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# EFFECT OF CONVECTION BOUNDARY CONDITION ON HYPERBOLIC HEAT CONDUCTION IN THERMOELASTIC MEDIUM

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**Abstract:** This paper aims to study the effect of heat transfer by convection on thermoelastic rectangular solid medium in the context of hyperbolic heat conduction. The studied geometry is a two dimensional finite thin rectangular plate without internal heat generation, which is initially at a uniform temperature and subjected to convection heat transfer from the extreme edge(y=a) while the opposite side is kept insulated and remaining two sides are at a constant temperature. The material properties are assumed to be constant. The differential transform method is applied to solve the hyperbolic heat conduction equation to obtain temperature distribution in the spatial and temporal domain. Then by applying the obtained temperature in the thermoelastic equation, the displacement component, and the stress field are calculated. Also, the effect of convection boundary conditions on temperature distribution and thermoelastic field are illustrated numerically and graphically for a copper plate. It is observed that the compressive and tensile stress occurs along y-direction due to heat transfer through convection at one end.

**Keywords:** hyperbolic heat conduction; thermal stress; convection boundary condition; differential transform method; displacement function**.**

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## 1. INTRODUCTION

As a result of increased usage of nanomaterials, laser heat sources, microwaves in industries and the medical field, the interest in the hyperbolic heat conduction model have grown considerably. Most of the practical applications are explained in the context of Classical theory of heat conduction which supports the infinite speed of heat propagation but in case of high temperature gradients, high heat fluxes or very short time duration where the speed of heat propagation is finite, the theory of Fourier heat conduction fails. To overcome these difficulties Cattaneo[1] and Vernotte[2] proposed the hyperbolic heat conduction model by introducing a new term called relaxation time or phase lag in heat flux in solids, which is given by the relation

$$q + \tau_q \frac{\partial q}{\partial t} = -\lambda \nabla T \tag{1}$$

Where $\tau_q$ is called relaxation time, $q$ is the heat flux and $\lambda$ is the thermal conductivity of the material.

Equation (1) along with energy conservation law gives the hyperbolic heat conduction equation in two dimensions as:

$$\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2} = k\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)T \tag{2}$$

Where $k = \frac{\lambda}{\rho c}$ , $\rho, c$ are the thermal diffusivity, mass density and specific heat capacity respectively.

Various researches have been done on thermoelastic behaviour of solids, associated with convection heat transfer in the Fourier heat conduction domain. Thermoelastic behaviour of isotropic rectangular plate with heat transfer by convection have studied, using the finite element method under the classical Fourier heat conduction model by Shubha and Kulkarni[3]. Chen[4] has studied the thermal stress in a rectangular plate subjected to non-uniform heat transfer in terms of power series using Lanczos-Chebyshev and discrete least square methods. Sugano[5] has investigated the thermal stresses in the orthotropic rectangular plate under different third kind boundary conditions at four edges. In their study, it is observed that the thermal stresses are dependent on the anisotropic properties of a material.

In the past, most of the research work has been reported on the study of the hyperbolic heat conduction equation for one or more variables under different boundary conditions. Paul J Antaki[6] have investigated the temperature distribution in the semi-infinite slab under hyperbolic heat conduction, considering heat transfer by convection for the case of cooling and heating and observed that the result obtained for temperature field have significantly different in case of HHC than parabolic. Glass et.al[7] applied the numerical method to solve hyperbolic heat conduction with convection type boundary condition for different types of an internal heat source and also observed that convection heat transfer does not affect the heat source if it considers at the right boundary. The effect of convection and radiation type boundary conditions on temperature distribution under hyperbolic heat conduction using heat flux balance and Newton's iteration method have studied by Shen and Han[8]. The temperature distribution in nanomaterials in the context of hyperbolic heat conduction theory have analyzed by Moran Wang et.al[9]. Yen and Wu[10] have investigated an analytical solution of non-Fourier heat conduction in a finite slab, which is subjected to surface radiation and periodic on-off heat flux. H.Rahideh et al. [11] applied DQM to obtain the solution of the hyperbolic heat conduction equation and also demonstrated the accuracy and convergence of the method for different parameters. A little work on the thermoelastic responses of solids in the context of the hyperbolic heat conduction model has been reported in the literature. Zhang et al.[12] have developed a coupled thermomechanical model for thermoelastic material, considering hyperbolic heat conduction model and obtained thermal stresses in the material which cannot be neglected in thermoelasticity. Deng and Liu [13] have shown the importance of non-Fourier heat conduction in the study of thermal stresses produced in skin tissues by cryopreservation. Recently, Yang and Chen[14] investigated the thermal stresses in FGM half-space using the hyperbolic heat conduction model in Fourier and Laplace domain.

The non-Fourier hyperbolic heat conduction generally appears in microscale applications, where convection heat transfer does not play any significant role. But in some experiment [15][16] it was observed that hyperbolic heat conduction is also associated with macroscale applications, involving relatively large time and length scale and in such applications the heat transfer by convection is important. For example, in the field of medical science burning of tissues by applying hot liquid [17]. Thus due to the advancement in the field of technology and medical science, it is essential to study the hyperbolic heat conduction with convective heat transfer in the field of thermoelasticity.

In the literature different analytical and numerical methods like Fourier transform, Laplace transform, Finite element method, Differential Quadrature method, and Differential transform method have been used to solve the hyperbolic heat conduction equation as well as an equation involving in the calculation of thermal stresses and displacement. In which, the differential transform method based on Taylor series is a semi-analytical method, which was first proposed by Zhou [18] in 1986 for the solution of linear and nonlinear initial value problems that appear in electrical circuits. Due to its effectiveness and simplicity in the past few years, many authors have applied this method to solve different types of partial and ordinary differential equations. Sobhan Mosayebidorcheha et al.[19] applied DTM to find the convergent solution of nonlinear heat conduction on a fin. Lo & Chen[20] presented the solution of HHCE by applying a hybrid differential transform and volume control method by choosing appropriate shape function and concluded that this method is easy and reliable for engineering problems. Yih et.al[21] have investigated Pennes bio-heat transfer equation using DTM along with finite difference method for temperature distribution in human eyes. Mahesh Kumar et.al [22] studied the temperature distribution under convective heat transfer over a stretching sheet by applying DTM. The n-dimensional differential transform of function T of n variables[23] as follows:

$$T\left(k_1,k_2,\ldots\ldots k_n\right)=\frac{1}{k_1!k_2!\ldots.k_n!}\left[\frac{\partial^{k_1+k_2+\ldots+k_n}T\left(x_1,x_2,\ldots\ldots x_n\right)}{\partial x_1^{k_1}\partial x_2^{k_2}\ldots.\partial x_n^{k_n}}\right]_{(0,0\ldots.0)} \tag{3}$$

Where $T(k_1, k_2, \ldots \ldots k_n)$ is the transformed function T.

The inverse differential transform of $T(k_1, k_2, \ldots \ldots k_n)$ is defined by

$$T\left(x_1,x_2,\ldots\ldots x_n\right)=\sum_{k_1,k_2,\ldots\ldots k_n=0}^{\infty}T\left(k_1,k_2,\ldots\ldots k_n\right)x_1^{k_1}x_2^{k_2}\ldots x_n^{k_n} \tag{4}$$

Further, from eq.(3) and (4) one can obtain

$$T\left(x_1, x_2, \ldots\ldots x_n\right)=\sum_{k_1, k_2, \ldots\ldots k_n=0}^{\infty}\left(\frac{1}{k_1!k_2!\ldots\ldots k_n!}\right) \left[\frac{\partial^{k_1+k_2+\ldots+k_n} T\left(x_1, x_2, \ldots\ldots x_n\right)}{\partial x_1^{k_1} \partial x_2^{k_2} \ldots\ldots \partial x_n^{k_n}}\right]_{(0,0\ldots\ldots0)} x_1^{k_1} x_2^{k_2} \ldots x_n^{k_n} \tag{5}$$

In this article, the temperature distribution and thermal stresses under the hyperbolic heat conduction model subjected to convection type heat transfer through the boundary surface are semi-analytically investigated. The displacement component is also calculated in a spatial direction. The governing, partial differential equations are solved by applying a semi-analytical differential transform method. The copper material is used to illustrate the results numerically and graphically.

## 2. MATHEMATICAL MODELING FOR TEMPERATURE FIELD

The present investigation concerns a finite thin rectangular plate, initially kept at constant temperature $T_0$ and subjected to convection type heat transfer at the edge y=b, the other side opposite to it is kept insulated and the other two parallel sides are at a constant temperature $T_\infty$. Here two-dimensional heat conduction and constant thermal properties have considered. A plate of rectangular shape occupying the space D: $\{(x, y): x \in [0, a], y \in [0, b]\}$ has taken into consideration. Where the unsteady state temperature distribution under the hyperbolic heat conduction in the plate with no internal heat generation given in Eq.(2) is subjected to initial and boundary conditions, which are given as:

$$T(x, y, 0)=T_0 \tag{6}$$

$$\frac{\partial T(x, y, 0)}{\partial t}=0 \tag{7}$$

$$T(0, y, t)=T_\infty \tag{8}$$

$$T\left(x,a,t\right)=T_{\infty} \tag{9}$$

$$\frac{\partial T\left(x,0,t\right)}{\partial y}=0 \tag{10}$$

$$k_{1}\frac{\partial T\left(x,b,t\right)}{\partial y}+h\left(T_{s}-T_{b}\right)=0 \tag{11}$$

For convenience we shall employing the following dimensionless variables:

$$\begin{gathered}\theta=\frac{T-T_{b}}{T_{0}-T_{b}},\ X=\frac{cx}{2k},\ Y=\frac{cy}{2k},\quad \tau=\frac{t}{2\tau_{q}},\quad \bar{a}=\frac{ca}{2k},\\ \bar{b}=\frac{cb}{2k},\quad B_{i}=\frac{2kh}{k_{1}c},\quad A_{i}=\frac{T_{\infty}-T_{b}}{T_{0}-T_{b}}\end{gathered} \tag{12}$$

Using the above dimensionless variables, the governing heat conduction equation becomes:

$$2\frac{\partial\theta}{\partial\tau}+\frac{\partial^{2}\theta}{\partial\tau^{2}}=\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)\theta \tag{13}$$

And the initial and boundary conditions are transformed as:

$$\begin{gathered}\theta\left(X,Y,0\right)=1,\quad \frac{\partial\theta\left(X,Y,0\right)}{\partial\tau}=0,\quad \theta\left(0,Y,\tau\right)=\theta\left(\bar{a},Y,\tau\right)=A_{i},\\ \frac{\partial\theta\left(X,0,\tau\right)}{\partial Y}=0,\quad \frac{\partial\theta\left(X,\bar{b},\tau\right)}{\partial Y}=-B_{i}\theta\left(X,Y,\bar{b}\right)\end{gathered} \tag{14}$$

## 3. Solution for Temperature Field

Applying differential transform method defined in Eq.(3) on non-dimensional heat transfer Eq. (13), we get

$$\begin{gathered}\theta\left(\zeta,\eta,\Theta+2\right)=\frac{1}{\tau_{q}\left(\Theta+1\right)\left(\Theta+2\right)}\Big[\kappa\left(\zeta+1\right)\left(\zeta+2\right)\theta\left(\zeta+2,\eta,\Theta\right)\\ +\kappa\left(\eta+1\right)\left(\eta+2\right)\theta\left(\zeta,\eta+2,\Theta\right)-2\left(\zeta+1\right)\theta\left(\zeta,\eta,\Theta+1\right)\Big]\end{gathered} \tag{15}$$

To find the differential transform of initial and boundary condition, here first initial condition and boundary condition at x=0 can be represented by using Fourier sine series as:

$$1=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}C_{nm}\sin\left(\frac{\pi X}{\bar{a}}\right)sin\left(\frac{m\pi Y}{\bar{b}}\right)$$

$$A_i=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}B_{nm}\sin\left(\frac{n\pi X}{\bar{a}}\right)sin\left(\frac{m\pi\tau}{\tau_k}\right)$$

Where $C_{nm}, B_{nm}$ can be determined by

$$C_{nm}=\frac{4}{\bar{a}\bar{b}}\int_0^{\bar{a}}\int_0^{\bar{b}}\sin\left(\frac{n\pi X}{\bar{a}}\right)sin\left(\frac{m\pi Y}{\bar{b}}\right)dydx$$

$$=16/(nm\pi^2), where\, n, m \text{ both are odd}$$

$$B_{nm}=\frac{4}{\bar{b}\tau_k}\int_0^{\tau_k}\int_0^{\bar{b}}A_i\sin\left(\frac{n\pi\tau}{\tau_k}\right)sin\left(\frac{m\pi Y}{\bar{b}}\right)dyd\tau$$

$$=\frac{16\,A_i}{nm\pi^2},\ \ where\ n, m\ \ both\ are\ odd.$$

Now using the above results, we can write the differential transform of initial and boundary conditions given in Eq. (14) as:

$$\theta(\zeta,\eta,0)=\frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\mathrm{m}^{\zeta}\mathrm{n}^{\eta}\pi^{\zeta+\eta}(-1)^{\zeta+\eta-2/2}}{(\mathrm{mn})\zeta!\eta!} \tag{16}$$

$$\theta(\zeta,\eta,1)=0 \tag{17}$$

$$\theta(\zeta,1,\Theta)=0 \tag{18}$$

$$\theta(0,\eta,\Theta)=\frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{A_i\mathrm{m}^{\Theta}\mathrm{n}^{\eta}\pi^{\Theta+\eta}(-1)^{\Theta+\eta-2/2}}{(\mathrm{mn})\Theta!\eta!} \tag{19}$$

Now assuming $\theta(\zeta,0,\Theta)=B$ and applying above transformed initial and boundary conditions from Eqs.(16)-(19) in Eq. (15) , one can obtain the transformed parameter for temperature using a recursive method and then applying inverse differential transform defined in Eq.(4) and neglecting the higher-order terms,we get the expression for temperature distribution as:

$$\theta\left(X,Y,\tau\right)=\frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{\sin\left(\frac{n\pi X}{\bar{a}}\right)}{nm}\left(sin\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi Y}{\bar{b}}\right)f\left(\tau\right)+\right.$$
$$\left.+\frac{A_i\left(sin\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi Y}{\bar{b}}\right)}{nm}\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)\right\}+\sum_{\zeta=1}^{\infty}\sum_{\Theta=1}^{\infty}BX^{\zeta}\tau^{\Theta} \tag{20}$$

Where

$$f\left(\tau\right)=\left[1-\frac{\pi^2}{4}\left(\frac{n^2}{\bar{a}^2}+\frac{m^2}{\bar{b}^2}\right)\left(e^{-2\tau}-1-2\tau\right)+\frac{\pi^4}{2^4}\left(\frac{n^2}{\bar{a}^2}+\frac{m^2}{\bar{b}^2}\right)^2\sum_{p=1}^{\infty}(-1)^{p+1}\frac{p\left(2\tau\right)^{p+3}}{\left(p+3\right)!}\right]\theta$$

Now applying the transformed boundary condition prescribed at y=b from Eq.(14), one can obtain the unknown constant B and substituting that value in Eq.(20), the expression for temperature field is given as:

$$\theta\left(X,Y,\tau\right)=\frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{\sin\left(\frac{n\pi X}{\bar{a}}\right)}{nm}\left(sin\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi Y}{\bar{b}}+m\pi\left(\frac{2}{\bar{b}B_i}+1\right)\right)f\left(\tau\right)+\right.$$
$$\left.\frac{A_i\left(sin\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi Y}{\bar{b}}\right)}{nm}\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)\right\} \tag{21}$$

## 4. MATHEMATICAL MODELING FOR THERMOELASTIC FIELD

Considering the thermal stress function $\chi$ defined as [24], in the rectangular coordinate system for the plane thermoelastic problem. The fundamental equation is given as:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\chi+\alpha E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\tau=0 \tag{22}$$

The general solution of Eq.(22) may be expressed as a sum of complementary function$\chi_c$ and the particular solution$\chi_p$ :

$$\chi = \chi_c + \chi_p \tag{23}$$

Where $\chi_c$ *and* $\chi_p$ are satisfied by the equations

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0 \tag{24}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\chi_p = -\alpha E T \tag{25}$$

And component of stress are related to stress function as follows:

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2}, \quad \sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \tag{26}$$

With the following mechanical boundary conditions:

$$\sigma_{xx} = 0, \sigma_{xy} = 0 \text{ on } x = 0, a \tag{27}$$

Also, the fundamental equation for displacement function defined as [24] for plane problem in a rectangular coordinate is given as:

$$U_x = \frac{1}{2G}\left[-\frac{\partial \chi}{\partial x} + \frac{1}{1+\nu}\frac{\partial \psi}{\partial y}\right] \tag{28}$$

$$U_y = \frac{1}{2G}\left[-\frac{\partial \chi}{\partial y} + \frac{1}{1+\nu}\frac{\partial \psi}{\partial x}\right] \tag{29}$$

Where G and $\nu$ are shear modulus of elasticity and poison’s ratio, and $\psi$ satisfy the following equation as:

$$\sigma_{xx} + \sigma_{yy} + \alpha E T \equiv \frac{\partial^2 \psi}{\partial x \partial y}, \text{ and } \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \psi = 0 \tag{30}$$

Now we introduce dimensionless quantities for stress function and displacement function as:

$$\bar{\chi} = \frac{\chi c^2}{4\alpha E k^2 \left(T_0 - T_b\right)} \quad , \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\alpha E \left(T_0 - T_b\right)} \quad , \quad \bar{U}_i = \frac{c U_i}{2\alpha E k \left(T_0 - T_b\right)} \quad , \bar{\psi} = \frac{\psi c^2}{4\alpha E k^2 \left(T_0 - T_b\right)} \tag{31}$$

Then, the fundamental equation of stress function and the component of stress and displacement given in Eqs.(22), (24),(25),(26), (28), (29) and (30) can be written in dimensionless form as:

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)^2 \bar{\chi} + \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)\left(\theta - 1\right) = 0 \tag{32}$$

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)^2 \bar{\chi}_c = 0 \tag{33}$$

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right) \bar{\chi}_p = -\left(\theta - 1\right) \tag{34}$$

$$\bar{\sigma}_{xx} = \frac{\partial^2 \bar{\chi}}{\partial Y^2}, \quad \bar{\sigma}_{yy} = \frac{\partial^2 \bar{\chi}}{\partial X^2}, \quad \bar{\sigma}_{xy} = -\frac{\partial^2 \bar{\chi}}{\partial X \partial Y} \tag{35}$$

$$\bar{U}_x = \frac{1}{2G}\left[-\frac{\partial \bar{\chi}}{\partial X} + \frac{1}{1+\nu}\frac{\partial \bar{\psi}}{\partial Y}\right] \tag{36}$$

$$\bar{U}_y = \frac{1}{2G}\left[-\frac{\partial \bar{\chi}}{\partial Y} + \frac{1}{1+\nu}\frac{\partial \bar{\psi}}{\partial X}\right] \tag{37}$$

$$\bar{\sigma}_{xx} + \bar{\sigma}_{yy} + \left(\theta - 1\right) \equiv \frac{\partial^2 \bar{\psi}}{\partial X \partial Y} \text{ and } \quad \frac{\partial^2}{\partial X \partial Y}\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)^2 \bar{\psi} = 0 \tag{38}$$

Furthermore, the mechanical boundary condition given in Eq.(27) are in dimensionless parameter as:

$$\bar{\sigma}_{xx} = 0, \ \bar{\sigma}_{xy} = 0 \ \text{ on } \ X = 0, \bar{a} \tag{39}$$

## 5. SOLUTION FOR THERMOELASTIC FIELD

The complementary function $\chi_c$, the general solution of Eq.(33) may be taken in the following form:

$$\begin{aligned}\bar{\chi}_c = &\left[\alpha_i \cosh\left(\frac{m\pi Y}{\bar{b}}\right) + \beta_i \frac{m\pi}{\bar{b}} Y \sinh\left(\frac{m\pi Y}{\bar{b}}\right)\right]\sin\left(\frac{n\pi X}{\bar{a}}\right) + \\ &\left[\gamma_i \cosh\left(\frac{n\pi X}{\bar{a}}\right) + \delta_i \frac{n\pi}{\bar{a}} X \sinh\left(\frac{n\pi X}{\bar{a}}\right)\right]\sin\left(\frac{m\pi Y}{\bar{b}}\right)\end{aligned} \tag{40}$$

Now the solution of particular integral $\bar{\chi}_p$ is obtained by applying differential transform and its inverse on Eq.(34).The transformed form of Eq.(34) is given by

$$\begin{aligned}&(\zeta+1)(\zeta+2)\bar{\chi}_p(\zeta+2,\eta,\Theta) + (\eta+2)(\eta+1)\bar{\chi}_p(\zeta,\eta+2,\Theta) = \\ &-\left(\theta(\zeta,\eta,\Theta) - \delta(\zeta,\eta,\Theta)\right)\end{aligned} \tag{41}$$

Where $\delta$ is a Dirac delta function.

Now using recursive method and applying inverse differential transform, one can obtain $\bar{\chi}_p$ as :

$$\begin{aligned}\bar{\chi}_p = \frac{16}{\pi^2}\Bigg\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg(&\frac{\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right) - \frac{n\pi\tau}{\tau_k}\right)A_i}{nm}\left[-\frac{m\pi Y}{\bar{b}}\frac{X^2}{2!} + \sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{X^2}{2!} + \left(\frac{m\pi}{\bar{b}}\right)^2\frac{X^4}{4!} + \dots\right)\right] + \\ &\frac{f(\tau)}{nm}\Bigg[\left(sin\left(\frac{m\pi Y}{\bar{b}}\right) - \frac{m\pi Y}{\bar{b}}\right)\left(\frac{n\pi}{\bar{a}}\frac{X^3}{3!} - \left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^5}{5!} + ..\right) + sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{m\pi}{\bar{b}}\right)^2\left(\frac{n\pi}{\bar{a}}\frac{X^5}{5!} + ..\right) + \\ &+\frac{m\pi\bar{a}^2}{(n\pi)^2}\left(\frac{2}{B_i\bar{b}} + 1\right)\left(\sin\left(\frac{n\pi X}{\bar{a}}\right) - \frac{n\pi X}{\bar{a}}\right)\Bigg]\Bigg)\Bigg\}\end{aligned} \tag{42}$$

Further using Eq.(35) and (39) one can obtain constant $\alpha_i, \beta_i, \delta_i, \gamma_i$ as:

$$\alpha_i = \beta_i = \gamma_i = 0$$

$$\begin{aligned}\delta_i = \frac{16}{\pi^3}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{n^2 msinh(n\pi)}\Bigg[&\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right) - \frac{n\pi\tau}{\tau_k}\right)A_i\left(\frac{\bar{a}^2}{2!} + \left(\frac{m\pi}{\bar{b}}\right)^2\frac{\bar{a}^4}{4!} + \dots\right) + \\ &+f(\tau)\left(\frac{n\pi}{\bar{a}}\frac{\bar{a}^3}{3!} - \left(\frac{n\pi}{\bar{a}}\right)^3\frac{\bar{a}^5}{5!} + \left(\frac{m\pi}{\bar{b}}\right)^2\frac{n\pi}{\bar{a}}\frac{\bar{a}^5}{5!}..\right)\Bigg]\end{aligned} \tag{43}$$

Now substituting these values, the expression for thermal stress components are as follows:

$$\begin{aligned}\bar{\sigma}_{xx} = \frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg\{ & -\delta_i \frac{n\pi}{\bar{a}} X\sinh\left(\frac{n\pi X}{\bar{a}}\right)\left(\frac{m\pi}{\bar{b}}\right)^2 sin\left(\frac{m\pi Y}{\bar{b}}\right)+ \\ & \left(\frac{m\pi}{\bar{b}}\right)^2\Bigg[\frac{1}{nm}\left(\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)A_i\right)\left(\frac{X^2}{2!}+\left(\frac{m\pi}{\bar{b}}\right)^2\frac{X^4}{4!}+\dots\right)+ \\ & +\frac{f(\tau)}{nm} sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{n\pi}{\bar{a}}\frac{X^3}{3!}-\left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^5}{5!}+\left(\frac{m\pi}{\bar{b}}\right)^2\frac{n\pi}{\bar{a}}\frac{X^5}{5!}..\right)\Bigg]\Bigg\}\end{aligned} \tag{44}$$

$$\begin{aligned}\bar{\sigma}_{yy} = \frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg\{ & \delta_i \frac{n\pi}{\bar{a}}\left(\frac{2n\pi}{\bar{a}} cosh\left(\frac{n\pi X}{\bar{a}}\right)+\left(\frac{n\pi}{\bar{a}}\right)^2 X\sinh\left(\frac{n\pi X}{\bar{a}}\right)\right) sin\left(\frac{m\pi Y}{\bar{b}}\right)- \\ & -\frac{1}{nm}\left(\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)A_i\right)\left(-\frac{m\pi Y}{\bar{b}}+sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(2+\left(\frac{m\pi}{\bar{b}}\right)^2\frac{X^2}{2!}+..\right)\right)+ \\ & +\frac{f(\tau)}{nm}\Bigg[\left(sin\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi Y}{\bar{b}}\right)\left(\frac{n\pi}{\bar{a}}X-\left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^3}{3!}+..\right)+ \\ & +sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{m\pi}{\bar{b}}\right)^2\left(\frac{n\pi}{\bar{a}}\frac{X^3}{3!}-\left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^5}{5!}+..\right)-\frac{\pi}{n}\left(\frac{2}{B_i\bar{b}}+1\right)\sin\left(\frac{n\pi X}{\bar{a}}\right)\Bigg]\Bigg\}\end{aligned} \tag{45}$$

$$\begin{aligned}\bar{\sigma}_{xy} = \frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg\{ & -\delta_i \frac{m\pi}{\bar{b}}\left(\frac{n\pi}{\bar{a}} sinh\left(\frac{n\pi X}{\bar{a}}\right)+\left(\frac{n\pi}{\bar{a}}\right)^2 X\cosh\left(\frac{n\pi X}{\bar{a}}\right)\right) cos\left(\frac{m\pi Y}{\bar{b}}\right)- \\ & -\frac{1}{nm}\left(\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)A_i\right)\left(\frac{m\pi X}{\bar{b}}+\frac{m\pi}{\bar{b}} cos\left(\frac{m\pi X}{\bar{b}}\right)\left(X+\left(\frac{m\pi}{\bar{b}}\right)^2\frac{X^3}{3!}+..\right)\right)+ \\ & +\frac{f(\tau)}{nm}\Bigg[\frac{m\pi}{\bar{b}}\left(cos\left(\frac{m\pi Y}{\bar{b}}\right)-1\right)\left(\frac{n\pi}{\bar{a}}\frac{X^2}{2!}-\left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^4}{4!}+..\right)+cos\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{m\pi}{\bar{b}}\right)^2\left(\frac{n\pi}{\bar{a}}\frac{X^4}{4!}+..\right)\Bigg]\Bigg\}\end{aligned} \tag{46}$$

Similarly, using Eqs.(36)~(38),we will get the solution for displacement components as:

$$
\begin{aligned}
\bar{U}_y = {} & \frac{16}{2G\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg\{-\delta_i \frac{n\pi}{\bar{a}} X \sinh\left(\frac{n\pi X}{\bar{a}}\right) cos\left(\frac{m\pi Y}{\bar{b}}\right)\left[\frac{\bar{b}}{m\pi(1+\nu)}\left(\left(\frac{n\pi}{\bar{a}}\right)^2-\left(\frac{m\pi}{\bar{b}}\right)^2\right)+\frac{m\pi}{\bar{b}}\right] - \\
& -\frac{D_i 2\bar{b}}{m\pi(1+\nu)}\left(\frac{n\pi}{\bar{a}}\right)^2 cosh\left(\frac{n\pi X}{\bar{a}}\right) cos\left(\frac{m\pi Y}{\bar{b}}\right)+\frac{1}{nm}\left(\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)A_i\right)\times \\
& \times\left(-\frac{m\pi}{\bar{b}}\frac{X^2}{2!}+cos\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{m\pi}{\bar{b}}\frac{X^2}{2!}+\left(\frac{m\pi}{\bar{b}}\right)^3\frac{X^4}{4!}+..+\frac{\bar{b}}{m\pi(1+\nu)}\right)\right)+ \\
& \frac{f(\tau)}{nm}\left[\left(\frac{m\pi}{\bar{b}} cos\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi}{\bar{b}}\right)\left(\frac{n\pi}{\bar{a}}\frac{X^3}{3!}-\left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^5}{5!}+..\right)+\right. \\
& \left. cos\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{m\pi}{\bar{b}}\right)^3\left(\frac{n\pi}{\bar{a}}\frac{X^5}{5!}+..\right)+\frac{2m\pi Y}{(1+\nu)}\left(\frac{2}{B_i\bar{b}}+1\right)\sin\left(\frac{n\pi X}{\bar{a}}\right)\right]\Bigg\}
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
\bar{U}_x = {} & \frac{16}{2G\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg\{-\delta_i \frac{n\pi}{\bar{a}} \sin\left(\frac{m\pi Y}{\bar{b}}\right)\left[\sinh\left(\frac{n\pi X}{\bar{a}}\right)\left(\frac{1}{(1+\nu)}\left(1+\left(\frac{m\bar{a}}{n\bar{b}}\right)^2\right)-1\right)+\right. \\
& \left. +Xcosh\left(\frac{n\pi X}{\bar{a}}\right)\left(\frac{1}{(1+\nu)}\left(\frac{n\pi}{\bar{a}}+\left(\frac{m\bar{a}}{\bar{b}}\right)^2\frac{\pi}{n}\right)-1\right)\right]+\frac{1}{nm}\left(\left(\sin\left(\frac{n\pi\tau}{\tau_k}\right)-\frac{n\pi\tau}{\tau_k}\right)A_i\right)\times \\
& \times\left(-\frac{m\pi}{\bar{b}}XY+sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(X+\left(\frac{m\pi}{\bar{b}}\right)^2\frac{X^3}{3!}+..-\frac{X}{(1+\nu)}\right)\right)+\frac{f(\tau)}{nm}\left[\left(sin\left(\frac{m\pi Y}{\bar{b}}\right)-\frac{m\pi}{\bar{b}}\right)\times\right. \\
& \times\left(\frac{n\pi}{\bar{a}}\frac{X^2}{2!}-\left(\frac{n\pi}{\bar{a}}\right)^3\frac{X^4}{4!}+..\right)+sin\left(\frac{m\pi Y}{\bar{b}}\right)\left(\frac{m\pi}{\bar{b}}\right)^3\left(\frac{n\pi}{\bar{a}}\frac{X^4}{4!}+..\right)+ \\
& \left.+\frac{\bar{a}m}{n}\left(\frac{2}{B_i\bar{b}}+1\right)\left(\left(\cos\left(\frac{n\pi X}{\bar{a}}\right)-1\right)-\frac{2}{(1+\nu)}\cos\left(\frac{n\pi X}{\bar{a}}\right)\right)\right]\Bigg\}
\end{aligned} \tag{48}
$$

## 6. NUMERICAL RESULT AND DISCUSSION

The thermoelastic natures of finite thin rectangular plate are illustrated here. For numerical purpose, values for $T_0 = 20℃$, $T_b = 60℃$ , $T_\infty = 10℃$ , $\bar{a} = 1, \bar{b} = 2$ , and $A = \frac{16\alpha E}{\pi^2}$ are considered. Also, $\tau_q = 0.02$, using values of material parameter of copper plate [25] :

$\alpha = 17 \times 10^{-6}{}^{\circ}\mathrm{C}^{-1}, k = 1.1283 \times 10^{-4} m^2/s$ , $\nu = 0.33$. The temperature field, thermal stresses and displacement along spatial direction are determined and plotted.

Temperature distribution in a finite plate for specific values of dimensionless time along spatial direction X and Y has shown in Fig.1 and Fig2. From Fig.1 the small variation in temperature is observed for a small part of the plate, but as move along y-direction and as reaches near to the boundary, large variation in temperature has observed due to heat transfer by convection at that end.

In Fig.2 variation of temperature along X-direction shows as the time reaches to double of relaxation time plate experienced a sudden increment in the value of temperature. The nature of temperature is sinusoidal. It is also clear from the figure that the behaviour of temperature distribution is opposite, as it was initially for an increment in time, due to the effect of relaxation time.

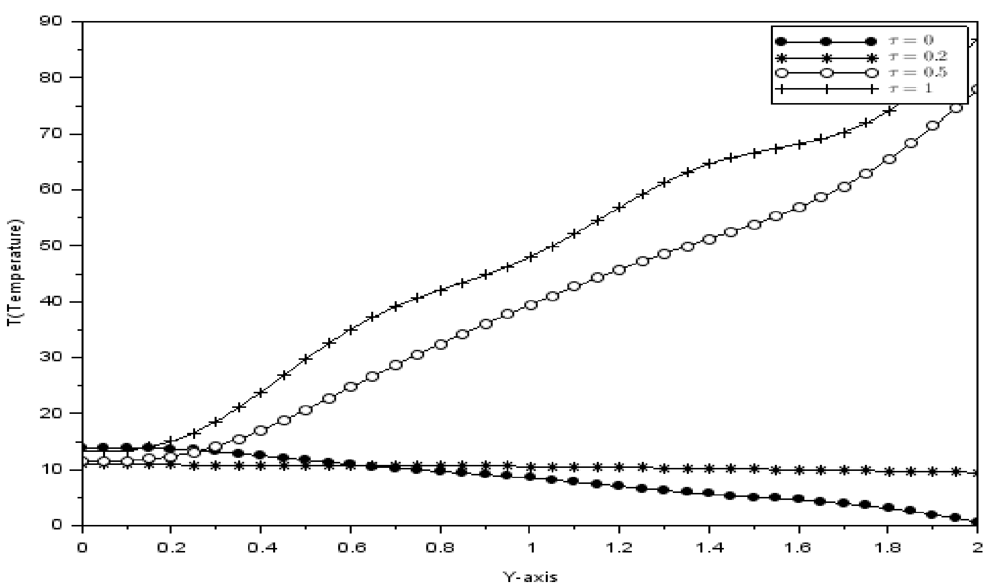


Figure 1. Temperature distribution along Y direction at X=0.5 for different values of $\tau$

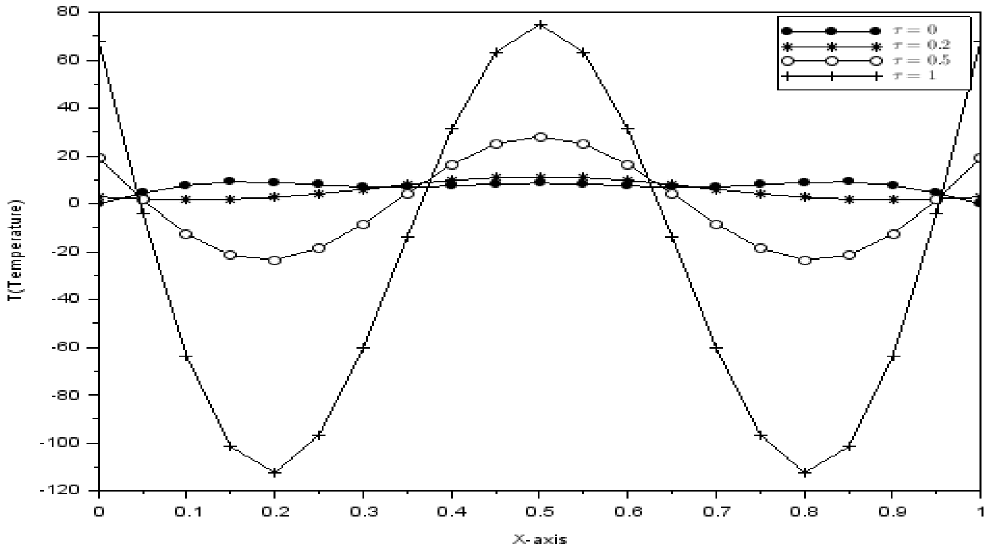


Figure 2. Temperature distribution along X direction at Y=1 for different values of $\tau$

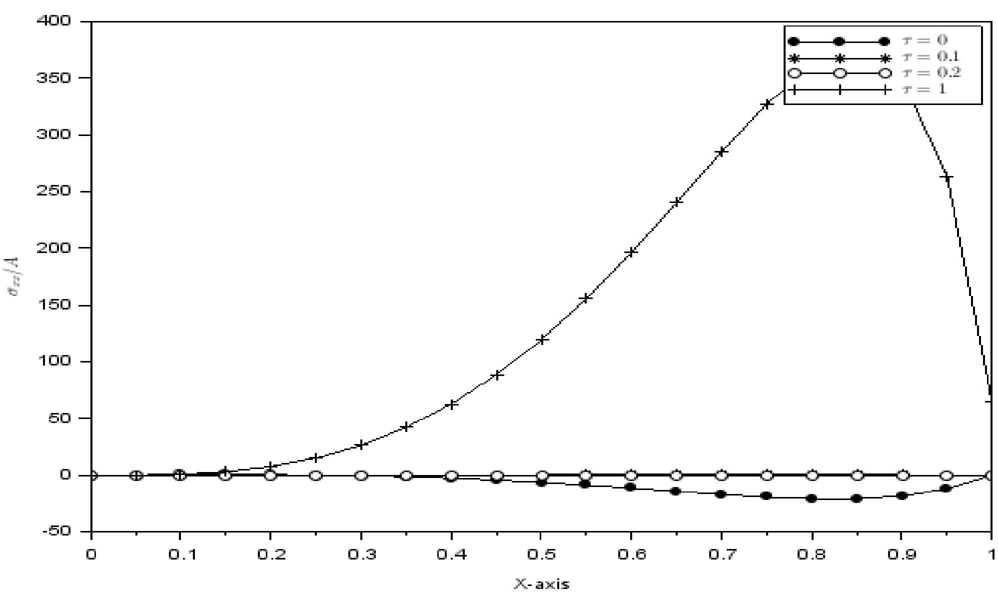


Figure3. Distribution of $\bar{\sigma}_{xx}$ along X direction at Y=1 for different values of $\tau$

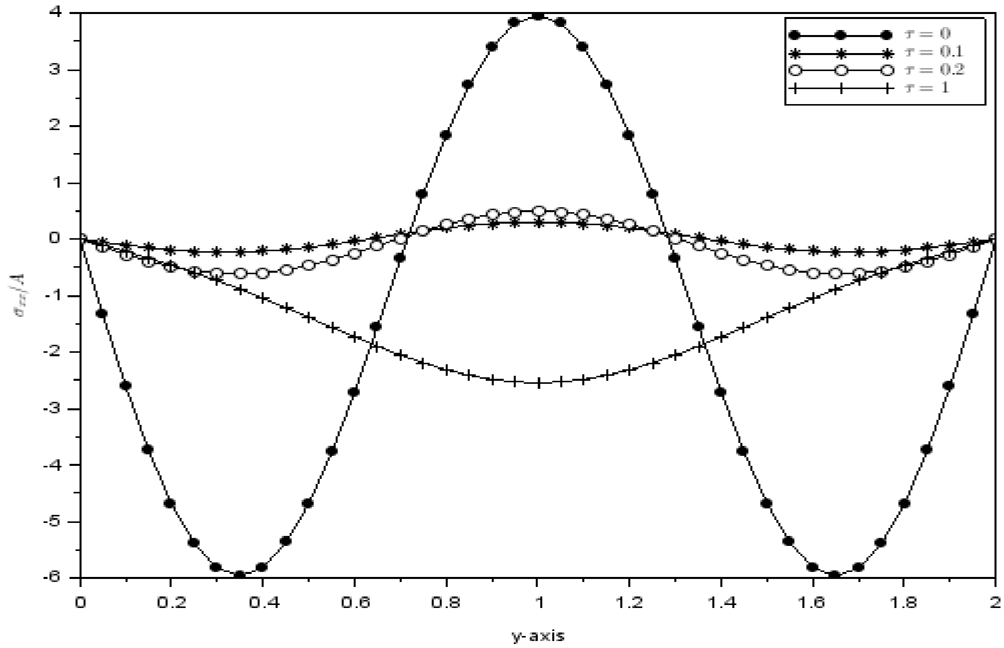


Figure 4. Distribution of $\bar{\sigma}_{xx}$ along Y direction at X=0.5 for different values of $\tau$

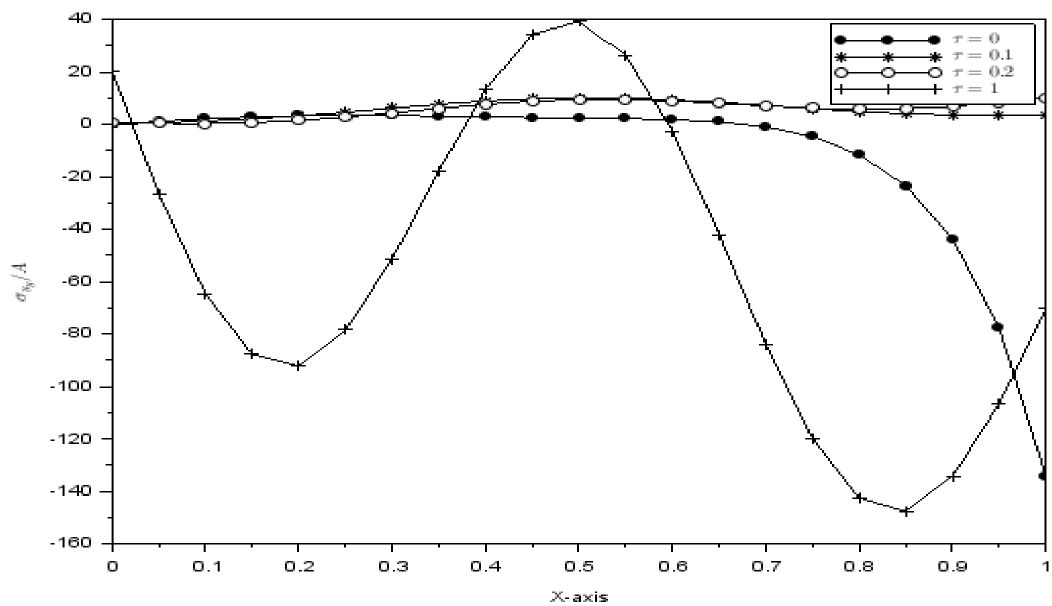


Figure 5. Distribution of $\bar{\sigma}_{yy}$ along X direction at Y=1 for different values of $\tau$

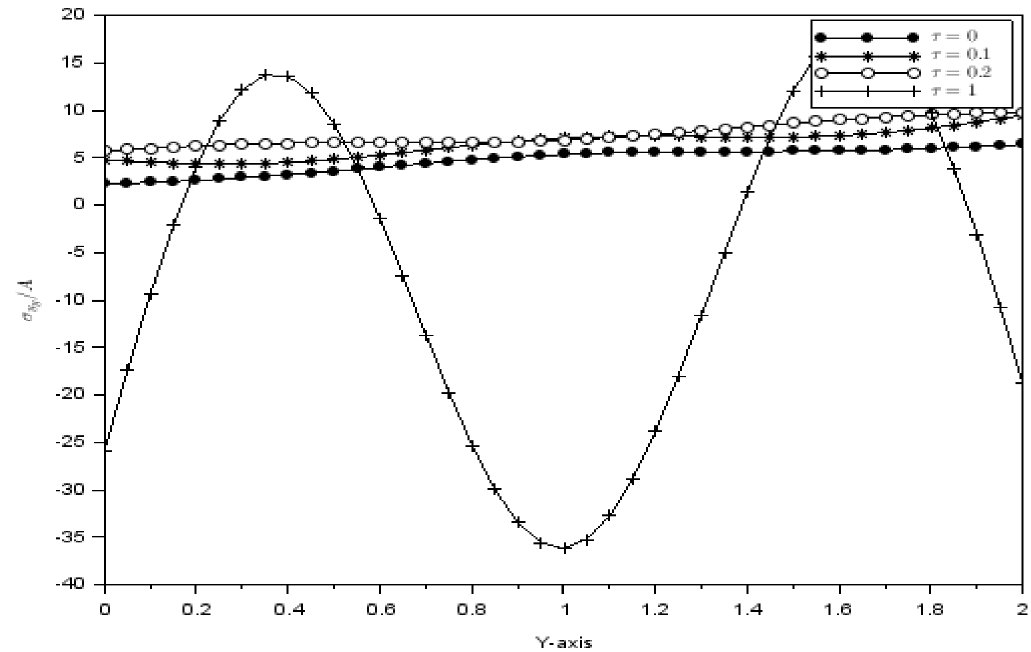


Figure 6. Distribution of $\bar{\sigma}_{yy}$ along Y direction at X=0.5 for different values of $\tau$

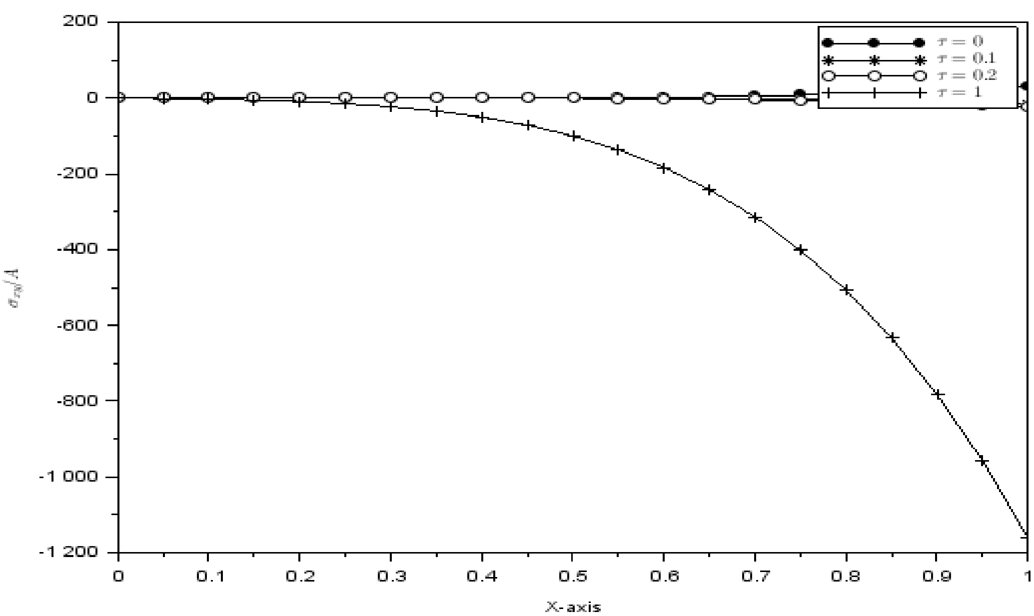


Figure 7. Distribution of $\bar{\sigma}_{xy}$ along X direction at Y=1 for different values of $\tau$

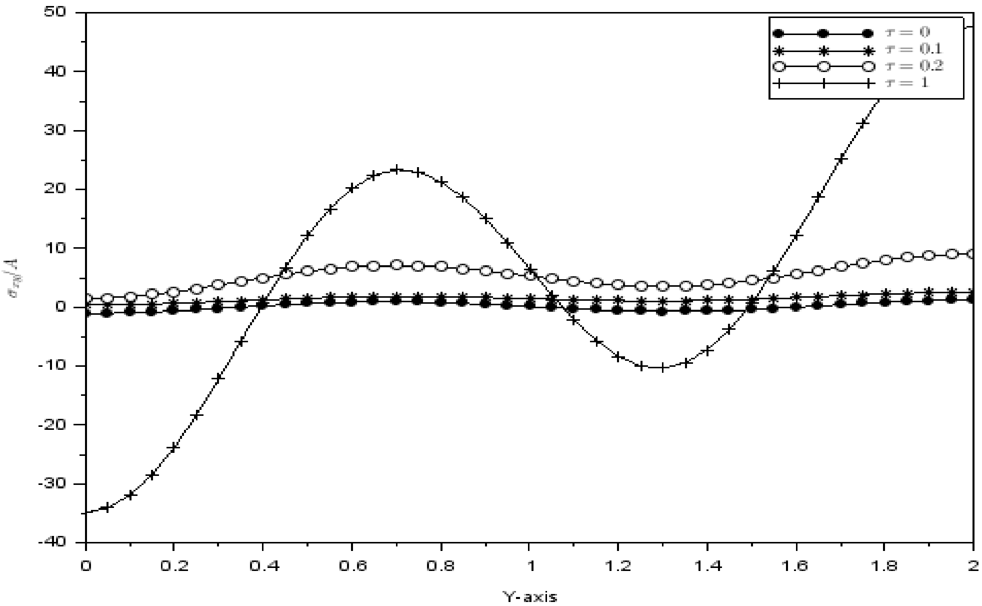


Figure 8. Distribution of $\bar{\sigma}_{xy}$ along Y direction at X=0.5 for different values of $\tau$

In Fig 3 to Fig 8 variation of stress components along spatial direction for different values of dimensionless time is shown. It is observed from Fig. 3 that the plate experienced a small amount of stress initially but as the value of dimensionless time increase, the amount of stress in X-direction also increases, and the left part of the plate experienced maximum stress. In Fig.4 variation of dimensionless stress component $\bar{\sigma}_{xx}$ along Y-direction is plotted. The stress fluctuates between compressive and tensile stress throughout the plate. It is clear from the graph that, the plate experienced peak stress at the mid of the plate initially and the compressive stress occurs in mid of the plate as the time increases, due to the phase lag in heat flux.

Fig.5 illustrating the variation of dimensionless thermal stress component $\bar{\sigma}_{yy}$ in X-direction. It is observed that the tensile stress occurs in a plate, suddenly after mid of the plate at $\tau = 0$. For the increasing values of time, the behaviour of stress changes due to the hyperbolic nature of temperature distribution. In Fig.6 variation of $\bar{\sigma}_{yy}$in Y-direction has been depicted. It can be easily seen that the amount of stress increase as increasing the value of time. Also, it is observed that the boundaries exert compressive stress in this case.

As shown in Fig.7, the thermal stress component $\bar{\sigma}_{xy}$ along x-direction does not occur up to mid part of the plate and then suddenly plate experienced the compressive stress at the boundary. In

Fig.8 variation of $\bar{\sigma}_{xy}$ along y-direction has been shown. It is clear from the figure that maximum stress occurs at the boundary of the plate.

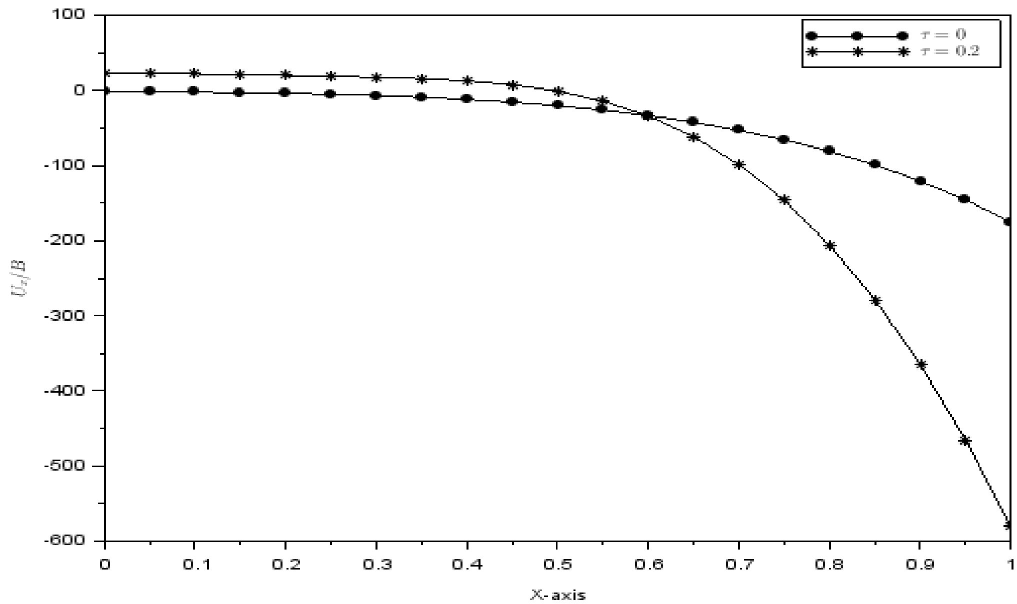


Figure 9. Distribution of displacement $\overline{U}_x$along X direction at Y=1 for different values of $\tau$

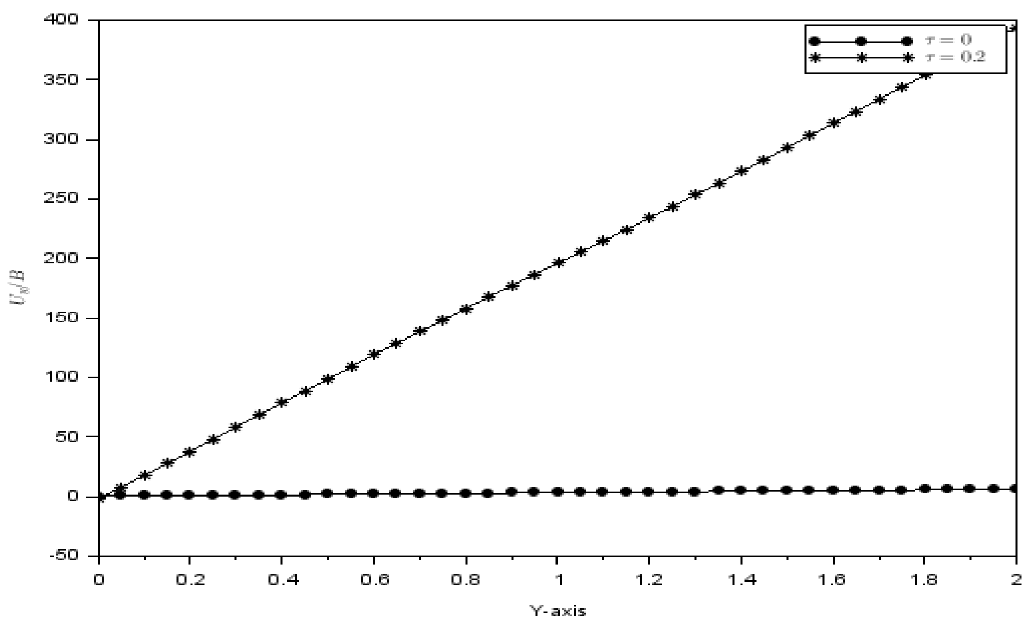


Figure10. Distribution of displacement $\overline{U}_y$along Y direction at X=0.5 for different values of $\tau$

In Fig. (9) and (10) variation of dimensionless displacement component $\overline{U}_x$ and $\overline{U}_y$ along x and y direction has been plotted. From both graphs, it is observed that the maximum displacement occurs at the boundary x=a and y=b. It is clear that the displacement along y-direction varies linearly.

## 7. CONCLUSION

In the present work, the effects of convection type boundary condition on thermoelastic behaviour of rectangular plate under the framework of the hyperbolic heat conduction model have been investigated. The distribution of temperature, thermal stresses, and the displacements in a plate is obtained in the differential transform domain. The solution is obtained in the form of an infinite series. From the study, it is observed that the peak temperature is experienced at the boundary where heat is transfer by convection. The study also shows that tensile and compressive stresses occur in the plate along Y-direction. Also, it is observed that the behaviour of stress changes as time increases.  This study may be useful for designing and engineering applications where the heat transfer by convection along with temperature gradient is high or the time duration is very short.

## CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

## REFERENCES

[1] C. Cattaneo. A form of heat conduction equation which eliminates the paradox of instantaneous propagation. Compte Rendus. 247 (1958), 431–433

[2] P. Vernotte. Les paradoxes de la théorie continue de l'équation de la chaleur. Comptes Rendus Acad. Sci. Paris 246 (1958), 3154–3155.

[3] S. Verma, V. S. Kulkarni, Analytical solution of rectangular plate due to convective heat transfer by dissipation, Int. J. Current Res. 6(10) (2014), 9142-9150.

[4] P.Y.P. Chen, Transient thermal stresses in rectangular plate due to nonuniform heat transfer coefficient, J. Therm. Stress. 11(2) (1988), 115-125.

[5] Y. Sugano, Transient thermal stresses in an orthotropic finite rectangular plate due to arbitrary surface heat-generations, Nuclear Eng. Des. 59 (1980), 379-393.

[6] P.J. Antaki, Analysis of hyperbolic heat conduction in a semi-infinite slab with surface convection, Int. J. Heat Mass Transfer. 40(13) (1997), 3247-3250.

[7] D.E. Glass, K.K. Tamma, S.B. Railkar, Hyperbolic heat conduction with convection boundary conditions and pulse heating effects, J. Thermophys. Heat Transfer. 5 (1991), 110–116.

[8] W. Shen, S. Han, A numerical solution of two-dimensional hyperbolic heat conduction with non-linear boundary conditions, Heat Mass Transfer. 39 (2003), 499–507.

[9] M. Wang, N. Yang, Z.Y. Guo, Non-Fourier heat conductions in nanomaterials, J. Appl. Phys. 110 (2011), 064310.

[10] C.C. Yen, C.Y. Wu, Modelling hyperbolic heat conduction in a finite medium with periodic thermal disturbance and surface radiation, Appl. Math. Model. 27 (2003), 397–408.

[11] H. Rahideh, P. Malekzadeh, M.R. Golbahar Haghighi, Non-Fourier Heat Conduction Analysis with Temperature-Dependent Thermal Conductivity, ISRN Mech. Eng. 2011 (2011), 321605.

[12] X. Zhang, L. Zhang, Z. Chu, Thermomechanical Coupling of non-Fourier Heat Conduction with the C-V Model: Thermal Propagation in Coating Systems, J. Therm. Stress. 38(10) (2015), 1104-1117.

[13] Z.S. Deng, J. Liu, Non-Fourier heat conduction effect on prediction of temperature transients and thermal stress in skin cryopreservation, J. Therm. Stress. 26(8) (2003), 779-798.

[14] W. Yang, Z. Chen, Investigation of the thermal-elastic problem in cracked semi-infinite FGM under thermal shock using hyperbolic heat conduction theory, J. Therm. Stress. 42(8) (2019), 993-1010.

[15] W. Kaminski, Hyperbolic Heat Conduction Equation for Materials with a Nonhomogeneous Inner Structure, J. Heat Transfer. 112 (1990), 555-560.

[16] K. Mitra, S. Kumar, A. Vedevarz, M.K. Moallemi, Experimental Evidence of Hyperbolic Heat Conduction in Processed Meat, J. Heat Transfer. 117 (1995), 568–573.

[17] K.R. Diller, Modeling of bioheat transfer processes at high and low temperatures, Adv. Heat Transfer. 22

(1992), 157-357.

[18] J.K. Zhou. Differential transformation and its applications for electrical circuits. Huazhong University Press, Wuhan, China. (1986).

[19] S. Mosayebidorcheh, D.D. Ganji, M. Farzinpoor, Approximate solution of the nonlinear heat transfer equation of a fin with the power-law temperature-dependent thermal conductivity and heat transfer coefficient, Propulsion Power Res. 3 (2014), 41–47.

[20] C.Y. Lo, B.Y. Chen, Application of Hybrid Differential Transform/Control-Volume Method to Hyperbolic Heat Conduction Problems, Numerical Heat Transfer, Part B: Fundamentals. 55 (2009) 219–231.

[21] S.Y. Lin, M.H. Shao, S.P. Shih, H.Y. Lu, Application of differential transformation method to the human eye heat transfer problems, in: 2018 IEEE International Conference on Applied System Invention (ICASI), IEEE, Chiba, 2018: pp. 1296–1299.

[22] M. Kumar, G.J. Reddy, N.N. Kumar, O.A. Bég, Application of differential transform method to unsteady free convective heat transfer of a couple stress fluid over a stretching sheet. Heat Transfer-Asian Res. 48 (2019), 582–600.

[23] A. Kurnaz, G. Oturanç, M.E. Kiris, n-Dimensional differential transformation method for solving PDEs, Int. J. Comput. Math. 82(3) (2005), 369-380.

[24] N. Noda, R.B. Hetnarski, Y. Tanigawa, Thermal Stresses, 2nd ed., Routledge, 2018.

[25] H.H. Sherief, M.N. Anwar, Two-dimensional generalized thermoelasticity problem for an infinitely long cylinder, J. Therm. Stress. 17(2) (1994), 213-227.