INVESTIGATION OF PLANE LONGITUDINAL WAVES IN A MICRO-ISOTROPIC, MICRO-ELASTIC SOLID

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Abstract. In the frame work of plane harmonic wave solution, we investigate the phase velocities of longitudinal micro-rotational (MR), micro-isotropical (MI), and micro strain (MS) waves. Two sets of MR waves, one set of MI and MS waves are obtained. These waves are dispersive in nature. Micro non-rotational (MNR) waves are obtained as a particular case of MR waves. Micro-isotropical shear (MIS) and micro strain shear (MSS) waves are obtained as a particular case of MI and MS waves. MNR waves are dispersive in nature, while MIS and MSS waves are non-dispersive and they depend only on elastic constants. With the help of MATLAB programme, numerical example is considered, and speeds are plotted against frequency ratios and wave number.

Keywords: micro-isotropic; micro-elasticity; plane longitudinal waves.

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1. INTRODUCTION

Eringen and his co-researchers [1, 2] are developed the non linear theory of micro elasticity. Eringen [3] modified the linear theory of micropolar elasticity. The basic difference between the theory of micropolar elasticity and classical theory of elasticity is only the introduction of

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an independent micro rotation vector. With this reason, the motion in the micropolar solids is characterized by six degrees of freedom, namely three of translation and three of rotation. The introduction between two parts of micropolar body is transmitted not only by a force vector but also by a couple resulting in a symmetric force stress tensor, and couple stress tensor. Propagation of waves in micro polar elastic solids was studied by many authors like Parfitt and Eringen [4], Tomar and Gogna [5], Tomer et.al. [6] and Tomer and Gogna [7, 8]. Singh and Tomer [9] are studied the plane wave propagation in thermo elastic solids with voids. Pochhmer [10] and Chree [11] discussed the longitudinal waves in a cylinder within the frame work of classical theory of elasticity. The velocity of longitudinal waves in a cylindrical bars was derived by Bancroft [12]. Oliver [13] investigated the wave propagation in a cylindrical rods under the pulse techniques. In recent, Somaiah [14] studied the plane wave propagation in micro stretch elastic solids.

2. BASIC EQUATIONS

The basic governing equations of displacements, micro-rotations and micro-strains for a micro-isotropic, micro-elastic solid in the absence of body forces and body couples are given by Nowacki [15] as follows:

$$(\mu + K)\nabla^2 \vec{u} + (\lambda + \mu)\nabla\nabla.\vec{u} + K\nabla \times \vec{\phi} = \rho\ddot{\vec{u}} \tag{1}$$

$$\gamma\nabla^2 \vec{\phi} + (\alpha + \beta)\nabla\nabla.\vec{\phi} + K\nabla \times \vec{u} - 2K\vec{\phi} = \rho J\ddot{\vec{\phi}} \tag{2}$$

$$\tau_1 \phi_{pp,kk}\delta_{ij} + \tau_2 \phi_{(ij),kk} - \sigma_1 \phi_{pp}\delta_{ij} - \sigma_2 \phi_{(ij)} = \frac{1}{2}\rho J\ddot{\phi}_{(ij)} \tag{3}$$

where λ, μ are Lames constants, K, τ_1, τ_2 are elastic constants,ρ is the density, J is the moment of micro inertia.α, β, γ are micro-rotational constants,σ_1, σ_2 are respectively isotropical and strain parameters, δ_{ij} is the Kronecker delta and double dot on right hand side is the second order partial derivative with respect to time t .

3. FORMULATION AND SOLUTION OF THE PROBLEM

Equations (1) and (2) can be written as

$$(4) \qquad c_1^2 \nabla^2 \vec{u} + c_2^2 \nabla \nabla . \vec{u} + \omega_0^2 \nabla \times \vec{\phi} = \frac{\partial^2 \vec{u}}{\partial t^2}$$

$$(5) \qquad c_4^2 \nabla^2 \vec{\phi} + c_5^2 \nabla \nabla . \vec{\phi} + c_6^2 \nabla \times \vec{u} - c_7^2 \vec{\phi} = \frac{\partial^2 \vec{\phi}}{\partial t^2}$$

While equation (3) written by Somaiah.K and Sambaiah.K [16] as follows:

$$(6) \qquad (3\tau_1 + 2\tau_2) \nabla^2 \phi_{pp} - (3\sigma_1 + \sigma_2) \phi_{pp} = \frac{\rho J}{2} \frac{\partial^2 \phi_{pp}}{\partial t^2}$$

and

$$(7) \qquad \tau_2 \nabla^2 \phi_{(ij)} - \sigma_2 \phi_{(ij)} = \frac{\rho J}{2} \frac{\partial^2 \phi_{(ij)}}{\partial t^2}$$

The equations(4) and (5) are coupled in $\vec{u}$ and $\vec{\phi}$, while equations (6) and (7) are uncoupled in ϕ_{pp} and $\phi_{(ij)}$,

where

$$(8) \quad c_1^2 = \frac{\mu + K}{\rho}; c_2^2 = \frac{\lambda + \mu}{\rho}; \omega_0^2 = \frac{K}{\rho}; c_4^2 = \frac{\gamma}{\rho J}; c_5^2 = \frac{\alpha + \beta}{\rho J}; c_6^2 = \frac{K}{\rho J} = \frac{\omega_0^2}{J}; C_7^2 = \frac{2K}{\rho J} = \frac{2\omega_0^2}{J}$$

and ∇^2 is Laplacian.

For plane harmonic solution in the positive direction of unit vector $\hat{n}$, we may seek the solution of equations (4) to(7) as,

$$(9) \qquad [\vec{u}, \vec{\phi}, \phi_{pp}, \phi_{(ij)}] = [\vec{A}, \vec{B}, C_{pp}, D_{(ij)}] exp[iq(\hat{n}.\vec{r} - vt)]$$

where $\vec{A}, \vec{B}$ are vector amplitudes, $C_{pp}, D_{(ij)}$ are scalar amplitudes, q is the wave number, $\vec{r}$ is the position vector, v is the phase velocity. Thus,

$$(10) \qquad q = \frac{2\pi}{l}; \vec{r} = x_k i_k; \omega = qv$$

where l is the wave length, x_k are the components of position vector and ω is the angular frequency of the solid.

4. LONGITUDINAL MICRO-ROTATIONAL WAVES

On using eq. (9) into eq. (4) and eq. (5) we obtain,

$$(11) \qquad q^2(c_1^2 - v^2)\vec{A} + c_2^2 q^2 \hat{n}(\hat{n}.\vec{A}) - \omega_0^2 iq(\hat{n} \times \vec{B}) = 0$$

and

$$(12) \qquad [q^2(c_4^2 - v^2) + c_7^2]\vec{B} + c_5^2 q^2 \hat{n}(\hat{n}.\vec{B}) - c_6^2 iq(\hat{n} \times \vec{A}) = 0$$

Taking scalar product of equation (11) with vector $\vec{A}$ we obtain,

$$(13) \qquad q^2(c_1^2 - v^2 + c_2^2)A^2 - \omega_0^2 iq\vec{A} \cdot (\hat{n} \times \vec{B}) = 0$$

where

$$(14) \qquad \vec{A}.\vec{A} = A^2$$

Solving eq. (12) for $\vec{B}$, we get

$$(15) \qquad \vec{B} = \frac{c_6^2 iq}{(c_4^2 + c_5^2 - v^2)q^2 + c_7^2}(\hat{n} \times \vec{A})$$

where

$$(16) \qquad \hat{n}(\hat{n} \cdot \vec{B}) = \vec{B}$$

On using eq.(15) into eq. (13) we obtain,

$$(17) \qquad [(c_1^2 + c_2^2 - v^2)(C_4^2 + c_5^2 - v^2)]q^2 + [(\frac{\omega_0^2}{2} - v^2) + (c_1^2 + c_2^2)]\frac{2\omega_0^2}{J} = 0$$

For plane longitudinal wave, take $q = \frac{\omega}{v}$ in equation(17) we obtain the following quadratic equation in v^2 ;

$$(18) \qquad P(v^2)^2 + Qv^2 + R = 0$$

where

$$(19) \; P = \frac{\omega^2}{\omega_0^2} - \frac{2}{J}, Q = \frac{2(c_1^2 + c_2^2) + \omega_0^2}{J} - (c_1^2 + c_2^2 + c_4^2 + c_5^2)\frac{\omega^2}{\omega_0^2} \; and \; R = (c_1^2 + c_2^2)(c_4^2 + c_5^2)\frac{\omega^2}{\omega_0^2}$$

The roots of equation (18) are given by

$$(20) \qquad v_{r_1}^2 = \frac{1}{2}[\frac{-Q}{P} + \frac{(Q^2 - 4PR)^{\frac{1}{2}}}{P}], v_{r_2}^2 = \frac{1}{2}[\frac{-Q}{P} - \frac{(Q^2 - 4PR)^{\frac{1}{2}}}{P}]$$

Equation(20) represents the speed of two sets longitudinal micro-rotational (MR) waves and they are not encountered in any classical theory of elasticity and they are influenced by micro-rotational parameters α, β, γ. Also they are dispersive in nature. The classical results can be obtained as a particular case if and only if $K \to 0 (i.e., \omega_0^2 = c_6^2 = c_7^2 = 0)$ in equation (17) we obtained the speeds of micro-rotational (MR) waves as,

$$(21) \qquad v_1^2 = \frac{\lambda + 2\mu}{\rho} \text{ or } v_2^2 = \frac{\alpha + \beta + \gamma}{\rho J}$$

and these are non dispersive in nature.

4.1. Particular case. In the absence of micro-rotational parameters (i.e., $\alpha = \beta = \gamma = 0$) in eq. (18), we obtain the speed of longitudinal wave as,

$$(22) \qquad v^2 = (c_1^2 + c_2^2) + [\frac{\omega_0^2}{(\frac{\omega}{\omega_0^2})^2 J - 2}]$$

Equation (22) is known as the speed of micro non rotational(MNR) waves and these waves are also dispersive in nature. The speed of waves shown in eq. (22) approaches the speed v_1^2 in the case of classical result.

5. LONGITUDINAL MICRO-ISOTROPICAL AND MICRO-STRAIN WAVES

On using eqution (9) in the equations (6) and (7), we obtained the speed of micro-isotropical (MI) wave v_p and the speed of micro strain (MS) wave $v_{(ij)}$ are given by

$$(23) \qquad v_p^2 = \frac{2}{\rho J}[(3\tau_1 + 2\tau_2) + (\frac{3\sigma_1 + \sigma_2}{q^2})]$$

and

$$(24) \qquad v_{(ij)}^2 = \frac{2}{\rho J}[\tau_2 + \frac{\sigma_2}{q^2}], \ i = 1, 2; j = 2, 3; i \neq j$$

From equations (23) and (24), we observed that the speed of two distinct micro-isotropical (MI) and two distinct micro-strain (MS) waves are also depends on the wave number, so they are dispersive in nature, and they are influenced by isotropical and strain parameters σ_1, σ_2.

5.1. Particular case. In the absence of isotropical and strain parameters$(i.e, \sigma_1 = 0; \sigma_2 = 0)$, the eq.(6) and eq.(7) are reduces to

$$(3\tau_1 + 2\tau_2)\nabla^2 \phi_{pp} = \frac{\rho J}{2}\frac{\partial^2 \phi_{pp}}{\partial t^2} \tag{25}$$

and

$$\tau_2 \nabla^2 \phi_{(ij)} = \frac{\rho J}{2}\frac{\partial^2 \phi_{(ij)}}{\partial t^2} \tag{26}$$

Equations (25) and (26) represents micro-isotropical shear (MIS) waves and micro-strain shear (MSS) waves respectively, and their speeds v_p^* and $v_{(ij)}^*$ are given by

$$v_p^{*2} = \frac{2(3\tau_1 + 2\tau_2)}{\rho J} \tag{27}$$

and

$$v_{(ij)}^{*2} = \frac{2\tau_2}{\rho J} \tag{28}$$

It is observed that isotropical and strain shear waves are non-dispersive and they are depends on elastic constants.

6. NUMERICAL RESULTS AND DISCUSSION

In order to study the computational results in great detail, we have taken the physical constants for aluminium epoxy material from [17] as: $\lambda = 7.59 \times 10^{10} N/m^2; \mu = 1.89 \times 10^{10} N/m^2;$ $J = 0.000196 m^2; K = 0.0149 \times 10^{10} N/m^2; \rho = 2190 kg/m^3; \gamma = 0.0268 \times 10^{10} N$. Micro-rotational parameters taken from [18] as: $\alpha = 0.036 \times 10^{10} N; \beta = 0.037 \times 10^{10} N$. Due to unavailability of isotropical and strain solids, we take the micro-isotropical and micro-strain parameters as: $\sigma_1 = 0.003 \times 10^3 N; \sigma_2 = 0.0012 \times 10^3 N; \tau_1 = 0.03 \times 10^3 N \text{ and } \tau_2 = 0.062 \times 10^3 N.$

To study the behavior of micro-rotational(MR) waves, we take the non-dimensional material frequencies ω with range $0.1 \times 10^4 \leq \omega \leq 10 \times 10^4$ .The variations of frequency ratios $\frac{\omega^2}{\omega_0^2}$ versus the speed $v_{r_2}^2$ of micro-rotational (MR) wave-A and the speed of $v_{r_2}^2$ micro-rotational wave-B are shown in figures (1) to (3). From comparative figures we observed that the speeds $v_{r_1}^2$ and $v_{r_2}^2$ are symmetric to each other and they are coincide at frequency ratio 1×10^4 . The comparative speeds v^2, $v_{r_1}^2$ and $v_{r_2}^2$ versus frequency ratios $\frac{\omega^2}{\omega_0^2}$ are shown in figures (4) to (6).

From these figures we observed that the MNR waves are fall down to low speed at frequency ratio less than 1×10^4 , and suddenly jumped to high speed at frequency ratio 1×10^4
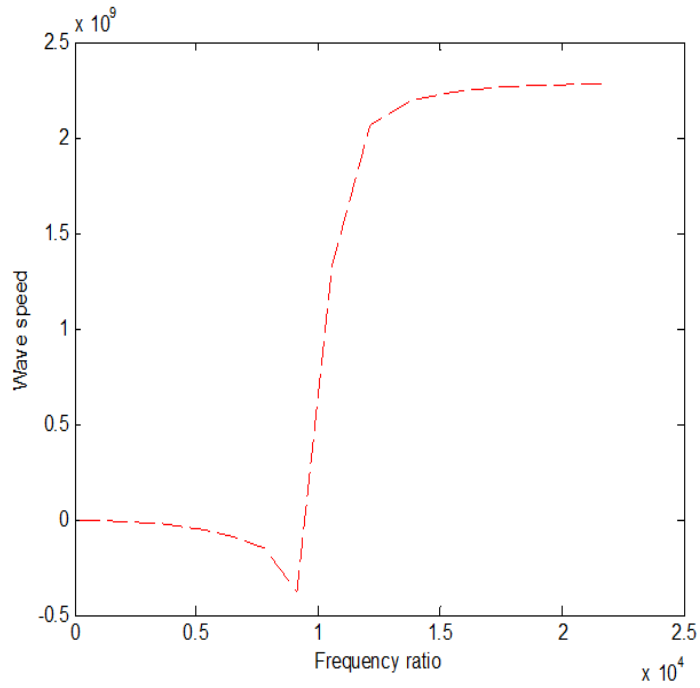

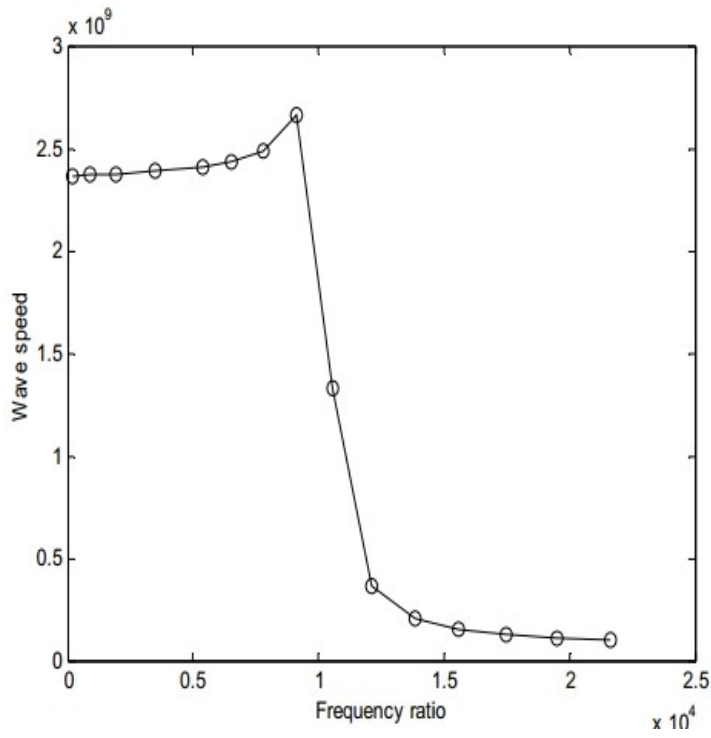FIGURE 1. Variation of frequency ratio versus wave speed

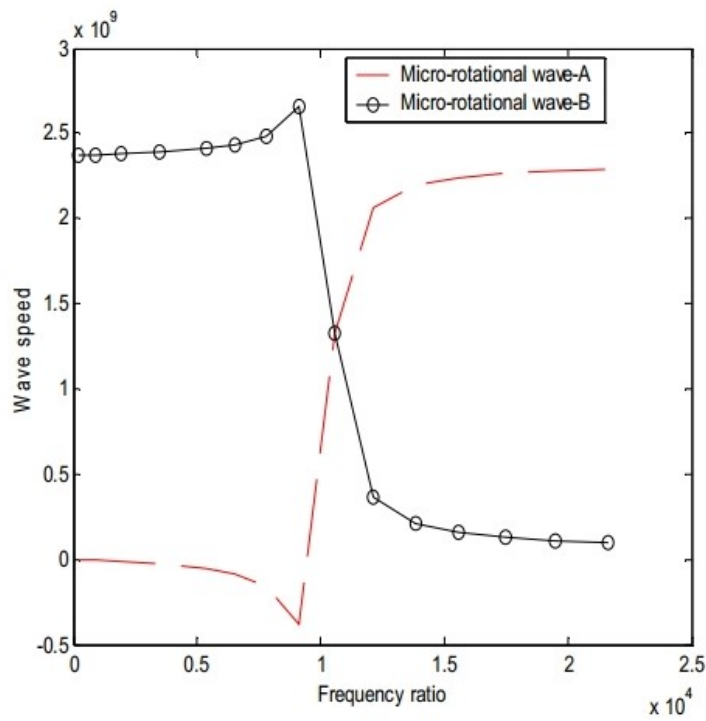FIGURE 2. Variation of frequency ratio versus wave speed

FIGURE 3. Variation of frequency ratio versus wave speed

The speed of micro-isotropical (MI) waves and micro-strain (MS) waves versus nondimensional wave number q with ratio $0.1 \leq q \leq 10$ are shown in figure (7), and we observed that micro-strain (MS) waves are slower than the micro-isotropical (MI) waves in the given range of wave number.
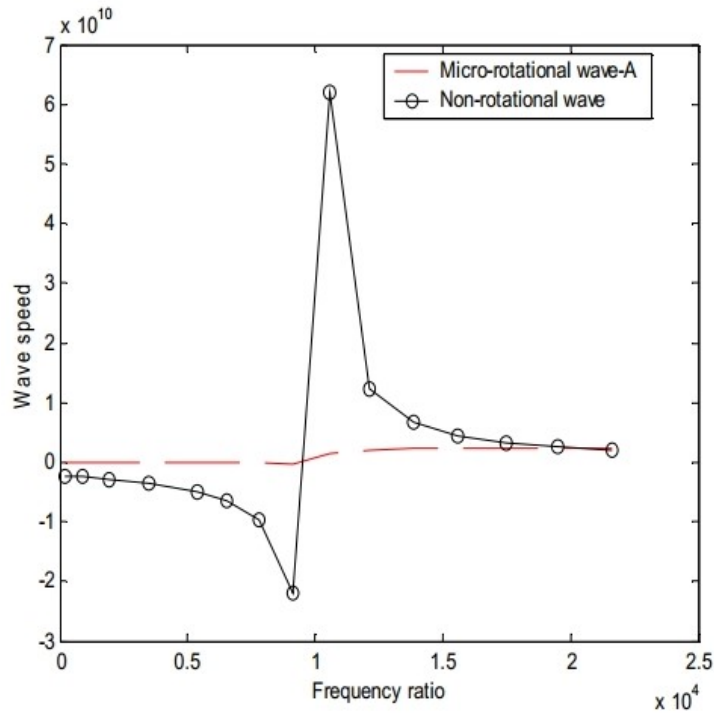

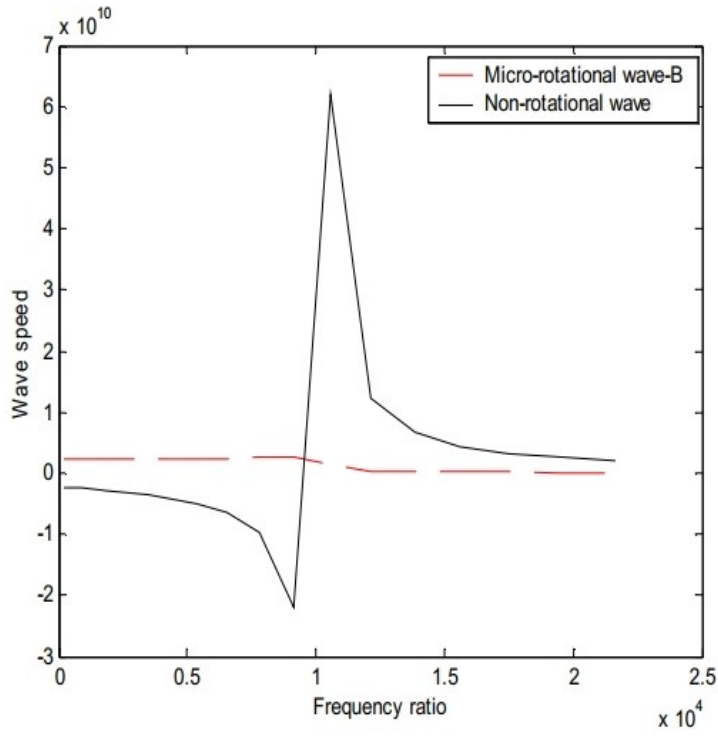FIGURE 4. Variation of frequency ratio versus wave speed

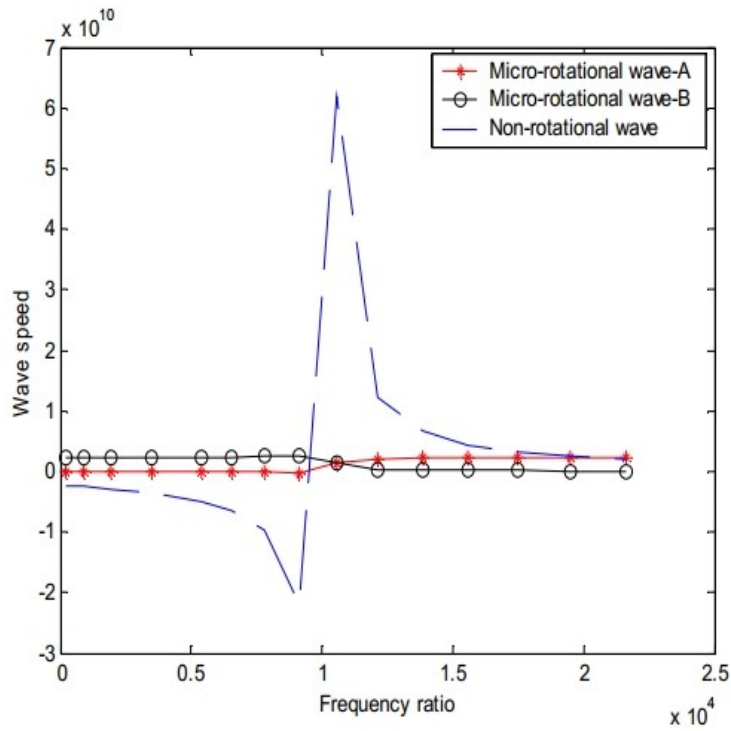FIGURE 5. Variation of frequency ratio versus wave speed

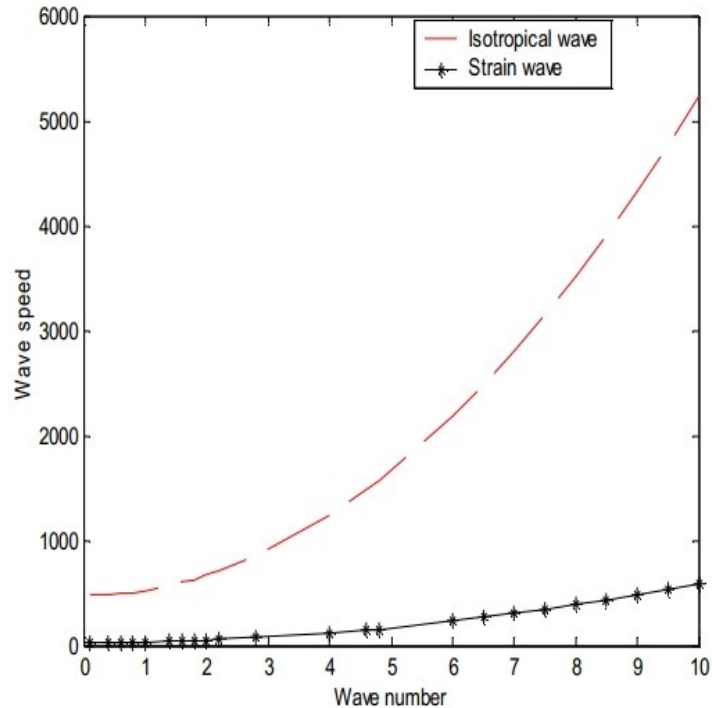FIGURE 6. Variation of frequency ratio versus wave speed

FIGURE 7. Variation of frequency ratio versus wave speed

7. CONCLUSIONS

Under the theoretical computations and numerical examples of plane longitudinal waves in a micro-isotropic, micro-elastic solid, we concluded that:

- Two sets of MR waves and one set of MI and MS waves are propagate with distinct speeds.
- MR, MNR, MI and MS waves are dispersive in nature, while MIS and MSS waves are non-dispersive.
- MIS and MSS waves are constant and they are only elastic constant dependent waves.
- MS waves slower than MI waves.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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