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AN ANALYTICAL STUDY OF REACTION DIFFUSION, (3 + 1)-DIMENSIONAL DIFFUSION EQUATIONS USING CAPUTO FABRIZIO FRACTIONAL DIFFERENTIAL OPERATOR

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Abstract. Motivated by the memory features and property to portray substance heterogeneities and arrangement with various sizes of Caputo Fabrizio operator of fractional order to investigate hidden dynamics of several non-linear differential systems. In the present work, we conduct analytical study and obtain numerical simulations of one, two and $(3+1)$-dimensional Caputo-Fabrizio reaction-diffusion equations. Hybrid Laplace transform-based iterative method is constructed to find approximate solutions of diffusion equations involving Caputo–Fabrizio derivative with the exponential kernel. We have also carried out comparative analysis between Caputo and Caputo Fabrizio fractional differential operator. Moreover, we have obtained absolute error between exact and approximate solutions. 2D and 3D plots are obtained to demonstrate the efficiency of method graphically.

Keywords: reaction diffusion equations; Caputo Fabrizio fractional differential operator; numerical simulations.

2010 AMS Subject Classification: 35A20, 35A22, 34A08, 35R11.

1. Introduction

The idea of fractional calculus is appeared more than 324 years prior, yet as of late it has draw the consideration of numerous researchers working in several areas of applied sciences and mathematical modeling. From the most recent couple of decades, fractional calculus turns

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into the most incredible asset to depict the nonlinear phenomena more accurately of numerous complex mathematical models, because of its good properties, such as memory effect, nonlocality, hereditary.[1, 2, 3]

Indeed, different models of mathematical physics involving classical derivative have been reached out to represent non-local effects. The purpose of formulating mathematical models using fractional differential equations is to improve and generalize several ordinary differential systems. Hence, demonstrating some real world phenomena using fractional derivative operator has fascinated the consideration of many researchers in the field of applied mathematics. [4, 5, 6, 7].

Nevertheless, it is complicated to investigate exact analytical solutions for these models represented by nonlinear differential equations of fractional order. To beat such shortcoming, many researchers across the globe have explored broadly reliable and powerful computational techniques to approximate their solutions turns into an essential problem of examination. Hence, our goal is to derive computational technique associated with the particular problem under examination and which protects the significant characteristic of solution of intrest such as boundedness, positivity,symmetry, convexity, monotonicity, among other properties [8, 9]. Some of the useful methods are Adomain decomposition method [10, 11], finite difference method [12], iterative Laplace transform method [13, 14], spectral collocation method [15], homotopy perturbation method[16] and many other [17, 18, 19, 20]

In this work, we consider a fractional linear and nonlinear differential equations with reaction. The term reaction means a generalized type of the reaction laws associated with the Hodgkin-Huxley and Fisher's models [21, 22], Normally, these equations emerge as description models of several evolution processes in diiferent fields of applied science [23, 24]. Several authors obtained approximate solutions of fractional reaction-diffusion equations [25, 26, 27, 28] Inspired by this literature, in the present work, we have obtained approximate solutions of one, two and $(3+1)$-dimensional nonlinear reaction- diffusion equations utilizing a novel and systematic procedure called as the iterative Laplace transform method (ILTM). The suggested method is a combination of iterative method (NIM) and the Laplace transform. Moreover, we have applied Caputo-Fabrizio operator commensurate with the exponential decay law. This derivative operator is reliable and more appropriate to obtain solutions of numerous mathematical models of physical systems. We have considered the generalized one, two and $(3+1)$-dimensional Caputo-Fabrizio reaction- diffusion equations as follows

$$ {}^{CF}D_t^{\kappa} v(x,t) = \lambda v_{xx} + \eta v^{\beta}(1-v^{\delta})(v-\phi) + g(x,t) \tag{1} $$

$$ {}^{CF}D_t^{\kappa} v(x,y,t) = \lambda (v_{xx} + v_{yy}) + \eta v^{\beta}(1-v^{\delta})(v-\phi) + g(x,y,t) \tag{2} $$

$$ {}^{CF}D_t^{\kappa} v(x,y,z,t) = \lambda (v_{xx} + v_{yy} + v_{zz}) + \eta v^{\beta}(1-v^{\delta})(v-\phi) + g(x,y,z,t) \tag{3} $$

where source term is denoted by the function g and λ is the diffusion coefficient.
After substituting $\lambda = 1,\ \eta = 1,\ \beta = 1,\ \delta = 1$ and $g(x,t) = 0$ in equation (1), it becomes a time fractional Fitzhugh-Nagumo (FN) equation which denotes the transmission of nerve driving forces.

The remaining part of this paper is arranged as follows. In Sections 2, useful preliminary of fractional calculus is presented. In Section 3, general iterative Laplace transforms method and approximate solution pertaining to general fractional reaction diffusion equation is discussed. In Sections 4, we center around the verification of existence and stability criteria by employing Picard successive approximation technique and fixed point theory of Banach. In Section 5, the efficiency of suggested technique is illustrated by applying it on some examples. Moreover, the numerical simulations are presented graphically and with the help of tables. Section 6 is conclusions.

2. Preliminaries

In this section, we present some useful definitions and lemmas of fractional calculus.

Definition 2.1. *([29]) The Caputo-Fabrizio fractional integral operator with order $0 < \kappa < 1$ is given by*

$$ {}^{CF}J_t^{\kappa} v(\bar{x},t) = \frac{2(1-\kappa)}{(2-\kappa)\mathscr{M}(\kappa)} u(t) + \frac{2\kappa}{(2-\kappa)\mathscr{M}(\kappa)} \int_0^t v(\bar{x},\tau)\,d\tau, \tag{4} $$

Definition 2.2. *([30]) Let $v \in H^1(0,a),\ a > 0, 0 < \kappa < 1,$ then the time fractional Caputo-Fabrizio differential operator is given as*

$$ {}^{CF}D_t^{\kappa} v(\bar{x},t) = \frac{(2-\kappa)\mathscr{M}(\kappa)}{2(1-\kappa)} \int_0^t exp\Big[-\frac{\kappa(t-s)}{1-\kappa}\Big] v'(\tau)\,d\tau,\ t \geq 0,\ 0 < \kappa < 1, \tag{5} $$

where $\mathscr{M}(\kappa)$ is a normalisation function depending on κ such that $\mathscr{M}(0) = \mathscr{M}(1) = 1$.

Similar to Caputo derivative operator, the CF operator gives ${}^{CF}D_t^{\kappa} v(\bar{x},t) = 0$, if v is a constant function.

The benefit of Caputo-Fabrizio operator is that there is no singularity for $t = s$ in the new kernel as compared to Caputo operator

Definition 2.3. *([30]) The Laplace transform for the Caputo-fabrizio fractional operator of order $0 < \kappa \leq 1$ and $m \in \mathbb{N}$ is given by*

$$\begin{aligned} L\big({}^{CF}D_t^{m+\kappa} v(\bar{x},t)\big)(s) &= \frac{1}{1-\kappa} L(v^{(m+1)}(\bar{x},t)) L\left(exp\left(-\frac{\kappa}{1-\kappa}t\right)\right) \\ &= \frac{s^{m+1}L(v(\bar{x},t)) - s^m v(\bar{x},0) - s^{m-1}v'(\bar{x},0) - \cdots - v^{(m)}(\bar{x},0)}{s+\kappa(1-s)}. \end{aligned} \tag{6}$$

In particular, we have

$$L\big({}^{CF}D_t^{\kappa} v(\bar{x},t)\big)(s) = \frac{sL(v(\bar{x},t))}{s+\kappa(1-s)}, \quad m = 0.$$

$$L\big({}^{CF}D_t^{\kappa+1} v(\bar{x},t)\big)(s) = \frac{s^2L(v(\bar{x},t)) - sv(\bar{x},0) - v'(\bar{x},0)}{s+\kappa(1-s)}, \quad m = 1.$$

3. Iterative Laplace Transform Method

In this section, a general nonhomogeneous Caputo-Fabrizio fractional differential equation is considered which is given as below

$$ {}^{CF}D_t^{\kappa} v(\bar{x},t) = g(\bar{x},t) + Rv(\bar{x},t) + Nv(\bar{x},t), \ 0 < \kappa \leq 1, \tag{7}$$

having initial condition

$$v(\bar{x},0) = \theta(\bar{x}),$$

where source term is denoted by $g(\bar{x},t)$ and linear and non-linear operators are shown by R and N respectively.

Employing the Laplace transform (6) on both sides of (7) gives

$$L(v(\bar{x},t)) = \omega(\bar{x},s) + \left(\frac{s+\kappa(1-s)}{s}\right) L\big(R(v(\bar{x},t)) + N(v(\bar{x},t))\big), \tag{8}$$

where

$$\omega(\bar{x},s) = \frac{s+\kappa(1-s)}{s}\widetilde{g}(\bar{x},s).$$

Next, we apply inverse laplace transform on (8) then we get

$$v(\bar{x},t) = \omega(\bar{x},t) + L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\big(R(v(\bar{x},t)) + N(v(\bar{x},t))\big)\right], \tag{9}$$

where $\omega(\bar{x},t)$ is the term derived from source term.

Further, we use new iterative method to obtain infinite series solution. This method is introduced by Daftardar-Gejji and Jafari [19].

$$v(\bar{x},t) = \sum_{n=0}^{\infty} v_n(\bar{x},t), \tag{10}$$

since R is linear,

$$R\left(\sum_{n=0}^{\infty} v_n(\bar{x},t)\right) = \sum_{n=0}^{\infty} R\big(v_n(\bar{x},t)\big). \tag{11}$$

The decomposition of nonlinear operator N is given as

$$N\left(\sum_{n=0}^{\infty} v_n\right) = N(v_0(\bar{x},t)) + \sum_{n=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} v_j(\bar{x},t)\right) - N\left(\sum_{j=0}^{i-1} v_j(\bar{x},t)\right)\right\}. \tag{12}$$

In view of (10), (11) and (12), the equation (9) is equivalent to

$$\begin{aligned}\sum_{i=0}^{\infty} v_i(\bar{x},t) = \omega(\bar{x},t) &+ L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\left(\sum_{i=0}^{\infty} R\big(v_i(\bar{x},t)\big)\right)\right] \\ &+ L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\left(N\big(v_0(\bar{x},t)\big) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} v_j(\bar{x},t)\right)\right.\right.\right. \\ &\left.\left.\left. - N\left(\sum_{j=0}^{i-1} v_j(\bar{x},t)\right)\right\}\right)\right],\end{aligned} \tag{13}$$

further, consider the recurrence relation as follows

$$v_0(\bar{x},t) = \omega(\bar{x},t)$$

$$v_1(\bar{x},t) = L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\Big(R\big(v_0(\bar{x},t)\big)+N\big(v_0(\bar{x},t)\big)\Big)\right]$$

$$\vdots \tag{14}$$

$$v_{p+1}(\bar{x},t) = L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\Bigg(R\big(v_p(\bar{x},t)\big)+\bigg\{N\Big(\sum_{j=0}^{p} v_j(\bar{x},t)\Big)\right.$$

$$\left. -N\Big(\sum_{j=0}^{p-1} v_j(\bar{x},t)\Big)\bigg\}\Bigg)\right] \tag{15}$$

The approximate solution with p−term is given as

$$v = v_0+v_1+v_2+\cdots+v_{p-1}. \tag{16}$$

The convergence condition of the above approximate solution is obtained in [31].

3.1. Derivation of the solution using iterative method. In this section, the derivation of approximate solution of Eq.(3) utilizing iterative Laplace transform method is presented. Let $\bar{x}=(x,y,z)$ and taking Laplace transform on both sides of Eq. (3), we obtain

$$L\Big({}^{CF}D_t^{\kappa}v(\bar{x},t)\Big) = L\big(\lambda(v_{xx}+v_{yy}+v_{zz})+\eta v^{\beta}(1-v^{\delta})(v-\phi)+g(\bar{x},t)\big) \tag{17}$$

$$\frac{sL(v(\bar{x},t))-v(\bar{x},0)}{s+\kappa(1-s)} = L\big(\lambda(v_{xx}+v_{yy}+v_{zz})+\eta v^{\beta}(1-v^{\delta})(v-\phi)+g(\bar{x},t)\big).$$

Rearranging, we obtain

$$L\big(v(\bar{x},t)\big) = \frac{v(\bar{x},0)}{s}+\left(\frac{s+\kappa(1-s)}{s}\right)L\Big(\lambda(v_{xx}+v_{yy}+v_{zz})$$

$$+\eta v^{\beta}(1-v^{\delta})(v-\phi)+g(\bar{x},t)\Big) \tag{18}$$

Next, applying inverse Laplace transform on equation (18) , gives

$$v(\bar{x},t) = v(\bar{x},0)+L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\Big(\lambda(v_{xx}+v_{yy}+v_{zz})\right.$$

$$\left.+\eta v^{\beta}(1-v^{\delta})(v-\phi)+g(\bar{x},t)\Big)\right]. \tag{19}$$

The obtained series solution is given by,

$$v(\bar{x},t) = \sum_{p=0}^{\infty} v_p(\bar{x},t). \tag{20}$$

The recursive formula by using initial conditions is obtained as below.

$$v_{p+1}(\bar{x},t) = v_p(\bar{x},t) + L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\Big(\lambda(v_{p_{xx}} + v_{p_{yy}} + v_{p_{zz}}) + \eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi) + g(\bar{x},t)\Big)\right]$$

where the nonlinear term $\eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi)$ is decomposed as follows

$$\eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi) = \eta\left[\sum_{j=0}^{p} v_j^{\beta}\left(1-\sum_{j=0}^{p} v_j^{\delta}\right)\left(\sum_{j=0}^{p} v_j - \phi\right) - \sum_{j=0}^{p-1} v_j^{\beta}\left(1-\sum_{j=0}^{p-1} v_j^{\delta}\right)\left(\sum_{j=0}^{p-1} v_j - \phi\right)\right]$$

4. Stability Analysis

Let $(\mathscr{B}, \|\cdot\|)$ as a Banach space. Further, define τ as self-map of $\mathscr{B}$. and $\xi_{p+1} = g(\tau, \xi_p)$ shows exact recurring process. The fixed-point set on τ is denoted by $\mathscr{F}(\tau)$. Moreover, τ has atleast one element such that ξ_p converges to $h \in \mathscr{F}(\tau)$. Let $\{\varsigma_p\} \subseteq \mathscr{B}$ and define $z_p = \|\varsigma_{p+1} - g(\tau, \varsigma_p)\|$. If $\lim_{p\to\infty} z^p = 0$ implies that $\lim_{p\to\infty} \varsigma^p = h$, then the iteration method $\xi_{p+1} = g(\tau, \xi_p)$ is called as τ-stable. Comparably, we think about that, this sequence $\{\varsigma_p\}$ has an upper bound. This iteration is called as Picard's iteration and it is $\tau-$ stable, if all these criterias are fulfilled for $\xi_{p+1} = \tau\xi_p$.

Theorem 4.1. *Consider a Banach space $(\mathscr{B}, \|\cdot\|)$ and define τ as self-map on $\mathscr{B}$ fulfilling*

$$\|\tau_p - \tau_q\| \leq \chi\|p - \tau_p\| + \rho\|p - q\|$$

for all $p, q \in \mathscr{B}$ where $0 \leq \chi,\ 0 \leq \rho < 1$. Assume that τ is Picard τ-stable. Let the following equation related to (7)

$$v_{p+1}(\bar{x},t) = v_p(\bar{x},t) + L^{-1}\left[\left(\frac{s+\kappa(1-s)}{s}\right)L\Big(\lambda(v_{p_{xx}} + v_{p_{yy}} + v_{p_{zz}}) + \eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi) + g(\bar{x},t)\Big)\right] \tag{21}$$

where $\dfrac{s+\kappa(1-s)}{s}$ is a fractional Lagrange multiplier.

Theorem 4.2. *Consider a self-map τ defined as*

$$\tau(v_p(\bar{x},t)) = v_{p+1}(\bar{x},t) = v_p(\bar{x},t) + L^{-1}\Bigg[\Big(\frac{s+\kappa(1-s)}{s}\Big)L\Big(\lambda(v_{p_{xx}} + v_{p_{yy}} + v_{p_{zz}}) + \eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi) + g(\bar{x},t)\Big)\Bigg].$$

is $\tau-$stable in $L^2(m,n)$ if

$$\Big\{1+\lambda\left(\sigma_1\sigma_2 f_1(\kappa)+\sigma_3\sigma_4 f_2(\kappa)+\sigma_5\sigma_6 f_3(\kappa)\right)\eta(\pi+\mu)^{\beta} f_4(\kappa) - \eta\phi(\pi+\mu)^{\beta-1} f_5(\kappa) - \eta(\pi+\mu)^{\beta+\delta} f_6(\kappa) + \eta\phi(\pi+\mu)^{\beta+\delta-1} f_7(\kappa)\Big\} < 1. \tag{22}$$

Proof. Here, we will show that τ consists a fixed point. Hence, for all $(p,q)\in\mathbb{N}\times\mathbb{N}$, we consider the following.

$$\begin{aligned}\tau(v_p(\bar{x},t)) - \tau(v_q(\bar{x},t)) = v_p(\bar{x},t) - v_q(\bar{x},t) &+ L^{-1}\Bigg[\Big(\frac{s+\kappa(1-s)}{s}\Big)L\Big(\lambda(v_{p_{xx}}+v_{p_{yy}} \\ &+ v_{p_{zz}}) + \eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi) + g(\bar{x},t)\Big)\Bigg] \\ &- L^{-1}\Bigg[\Big(\frac{s+\kappa(1-s)}{s}\Big)L\Big(\lambda(v_{q_{xx}}+v_{q_{yy}} \\ &+ v_{q_{zz}}) + \eta v_q^{\beta}(1-v_q^{\delta})(v_q-\phi) + g(\bar{x},t)\Big)\Bigg].\end{aligned} \tag{23}$$

By applying norm on both sides of (23) and without loss of generality, we obtain

$$\begin{aligned}\left\|\tau(v_p(\bar{x},t)) - \tau(v_q(\bar{x},t))\right\| = \Bigg\| v_p(\bar{x},t) - v_q(\bar{x},t) &+ L^{-1}\Bigg[\Big(\frac{s+\kappa(1-s)}{s}\Big)L\Big(\lambda(v_{p_{xx}} \\ &+ v_{p_{yy}} + v_{p_{zz}}) + \eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi) + g(\bar{x},t)\Big)\Bigg] \\ &- L^{-1}\Bigg[\Big(\frac{s+\kappa(1-s)}{s}\Big)L\Big(\lambda(v_{q_{xx}}+v_{q_{yy}} \\ &+ v_{q_{zz}}) + \eta v_q^{\beta}(1-v_q^{\delta})(v_q-\phi) + g(\bar{x},t)\Big)\Bigg]\Bigg\|.\end{aligned} \tag{24}$$

Next, utilizing triangular inequality and simplifying further (24) we get,

$$\begin{aligned}\left\|\tau(v_p(\bar{x},t))-\tau(v_q(\bar{x},t))\right\| \leq \|v_p(\bar{x},t)-v_q(\bar{x},t)\| + L^{-1}\Bigg[\left(\frac{s+\kappa(1-s)}{s}\right)L\Big[\|\lambda\big(v_{p_{xx}}\\ +v_{p_{yy}}+v_{p_{zz}}-v_{q_{xx}}-v_{q_{yy}}-v_{q_{zz}}\big)\|\Big]\\ +L\Big[\|\eta v_p^{\beta}(1-v_p^{\delta})(v_p-\phi)-\eta v_q^{\beta}(1-v_q^{\delta})(v_q-\phi)\|\Big]\Bigg].\end{aligned} \tag{25}$$

$$\begin{aligned}\left\|\tau(v_p(\bar{x},t))-\tau(v_q(\bar{x},t))\right\| \leq \|v_p(\bar{x},t)-v_q(\bar{x},t)\| + L^{-1}\Bigg[\left(\frac{s+\kappa(1-s)}{s}\right)L\Big[\|\lambda\big(v_{p_{xx}}-v_{q_{xx}}\big)\\ \lambda\big(v_{p_{yy}}-v_{q_{yy}}\big)+\lambda\big(v_{p_{zz}}-v_{q_{zz}}\big)\|\Big]+L\Big[\|\eta\big(v_p^{\beta+1}-v_q^{\beta+1}\big)\\ -\eta\phi\big(v_p^{\beta}-v_q^{\beta}\big)-\eta\big(v_p^{\beta+\delta+1}-v_q^{\beta+\delta+1}\big)+\eta\phi\big(v_p^{\beta+\delta}-v_q^{\beta+\delta}\big)\|\Big]\Bigg].\end{aligned} \tag{26}$$

Each term of Equation (26) can be evaluated as follows

$$\begin{aligned}\|\lambda\big(v_{p_{xx}}(\bar{x},t)-v_{q_{xx}}(\bar{x},t)\big)\| &\leq \lambda\sigma_1\sigma_2\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\ \|\lambda\big(v_{p_{yy}}(\bar{x},t)-v_{q_{yy}}(\bar{x},t)\big)\| &\leq \lambda\sigma_3\sigma_4\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\ \|\lambda\big(v_{p_{zz}}(\bar{x},t)-v_{q_{zz}}(\bar{x},t)\big)\| &\leq \lambda\sigma_5\sigma_6\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|.\end{aligned} \tag{27}$$

Further, this follows

$$\begin{aligned}\|\eta\big(v_p^{\beta+1}(\bar{x},t)-v_q^{\beta+1}(\bar{x},t)\big)\| &\leq \left\|\sum_{i=0}^{\beta}C_{\beta}^{i}\big(v_p(\bar{x},t)\big)^{i}\big(v_q(\bar{x},t)\big)^{\beta-i-1}\right\|\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\ \|\eta\phi\big(v_p^{\beta}(\bar{x},t)-v_q^{\beta}(\bar{x},t)\big)\| &\leq \left\|\sum_{i=0}^{\beta-1}D_{\beta-1}^{i}\big(v_p(\bar{x},t)\big)^{i}\big(v_q(\bar{x},t)\big)^{\beta-i-2}\right\|\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\ \|\eta\big(v_p^{\beta+\delta+1}(\bar{x},t)-v_q^{\beta+\delta+1}(\bar{x},t)\big)\| &\leq \left\|\sum_{i=0}^{\beta+\delta}E_{\beta+\delta}^{i}\big(v_p(\bar{x},t)\big)^{i}\big(v_q(\bar{x},t)\big)^{\beta+\delta-i-1}\right\|\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\ \|\eta\phi\big(v_p^{\beta+\delta}(\bar{x},t)-v_q^{\beta+\delta}(\bar{x},t)\big)\| &\leq \left\|\sum_{i=0}^{\beta+\delta-1}F_{\beta+\delta-1}^{i}\big(v_p(\bar{x},t)\big)^{i}\big(v_q(\bar{x},t)\big)^{\beta+\delta-i-2}\right\|\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|.\end{aligned} \tag{28}$$

The boundedness of $v_p(\bar{x},t)$ and $v_q(\bar{x},t)$ implies existence of two distinct positive constants, π,μ such that for all $(\bar{x},t)$,

$$\|v_p(\bar{x},t)\| < \pi,\ \|v_q(\bar{x},t)\| < \mu,\ (p,q)\in\mathbb{N}\times\mathbb{N}$$

Hence, applying triangular inequality on (28) and using above positive constants we get

$$
\begin{aligned}
\|\eta\left(v_p^{\beta+1}(\bar{x},t)-v_q^{\beta+1}(\bar{x},t)\right)\| &\leq (\pi+\mu)^{\beta}\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\
\|\eta\phi\left(v_p^{\beta}(\bar{x},t)-v_q^{\beta}(\bar{x},t)\right)\| &\leq (\pi+\mu)^{\beta-1}\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\
\|\eta\left(v_p^{\beta+\delta+1}(\bar{x},t)-v_q^{\beta+\delta+1}(\bar{x},t)\right)\| &\leq (\pi+\mu)^{\beta+\delta}\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|,\\
\|\eta\phi\left(v_p^{\beta+\delta}(\bar{x},t)-v_q^{\beta+\delta}(\bar{x},t)\right)\| &\leq (\pi+\mu)^{\beta+\delta-1}\left\|v_p(\bar{x},t)-v_q(\bar{x},t)\right\|.
\end{aligned}
\tag{29}
$$

Simplifying (26), by using equations (27) and (29), we obtain

$$
\begin{aligned}
\left\|\tau(v_p(\bar{x},t))-\tau(v_q(\bar{x},t))\right\| \leq \Big\{&1+\lambda\left(\sigma_1\sigma_2 f_1(\kappa)+\sigma_3\sigma_4 f_2(\kappa)+\sigma_5\sigma_6 f_3(\kappa)\right)\\
&\eta(\pi+\mu)^{\beta}f_4(\kappa)-\eta\phi(\pi+\mu)^{\beta-1}f_5(\kappa)-\eta(\pi+\mu)^{\beta+\delta}f_6(\kappa)\\
&+\eta\phi(\pi+\mu)^{\beta+\delta-1}f_7(\kappa)\Big\}\|v_p(\bar{x},t)-v_q(\bar{x},t)\|.
\end{aligned}
\tag{30}
$$

where $f_1,\ f_2,\ f_3,\ f_4,\ f_5,\ f_6$ and f_7 are functions of $L^{-1}\left\{L\left(\dfrac{s+\kappa(1-s)}{s}\right)\right\}$.

Hence, the self-mapping τ has a fixed point. This completes the proof.

Further, we prove that τ satisfies all the criterias in Theorem 3.1. Let (30) holds then using

$$
\begin{aligned}
\rho=0,\quad \chi=\Big\{&1+\lambda\left(\sigma_1\sigma_2 f_1(\kappa)+\sigma_3\sigma_4 f_2(\kappa)+\sigma_5\sigma_6 f_3(\kappa)\right)\eta(\pi+\mu)^{\beta}f_4(\kappa)-\\
&\eta\phi(\pi+\mu)^{\beta-1}f_5(\kappa)-\eta(\pi+\mu)^{\beta+\delta}f_6(\kappa)+\eta\phi(\pi+\mu)^{\beta+\delta-1}f_7(\kappa)\Big\}
\end{aligned}
\tag{31}
$$

Thus, all the conditions in Theorem 3.2 are satisfied by τ. Therefore, τ is Picard $\tau-$ stable.

5. NUMERICAL SIMULATIONS AND DISCUSSION

In the section, we exhibit the applicability of iterative Laplace transform method using numerical and graphical simulations of Caputo Fabrizio fractional reaction-diffusion equations.

Example 5.1. *Consider the time fractional Fitzhugh– Nagumo equation*

$$
\frac{{}^{CF}\partial^{\kappa}v(x,t)}{\partial t^{\kappa}}=\frac{\partial^2 v}{\partial x^2}+v(1-v)(v-\phi)
\tag{32}
$$

subject to the initial condition

$$
v(x,0)=\frac{1}{1+e^{\frac{-x}{\sqrt{2}}}}.
\tag{33}
$$

This equation is obtained by putting $\lambda = 1,\ \eta = 1,\ \beta = 1\ \delta = 1$ alongwith $g(x,t) = 0$ in equation (1) The exact solution of Eq. (32) for $\kappa = 1$ is given as [32]

$$v(x,t) = \frac{1}{1+e^{\frac{1}{2}\left(-\frac{t(2\phi+1)}{\sqrt{2}}-x\right)}} \tag{34}$$

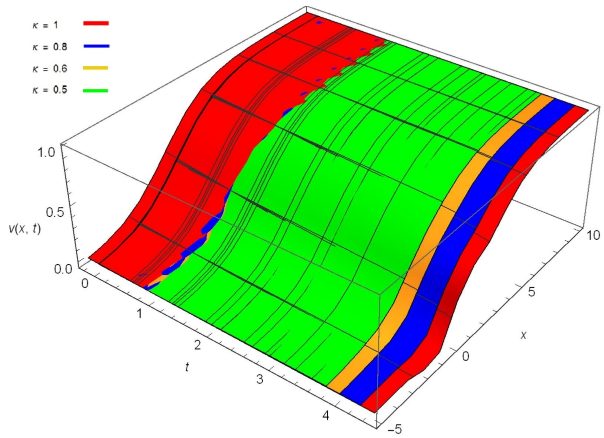
$$v_1 = -\frac{e^{\frac{x}{\sqrt{2}}}(2\phi-1)(\kappa(t-1)+1)}{2\left(e^{\frac{x}{\sqrt{2}}}+1\right)^2}$$

$$v_2 = \frac{\kappa^4 t^4 e^{\sqrt{2}x+\frac{x}{\sqrt{2}}}(1-2\phi)^2(2\phi-1)}{32\left(e^{\frac{x}{\sqrt{2}}}+1\right)^6} + \frac{t^3 e^{\frac{x}{\sqrt{2}}}(1-2\phi)^2\left(-24\kappa^4 e^{\sqrt{2}x}(2\phi-1)\right)}{96\left(e^{\frac{x}{\sqrt{2}}}+1\right)^6}$$
$$+\frac{t^3 e^{\frac{x}{\sqrt{2}}}(1-2\phi)^2\left(8\kappa^3 e^{\frac{3x}{\sqrt{2}}}(\phi-2)+8\kappa^3 e^{\frac{x}{\sqrt{2}}}(\phi+1)+32\kappa^3 e^{\sqrt{2}x}(2\phi-1)\right)}{96\left(e^{\frac{x}{\sqrt{2}}}+1\right)^6}+\cdots$$

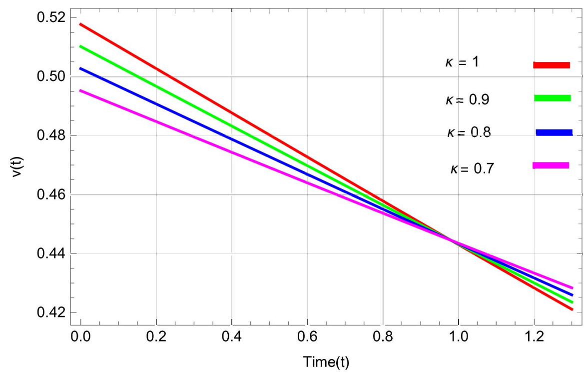
$$v_3 = \frac{1}{\left(1.+1.e^{\frac{x}{\sqrt{2}}}\right)^{18}} e^{\frac{x}{\sqrt{2}}}\left(-0.027+\kappa(0.081-0.081t)+\kappa^2\left(-0.0405t^2+0.162t-0.081\right)\right.$$
$$+\kappa^3\left(-0.0045t^3+0.0405t^2-0.081t+0.027\right)+e^{8\sqrt{2}x}\big(-0.027+\kappa(0.081-0.081t)$$
$$+\kappa^2\big(-0.0405t^2+0.162t-0.081\big)+\big)+\kappa^3\left(-0.0045t^3+0.0405t^2-0.081t+0.027\right)+\cdots$$

t	**Caputo Fabrizio operator**			**Absolute Error** for $\kappa = 1$	
	$\kappa = 0.7$	$\kappa = 0.9$	$\kappa = 1$	$\lvert v_{CF} - v_{Caputo}\rvert$	$\lvert v_{Exact} - v_{Apprx}\rvert$
0.01	0.49466	0.509499	0.516921	1.11032×10^{-16}	0.002127
0.05	0.492566	0.506799	0.513924	3.33067×10^{-16}	0.010048
0.1	0.489951	0.503424	0.510176	5.55152×10^{-16}	0.025240
0.15	0.487339	0.500050	0.506427	8.88178×10^{-16}	0.040464
0.2	0.484729	0.496676	0.502678	1.22125×10^{-15}	0.055519

TABLE 1. Approximate solution with different values of κ using CF operator and absolute error of (32)



(A)



(B)

FIGURE 1. a) The surface plot for ($\kappa = 1, 0.8, 0.6, 0.5,\ \phi = 0.8$) and b) 2d plot for ($\kappa = 1, 0.9, 0.8, 0.7$ and $x = 0.1,\ \phi = 0.8$) of approximate solution of equation (32)

Example 5.2. *Consider time fractional nonhomogeneous reaction-diffusion equation*

$$\frac{{}^{CF}\partial^{\kappa} v(x,t)}{\partial t^{\kappa}} = \frac{\partial^2 v}{\partial x^2} + v(1-v) + sinx + 2sinx\frac{t^{\kappa}}{\Gamma(1+\kappa)} + sin^2x\frac{t^{2\kappa}}{\big(\Gamma(1+\kappa)\big)^2}. \tag{35}$$

subject to the initial condition $v(x,0) = 1$.

This equation is obtained by putting $\lambda = 1,\ \eta = 1,\ \beta = 1\ \delta = 1$ and $\phi = 0$ in equation (1) The exact solution of Eq. (35) for $\kappa = 1$ is given as [32]

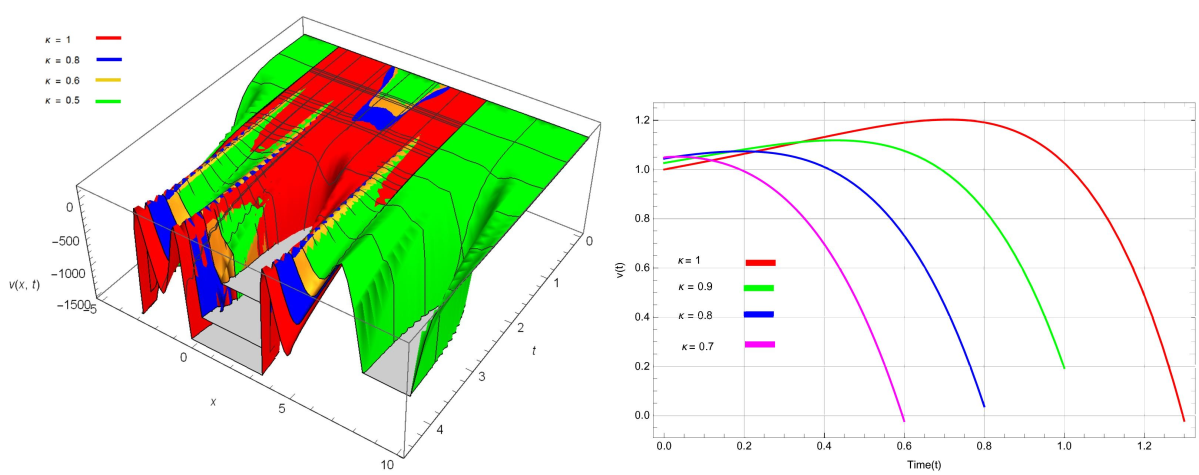
$$v(x,t) = 1 + sinx\frac{t^{\kappa}}{\Gamma(1+\kappa)} \tag{36}$$

$$\begin{aligned} v_1 = \sin(x)\Bigg(&1 - \kappa + \frac{2\kappa t^{\kappa+1}}{\Gamma(\kappa+2)} + \frac{t^{2\kappa}\sin(x)(\kappa t\Gamma(2\kappa+1) - (\kappa-1)\Gamma(2\kappa+2))}{\Gamma(\kappa+1)^2\Gamma(2\kappa+2)} \\ &- \frac{2(\kappa-1)t^{\kappa}}{\Gamma(\kappa+1)} + \kappa t\Bigg) \\ v_2 = &\frac{2\kappa^2 t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} - \frac{4\kappa t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} + \frac{2t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} + \frac{2\kappa\Gamma(2\kappa+1)t^{2(\kappa+1)-1}\cos^2(x)}{\Gamma(\kappa+1)^2\Gamma(2(\kappa+1))} \\ &- \frac{2\kappa^2\Gamma(2\kappa+1)t^{2(\kappa+1)-1}\cos^2(x)}{\Gamma(\kappa+1)^2\Gamma(2(\kappa+1))} + \frac{2\kappa\Gamma(2\kappa+1)t^{2\kappa+1}\cos^2(x)}{\Gamma(\kappa+1)^2\Gamma(2\kappa+2)} - \cdots \end{aligned}$$

$$v_3 = 24\kappa^2 t\cos^2(x) - 8\kappa t\cos^2(x) - 12\kappa^2\cos^2(x) + 8\kappa\cos^2(x) - 2\cos^2(x)8\kappa^3\cos^2(x)$$
$$-7\kappa^2t^2\cos^2(x) + \frac{40\kappa^3t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{40\kappa^2t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} + \frac{20\kappa t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{4t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4}$$
$$\frac{4\kappa^5t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{20\kappa^4t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{82\kappa^2t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} + \frac{64\kappa t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} - \frac{18t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} + \cdots$$

t	Caputo Fabrizio operator			Absolute Errorfor $\kappa = 1$	
	$\kappa = 0.7$	$\kappa = 0.9$	$\kappa = 1$	$\lvert v_{CF} - v_{Caputo}\rvert$	$\lvert v_{Exact} - v_{Apprx}\rvert$
0.1	0.946523	1.00409	1.003110	3.10862×10^{-15}	0.002106
0.2	0.866732	1.003640	1.006490	9.32587×10^{-15}	0.004487
0.3	0.732512	0.997939	1.010070	1.86517×10^{-14}	0.007072
0.4	0.524205	0.981651	1.013140	3.06422×10^{-14}	0.009138
0.5	0.218612	0.946716	1.013690	4.41869×10^{-14}	0.008686

TABLE 2. Approximate solution with different values of κ using CF operator and abosolute error of (35)



(A) (B)

FIGURE 2. a) The surface plot for $\kappa = 1, 0.8, 0.6, 0.5$ and b) 2d plot for $\kappa = 1, 0.9, 0.8, 0.7$ and $x = 0.1$ of approximate solution of equation (35)

Example 5.3. *Consider two-dimensional time fractional Fisher equation*

$$\frac{{}^{CF}\partial^{\kappa} v(x,y,t)}{\partial t^{\kappa}} = \frac{1}{2}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + v^2(1-v), \qquad 0 < \kappa \leq 1, \quad t > 0, \tag{37}$$

$$\varphi = \{(x,y)/0 \leq x \leq 1,\ 0 \leq y \leq 1\},$$

subject to the initial condition

$$v(x,y,0) = \frac{1}{1 + e^{\frac{x+y}{\sqrt{2}}}} \tag{38}$$

This equation is obtained by putting $\lambda = \frac{1}{2},\ \eta = 1,\ \beta = 2,\ \delta = 1\ \phi = 0$ and $g(x,y,t) = 0$ in equation (2) The exact solution of Eq. (37) for $\kappa = 1$ is given as [32]

$$v(x,y,t) = \frac{1}{1 + e^{\frac{x+y-\left(\frac{t}{\sqrt{2}}\right)}{\sqrt{2}}}} \tag{39}$$

$$v_1 = -\kappa + \kappa t + 1\left(\frac{1}{2}\left(\frac{e^{\sqrt{2}(x+y)}}{\left(e^{\frac{x+y}{\sqrt{2}}}+1\right)^3} - \frac{e^{\frac{x+y}{\sqrt{2}}}}{2\left(e^{\frac{x+y}{\sqrt{2}}}+1\right)^2} - \frac{e^{-\frac{x}{\sqrt{2}}}}{2\left(e^{-\frac{x}{\sqrt{2}}}+1\right)^2} + \frac{e^{-\sqrt{2}x}}{\left(e^{-\frac{x}{\sqrt{2}}}+1\right)^3}\right)\right.$$

$$\left.(-\kappa + \kappa t + 1)\left(\left(\frac{1}{e^{\frac{x+y}{\sqrt{2}}}+1}\right)^2\left(1 - \frac{1}{e^{\frac{x+y}{\sqrt{2}}}+1}\right)\right)\right)$$

$$v_2 = \frac{1}{768\left(e^{\frac{x}{\sqrt{2}}}+1\right)^9\left(e^{\frac{x+y}{\sqrt{2}}}+1\right)^9}\left(-144e^{\frac{x}{\sqrt{2}}} - 3132e^{\frac{3x}{\sqrt{2}}} + 2028e^{\frac{5x}{\sqrt{2}}} + 1200e^{\frac{7x}{\sqrt{2}}} - 1392e^{\sqrt{2}x}\right.$$

$$-288e^{\frac{x+y}{\sqrt{2}}} - 5748e^{\frac{3(x+y)}{\sqrt{2}}} + 2772e^{\frac{5(x+y)}{\sqrt{2}}} + 1644e^{2\sqrt{2}x} + 2940e^{3\sqrt{2}x} + 144e^{4\sqrt{2}x} + \cdots$$

$$v_3 = 24\kappa^2 t\cos^2(x) - 8\kappa t\cos^2(x) - 12\kappa^2\cos^2(x) + 8\kappa\cos^2(x) - 2\cos^2(x)8\kappa^3\cos^2(x)$$

$$-7\kappa^2 t^2\cos^2(x) + \frac{40\kappa^3 t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{40\kappa^2 t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} + \frac{20\kappa t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{4t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4}$$

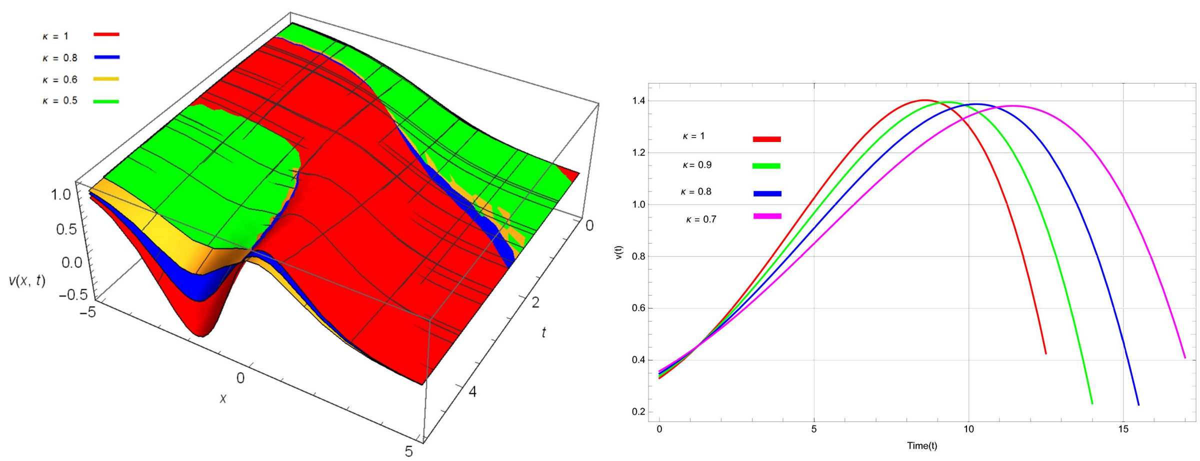
$$\frac{4\kappa^5 t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{20\kappa^4 t^{4\kappa}\cos^4(x)}{\Gamma(\kappa+1)^4} - \frac{82\kappa^2 t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} + \frac{64\kappa t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} - \frac{18t^{2\kappa}\cos^2(x)}{\Gamma(\kappa+1)^2} + \cdots$$

Example 5.4. *Let the (3+1)-D diffusion equation in fractal 2D space given below*

$$\frac{{}^{CF}\partial^{\kappa} v(\bar{x},t)}{\partial t^{\kappa}} = \frac{\partial^2 v(\bar{x},t)}{\partial x^2} + \frac{\partial^2 v(\bar{x},t)}{\partial y^2} + \frac{\partial^2 v(\bar{x},t)}{\partial z^2}, \quad \bar{x} = (x,y,z),\ 0 < \kappa \leq 1, \quad t > 0, \tag{40}$$

t	**Caputo Fabrizio operator**			**Absolute Error** for $\kappa = 1$	
	$\kappa = 0.7$	$\kappa = 0.9$	$\kappa = 1$	$\lvert v_{CF} - v_{Caputo} \rvert$	$\lvert v_{Exact} - v_{Apprx} \rvert$
0.01	0.357955	0.339441	0.331052	0	0.000293
0.05	0.360742	0.342607	0.334336	0	0.00145502
0.1	0.364257	0.346617	0.338508	0	0.0018819
0.15	0.367809	0.350687	0.342754	4.49179×10^{-17}	0.0042794
0.2	0.371395	0.354817	0.347073	5.55117×10^{-17}	0.0056464

TABLE 3. Approximate solution with different values of κ using CF operator and abosolute error of (37).



(A) (B)

FIGURE 3. a) The surface plot for $\kappa = 1, 0.8, 0.6, 0.5$ and $y = 0.5$ b) 2d plot for $\kappa = 1, 0.9, 0.8, 0.7$ and $x = y = 0.5$ of approximate solution of equation (37)

subject to initial condition

$$v(\bar{x}, 0) = (1 - y)e^{z} E_m(x^m) \tag{41}$$

where

$$E_m(x^m) = \sum_{i=0}^{\infty} \frac{x^{im}}{\Gamma(im+1)}$$

is the one-parameter Mittag–Leffler function

This equation is obtained by putting $\lambda = 1,\ \eta = 0$ and $g(x,y,z,t) = 0$ in equation (3) The exact solution of Eq. (40) for $\kappa = 1$ is given as [33]

$$v(\bar{x},t) = (1-y)e^{x+z+2t} \tag{42}$$

$$v_1 = 2.19852(-\kappa + \kappa t + 1)$$

$$v_2 = \kappa^2\left(2.19852t^2 - 8.7941t + 4.39705\right) + \kappa(8.7941t - 8.7941) + 4.39705$$

$$\begin{aligned} v(\bar{x},t) &= v_0 + v_1 + v_2 \\ &= 4.39705\kappa^2 - 10.9926\kappa + 2.19852\kappa^2 t^2 + \left(10.9926\kappa - 8.7941\kappa^2\right)t + 7.69484 \end{aligned}$$

t	**Caputo Fabrizio operator**			**Absolute Error**for $\kappa = 1$	
	$\kappa = 0.7$	$\kappa = 0.9$	$\kappa = 1$	$\lvert v_{CF} - v_{Caputo}\rvert$	$\lvert v_{Exact} - v_{Apprx}\rvert$
0.1	2.50390	1.65791	1.34110	0.001465	0.001542
0.2	2.87479	1.98835	1.62691	0.011725	0.012998
0.3	3.26723	2.35440	1.95669	0.039573	0.046299
0.4	3.68121	2.75607	2.33044	0.093803	0.116017
0.5	4.11674	3.19336	2.74816	0.183210	0.239949

TABLE 4. Approximate solution with different values of κ using CF operator and abosolute error of (40)

5.1. Discussion.

Figures 1a and 1b shows the surface plot and 2d plot behavior respectively of fourth term series solution of fractional Fitzhugh- Nagumo equation (32) for various values of parameter κ and $\phi = 0.8$. The surface ($\kappa = 1, 0.8, 0.6, 0.5$) and 2d plot ($\kappa = 1, 0.9, 0.8, 0.7$) behavior of fourth term approximate series solution of fractional nonhomogeneous reaction–diffusion equation (35) is given Figures 2a and 2b respectively. Figures 3a and 3b demonstrates the 3d $\kappa = 1, 0.8, 0.6, 0.5$ and $y = 0.5$) and 2d plot ($\kappa = 1, 0.9, 0.8, 0.7$ and $x = y = 0.5$) dynamics of fourth term approximate series solution of two-dimensional fractional Fisher equation (37) respectively.

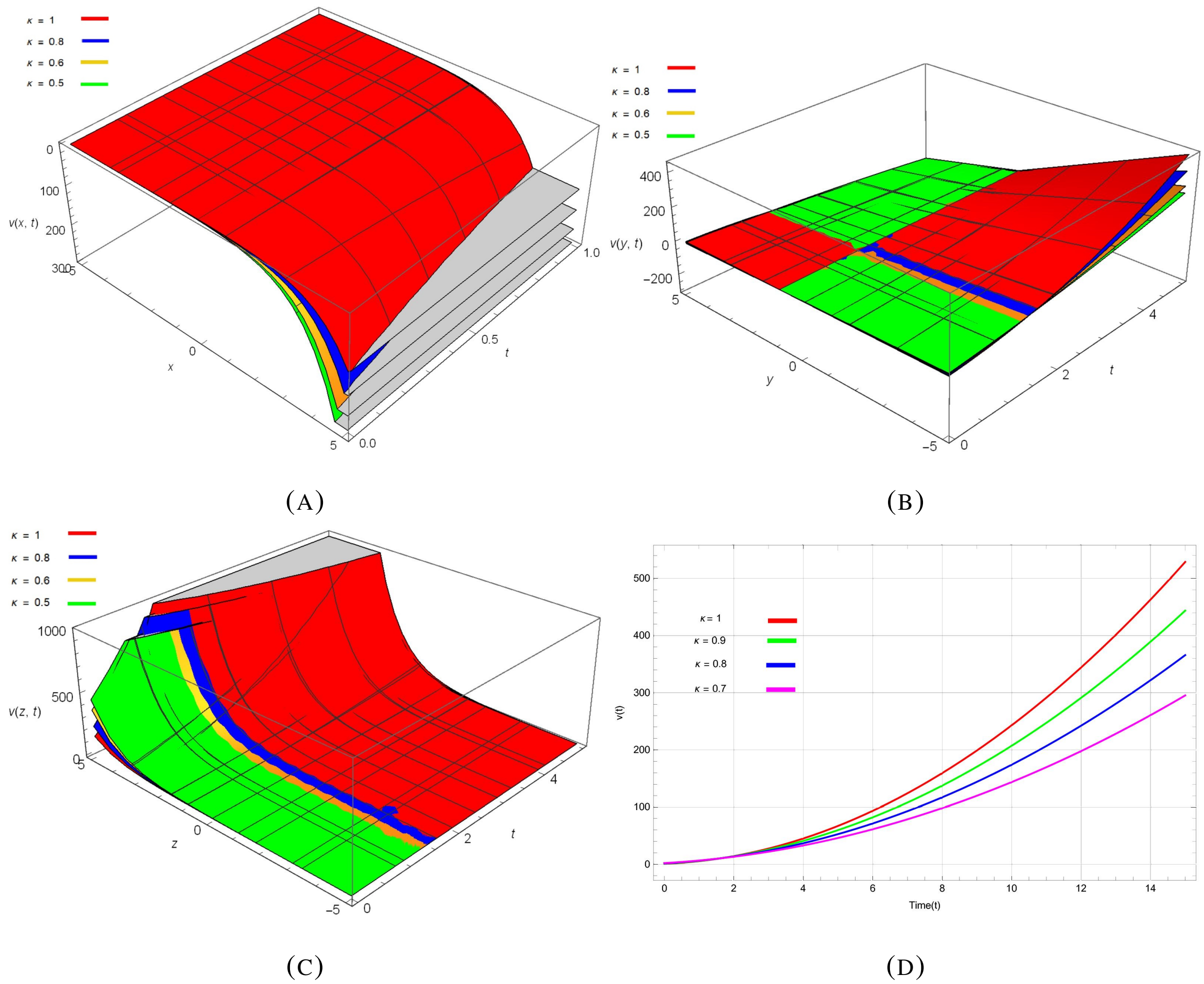


FIGURE 4. a) The surface plot for $(y = z = 0.5)$ b) The surface plot for $(x = z = 0.5)$ c) The surface plot for $(x = y = 0.5)$ and $\kappa = 1, 0.8, 0.6, 0.5$. d) 2d plot for $(x = 0.1, y = 0.1, z = 0.1)$ and $\kappa = 1, 0.9, 0.8, 0.7$ of approximate solution of equation (40)

Figures 4a, 4b and 4c shows the surface plot for different combination values of x, y and z and figure 4d gives 2d plot behavior of third approximate solution of $3+1$-D diffusion equation in fractal 2D space given by equation (40) for different values of fractional order κ. It is observed that these plots are connected sequentially and approaches towards exact solution of the related integer-order case as the fractional derivative parameters increases. Apparently, when $\kappa = 1$ the approximate solution coincide with the exact solution.

The numerical values in Tables 1 to 4 shows the approximate solution of (32), (35), (37) and (40) respectively for $\kappa = 0.7,\ 0.9,\ 1$. In addition, it shows comparision between approximate solutions obtained by using Caputo-Fabrizio derivative and Caputo derivative for $\kappa = 1$. Moreover, it shows absolute error between Caputo-Fabrizio and Caputo derivative operator as well as Caputo-Fabrizio operator and corresponding exact solution. It is observed that Caputo-Fabrizio fractional derivative demonstrates new nature contrasted with the Caputo fractional derivative.

It is noted from figures and tables that there is a remarkable difference at various estimations of κ and these equations depend continuously on the time-fractional derivative. From the results, it is clear that this method yields very accurate and convergent approximate solutions using only a few iterations in mathematical models of fractional order.

6. CONCLUSIONS

In this work, we have conducted analytical study of one, two and $(3+1)$-dimensional Caputo-Fabrizio reaction-diffusion equations using iterative Laplace transform method. Moreover, by utilizing Banach theorem, the existence and stability criteria for steady solutions have been demonstrated. The approximate series solutions obtained by this efficient approach shows a reasonable consent to control the significant effect of diffusion dynamics for the different time period. The adequacy of this procedure can be radically improved by lessening steps and computing more components. Also, Caputo-Fabrizio fractional operator and the methodology presented in this work shall be appropriate for modeling other real-world problems.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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