.9989	0.9991	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999
3.7	0.9994	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1	1
5.0152	1	1	1	1	1	1	1	1	1	1
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1

$k = 13, N = 283, \alpha^* = 5.0399$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2274	0.2665	0.3149	0.3752	0.4498	0.5407	0.6478	0.7658	0.8803	0.9671
1.3	0.7070	0.7487	0.7904	0.8312	0.8701	0.9061	0.9378	0.9640	0.9837	0.9959
1.6	0.9277	0.9406	0.9525	0.9631	0.9727	0.9809	0.9877	0.9930	0.9969	0.9992
1.9	0.9809	0.9845	0.9877	0.9905	0.9930	0.9951	0.9968	0.9983	0.9992	0.9998
2.2	0.9941	0.9951	0.9962	0.9971	0.9978	0.9985	0.9990	0.9995	0.9998	0.9999
2.5	0.9978	0.9982	0.9986	0.9989	0.9992	0.9995	0.9997	0.9998	0.9999	0.9999
2.8	0.9990	0.9992	0.9994	0.9995	0.9996	0.9998	0.9998	0.9999	0.9999	0.9999
3.1	0.9994	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999
3.4	0.9992	0.9993	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
3.7	0.9992	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1
5.0399	1	1	1	1	1	1	1	1	1	1
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1	1

$k = 14$, $N = 285$, $\alpha^* = 5.0674$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2253	0.2642	0.3124	0.3725	0.4469	0.5377	0.6451	0.7637	0.7891	0.9667
1.3	0.7047	0.7466	0.7885	0.8296	0.8689	0.9051	0.9372	0.9637	0.9835	0.9958
1.6	0.9269	0.9400	0.9520	0.9628	0.9724	0.9806	0.9875	0.9929	0.9968	0.9992
1.9	0.9808	0.9844	0.9876	0.9905	0.9929	0.9951	0.9969	0.9982	0.9992	0.9998
2.2	0.9941	0.9952	0.9962	0.9971	0.9979	0.9985	0.9990	0.9995	0.9998	0.9999
2.5	0.9979	0.9983	0.9986	0.9989	0.9992	0.9995	0.9997	0.9998	0.9999	0.9999
2.8	0.9991	0.9993	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
3.1	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9993	0.9994	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999
4	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1
5.0674	1	1	1	1	1	1	1	1	1	1
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1	1

$k = 15, N = 287, \alpha^* = 5.0978$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2236	0.2622	0.3103	0.3702	0.4444	0.5353	0.6428	0.7619	0.8780	0.9664
1.3	0.7026	0.7447	0.7869	0.8283	0.8678	0.9043	0.9365	0.9633	0.9833	0.9958
1.6	0.9264	0.9395	0.9516	0.9625	0.9721	0.9805	0.9874	0.9929	0.9968	0.9992
1.9	0.9807	0.9843	0.9875	0.9904	0.9929	0.9951	0.9969	0.9982	0.9992	0.9998
2.2	0.9941	0.9952	0.9962	0.9970	0.9978	0.9985	0.9990	0.9994	0.9997	0.9999
2.5	0.9979	0.9983	0.9986	0.9989	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999
2.8	0.9991	0.9993	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
3.1	0.9996	0.9997	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9997	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1
5.0978	1	1	1	1	1	1	1	1	1	1
5.2	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1	1	1

$k = 16, N = 289, \alpha^* = 5.1313$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2221	0.2606	0.3084	0.3681	0.4422	0.5331	0.6408	0.7602	0.8771	0.9661
1.3	0.7009	0.7431	0.7855	0.8271	0.8668	0.9036	0.9361	0.9630	0.9832	0.9958
1.6	0.9258	0.9391	0.9512	0.9622	0.9719	0.9803	0.9874	0.9928	0.9968	0.9992
1.9	0.9805	0.9841	0.9875	0.9903	0.9929	0.9950	0.9968	0.9982	0.9992	0.9998
2.2	0.9941	0.9951	0.9962	0.9971	0.9979	0.9985	0.9990	0.9995	0.9997	0.9999
2.5	0.9979	0.9983	0.9987	0.9989	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999
2.8	0.9992	0.9993	0.9994	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999
3.1	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.1313	1	1	1	1	1	1	1	1	1	1
5.2	0.9999	0.9999	1	1	1	1	1	1	1	1

$k = 17, N = 291, \alpha^* = 5.6149$										
ρ / α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.2207	0.2591	0.3067	0.3663	0.4403	0.5311	0.6390	0.7588	0.8762	0.9658
1.3	0.6993	0.7416	0.7842	0.8259	0.8659	0.9029	0.9356	0.9627	0.9830	0.9957
1.6	0.9253	0.9387	0.9509	0.9619	0.9717	0.9802	0.9872	0.9927	0.9967	0.9991
1.9	0.9804	0.9840	0.9873	0.9903	0.9928	0.9950	0.9968	0.9982	0.9992	0.9998
2.2	0.9941	0.9951	0.9962	0.9971	0.9979	0.9985	0.9990	0.9995	0.9997	0.9999
2.5	0.9979	0.9983	0.9986	0.9989	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999
2.8	0.9992	0.9993	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
3.1	0.9996	0.9997	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9997	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
5.2	1	1	1	1	1	1	1	1	1	1
5.6149	1	1	1	1	1	1	1	1	1	1

Table 2: Values of WSRRs $R_{D(0.95)}(\rho)$ for second order slope rotatable designs under intra-class correlated structure of errors using CCD for $\rho(0 \le \rho \le 0.9)$ and for $2 \le k \le 17$ (k number of factors)

α / k	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.6	4.9	5.2
2	0.9	0.9	0.7-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
3	0.9	0.7-0.9	0.6-0.9	0.3-0.9	0.3-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
4	0.9	0.6-0.9	0.3-0.9	0.3-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
5	0.9	0.6-0.9	0.1.-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
6	0.9	0.6-0.9	0.1-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
7	0.9	0.6-0.9	0.1-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
8	0.9	0.6-0.9	0.1-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
9	0.9	0.6-0.9	0.1-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
10	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
11	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
12	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
13	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
14	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
15	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
16	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
17	0.9	0.7-0.9	0.2-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9

CONCLUSION

In this paper, the measure of slope rotatability for second order response surface designs with intra-class correlated structure of errors using CCD is studied. The degree of slope rotatability of the given design can be calculated for different values of $\rho(0 \le \rho \le 0.9)$ and for $2 \le k \le 17$ (k number of factors). By increasing α and ρ values for different factors (k) the measure of slope rotatability values for second order response surface design under intra-class correlated structure of errors using CCD are increased.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] G.E.P. Box, J.S. Hunter, Multifactor experimental designs for exploring response surfaces. Ann. Math. Stat. 28 (1957), 195-241.

[2] M.N. Das, V.L. Narasimham, Construction of rotatable designs through balanced incomplete block designs, Ann. Math. Stat. 33 (1962), 1421-1439.

[3] R.N. Das, Robust second order rotatable designs (part-I). Cal. Stat. Asso. Bull. 47 (1997), 199-214.

[4] R.N. Das, Robust second order rotatable designs (part-II). Cal. Stat. Asso. Bull, 49 (1999), 193-194.

[5] R.N. Das, Robust second order rotatable designs (part-III). J. Indian Soc. Agric. Stat. 56 (2003), 117-130.

[6] R.N. Das, Slope rotatability with correlated errors. Cal. Stat. Asso. Bull. 54 (2003), 58-70.

[7] Das, R.N. Das, Robust response surfaces, Regression, and Positive data analysis. CRC Press, Taylor and Francis Group, New York. (2014).

[8] R.N. Das, S.H. Park, Slope rotatability over all directions with correlated errors. Appl. Stoch. Models Bus. Ind. 22 (2006), 445-457.

[9] Das, R.N. Das, S.H. Park, A measure of robust rotatability for second order response surface designs. J. Korean Stat. Soc. 36 (2007), 557-578.

[10] S.H. Park, H.J. Kim, A measure of slope-rotatability for second order response surface experimental designs, J. Appl. Stat. 19 (1992) 391–404.

[11] R.J. Hader, S.H. Park, Slope rotatable central composite designs. Technometrics, 20 (1978), 413-417.

[12] K. Rajyalakshmi, Some contributions to second order rotatable and slope rotatable designs under different correlated error structures. Ph.D Thesis. Acharya Nagarjuna University, Guntur-522510, India. (2014).

[13] K. Rajyalakshmi, B. Re. Victorbabu, An empirical study on robustness of slope rotatable central composite designs. J. Stat. 21 (2014), 1-14.

[14] K. Rajyalakshmi, B. Re. Victorbabu, Second Order slope rotatable designs under intra-class correlation structure of errors using symmetrical unequal block arrangements with two unequal block arrangements. Thai. Stat. 12 (2014), 71-82.

[15] K. Rajyalakshmi, B. Re. Victorbabu, Second Order slope rotatable designs under intra-class correlated error structure using balanced incomplete block designs. Thai. Stat. 13 (2015), 169-183.

[16] S.H. Park, A class of multifactor designs for estimating the slope of response surfaces. Technometrics, 29 (1987), 449–453.

[17] S.H. Park, H.J. Kim, A measure of slope rotatability for second order response surface experimental designs. J. App. Stat. 19 (1992), 391 – 404.

[18] S.H. Park, J.H. Lim, Y.A. Baba, measure of rotatability for second order response surface designs. Ann. Inst. Stat. Math.45 (1993), 655–664.

[19] B. Re. Victorbabu, On second order slope rotatable designs – A review. J. Korean Stat. Soc. 36 (2007), 373-386.

[20] B. Re. Victorbabu, V.L. Narasimham, Construction of second order slope rotatable designs through balanced incomplete block designs. Commu. Stat., Theory Meth. 20 (1991), 2467-2478.

[21] B. Re. Victorbabu, Ch. V. V.S. Surekha, Construction of measure of second order slope rotatable designs using central composite designs. Int. J. Agric Stat. Sci. 7 (2011), 351-360.