DETERMINATION OF THERMAL CONDUCTIVITY OF HOLLOW GLASS MICROSPHERES (HGM) USING GRAPH THEORY AND MATRIX APPROACH

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Abstract. The method named "Graph Theory and Matrix Approach" has been well used in numerous research studies to carry out decision-making when the problem becomes perplexed form or when the relative importance of one parameter over another is quite high. In such instances, the said Theory of graphical and matrix approach offers very suitable and fruitful solutions to efficiently render the decision. Using combined application of theoretical graph method findings coupled with certain artificial intelligence-inspired logics and practises like fuzzy logic, artificial neural network, etc., more developments and outcome enhancement can also be disclosed. Such method's relevance and applicability in large fields of technology, engineering, and analysis are also proven. Today, our industrial industries are upgrading by artificial intelligence applications and other software-based directions. In this research, graph theory is used to test hollow glass microspheres (HGM) performance by evaluating thermal conductivity based on pressure, evaporation rate and heat leakage. 16 Tests were conducted with four samples A, B, C and D with four different pressure variations.

Keywords: graph theory approach; diagraph; variable permanent matrix; thermal conductivity; evaporation rate; heat leak.

2010 AMS Subject Classification: 94C15.

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1. INTRODUCTION

Hollow glass spheres are a kind of particles in the air substance that has seen large applications for low density, high thermal resistance, and good wear resistance in paint and coating systems. It can be anticipated that if the hollow glass spheres were covered with a sheet of magnetic metal, broader usage will be formed in some areas, such as immunoassay, catalyst and magnetorheological fluids, as they incorporate the properties of hollow glass spheres and metal together(1). Several deposition processes may be preferred to protect hollow glass spheres, such as physical and chemical vapour deposition, sputtering or electroless plating. Electroless putting is a favoured approach for its above-described benefits. But we know that coating hollow glass microspheres with metal through electroless plating is rarely mentioned. The potential explanation for this may be that owing to their microspheric forms and very low density, there are certain difficulties receiving active care on hollow glass microspheres(2).
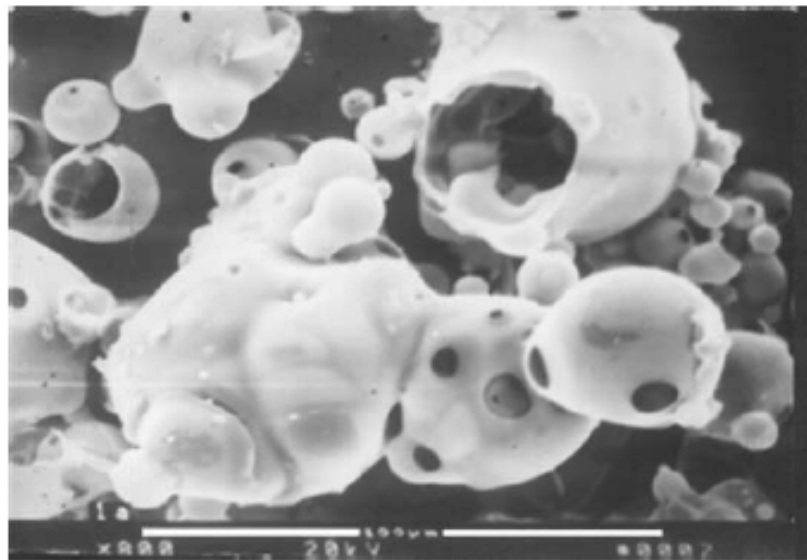

FIGURE 1. Hollow glass spheres etched by HF solution

HGM was produced from high-silica (P70 mol percent) glasses, but lithium aluminoborate glasses provide unique radiation shielding features. The existence of 6 Li and 10B (7.5% and 19.9% natural abundance respectively in the glass can help capture high-energy fragmentation

neutrons in conjunction with previous findings on boronated or lithiated polyethylene(3). Additionally, such glasses can be doped with elemental isotopes featuring large thermal neutron-capture cross-sections, such as Sm or Gd, for improved capture of high-energy neutrons.

Human behaviour was mimicked successfully and sewn by artificial intelligence models. Artificial neural networks, evolutionary computation, fuzzy logic, probabilistic analytical structures, smart agents, etc. are several major resources that usually describe the simple artificial intelligence framework. There are too many AI strategies that have been investigated in previous studies for thermal conductivity measurement purposes. These methodologies are data pattern analysis, data distribution coordination, forecasting, data accuracy creation, uncertainty quantification, etc. The well-known decision-making technique, named graph theory, is a comprehensive and analytical methodology that has often proved effective in evaluating and modelling a broad variety of engineering applications and many other fields. This approach is especially focused on advanced graph theory, and hence its implementations are well established. Demonstration by graph "or" directed graph model has proved useful for modelling and evaluating various device varieties and complications in various engineering and technology fields (4). The matrix-based method is helpful in evaluating graph / directed graph models easily to extract machine feature and guide to accomplish the objective. Graph theory is a topic emerged from combinatorial mathematics, drawing much from matrix concept. Graph representation matrix moulds the issue of using computers for different dynamic operations (5). Methods of GTMM (graph theory and matrix) comprises the representations of directed graph, matrix and mathematics, i.e., permanent function. The directed graph is the optical depiction and interdependency of variable quantity. The matrix transforms the directed graph into a graphical shape, and the permanent function is a mathematical representation that helps to define the index.

The method of graph theory is a systematic technique for converting qualitative variables into quantitative quantities, and statistical modelling offers an advantage over conventional methods such as cause-effect diagrams, flow maps, etc. This process results can now be finalized in a range of artificial intelligence-based approaches and initiatives intended to create more competitive and smarter solutions (6). Specifically, the fuzzy logic-based method may be further applied to use individual thoughts on the subject under consideration. Logic generation will
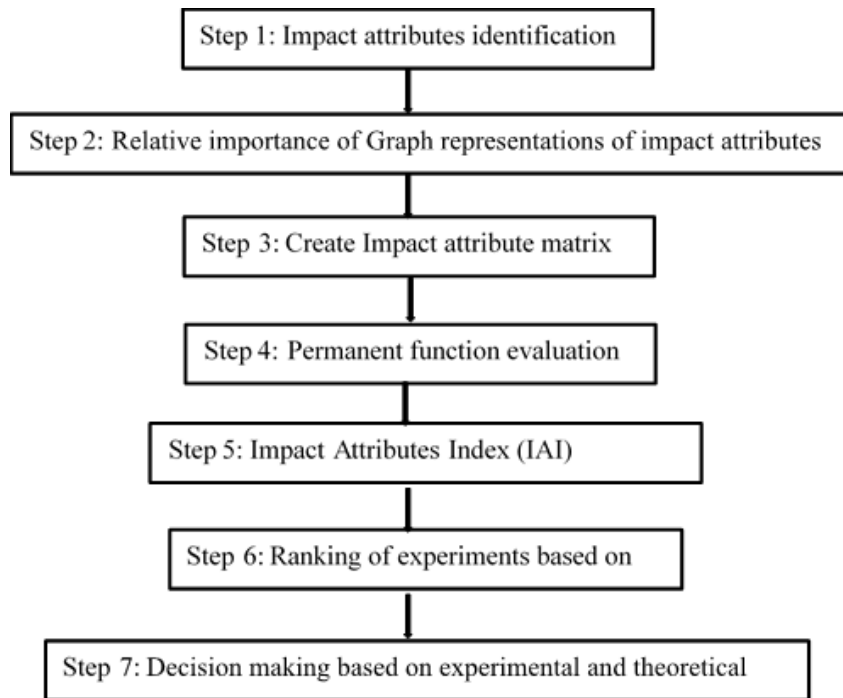
FIGURE 2. Flowchart of GMA (Graph Matrix Approach)

rely on pure human experiences and would aim to bring a more logical and realistic approach to artificial intelligence-based graph theory (7). The AI, generally described as the discipline of designing machines that do things that require human astuteness, is often perplexed by robotics, in particular humanoid robots, as they are contiguous to "human" intelligence. Graph theory has many uses in architecture and many other areas (8). The basic steps for the above technique are stated in flowchart.

2. ATTRIBUTES SELECTION

If each edge is connected with an ordered pair of vertices in graph G = (V, A), G is referred to as a digraph or a directed graph. Performance attributes directed graph signifies performance attributes and their node and edge interrelationship (9). The directed graph consists of a set of nodes N=Ui, i= 1,2, 3, ... M and E=eij. A node Ui represents the parameter measurement and edge eij represents the interdependence of parameters. Total number of nodes, M, is equivalent to output attributes. If a node I has relative value over node j, an edge is drawn from node I to node j (eij). If node j has relative importance over node i an edge is drawn from node j to node i (eji). HGMs (Hollow glass microspheres) were added as effective and low-maintenance

thermal insulation products to the cryogenic thermal insulation device. HGMs' physical properties affect their thermal insulation efficiency, which has been overlooked in recent decades (10). A new system focused on a steady-state liquid nitrogen evaporation rate approach was developed and manufactured to define thermal resisting properties for four forms of Hollow glass microspheres at boundary temperatures of 77 K–293 K and varying cold vacuum pressures. The performance attributes considered in this study are evaporation rate, heat leakage

FIGURE 3. The image showing a device to measure thermal conductivity

and experimental thermal conductivity. In recent decades, cryogenic thermal insulation systems have gained substantial interest due to increasingly evolving cryogenic technology. To satisfy ever-increasing demand for durable and low maintenance of the cryogenic thermal resistance system, highly effective and stabilised cryogenic thermal resistant materials are of significant importance. HGMs (Hollow glass microspheres) with a thin wall (0.5–2.0 lm) and 10–120 lm diameter have exceptional properties such as sphere-shape, manageable scale, low density, free-flowing, high strength, non-combustible and strong soft vacuum thermal insulation performance (11).

HGM heat transmission comprises primarily of strong conduction, gas conduction, and radiation. HGMs can provide a lightweight, long-lasting and low-maintenance cryogenic insulation

system (12). To describe the cryogenic thermal insulation properties of four forms of HGMs, an easy and realistic system based on steady-state liquid nitrogen evaporation rate method was developed and produced. To evaluate the influence of physical properties on its thermal insulation properties, each heat transfer mode, including solid conduction, gas conduction and radiation, measured thermal conductivities of four forms of HGMs. Analytical method and simulation of ANSYS heat flow, heat leakage, regular evaporation rate and thermal conductivity were obtained. Among them, thermal conductivity is the most fundamental and significant physical quantity to describe materials' thermal insulation properties. Higher thermal conductivity levels are optimal in an output assessment HGM. As three efficiency characteristics are considered,
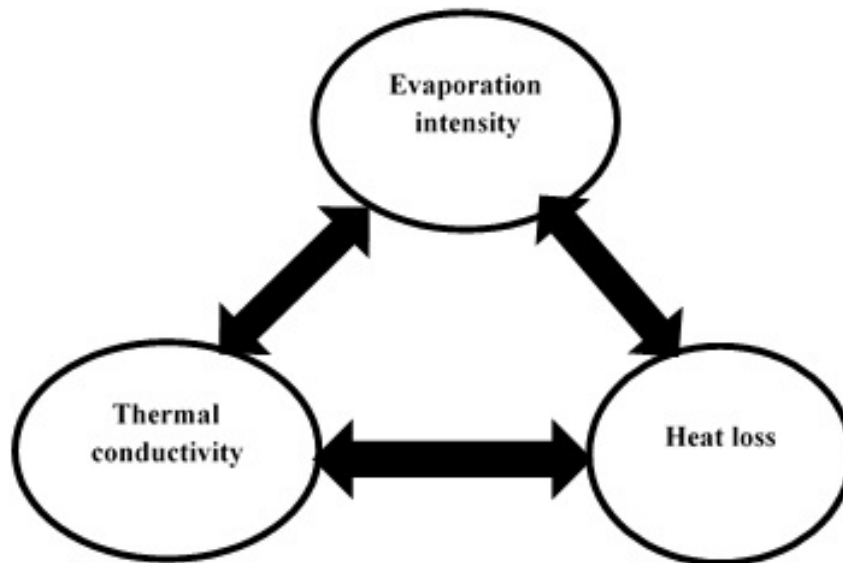

FIGURE 4. Impact assessment attributes graph

there are three nodes with 1,2 and 3 representing evaporation intensity, heat loss and investigational thermal conductivity, respectively. Thermal conductivity is more critical than other attributes. Directed edges are drawn to the other attributes (nodes 2 and 3) for attribute evaporation intensity (node 1). Thermal conductivity attribute is also significant in evaluating HGM performance. For the attribute Pressure (node 2) is drawn to the other attributes (nodes 3 and 1). Thermal conductivity is also significant, too. Directed edges are drawn to other attributes (node 3) (nodes 1 and 2). Figure 3 indicates the graph of the Performance attributes graph (PAG). If

the number node increases, the graph will become more complicated [7]. The directed graph is in matrix form to solve this problem.

3. MATRIX REPRESENTATION

The matrix has a simple and elegant way of expressing the diagram, as it gives one-to-one representation. A matrix named the Performance Matrix (PAM) is defined as, which consists of all attributes (Ri) and their relative value (aij). This is an NXN matrix in Eq. (1).The output matrix attribute is identical to the graph theory matrix.

$$\text{IAM} = \text{A} = \begin{bmatrix} I_i & a_{ij} & a_{ik} \\ a_{ji} & I_j & a_{jk} \\ a_{ki} & a_{kj} & I_k \end{bmatrix} = \begin{bmatrix} I_1 & a_{12} & a_{13} \\ a_{21} & I_2 & a_{23} \\ a_{31} & a_{32} & I_3 \end{bmatrix} \quad (1)$$

Ri is the value of the ith attribute stated by the node v of i and aij is the relative significance of the ith attribute over the jth attribute expressed by the edge eij. The permanent matrix is used to describe the configuration of the device or the composition of the graph and to create a special representation that is independent of the mark (13). It is a standard matrix function that leads to greater appreciation since no detail is lost in the assessment.

$$Per\,(A) = \prod_1^5 Pi + \sum_i \sum_j \sum_k \sum_l \sum_m (a_{ij}.a_{ji}) A_k A_l....A_m + \sum_i \sum_j \sum_k \sum_l \sum_m (a_{ij}.a_{ji}.a_{ki} + a_{ik}.a_{kj}.a_{ji}) A_l A_n....A_m$$

$$+ \left(\sum_i \sum_j \sum_k \sum_m (a_{ij}.a_{ji})(a_{kl}.a_{lk}) A_m A_n....A_m + \sum_i \sum_j \sum_k \sum_m [(a_{ij}.a_{jk}.a_{kl}.a_{li}) + (a_{il}.a_{lk}.a_{kj}.a_{ji})] A_m A_n....A_m \right)$$

$$+ \left(\sum_i \sum_j \sum_k \sum_m (a_{ij}.a_{ji})(a_{kl}.a_{lm}.a_{mk} + a_{km}.a_{ml}.a_{lk}) A_o A_n....A_m + \sum_i \sum_j \sum_k \sum_m [(a_{ij}.a_{jk}.a_{kl}.a_{lm}.a_{mi}) \right.$$

$$+ (a_{im}.a_{ml}.a_{lk}.a_{kj}.a_{ji})] A_o A_n....A_m \Bigg) \quad (2)$$

The permanent function of the output attribute matrix (3x3), Per (A) is written as follows:

$$Per\,(A) = \prod_1^5 Pi + \sum_{ijk}(a_{ij} * a_{ji}) A_k + \sum_{ijk}(a_{ij} * a_{jk} * a_{kl} + a_{ik} * a_{kj} * a_{ji}) \text{ Eq, in a simpler form. (2) can be}$$

written as follows:

$$Per\,(A) = A_1 * A_2 * A_3 + a_{12} * a_{23} * a_{31} + a_{13} *_{21} * a_{32} + a_{13} * A_2 * a_{31} + a_{12} * a_{21} * A_3 + A_1 *$$

$$a_{23} * a_{32}$$

4. EFFICIENCY ATTRIBUTE INDEX

Permanent performance attribute matrix in Eq. (2) is referred to as the Efficiency Attribute Index (EAI) (14). The values of Ri and aij are required to determine the EAI value. The values of Ri are taken from the experimental findings and are normalized to the same scale. If the quantitative values of Ri are not available, a graded value judgment on a scale of 0 to 10 can be adopted and shown in Table 1. The value of relative significance between two attributes (aij) is

TABLE 1. Value of impact attributes

S.No.	Quality measure of impact attribute	Assigned value
1	Extremely low	0
2	Very low	0.1
3	Average	0.2
4	Above average	0.3
5	High	0.4
6	Very high	0.5

also assigned to scales 0 to 1 and is seen in Table 2. If the ij reflects the relative value of the ith attribute over the jth attribute, the relative importance of the jth attribute over the ith attribute is calculated using the Eq. (3). The relative importance of I j and j, I is given as

$a_{ji} = 1 - a_{ij}$ (3)

The index of output attributes for each experiment is assessed using the Eq. (2) by substituting the values of Ri and Aij;(15). Evaluated EAI values are organised to be graded in descending order. The experiment for which the maximum EAI value has been reported is the optimum combination to achieve the best output from the engine under a given set of conditions.

TABLE 2. Relative importance of impact attributes

Description	Relative importance of attributes	
	aij	1-aj
Two attributes are equally important	0.5	0.5
One attribute is slightly more important than the other	0.6	0.4
One attribute is strongly more important than the other	0.7	0.3
One attribute is very strongly more important than the other	0.8	0.2
One attribute is extremely more important than the other	0.9	0.1
One attribute is exceptionally more important than the other	1	0

5. METHODOLOGY

Graph theory and matrix solution were proposed to select the best mix of evaporation rate and HGM heat leakage. The key steps in the technique are as follows:

1. Quality attributes and relative significance parameters are selected as seen in Step 1 of Figure 2.

2. Create a graph of output attributes dependent on attributes and their relative value as seen in section 3.0. The number of nodes must be the same as the number of attributes.

3. The matrix of performance attributes is constructed from the diagram. The dimension of the matrix is N X N.

4. Assess the lasting feature of the output attribute matrix.

5. Replace the values of Ri and aij to achieve the performance attribute index for the tests carried out.

6. Rank the experiments based on the efficiency attribute index.

7. Decision making – Choose the optimal combination which has the highest value of efficiency attribute index.

6. EXPERIMENTAL

TABLE 3. Relative importance of impact attributes

Exp. No.	Samples	Pressure (Pa)	Evaporation rate (L/min)	Heat Leak Q (W)	Experimental Thermal Conductivity X 10-4(W/m.K)
1	A	0.001	573.41	1.0142	5.0465
2	A	0.01	547.99	1.0157	5.0511
3	A	0.1	535.1	1.1012	5.4358
4	A	1	529.6	1.2188	6.007
5	B	0.001	666.92	1.0593	5.215
6	B	0.01	645.55	1.0594	5.206
7	B	0.1	633.17	1.1453	5.697
8	B	1	629.56	1.368	6.241
9	C	0.001	651.28	1.316	6.54
10	C	0.01	625.22	1.3172	6.52
11	C	0.1	620.03	1.4022	7.028
12	C	1	616.97	1.5027	7.62
13	D	0.001	643.74	1.628	8.0622
14	D	0.01	617.94	1.637	8.0578
15	D	0.1	616.52	1.705	8.544
16	D	1	604.05	1.823	8.35

The L16 orthogonal array is selected according to the Taguchi specification and 16 experiments have been performed. Experimental thermal conductivity, heat leakage and evaporation concentrations were calculated for four cold vacuum pressures of 0.001 Pa, 0.01 Pa, 0.1 Pa and 1 Pa. The experimental findings are displayed and tabulated in Table 3.

The different steps of the technique have been carried out as described below:

The efficiency qualities selected for this work are thermal conductivity, heat leakage and evaporation rate. The quantitative values for these attributes are shown in Table 3. The normalized values for these attributes are displayed in Table 4.

The efficiency attributes graph showing the attributes and their relative value as seen in Figure 2. Nodes 1, 2 and 3 are evaporation rate, heat leakage and experimental thermal conductivity.

$$\text{IAM} = A = \begin{bmatrix} I_i & a_{12} & a_{13} \\ a_{21} & I_j & a_{23} \\ a_{31} & a_{32} & I_k \end{bmatrix} = \begin{bmatrix} I_1 & 0.2 & 0.1 \\ 0.8 & I_2 & 0.2 \\ 0.9 & 0.8 & I_3 \end{bmatrix} \quad (4)$$

In a simpler form, equation (4) can be written as follows:

$$Per\ (A) = \prod_{i=1}^{3} \mathrm{I}i + \sum_{ijk}(a_{ij}*a_{ji})A_k + \sum_{ijk}(a_{ij}*a_{jk}*a_{kl} + a_{ik}*a_{kj}*a_{ji})$$

$$Per\ (A) = I_1*I_2*I_3 + a_{12}*a_{23}*a_{31} + a_{13}*_{21}*a_{32} + a_{13}*A_2*a_{31} + a_{12}*a_{21}*A_3 + A_1*a_{23}*a_{32}$$

The values of the Impact evaluation attribute index were determined for different Ri and aij values for 16 experiments for which an MS-office excel prototype was developed. The values of the Effect Evaluation attribute index for 16 studies are ranked and shown in Table 5.

TABLE 4. Impact assessment ranking

Exp. No.	Samples	Pressure (Pa)	IAI	Rank
16	D	1	0.2543	1
15	D	0.1	0.2543	2
14	D	0.01	0.2502	3
13	D	0.001	0.2446	4
11	C	0.1	0.2359	5
12	C	1	0.2347	6
10	C	0.01	0.2337	7
9	C	0.001	0.2301	8
7	B	0.1	0.2204	9
6	B	0.01	0.2195	10
8	B	1	0.2178	11
5	B	0.001	0.2172	12
2	A	0.01	0.2053	13
3	A	0.1	0.2049	14
1	A	0.001	0.2042	15
4	A	1	0.2011	16

Table 5 indicates that Exp. No.16 has the largest value (0.2543) of a permanent index. This shows that the optimum combination of sample 4 with a pressure of 1 Pa is the correct option for determining the best value of Hollow glass microspheres.

7. Conclusion

Unlike conventional methods, the Matrix and Performance Attribute Index of Effect Assessment provides accurate and rigorous data assessment. Graph Theory Matrix Methodology allows to pick the right operating parameters. It is used in this research to pick the optimal parameters for lower thermal conductivity. For lower thermal conductivity, the evaporation rate and heat leakage of the sample D at a pressure of 1 Pa are ideal.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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