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ST-COLORING OF SOME PRODUCTS OF GRAPHS

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Abstract. For a finite set T of non negative integers containing zero, a function $c : V(G) \rightarrow Z^+ \cup \{0\}$ is said to be a ST -coloring of the graph $G = (V, E)$, if $|c(x) - c(y)|$ is not in T for any any edge (x, y) and for any two distinct edges (x, y) and (u, v) , $|c(x) - c(y)| \neq |c(u) - c(v)|$. $sp_{ST}(G)$ is the minimum of the difference between the largest and smallest colors assigned over all the vertices and $esp_{ST}(G)$ is the minimum of the maximum difference between the colors assigned to the vertices of an edge over all the edges of the graph, where the minimum is taken over all ST -coloring c . Here we establish some results related to ST -chromatic number, span and edge span of some graph products namely, Tensor product, Cartesian product and Corona product of graphs.

Keywords: ST -chromatic number; $sp_{ST}(G)$; $esp_{ST}(G)$; Tensor product; Cartesian product; Corona product.

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1. INTRODUCTION

Graph coloring is one of the most important and extensively well known researched area in the field of graph theory. It is an important subfield of graph theory having various applications. Assignment of frequencies to various channels is one of such famous and well known problems in the field of telecommunication. Channel assignment problem [7] can be modelled with the

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help of graph, where transmitter channels will be considered as some vertices and if there is any interference between any two transmitters, then that interference can be considered as an edge. T -coloring is one kind of vertex coloring, which was introduced by W. K. Hale [3] by setting up an interrelation between graph coloring and the channel assignment problem. For a fixed set T of non negative integers containing zero, T -coloring of a graph $G = (V, E)$, is a function $f : V(G) \rightarrow \mathbb{Z}^+ \cup \{0\}$, such that the absolute values of the differences of the colors or non negative integers assigned to any two distinct vertices must not be in the fixed set T . T -chromatic number, T -span and T -edge span are some important measures of a T -coloring. T -chromatic number is the minimum number of colors or non negative integers required for an efficient T -coloring or the order of the T -coloring f and T -span is the maximum absolute differences of the non negative integers or colors assigned to any two distinct vertices. Whereas, T -edge span is the maximum absolute differences of the non-negative integers or colors assigned to two vertices of all the edges. For more about T -colorings, we refer to [1, 2, 5, 7, 9, 11–13]. A particular type of T -coloring is ST -coloring of graphs. A Strong T -coloring of $G = (V, E)$ is a function $c : V(G) \rightarrow \mathbb{Z}^+ \cup \{0\}$ such that for all $u \neq w$ in $V(G)$, (i) $(u, w) \in E(G)$ then $|c(u) - c(w)| \notin T$ and (ii) $|c(u) - c(w)| \neq |c(x) - c(y)|$ for any two distinct edges (u, w) and (x, y) in $E(G)$.

There are various types of graph products, such as, Cartesian product, Tensor product also called as direct product, Lexicographic product also called as graph composition, Strong product, etc. For more details on products of graph, see [4]. All graphs considered in this paper are finite, simple and undirected.

In this paper, we consider strong T -colorings on some graph products, viz. Tensor product, Cartesian product and Corona product of graphs. We start with some preliminary results in the next section, followed by the main results of ST -coloring of Tensor product, Cartesian product and Corona product in section 3. In section 4, the conclusion of the paper is drawn.

2. PRELIMINARIES

Theorem 2.1. [10] *Let H be a subgraph of a graph G . For each finite set T of positive integers containing zero,*

$$(i) \text{ } sp_{ST}(H) \leq sp_{ST}(G) \quad (ii) \text{ } esp_{ST}(H) \leq esp_{ST}(G).$$

Theorem 2.2. [10] For all graphs G , (i) $sp_T(G) \leq sp_{ST}(G)$ (ii) $esp_T(G) \leq esp_{ST}(G)$.

Observation 1: [10] $\chi_{ST}(G) \geq \chi(G) = \chi_T(G)$.

For more results of ST -colorings, we refer to [6, 10].

3. MAIN RESULTS

3.1. ST -coloring of Tensor Product of Graphs.

Theorem 3.1. For all finite set T of positive integers containing zero and for any two graphs G_1 and G_2

- (i) $\chi_{ST}(G_1 \times G_2) \geq \min\{\chi(G_1), \chi(G_2)\}$
- (ii) $sp_{ST}(G_1 \times G_2) \leq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}$
- (iii) $esp_{ST}(G_1 \times G_2) \leq \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}$.

Proof. (i) Since, $\chi_{ST}(G) \geq \chi_T(G) = \chi(G)$, by using observation 1 $\Rightarrow \chi_{ST}(G_1 \times G_2) \geq \chi_T(G_1 \times G_2) = \chi(G_1 \times G_2) = \min\{\chi(G_1), \chi(G_2)\}$. Hence,

$$(1) \quad \chi_{ST}(G_1 \times G_2) \geq \min\{\chi(G_1), \chi(G_2)\}.$$

(ii) Let f and g are two ST -colorings of the graphs G_1 and G_2 respectively, such that $sp_{ST}^f(G_1) = sp_{ST}(G_1)$ and $sp_{ST}^g(G_2) = sp_{ST}(G_2)$. Define a ST -coloring of $G_1 \times G_2$ as $c(u, v) = f(u)$. Let, $(u_1, v_1), (u_2, v_2) \in V(G_1 \times G_2)$, such that $u_1, u_2 \in V(G_1)$, $v_1, v_2 \in V(G_2)$. Then,

$$|c(u_1, v_1) - c(u_2, v_2)| = |f(u_1) - f(u_2)| \notin T \text{ and}$$

$|c(u_1, v_1) - c(u_2, v_2)|$ is distinct for any two edges. Hence, c is a ST -coloring of $G_1 \times G_2$, therefore, we have,

$$(2) \quad |c(u_1, v_1) - c(u_2, v_2)| = |f(u_1) - f(u_2)| \leq sp_{ST}(G_1)$$

Define, $c(u, v) = g(v)$. Let, $(u_1, v_1), (u_2, v_2) \in V(G_1 \times G_2)$, such that $u_1, u_2 \in V(G_1)$, $v_1, v_2 \in V(G_2)$. Then,

$$|c(u_1, v_1) - c(u_2, v_2)| = |g(v_1) - g(v_2)| \notin T \text{ and}$$

$|c(u_1, v_1) - c(u_2, v_2)|$ is distinct for any two edges. Hence, c is a ST -coloring of $G_1 \times G_2$.

Therefore, we have

$$(3) \quad |c(u_1, v_1) - c(u_2, v_2)| = |g(v_1) - g(v_2)| \leq sp_{ST}(G_2)$$

From equations (2) and (3)

$$sp_{ST}(G_1 \times G_2) \leq sp_{ST}^c(G_1 \times G_2) \leq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

$$(4) \quad sp_{ST}(G_1 \times G_2) \leq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}.$$

(iii) The proof can be obtained by using the definition of *ST – edge span* and proceeding in the similar to the proof of (ii). \square

Corollary 3.1. *For any T-set, If G_1 and G_2 are two subgraphs of their Tensor products, $(G_1 \times G_2)$, then*

$$(i) \quad sp_{ST}(G_1 \times G_2) = \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

$$(ii) \quad esp_{ST}(G_1 \times G_2) = \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}.$$

Proof. (i) If G_1 and G_2 are subgraphs of $(G_1 \times G_2)$, then by using theorem 2.1 (i)

$sp_{ST}(G_1) \leq sp_{ST}(G_1 \times G_2)$ and $sp_{ST}(G_2) \leq sp_{ST}(G_1 \times G_2)$. Thus,

$$(5) \quad sp_{ST}(G_1 \times G_2) \geq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

Hence, by using theorem 3.1 (ii) and equation (5),

$$sp_{ST}(G_1 \times G_2) = \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}.$$

(ii) If G_1 and G_2 are subgraphs of $(G_1 \times G_2)$, then by using theorem 2.1 (ii)

$esp_{ST}(G_1) \leq esp_{ST}(G_1 \times G_2)$ and $esp_{ST}(G_2) \leq esp_{ST}(G_1 \times G_2)$. Thus,

$$(6) \quad esp_{ST}(G_1 \times G_2) \geq \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}$$

Hence, by using theorem 3.1 (iii) and equation (6),

$$esp_{ST}(G_1 \times G_2) = \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}.$$

\square

3.2. ST-coloring of Cartesian Product of Graphs.

Theorem 3.2. *For any T -sets of positive integers containing zero and for any two graphs G_1 and G_2*

- (i) $\chi_{ST}(G_1 \square G_2) \geq \max\{\chi_{ST}(G_1), \chi_{ST}(G_2)\}$
- (ii) $sp_{ST}(G_1 \square G_2) = \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}$
- (iii) $esp_{ST}(G_1 \square G_2) = \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}$.

Proof. (i) Since the Cartesian product of two graphs G_1 and G_2 , $G_1 \square G_2$ contains subgraphs that are isomorphic to both G_1 and G_2 . Hence, $\chi_{ST}(G_1 \square G_2) \geq \chi_{ST}(G_1)$ and $\chi_{ST}(G_1 \square G_2) \geq \chi_{ST}(G_2)$. Hence,

$$(7) \quad \chi_{ST}(G_1 \square G_2) \geq \max\{\chi_{ST}(G_1), \chi_{ST}(G_2)\}.$$

(ii) Since the Cartesian product of two graphs G_1 and G_2 , $G_1 \square G_2$ contains subgraphs that are isomorphic to both G_1 and G_2 . Hence, by using theorem 2.1 (i) $sp_{ST}(G_1) \leq sp_{ST}(G_1 \square G_2)$ and $sp_{ST}(G_2) \leq sp_{ST}(G_1 \square G_2)$. Hence,

$$(8) \quad sp_{ST}(G_1 \square G_2) \geq \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

Let f and g are two ST -colorings of G_1 and G_2 respectively such that $sp_{ST}^f(G_1) = sp_{ST}(G_1)$ and $sp_{ST}^g(G_2) = sp_{ST}(G_2)$. Let, c be a coloring on $(G_1 \square G_2)$ defined as

$$c(u, v) = f(u) + g(v), \quad (u, v) \in V(G_1 \square G_2)$$

Let, $(u_1, v_1), (u_2, v_2) \in E(G_1 \square G_2)$. Then we have two cases: either $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2 or $v_1 = v_2$ and u_1 is adjacent to u_2 in G_1 .

Case I: $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2

$$\begin{aligned} |c(u_1, v_1) - c(u_2, v_2)| &= |f(u_1) + g(v_1) - f(u_2) - g(v_2)| \\ &= |f(u_1) + g(v_1) - f(u_1) - g(v_2)| \\ &= |g(v_1) - g(v_2)| \notin T \end{aligned}$$

and $|c(u_1, v_1) - c(u_2, v_2)|$ is distinct for any two edges. Hence, c is a ST -coloring of $G_1 \square G_2$.
Hence,

$$(9) \quad |c(u_1, v_1) - c(u_2, v_2)| = |g(v_1) - g(v_2)| \leq sp_{ST}(G_2).$$

Case II: $v_1 = v_2$ and u_1 is adjacent to u_2 in G_1

$$\begin{aligned} |c(u_1, v_1) - c(u_2, v_2)| &= |f(u_1) + g(v_1) - f(u_2) - g(v_2)| \\ &= |f(u_1) + g(v_1) - f(u_2) - g(v_1)| \\ &= |f(u_1) - f(u_2)| \notin T, \end{aligned}$$

and $|c(u_1, v_1) - c(u_2, v_2)|$ is distinct for any two edges. Hence, c is a ST -coloring of $G_1 \square G_2$.
Hence,

$$(10) \quad |c(u_1, v_1) - c(u_2, v_2)| = |f(u_1) - f(u_2)| \leq sp_{ST}(G_1)$$

Hence, from equations (9) and (10)

$$|c(u_1, v_1) - c(u_2, v_2)| \leq \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

$$(11) \quad \Rightarrow sp_{ST}(G_1 \square G_2) \leq sp_{ST}^c(G_1 \square G_2) = \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

Thus, from equations (8) and (11)

$$(12) \quad sp_{ST}(G_1 \square G_2) = \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}.$$

(iii) The proof can be obtained by using the definition of ST – edge span and proceeding in the similar to the proof of (ii). □

Corollary 3.2. *If G_1 and G_2 are two graphs such that G_1 is a subgraph of G_2 , then*

$$(i) \quad sp_{ST}(G_1 \square G_2) = sp_{ST}(G_2)$$

$$(ii) \quad esp_{ST}(G_1 \square G_2) = esp_{ST}(G_2).$$

Proof. (i) Since, G_1 is a subgraph of G_2 , then, by using theorem 2.1 (i), $sp_{ST}(G_1) \leq sp_{ST}(G_2)$.

Thus,

$\max\{sp_{ST}(G_1), sp_{ST}(G_2)\} = sp_{ST}(G_2)$, Hence,

$$sp_{ST}(G_1 \square G_2) = sp_{ST}(G_2).$$

(ii) Since, G_1 is a subgraph of G_2 , then, by using theorem 2.1 (ii), $esp_{ST}(G_1) \leq esp_{ST}(G_2)$.

Thus,

$\max\{esp_{ST}(G_1), esp_{ST}(G_2)\} = esp_{ST}(G_2)$, Hence,

$$esp_{ST}(G_1 \square G_2) = esp_{ST}(G_2)$$

□

Now the following is the generalization of theorem 3.2 (ii) and 3.2 (iii) to Cartesian product of n graphs $G_1, G_2, G_3, \dots, G_n$ as follows:

Theorem 3.3. (i) $sp_{ST}(G_1 \square G_2 \square G_3 \dots \square G_n) = \max\{sp_{ST}(G_1), sp_{ST}(G_2), sp_{ST}(G_3), \dots, sp_{ST}(G_n)\}$

(ii) $esp_{ST}(G_1 \square G_2 \square G_3 \square G_3 \dots \square G_n) = \max\{esp_{ST}(G_1), esp_{ST}(G_2), esp_{ST}(G_3), \dots, esp_{ST}(G_n)\}$

Proof. (i) For any T-sets of positive integers containing zero, We shall prove this by the method of induction. Let, G_1 and G_2 are two graphs, then by using theorem 3.2 (ii), $sp_{ST}(G_1 \square G_2) = \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}$. Let, the result is true for any $k \leq n - 1$ such graphs. Then, $sp_{ST}(G_1 \square G_2 \square G_3 \square G_3 \dots \square G_{n-1}) = \max\{sp_{ST}(G_1), sp_{ST}(G_2), sp_{ST}(G_3), \dots, sp_{ST}(G_{n-1})\}$. Let, $H = G_1 \square G_2 \square G_3 \square G_3 \dots \square G_{n-1}$. Then,

$$\begin{aligned} sp_{ST}(G_1 \square G_2 \square G_3 \dots \square G_{n-1} \square G_n) &= sp_{ST}(H \square G_n) = \max\{sp_{ST}(H), sp_{ST}(G_n)\} \\ &= \max\{sp_{ST}(G_1), sp_{ST}(G_2), \dots, sp_{ST}(G_n)\}. \end{aligned}$$

(ii) For any T-sets of positive integers containing zero, We shall prove this by the method of induction. Let, G_1 and G_2 are two graphs, then by using theorem 3.2 (iii), $esp_{ST}(G_1 \square G_2) = \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}$. Let, the result is true for any $k \leq n - 1$ such graphs. Then,

$$esp_{ST}(G_1 \square G_2 \square G_3 \square G_3 \dots \square G_{n-1}) = \max\{esp_{ST}(G_1), esp_{ST}(G_2), esp_{ST}(G_3), \dots, esp_{ST}(G_{n-1})\}.$$

Let, $H = G_1 \square G_2 \square G_3 \square G_3 \dots \square G_{n-1}$. Then,

$$\begin{aligned} esp_{ST}(G_1 \square G_2 \square G_3 \dots \square G_{n-1} \square G_n) &= esp_{ST}(H \square G_n) = \max\{esp_{ST}(H), esp_{ST}(G_n)\} \\ &= \max\{esp_{ST}(G_1), esp_{ST}(G_2), \dots, esp_{ST}(G_n)\} \end{aligned}$$

□

3.3. ST-chromatic number of Corona Product of Graphs.

Theorem 3.4. For any two vertex disjoint graph G_1 and G_2 and for any T -set,

$$n + \chi_{ST}(G_1) \leq \chi_{ST}(G_1 \circ G_2) \leq m + n.$$

Proof. Let T be a set of positive integers containing zero. Let, $V(G_1) = \{v_1, v_2, v_3, \dots, v_m\}$ and $V(G_2) = \{v_{m+1}, v_{m+2}, v_{m+3}, \dots, v_{m+n}\}$. Now, let us rename the vertices of G_1 as $\{v_{ij} : j = 0\}$ and the vertices of i^{th} copy of G_2 after Corona product with G_1 as $\{v_{ij} \mid j = m + 1 \text{ to } m + n\}$, where, $i = 1$ to m .

Let, c be a coloring defined on $G_1 \circ G_2$ as,

$$c(v_{ij}) = (k + 2)^{i+j}.$$

Now, we need to show that

$$(13) \quad |c(v_{ij}) - c(v_{lm})| \neq |c(v_{ab}) - c(v_{cd})|$$

where, $(v_{ij}, v_{lm}), (v_{ab}, v_{cd}) \in E(G_1 \circ G_2)$.

If $(v_{ij}, v_{lm}), (v_{ab}, v_{cd})$ are adjacent then equation (13) holds. Hence assume that, $(v_{ij}, v_{lm}), (v_{ab}, v_{cd})$ are non-adjacent edges. Hence, $i + j, l + m, a + b, c + d$ are distinct positive integers. Without loss of generality, let $i + j$ is the largest and $c + d$ is the smallest integer. Then, we have either $c + d < a + b < l + m < i + j$ or $c + d < l + m < a + b < i + j$.

Let us consider, $c + d < a + b < l + m < i + j$.

If possible equation (13) is not true. Then,

$$\begin{aligned}
& |c(v_{ij}) - c(v_{lm})| = |c(v_{ab}) - c(v_{cd})| \\
\Rightarrow & |(k+2)^{i+j} - (k+2)^{l+m}| = |(k+2)^{a+b} - (k+2)^{c+d}| \\
\Rightarrow & (k+2)^{i+j} - (k+2)^{l+m} = (k+2)^{a+b} - (k+2)^{c+d} \\
\Rightarrow & (k+2)^{(i+j)-(c+d)} - (k+2)^{(l+m)-(c+d)} = (k+2)^{(a+b)-(c+d)} - 1 \\
\Rightarrow & (k+2)^{(i+j)-(c+d)} + 1 = (k+2)^{(l+m)-(c+d)} + (k+2)^{(a+b)-(c+d)} \\
\Rightarrow & (k+2)^{(l+m)-(c+d)} + (k+2)^{(a+b)-(c+d)} - (k+2)^{(i+j)-(c+d)} = 1
\end{aligned}$$

which is not true for any k , as $(i+j) - (c+d) > (l+m) - (c+d) > (a+b) - (c+d)$, Hence,

$$(14) \quad |c(v_{ij}) - c(v_{lm})| \neq |c(v_{ab}) - c(v_{cd})|$$

where, $(v_{ij}, v_{lm}), (v_{ab}, v_{cd}) \in E(G_1 \circ G_2)$

The proof for $c+d < l+m < a+b < i+j$ is analogous to the proof for $c+d < a+b < l+m < i+j$.

Hence, c is a ST-coloring. In $G_1 \circ G_2$, each of u_{ij} 's of G_1 are adjacent to all v_{ij} 's of i^{th} copy of G_2 . Then for all $j = 1$ to n ,

$$\begin{aligned}
& |c(u_{ij} - c(v_{i1}))| \neq |c(u_{ij} - c(v_{i2}))| \neq \dots \neq |c(u_{ij} - c(v_{in}))| \\
\Rightarrow & c(v_{i1}) \neq c(v_{i2}) \neq c(v_{i3}) \neq \dots \neq c(v_{in})
\end{aligned}$$

\Rightarrow all the vertices of i^{th} copies of G_2 in $G_1 \circ G_2$ will have distinct colors. Hence $|v(G_2)| = n$ no's of colors will be required to color the vertices of i^{th} copy of G_2 in $G_1 \circ G_2$ for ST-coloring and $\chi_{ST}(G_1)$ is the minimum number of colors for ST-coloring of G_1 . Hence, $\chi_{ST}(G_1 \circ G_2) \geq n + \chi_{ST}(G_1)$. But in $G_1 \circ G_2$, there will be m copies of G_2 . Now to minimize the number of positive integers (or colors) of the graph after Corona product, we can assign m numbers of positive integers (or colors) to the each vertices of G_1 , then every copies of G_2 can have same set of positive integers (or colors) for the vertices, because, in G_1 all the vertices are of different positive integers (or colors). Hence, $\chi_{ST}(G_1 \circ G_2) \leq m + n$ and

$$\chi_{ST}(G_1) + n \leq \chi_{ST}(G_1 \circ G_2) \leq m + n.$$

□

Corollary 3.3. *If K_m is a complete graph with m vertices and G be any graph with n vertices, then ST -chromatic number of their Corona product $K_m \circ G$ is $m + n$, i.e., $\chi_{ST}(K_m \circ G) = m + n$*

Proof. Let T be a set of positive integers containing zero with the largest element k . Let the vertices of K_m are $\{v_{ij} : j = 0\}$ and the vertices of i^{th} copy of G after Corona product with K_m as $\{v_{ij} \mid j = m + 1 \text{ to } m + n\}$, where $i = 1 \text{ to } m$

Let, c be a coloring defined on $G_1 \circ G_2$ as,

$$c(v_{ij}) = (k + 2)^{i+j}.$$

Then, clearly c is a ST -coloring of $(K_m \circ G)$.

Now, the vertices of K_m , are adjacent to each other. Hence m positive integers (or colors) will be required for ST -coloring. Moreover, In $K_m \circ G$, each of v_{ij} 's of K_m are adjacent to all v_{ij} 's of i^{th} copy of G . Then, for all $j = 1 \text{ to } n$,

$$\begin{aligned} & |c(u_{ij} - c(v_{i1}))| \neq |c(u_{ij} - c(v_{i2}))| \neq \dots \neq |c(u_{ij} - c(v_{in}))| \\ \Rightarrow & c(v_{i1}) \neq c(v_{i2}) \neq c(v_{i3}) \neq \dots \neq c(v_{in}) \end{aligned}$$

\Rightarrow all the vertices of i^{th} copies of G in $K_m \circ G$ will have distinct positive integers or colors. Hence $|v(G)| = n$ no's of positive integers or colors will be required to color the vertices of i^{th} copy of G in $K_m \circ G_2$ for ST -coloring and every copies of G can have the same set of positive integers (or colors) for the vertices, because, in K_m all the vertices are of different positive integers (or colors). Hence, $\chi_{ST}(K_m \circ G) = m + n$, which verifies the lower bound of the above theorem 3.4. □

Example 3.3.1. For any T -set, $\chi_{ST}(K_4 \circ P_3) = 7$.

Proof. Since, $\chi_{ST}(K_4) = 4$ and $|V(P_3)| = 3$. Hence by using corollary 3.3, $\chi_{ST}(K_4 \circ P_3) = \chi_{ST}(K_4) + |V(P_3)| = 4 + 3 = 7$. □

4. CONCLUSION

In the paper, we establish some results related to ST -chromatic number of some graph products, viz. Tensor products, Cartesian products and Corona products. We also find few results

related to ST -span and ST -edge span of Tensor products and Cartesian products of graphs. In continuation, one can consider other products of graphs for future research. Moreover, in this communication, we find closed expressions related to ST -span and ST -edge span of Tensor product and Cartesian product only. So, finding ST -span and ST -edge span of Corona product and other graph products can be considered for future study.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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