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ON CONJECTURE 2 OF NADA AND ZOHNY FROM THE PERSPECTIVE OF BITOPOLOGICAL DYNAMICAL SYSTEMS

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Abstract. Bitopological dynamical system is a recently explored area of dynamical system to investigate dynamical properties in terms of a bitopological space. Nada and Zohny [S.I. Nada, H. Zohny, An application of relative topology in biology, Chaos, Solitons and Fractals. 42 (2009), 202–204] explored the use of topological dynamical system in the development of an organism from zygote until birth and they made three conjectures regarding the development of child growth from zygote to till its birth. In this paper, we disprove the conjecture 2 of Nada and Zohny [S.I. Nada, H. Zohny, An application of relative topology in biology, Chaos, Solitons and Fractals. 42 (2009), 202–204] by applying some mathematical results from bitopological dynamical system, which was recently introduced by Acharjee et al. [S. Acharjee, K. Goswami, H.K. Sarmah, Transitive map in bitopological dynamical systems, (communicated)] with medical evidences. Also, we introduce forward iterated connected space, backward iterated connected space, pairwise iterated connected space and establish some of their relations with pairwise connectedness in bitopological dynamical system. We show that during the development of an organism from zygote until birth, the developing stage after gastrulation is pairwise disconnected and forward iterated disconnected.

Keywords: bitopological dynamical system; pairwise connected; forward iterated connected; embryo; mitosis; gene mutation.

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1. INTRODUCTION

Bitopological dynamical system is a recently explored area of dynamical system to investigate dynamical properties in terms of a bitopological space. Because of its applications in human embryo development, it shows the potential to be a promising area of dynamical system. Bitopological dynamical system, introduced by Acharjee et al. [1], overcomes the limitations of topological dynamical system. In [1], authors disproved the conjecture 1 of Nada and Zohny [2] by using results of bitopological dynamical system and introduced some new notions like transitive map, pairwise iterated compactness, etc. Moreover, in another paper [3], they introduced forward iterated Hausdorffness and showed that the developing stage after gastrulation is forward iterated Hausdorff in the development of an organism from zygote until birth. It motivated us for this paper on viewing the possibilities to use bitopological dynamical system to study the developmental anomalies of human brain.

Kelly [4] introduced the concept of bitopological space. Later, bitopological space was extensively studied by many researchers of various branches and the study is still going on. For recent theoretical works in bitopological space, one may refer to Acharjee and Tripathy [5], Acharjee et al. [6], Acharjee et al. [7] and many others. Recently, bitopological space has been applied in many areas of science and social science, viz. medical science [8], economics ([9], [10]), computer science [11], human embryo development ([1], [3]), etc. Moreover, entropy of pairwise continuous map in bitopological dynamical system was recently introduced by Acharjee et al. [29].

Reilly [12] introduced the concept of pairwise connectedness in a bitopological space. After that many researchers studied the concept of pairwise connectedness because of its fundamental importance in the theory of bitopological space. Recently, Mishra and Singh [13] introduced the concept of pairwise connectedness for hyperspace of a bitopological space (briefly, called bihypertopological space).

Mutation is a change in the DNA sequence of a gene. Many researchers ([14], [15], [16]) have studied the mutations of certain genes as causes of various anomalies of brain development.

This paper is divided into three sections. In the preliminary section, we recall some existing definitions of bitopological space as well as bitopological dynamical system. In the next section,

we introduce forward iterated connected space, backward iterated connected space and pairwise iterated connected space; which generalize the notion of pairwise connectedness in a bitopological dynamical system. Also, we establish some relationships among these generalizations. Then, we show that the developing stage after gastrulation is forward iterated disconnected in the development of an organism from zygote until birth. Finally, in the last section we disprove the conjecture 2 of Nada and Zohny [2] by using the concept of pairwise Hausdorffness and gene mutation. Our theory is supported by medical evidences.

2. PRELIMINARY DEFINITIONS

In this section, we recall some existing definitions of bitopological space and bitopological dynamical system.

Definition 2.1.[4] A quasi-pseudo-metric on a set X is a non-negative real-valued function $p(,)$ on the product $X \times X$ such that

- (i) $p(x, x) = 0$, where $x \in X$;
- (ii) $p(x, z) \leq p(x, y) + p(y, z)$, where $x, y, z \in X$.

Definition 2.2.[4] Let $p(,)$ be a quasi-pseudo-metric on X , and let $q(,)$ be defined by $q(x, y) = p(y, x)$, where $x, y \in X$. Then, $q(,)$ is also a quasi-pseudo-metric on X . We say that $p(,)$ and $q(,)$ are conjugate, and denote the set X with the structure by (X, p, q) .

If $p(,)$ is a quasi-pseudo-metric on a set X , then the open p -sphere with centre x and radius $\varepsilon > 0$ is the set $S_p(x, \varepsilon) = \{y : p(x, y) < \varepsilon\}$. The collection of all open p -spheres forms a base for a topology. Similarly, $q(,)$ determines a topology for X . We shall denote the topology determined by $p(,)$ by τ_1 and the topology that of $q(,)$ by τ_2 .

Definition 2.3.[4] A space X on which are defined two (arbitrary) topologies τ_1 and τ_2 is called a bitopological space and denoted by (X, τ_1, τ_2) .

Definition 2.4.[12] A function f from a bitopological space (X, τ_1, τ_2) into a bitopological space (Y, ψ_1, ψ_2) is said to be pairwise continuous (respectively, a pairwise homeomorphism) iff the induced functions $f : (X, \tau_1) \rightarrow (Y, \psi_1)$ and $f : (X, \tau_2) \rightarrow (Y, \psi_2)$ are continuous (respectively, homeomorphisms).

Pervin [17] called this a continuous map. However, we call this as pairwise continuous map, due to Reilly [12].

Definition 2.5.[12] A bitopological space (X, τ_1, τ_2) is pairwise connected iff X cannot be expressed as the union of two non-empty disjoint sets A and B such that $(A \cap \tau_1\text{-cl}B) \cup (\tau_2\text{-cl}A \cap B) = \phi$; where $\tau_1\text{-cl}A$ denotes the τ_1 -closure of the set A . If X can be so expressed, we write $X = A|B$ and this is a separation of X , which is then pairwise disconnected.

Pervin [17] called this as connectedness. However, we call this as pairwise connectedness, due to Reilly [12].

Definition 2.6.[4] A bitopological space (X, τ_1, τ_2) is said to be pairwise Hausdorff if, for each two distinct points x and y , there are a τ_1 -neighbourhood U of x and a τ_2 -neighbourhood V of y such that $U \cap V = \phi$.

Definition 2.7.[18] A set $A \subset X$ is +invariant when $f(A) \subset A$ and A is –invariant when $A \subset f(A)$. A is called invariant when $f(A) = A$.

Definition 2.8.[3] A map $f : X \rightarrow X$ is called +invariant if for all $A \subset X$, $f(A) \subset A$ and –invariant when $A \subset f(A)$. The map f is invariant when $f(A) = A$, for all $A \subset X$.

In [1], we considered \mathbb{N} , \mathbb{Z} and \mathbb{R} as the set of non-negative integers, the set of integers and the set of real numbers, respectively.

Definition 2.9.[1] Let (X, τ_1, τ_2) be a bitopological space. A bitopological dynamical system is a pair (X, f) , where (X, τ_1, τ_2) is a bitopological space and $f : X \rightarrow X$ is a pairwise continuous map. The dynamics is obtained by iterating the map.

The forward orbit of a point $x \in X$ under f is defined as $O_+(x) = \{f^n(x) : n \in \mathbb{N}\}$, where f^n denotes the n^{th} iteration of the map f . If f is a homeomorphism, then the backward orbit of x is the set $O_-(x) = \{f^{-n}(x) : n \in \mathbb{N}\}$ and the full orbit of x (or simply orbit of x) is the set $O(x) = \{f^n(x) : n \in \mathbb{Z}\}$.

Definition 2.10.[3] Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \rightarrow X$ is a pairwise continuous map. We call (X, f) as (m, n) -forward iterated Hausdorff if for each two distinct points x and y of X , there exist $m, n \in \mathbb{N}$, $U \in \tau_1$ and $V \in \tau_2$, such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$.

Theorem 2.1.[17] The following conditions are equivalent for any bitopological space (X, τ_1, τ_2) :

- (a) (X, τ_1, τ_2) is connected,
- (b) X cannot be expressed as the union of two non-empty disjoint sets A and B such that A is

τ_1 -open and B is τ_2 -open (hence A is τ_2 -closed and B is τ_1 -closed),

(c) X contains no non-empty proper subset which is both τ_1 -open and τ_2 -closed (hence none which is τ_1 -closed and τ_2 -open).

3. MAIN RESULTS

In this section, our main aim is to introduce forward iterated connectedness, backward iterated connectedness, pairwise iterated connectedness and some related results. Moreover, we show that the developing stage after gastrulation is forward iterated disconnected in the development of an embryo from zygote until birth.

Let \mathbb{N} , \mathbb{Z} and \mathbb{R} denote the set of non-negative integers, the set of integers and the set of real numbers, respectively.

Pairwise connectedness is a basic concept in bitopological space. So, in next few definitions we try to define some generalized versions of pairwise connectedness in bitopological dynamical system with respect to iteration.

Definition 3.1. Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \rightarrow X$ is a pairwise continuous map. We call (X, f) as forward iterated connected if for all $m, n \in \mathbb{N}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$; $X \neq f^m(U) \cup f^n(V)$ and $f^m(U) \cap f^n(V) = \phi$.

If there exist $m, n \in \mathbb{N}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$, such that $X = f^m(U) \cup f^n(V)$ and $f^m(U) \cap f^n(V) = \phi$, then we call (X, f) as (m, n) -forward iterated disconnected.

Definition 3.2. Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \rightarrow X$ is a pairwise continuous map. We call (X, f) as backward iterated connected if for all $m, n \in \mathbb{N}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$; $X \neq f^{-m}(U) \cup f^{-n}(V)$ and $f^{-m}(U) \cap f^{-n}(V) = \phi$.

If there exist $m, n \in \mathbb{N}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$, such that $X = f^{-m}(U) \cup f^{-n}(V)$ and $f^{-m}(U) \cap f^{-n}(V) = \phi$, then we call (X, f) as (m, n) -backward iterated disconnected.

Definition 3.3. Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \rightarrow X$ is a pairwise continuous map. We call (X, f) as pairwise iterated connected if for all $m, n \in \mathbb{Z}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$; $X \neq f^m(U) \cup f^n(V)$ and $f^m(U) \cap f^n(V) = \phi$.

If there exist $m, n \in \mathbb{Z}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$, such that $X = f^m(U) \cup f^n(V)$ and $f^m(U) \cap f^n(V) = \phi$, then we call (X, f) as (m, n) -pairwise iterated disconnected.

The following theorem establishes the relation between pairwise connectedness, forward iterated connectedness, backward iterated connectedness and pairwise iterated connectedness without considering any extra condition.

Theorem 3.1. Let (X, f) be a bitopological dynamical system. If (X, f) is forward iterated connected or backward iterated connected or pairwise iterated connected, then X is pairwise connected.

Now, we get the following relation between pairwise connectedness and forward iterated connectedness in case of a +invariant map.

Theorem 3.2. Let (X, f) be a bitopological dynamical system, where X is pairwise connected. If f is pairwise open and +invariant, then (X, f) is forward iterated connected.

Proof: Since X is pairwise connected, so X cannot be expressed as the union of two non-empty disjoint sets U and V such that U is τ_1 -open and V is τ_2 -open. Let us consider $m, n \in \mathbb{N}$ be arbitrary. Since f is pairwise open, so $f^m(U) = U_1 \in \tau_1$ and $f^n(V) = V_1 \in \tau_2$. Also, $f^m(U) = U_1 \subset U$ and $f^n(V) = V_1 \subset V$ as f is +invariant. Again, $f^m(U) \cup f^n(V) \subset U \cup V \neq X$ and $f^m(U) \cap f^n(V) \subset U \cap V = \phi$. Thus, for all $m, n \in \mathbb{N}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$; $X \neq f^m(U) \cup f^n(V)$ and $f^m(U) \cap f^n(V) = \phi$. Hence, (X, f) is forward iterated connected.

For a –invariant map, we get the following result.

Theorem 3.3. Let (X, f) be a bitopological dynamical system, where X is pairwise connected. If f is –invariant, then (X, f) is backward iterated connected.

Proof: The pairwise connectedness of the bitopological space X implies that X cannot be expressed as the union of two non-empty disjoint sets U and V such that U is τ_1 -open and V is τ_2 -open. Let us consider $m, n \in \mathbb{N}$ be arbitrary. Now, the pairwise continuity of the map f implies that $f^{-m}(U) = U_1 \in \tau_1$ and $f^{-n}(V) = V_1 \in \tau_2$. Also, $f^{-m}(U) = U_1 \subset U$ and $f^{-n}(V) = V_1 \subset V$ as f is –invariant. Again, $f^{-m}(U) \cup f^{-n}(V) \subset U \cup V \neq X$ and $f^{-m}(U) \cap f^{-n}(V) \subset U \cap V = \phi$. Thus, for all $m, n \in \mathbb{N}$, $U (\neq \phi) \in \tau_1$ and $V (\neq \phi) \in \tau_2$; $X \neq f^{-m}(U) \cup f^{-n}(V)$ and $f^{-m}(U) \cap f^{-n}(V) = \phi$. Hence, (X, f) is backward iterated connected.

Theorem 3.4. Let (X, f) be a bitopological dynamical system, where X is pairwise connected. If f is invariant, then (X, f) is pairwise iterated connected.

Proof: Since X is pairwise connected, so X cannot be expressed as the union of two non-empty disjoint sets U and V such that U is τ_1 -open and V is τ_2 -open. Let us consider $m, n \in \mathbb{Z}$ be arbitrary. Now, $f^m(U) = U$ and $f^n(V) = V$ as f is invariant. Again, $f^m(U) \cup f^n(V) = U \cup V \neq X$ and $f^m(U) \cap f^n(V) = U \cap V = \phi$. Thus, for all $m, n \in \mathbb{Z}$, $U(\neq \phi) \in \tau_1$ and $V(\neq \phi) \in \tau_2$; $X \neq f^m(U) \cup f^n(V)$ and $f^m(U) \cap f^n(V) = \phi$. Hence, (X, f) is pairwise iterated connected.

Now, we recall some biological terms and try to use some of the above results in the growth process of an embryo from zygote until birth.

Definition 3.4.[19] Mitosis is the process whereby one cell divides giving rise to two daughter cells each with 46 chromosomes.

Definition 3.5.[19] Gastrulation is the process of forming the three primary germ layers from the epiblast involving movement of cells through the primitive streak to form endoderm and mesoderm.

We consider the bitopological space (R, τ_1, τ_2) as defined by Acharjee et al. [1]; where:

$$\tau_1 = \{(\phi, \tau_1(t_0)), (U_1, \tau_1(t_1)), (U_2, \tau_1(t_2)), \dots, (U_m, \tau_1(t_m)), (R, \tau_1(T))\} \text{ and}$$

$$\tau_2 = \{(\phi, \tau_2(t_0)), (V_1, \tau_2(t_1)), (V_2, \tau_2(t_2)), \dots, (V_n, \tau_2(t_n)), (R, \tau_2(T))\}.$$

Here; $\phi = Z = U_0 = V_0$, $U_m = X$ is the brain together with central nervous system of the whole organism and $V_n = Y$ is the other body parts of the whole organism except the brain and the central nervous system. Also, U_1, U_2, \dots represent different development stages of the brain and the central nervous system; and V_1, V_2, \dots represent different development stages of the other body parts except the brain and the central nervous system. Here, t_0 is the time of fertilization and T is the time of birth. It is important to note that before gastrulation, we have $U_i = V_i$. At the end; X and Y together form the whole organism, the baby R , i.e. $X \cup Y = R$. Figure 1. represents it and it is due to Acharjee et al. [1]. Figure 1. is the modified version of the given figure by Nada and Zohny [2].

Acharjee et al. [3] showed that (R, h) is a bitopological dynamical system, where the map $h : R \rightarrow R$ (it is the map that represents mitosis process) is defined by

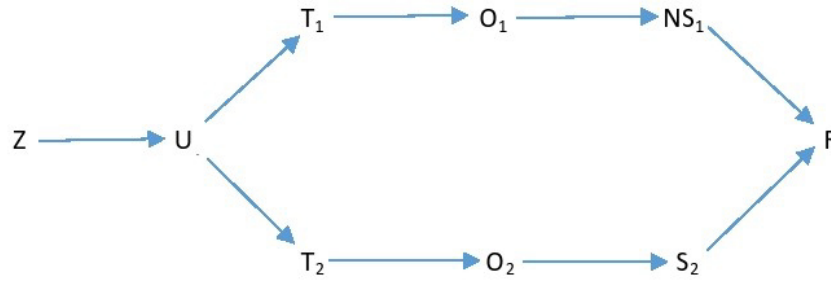


FIGURE 1. Here, Z- the zygote, U- development of the zygote just before gastrulation, T₁- Neural tissues, T₂- Non-neural tissues, O₁- Neural organs, O₂- Non-neural organs, NS₁- Neural organ systems, S₂- Non-neural organ systems and R- the baby at the time of birth.

$$h(x_i^j) = \{x_i^{j1}, x_i^{j2}\},$$

where x_i^j is the mother cell, x_i^{j1} and x_i^{j2} are the daughter cells.

Let R^* be the postgastrulation part of the whole organism R , τ_1^* and τ_2^* are the relative topologies on R^* and h^* is the restriction of the map h on R^* i.e. $h^* = h|_{R^*}$. Then according to [3], $(R^*, \tau_1^*, \tau_2^*)$ is a subspace of (R, τ_1, τ_2) and the mapping $h^* : (R^*, \tau_1^*, \tau_2^*) \rightarrow (R^*, \tau_1^*, \tau_2^*)$ is a pairwise open map. In this section, we mainly focus on the bitopological dynamical system (R^*, h^*) .

Now, $U_m = X \in \tau_1$ and $V_n = Y \in \tau_2$ are non-empty τ_1 -open set and τ_2 -open set respectively. Therefore, $X^* = R^* \cap X$ and $Y^* = R^* \cap Y$ are non-empty τ_1^* -open set and τ_2^* -open set respectively. Also,

$$\begin{aligned} X^* \cup Y^* &= (R^* \cap X) \cup (R^* \cap Y) \\ &= R^* \cap (X \cup Y) \\ &= R^* \cap R \\ &= R^* \end{aligned}$$

Again, the set X^* and Y^* are disjoint; since X^* consists of brain and central nervous system cells whereas Y^* consists of cells from other body parts except the brain and central nervous system. Thus, R^* is expressed as the union of two non-empty disjoint sets X^* and Y^* . Hence, by theorem 2.1, the bitopological space $(R^*, \tau_1^*, \tau_2^*)$ is pairwise disconnected.

Now, let $U^*, V^* \subset R^*$ be two developing stages of the embryo just after gastrulation such that $U^* \in \tau_1^*$ and $V^* \in \tau_2^*$. After a certain time, U^* will develop into X^* and V^* will develop into Y^* . Let us consider $a, b \in \mathbb{N}$ be such that $(h^*)^a(U^*) = X^*$ and $(h^*)^b(V^*) = Y^*$. Also, the set X^* and Y^* are disjoint due to the above paragraph. Thus, there exists $a, b \in \mathbb{N}$, $U^* \in \tau_1^*$ and $V^* \in \tau_2^*$ such that $R^* = (h^*)^a(U^*) \cup (h^*)^b(V^*)$ and $(h^*)^a(U^*) \cap (h^*)^b(V^*) = \emptyset$. Hence, the bitopological dynamical system (R^*, h^*) is forward iterated disconnected.

4. ON CONJECTURE 2 OF NADA AND ZOHNY

In conjecture 2 of [2], Nada and Zohny conjectured that a medical treatment should be started to stop cognitive anomalies at any step of growth based on the properties of local topological subspaces for the dynamical topology. In this section, we disprove their conjecture by using some new notions of Acharjee et al. [3] with medical evidences.

First we recall some biological terms.

Definition 4.1.[20] Cell cycle refers to the ordered series of events that lead to cell division and the production of two daughter cells, each containing chromosomes identical to those of the parent cell. During each cycle, DNA replication leads to the creation of two identical DNA molecules, which are compacted and structured for their segregation into daughter cells. The cell cycle is divided into four major phases. Cycling (replicating) mammalian somatic cells grow in size and synthesize the RNAs and proteins required for DNA synthesis during the G1 (first gap) phase. When cells reach the appropriate size and synthesize the required proteins, they enter the cell cycle by traversing a point in G1 known as START in yeast and the restriction point in mammals. Once this point is crossed, cells are committed to cell division. The first step towards successful cell division is to enter into the S (synthesis) phase, the period in which cells actively replicate their chromosomes. After progressing through a second gap phase, the G2 phase, cells begin the complicated process of mitosis, also called the M (mitotic) phase, which is divided into several stages.

Definition 4.2.[21] Mutation is a change in the DNA sequence of a gene. Gene mutations can arise spontaneously or they can be induced. Spontaneous mutations are naturally occurring mutations and arise in all cells. Induced mutations arise through the action of certain agents called mutagens that increase the rate at which mutations occur. Spontaneous mutations arise from a

variety of sources. One source is the DNA replication process. Although DNA replication is a remarkably accurate process, mistakes are made in the copying of the millions, even billions, of base pairs in a genome. Spontaneous mutations also arise in part because DNA is a very labile molecule and the cellular environment itself can damage it.

Definition 4.3.[22] Mental Retardation (MR) is significantly subaverage general intellectual functioning (Criterion A) that is accompanied by significant limitations in adaptive functioning in at least two of the following skill areas: communication, self-care, home living, social/interpersonal skills, use of community resources, self-direction, functional academic skills, work, leisure, health, and safety (Criterion B). The onset must occur before age 18 years (Criterion C). Mental Retardation has many different etiologies and may be seen as a final common pathway of various pathological processes that affect the functioning of the central nervous system.

According to [23], narrow definitions of MR restrict it to cases of non-progressive cognitive impairment detectable early after birth. Schaefer and Bodensteiner [24] found that a significant group of patients with MR have readily observable malformations of the brain. These malformations have in common the disruption of primary organizational processes in brain formation that occur very early in embryogenesis. Two of the most common anomalies listed by Schaefer and Bodensteiner [24] are Holoprosencephaly and Lissencephaly.

Definition 4.4.[19] Holoprosencephaly is a developmental defect where so much midline tissue for the face and brain has been lost that the two lateral ventricles fuse together and appear as one.

Definition 4.5.[25] Classical lissencephaly (previously type I) or generalized agyria-pachygyria is a severe brain malformation manifested by a smooth cerebral surface, abnormally thick cortex with four abnormal layers, widespread neuronal heterotopia, enlarged ventricles, and often agenesis or malformation of the corpus callosum.

Recently, Roessler et al. [16] found that mutations in the human Sonic Hedgehog gene cause Holoprosencephaly. Also, Lo Nigro et al. [15] and Reiner et al. [14] showed that classical or type I lissencephaly results from sporadic mutations in the LIS1 gene.

According to Oxford Dictionary of English [26], anomaly is something that deviates from what is standard, normal or expected. Thus, cognitive anomaly is synonymous with cognitive impairment.

Let x be a brain cell or central nervous system cell. Suppose, during the cell cycle of the cell x ; a gene is mutated. The mitosis of this cell will produce two daughter cells y and z which are defected (here, by a defected cell we mean a cell that contains atleast one mutated gene). Thus, in this way a defected cell will lead to a group of defected cells ultimately leading to cognitive anomalies [14-16]. Acharjee et al. [3] showed that the bitopological space $(R^*, \tau_1^*, \tau_2^*)$ is pairwise Hausdorff and therefore there is no structural relationship between brain cells together with central nervous system cells and cells from other body parts. Thus, to cure cognitive anomalies, medical treatments should be started on the brain cells or on the central nervous system cells. In this case, medical treatments on body cells will not cure cognitive anomalies as there is no structural relationship between brain cells together with central nervous system cells and cells from other body parts. Similar argument follows for defected body cells.

Thus, to cure cognitive anomaly, medical treatments should be done in the local bitopological subspace $(S^*, \tau_1^*|_{S^*}, \tau_2^*|_{S^*})$, where $S^* = U \cup V \subset R^*$, U is a developing stage of brain and central nervous system and V is a developing stage of other body parts without brain and central nervous system and the treatments must be particularly done in U . In this case, there may not be any role of the developing stage of V . Thus, we disprove conjecture 2 of Nada and Zohny [2], in the sense that to cure cognitive anomalies; medical treatments should be done on local bitopological subspace; not on local topological subspace as conjectured by Nada and Zohny [2].

Although, in the previous paragraph we claimed that to cure cognitive anomalies; medical treatments should be started on the brain cells or on the central nervous system cells, yet in some recent papers [30, 31] researchers have developed methods that can convert adult skin cells directly to brain cells by reprogramming the cells in postnatal stages. These cells functionally integrate into the local circuit of brain cells and thus these methods become promising areas of treating brain diseases. But it is interesting to note that to convert a skin cell to a brain cell, the cell is needed to be reprogrammed. These experiments and methods partly verify our theories and claims. Since, biological science is a rapidly growing area, thus we do not nullify

the possibilities of uses of reprogrammed cells as brain cells in prenatal stages to treat cognitive anomalies. Mostly, cognitive anomalies can be treated by the treatments of brain cells or central nervous system cells. We hope that our theoretical claims may be found to be true in future since Optogenetics [27], a new brainstorming research area in neuroscience, uses lights on neurons to study various brain activities for Parkinson's disease.

5. OPEN QUESTION

In this section, we state a question which is open to the readers of this paper.

Q. Is there any example that pairwise connectedness of a bitopological space (X, τ_1, τ_2) does not imply forward iterated connectedness or backward iterated connectedness or pairwise iterated connectedness in a bitopological dynamical system (X, f) ?

6. CONCLUSION

In this paper, we introduced the concepts of forward iterated connectedness, backward iterated connectedness and pairwise iterated connectedness in a bitopological dynamical system and we established some relationships with pairwise connectedness. Also, we proved that the developing stage after gastrulation is forward iterated disconnected in the development of a human embryo from the zygote. Later, we disproved the conjecture 2 of Nada and Zohny [2] by using the concept of pairwise Hausdorffness together with medical evidences of gene mutation. According to Adolphs [28], the cure of psychiatric and neurological diseases is one of the unsolved problems of neuroscience. In this case, in near future our theories may become the backbone of treatments for developmental anomalies of human brain. To make this a reality, we need deep research in the area of human embryo development process through the process of mitosis and bitopological dynamical system.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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