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EVALUATION OF NUMERICAL INTEGRATION BY USING HERMITE WAVELETS

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Abstract. In this research, we present a numerical scheme to evaluate the numerical integration of a given function using Hermite wavelets. The proposed technique is based on the expansion of the given function into a series of Hermite wavelets basis functions. Some numerical experiments have been performed to illustrate the accuracy of the proposed method.

Keywords: numerical integration; Hermite wavelets; function approximation; numerical examples.

2010 AMS Subject Classification: 65N99.

1. INTRODUCTION

The limitations of analytical methods have led the engineers and scientists to evolve graphical and numerical methods. As we know the graphical methods, though simple, give results to a low degree of accuracy. Numerical methods can, however, be applied which are more accurate. With the advent of high speed digital computers and increasing demand for numerical answers to various problems, numerical techniques have become indispensible tool in the hands of engineers. The process of evaluating a definite integral from a set of tabulated values of the integrand is called numerical integration. This process when applied to a function of a single variable,

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is called numerical quadrature . When applied to compute double integral of a function of two independent variables, the process is called numerical cubature. Many numerical techniques or rules such as Trapezoidal rule, Simpson's rule, Weddle rule and Gauss-quadrature methods, have been developed to find the numerical integration. In the recent years, the different types of wavelet methods have found their way for the numerical solution of different kinds of integral equations arising in mathematical physics models and many other scientific and engineering problems. Wavelets are mathematical functions which have been widely used in digital signal processing for waveform representation and segmentations, image compression, time-frequency analysis, quick algorithms for easy implementations and many other fields of pure and applied mathematics. Numerical integration has been used for solving various differential and integral equations. Haar wavelets methods have been used for solving differential equations in [1], [2], [3], [4], [5], [11] and [13]. Hermite wavelets have been applied to find the numerical solutions of differential equations in [6], [7], [8], [9], [10] and [12]. Hermite wavelets based technique has been developed to evaluate the numerical differentiation in [14].

In this research paper, we have developed a numerical technique to find the numerical integration of the given function with the help of Hermite wavelets. This research paper is arranged as: In Section 2, Hermite wavelets and its properties have been discussed. Operational matrices of integration have been discussed in Section 3. Function approximation has been explained in Section 4. In Section 5, proposed numerical scheme has been developed to find numerical integration. Some numerical examples have been presented in Section 6, to illustrate the accuracy of the proposed numerical scheme.

2. HERMITE WAVELET AND ITS PROPERTIES

Wavelets constitute a family of functions from dilation and translation of a single function known as mother wavelet. The continuous variation of dilation parameter α and translation parameter β, form a family of continuous wavelets as:

$$\psi_{\alpha,\beta}(x) = \mid \alpha \mid^{-\frac{1}{2}} \psi\Big(\frac{x-\beta}{\alpha}\Big),\ \alpha,\beta \in R,\ \alpha \neq 0, \tag{1}$$

If the dilation and translation parameters are restricted to discrete values by setting $\alpha = {\alpha_0}^{-k}$, $\beta = n\beta_0{\alpha_0}^{-k}$, $\alpha_0 > 1$, $\beta_0 > 0$, we obtain the following family of discrete wavelets:

$$\psi_{k,n}(x) = \mid \alpha \mid^{-\frac{1}{2}} \psi({\alpha_0}^k x - n\beta_0),\ \alpha, \beta \in R,\ \alpha \neq 0, \tag{2}$$

where $\psi_{k,n}$, form a wavelet basis for $L^2(R)$. For special case, if $\alpha_0 = 2$ and $\beta_0 = 1$, then $\psi_{k,n}(x)$ forms an orthonormal basis. Hermite wavelets are defined as:

$$\psi_{n,m}(x) = \begin{cases} \frac{2^{\frac{k+1}{2}}}{\sqrt{\pi}} H_m(2^k m - 2n + 1), & \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}}, \\ 0, & \text{Otherwise}, \end{cases} \tag{3}$$

where $m = 0, 1, 2, 3, ..., M-1$ and $n = 1, 2, 3, ..., 2^{k-1}$ and k is assumed any positive integer. Also, H_m are Hermite polynomials of degree m with respect to weight function $W(x) = \sqrt{1-x^2}$ on the real line R and satisfies the following recurrence relation

$$H_{m+2}(x) = 2xH_{m+1}(x) - 2(m+1)H_m(x), \tag{4}$$

where $m = 0, 1, 2, ..., H_0(x) = 1$ and $H_1(x) = 2x$.

3. Operational Matrices of Integration [12]

For $k = 1$ and $M = 6$, Assume the six basis functions on $[0, 1]$ as:

$$\begin{cases} \psi_{1,0}(x) = \frac{2}{\sqrt{\pi}}, \\ \psi_{1,1}(x) = \frac{2}{\sqrt{\pi}}(4x-2), \\ \psi_{1,2}(x) = \frac{2}{\sqrt{\pi}}(16x^2 - 16x + 2), \\ \psi_{1,3}(x) = \frac{2}{\sqrt{\pi}}(64x^3 - 96x^2 + 36x - 2), \\ \psi_{1,4}(x) = \frac{2}{\sqrt{\pi}}(256x^4 - 512x^3 + 320x^2 - 64x + 2), \\ \psi_{1,5}(x) = \frac{2}{\sqrt{\pi}}(1024x^5 - 2560x^4 + 2240x^3 - 800x^2 + 100x - 2). \end{cases} \tag{5}$$

Let $\psi_6(x) = [\psi_{1,0}(x), \psi_{1,1}(x), \psi_{1,2}(x), \psi_{1,3}(x), \psi_{1,4}(x), \psi_{1,5}(x)]^T$. Integrating the above equations with respect to x, from 0 to x and after expressing in the matrix form, we obtain

(6)
$$\begin{cases} \int_0^x \psi_{1,0}(x)dx = \frac{2}{\sqrt{\pi}}x = \left[\frac{1}{2}, \frac{1}{4}, 0, 0, 0, 0\right]\psi_6(x), \\ \int_0^x \psi_{1,1}(x)dx = \frac{2}{\sqrt{\pi}}(2x^2 - 2x) = \left[-\frac{1}{4}, 0, \frac{1}{8}, 0, 0, 0\right]\psi_6(x), \\ \int_0^x \psi_{1,2}(x)dx = \frac{2}{\sqrt{\pi}}(\frac{16}{3}x^3 - 8x^2 + 2x) = \left[-\frac{1}{3}, -\frac{1}{4}, 0, \frac{1}{12}, 0, 0\right]\psi_6(x), \\ \int_0^x \psi_{1,3}(x)dx = \frac{2}{\sqrt{\pi}}(16x^4 - 32x^3 + 18x^2 - 2x) = \left[\frac{1}{8}, 0, -\frac{1}{8}, 0, \frac{1}{16}, 0\right]\psi_6(x), \\ \int_0^x \psi_{1,4}(x)dx = \frac{2}{\sqrt{\pi}}(\frac{256}{5}x^5 - \frac{512}{4}x^4 + \frac{320}{3}x^3 - 32x^2 + 2x) = \left[-\frac{1}{15}, 0, 0, -\frac{1}{12}, 0, \frac{1}{20}\right]\psi_6(x), \\ \int_0^x \psi_{1,5}(x)dx = \frac{2}{\sqrt{\pi}}(\frac{512}{3}x^6 - \frac{2560}{5}x^5 + \frac{1120}{2}x^4 - \frac{800}{3}x^3 + 50x^2 - 2x) = \left[\frac{1}{24}, 0, 0, 0, -\frac{1}{16}, 0\right]\psi_6(x). \end{cases}$$

Therefore,

$$\int_0^x \psi_6(x)dx = P_{6\times 6}\psi_6(x) + \overline{\psi_6}(x), \tag{7}$$

where

$$P_{6\times 6} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & \frac{1}{8} & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{4} & 0 & \frac{1}{12} & 0 & 0 \\ \frac{1}{8} & 0 & -\frac{1}{8} & 0 & \frac{1}{16} & 0 \\ -\frac{1}{15} & 0 & 0 & -\frac{1}{12} & 0 & \frac{1}{20} \\ \frac{1}{24} & 0 & 0 & 0 & -\frac{1}{16} & 0 \end{pmatrix} \tag{8}$$

and

$$\overline{\psi_6}(x) = \left(0, 0, 0, 0, 0, \frac{1}{24}\psi_{1,6}(x)\right)^T. \tag{9}$$

Similarly integrating (7) with respect to x, from 0 to x, we obtain

$$\int_0^x \int_0^x \psi_6(x)dxdx = Q_{6\times 6}\psi_6(x) + \overline{\overline{\psi_6}}(x), \tag{10}$$

where

$$Q_{6\times 6} = \begin{pmatrix} \frac{3}{16} & \frac{1}{8} & \frac{1}{32} & 0 & 0 & 0 \\ -\frac{1}{6} & -\frac{3}{32} & 0 & \frac{1}{96} & 0 & 0 \\ -\frac{3}{32} & -\frac{1}{12} & -\frac{1}{24} & 0 & \frac{1}{192} & 0 \\ \frac{1}{10} & \frac{1}{16} & 0 & -\frac{1}{64} & 0 & \frac{1}{320} \\ -\frac{1}{24} & -\frac{1}{60} & \frac{1}{96} & 0 & -\frac{1}{120} & 0 \\ \frac{1}{42} & \frac{1}{96} & 0 & \frac{1}{192} & 0 & -\frac{1}{192} \end{pmatrix} \tag{11}$$

and

$$\overline{\overline{\psi_6}}(x) = \left(0,0,0,0,\frac{1}{480}\psi_{1,6}(x),\frac{1}{672}\psi_{1,7}(x)\right)^T. \tag{12}$$

4. Function Approximation

Consider any square integrable function $u(x)$ can be expanded in terms of infinite series of Hermite basis functions as:

$$u(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{n,m}\psi_{n,m}(x), \tag{13}$$

where $C_{n,m}$ are constants of this infinite series, known as Hermite wavelet coefficients. For numerical approximation the above infinite series is truncated upto finite terms as:

$$u(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} C_{n,m}\psi_{n,m}(x) = C^T\Psi(x), \tag{14}$$

where C and Ψ are $2^{k-1}M \times 1$ matrices and are given by

$$C^T = [C_{1,0}, ..., C_{1,M-1}, ..., C_{2^{k-1},0}, ..., C_{2^{k-1},M-1}] \tag{15}$$

and

$$\Psi = [\psi_{1,0}, ..., \psi_{1,M-1}, ..., \psi_{2^{k-1},0}, ..., \psi_{2^{k-1},M-1}]^T \tag{16}$$

5. Proposed Scheme for Numerical Integration

Divide the interval $[a,b]$ into n equal parts, each of length $h = \frac{b-a}{n}$. Let $[a,b]$ be divided into subintervals such as $[a,a+h], [a+h,a+2h], [a+2h,a+3h], [a+3h,a+4h], ..., [a+(n-1)h, b=a+nh]$, where $a = x_0, a+h = x_1, a+2h = x_2, ..., b = x_n$. Expand the unknown function $f(x)$ into a series of Hermite wavelets basis functions as follows:

$$f(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{n,m} \psi_{n,m}(x) \tag{17}$$

Substituting the values of nodes $x_0, x_1, x_2, ..., x_n$ in (17), we obtain

$$f(x_0) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{n,m} \psi_{n,m}(x_0), \tag{18}$$

$$f(x_1) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{n,m} \psi_{n,m}(x_1), \tag{19}$$

$$f(x_2) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{n,m} \psi_{n,m}(x_2), \tag{20}$$

$$\vdots$$

$$f(x_n) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{n,m} \psi_{n,m}(x_n). \tag{21}$$

Solving the above system of equations, we obtain the wavelets coefficients. Integrating (17) one time w.r.t x, from a to b, we obtain

$$\int_a^b f(x)dx = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{n,m} \int_a^b \psi_{n,m}(x)dx. \tag{22}$$

Substituting the values of wavelet coefficients into (22), we obtain the required result.

6. NUMERICAL RESULTS

In this section, some numerical examples have been presented for solving numerical integration of some functions. To illustrate the accuracy of the proposed scheme, we compare the numerical results obtained by proposed scheme with exact results. First of all, change the limits of integration from $[a,b]$ into $[0,1]$ by using the following procedure:
Consider the integration

$$I = \int_a^b g(x)dx, \tag{23}$$

where a and b are any constants. The basic requirement of the Hermite wavelet method is that the integral should be of the form $\int_0^1 g(x)dx$. Using the transformation

$$x = AX + B, \tag{24}$$

where A and B are unknowns satisfy the conditions $x = a, X = 0$ and $x = b, X = 1$. Therefore from (24), we obtain

$$\begin{cases} a = A(0) + B, \\ b = A(1) + B. \end{cases} \tag{25}$$

Solving these equations, we obtain $A = b - a$ and $B = a$. From (24), we obtain

$$\begin{cases} x = (b-a)X + a, \\ dx = (b-a)dX. \end{cases} \tag{26}$$

Substituting these values in (23), we obtain

$$I = (b-a)\int_0^1 g((b-a)X + a)dX, \tag{27}$$

This implies

$$I = (b-a)\int_0^1 f(X)dX, \tag{28}$$

where $f(X) = g((b-a)X + a)$.

Example 1: Consider the integration

$$\int_0^1 f(x)dx,\ f(x)=\frac{1}{x^2+1} \tag{29}$$

Divide the interval $[0,1]$ into 5 equal sub-intervals, each of length $h=\frac{1-0}{5}$. Let $[0,1]$ be divided into $[0,1/5],[1/5,2/5]...,[4/5,1]$. Expand the given function $f(x)=\frac{1}{x^2+1}$ into a series of Hermite wavelets basis functions by taking $k=1,M=6$ as follows:

$$\frac{1}{x^2+1}=\sum_{m=0}^{5} C_{1,m}\psi_{1,m}(x) \tag{30}$$

Substituting the values of nodes $x=0,1/5,2/5,3/5,4/5,1$ in (30), we obtain

$$1=C_{1,0}\psi_{1,0}(0)+C_{1,1}\psi_{1,1}(0)+C_{1,2}\psi_{1,2}(0)+C_{1,3}\psi_{1,3}(0)+C_{1,4}\psi_{1,4}(0) \tag{31}$$
$$+C_{1,5}\psi_{1,5}(0),$$

$$\frac{25}{26}=C_{1,0}\psi_{1,0}(1/5)+C_{1,1}\psi_{1,1}(1/5)+C_{1,2}\psi_{1,2}(1/5)+C_{1,3}\psi_{1,3}(1/5) \tag{32}$$
$$+C_{1,4}\psi_{1,4}(1/5)+C_{1,5}\psi_{1,5}(1/5),$$

$$\frac{25}{29}=C_{1,0}\psi_{1,0}(2/5)+C_{1,1}\psi_{1,1}(2/5)+C_{1,2}\psi_{1,2}(2/5)+C_{1,3}\psi_{1,3}(2/5) \tag{33}$$
$$+C_{1,4}\psi_{1,4}(2/5)+C_{1,5}\psi_{1,5}(2/5),$$

$$\frac{25}{34}=C_{1,0}\psi_{1,0}(3/5)+C_{1,1}\psi_{1,1}(3/5)+C_{1,2}\psi_{1,2}(3/5)+C_{1,3}\psi_{1,3}(3/5) \tag{34}$$
$$+C_{1,4}\psi_{1,4}(3/5)+C_{1,5}\psi_{1,5}(3/5),$$

$$\frac{25}{41}=C_{1,0}\psi_{1,0}(4/5)+C_{1,1}\psi_{1,1}(4/5)+C_{1,2}\psi_{1,2}(4/5)+C_{1,3}\psi_{1,3}(4/5) \tag{35}$$
$$+C_{1,4}\psi_{1,4}(4/5)+C_{1,5}\psi_{1,5}(4/5),$$

and

$$\frac{1}{2}=C_{1,0}\psi_{1,0}(1)+C_{1,1}\psi_{1,1}(1)+C_{1,2}\psi_{1,2}(1)+C_{1,3}\psi_{1,3}(1) \tag{36}$$
$$+C_{1,4}\psi_{1,4}(1)+C_{1,5}\psi_{1,5}(1).$$

Solving the above system of equations, we obtain the wavelets coefficients. The wavelet coefficients are

$$\begin{cases} 0.688591199285961, \\ -0.118136062261244, \\ -0.011096961844717, \\ 0.007556856455451, \\ -0.000863540753480, \\ -0.000199159875803. \end{cases} \tag{37}$$

Integrating (30) one time w.r.t x, from 0 to 1, we obtain

$$\int_0^1 \frac{1}{x^2+1}dx = \sum_{m=0}^{5} C_{1,m}\int_0^1 \psi_{1,m}(x)dx. \tag{38}$$

Substituting the values of wavelet coefficients into (38), we obtain

$$\int_0^1 \frac{1}{x^2+1}dx = 0.785469604481503. \tag{39}$$

which is nearly same as the exact solution.

Example 2: Consider the integration

$$\int_0^1 f(x)dx,\ f(x) = \frac{1}{1+x} \tag{40}$$

Divide the interval $[0,1]$ into 5 equal sub-intervals, each of length $h = \frac{1-0}{5}$. Let $[0,1]$ be divided into $[0,1/5],[1/5,2/5]...,[4/5,1]$. Expand the given function $f(x) = \frac{1}{1+x}$ into a series of Hermite wavelets basis functions by taking $k = 1, M = 6$ as follows:

$$\frac{1}{1+x} = \sum_{m=0}^{5} C_{1,m}\psi_{1,m}(x) \tag{41}$$

Substituting the values of nodes $x = 0, 1/5, 2/5, 3/5, 4/5, 1$ in (41), we obtain

$$1 = C_{1,0}\psi_{1,0}(0) + C_{1,1}\psi_{1,1}(0) + C_{1,2}\psi_{1,2}(0) + C_{1,3}\psi_{1,3}(0) + C_{1,4}\psi_{1,4}(0) + C_{1,5}\psi_{1,5}(0), \tag{42}$$

$$\frac{5}{6} = C_{1,0}\psi_{1,0}(1/5) + C_{1,1}\psi_{1,1}(1/5) + C_{1,2}\psi_{1,2}(1/5) + C_{1,3}\psi_{1,3}(1/5) + C_{1,4}\psi_{1,4}(1/5) + C_{1,5}\psi_{1,5}(1/5), \tag{43}$$

$$\frac{5}{7} = C_{1,0}\psi_{1,0}(2/5) + C_{1,1}\psi_{1,1}(2/5) + C_{1,2}\psi_{1,2}(2/5) + C_{1,3}\psi_{1,3}(2/5) + C_{1,4}\psi_{1,4}(2/5) + C_{1,5}\psi_{1,5}(2/5), \tag{44}$$

$$\frac{5}{8} = C_{1,0}\psi_{1,0}(3/5) + C_{1,1}\psi_{1,1}(3/5) + C_{1,2}\psi_{1,2}(3/5) + C_{1,3}\psi_{1,3}(3/5) + C_{1,4}\psi_{1,4}(3/5) + C_{1,5}\psi_{1,5}(3/5), \tag{45}$$

$$\frac{5}{9} = C_{1,0}\psi_{1,0}(4/5) + C_{1,1}\psi_{1,1}(4/5) + C_{1,2}\psi_{1,2}(4/5) + C_{1,3}\psi_{1,3}(4/5) + C_{1,4}\psi_{1,4}(4/5) + C_{1,5}\psi_{1,5}(4/5), \tag{46}$$

and

$$\frac{1}{2} = C_{1,0}\psi_{1,0}(1) + C_{1,1}\psi_{1,1}(1) + C_{1,2}\psi_{1,2}(1) + C_{1,3}\psi_{1,3}(1) + C_{1,4}\psi_{1,4}(1) + C_{1,5}\psi_{1,5}(1). \tag{47}$$

Solving the above system of equations, we obtain the wavelets coefficients. The wavelet coefficients are

$$\begin{cases} 0.626677706789449, \\ -0.107522888897875, \\ 0.018459626597798, \\ -0.003166040608343, \\ 0.000536617052261, \\ -0.000089436175377. \end{cases} \tag{48}$$

Integrating (41) one time w.r.t x, from 0 to 1, we obtain

$$\int_0^1 \frac{1}{1+x}dx = \sum_{m=0}^{5} C_{1,m}\int_0^1 \psi_{1,m}(x)dx. \tag{49}$$

Substituting the values of wavelet coefficients into (49), we obtain

$$\int_0^1 \frac{1}{x+1}dx = 0.693163029100529. \tag{50}$$

which is nearly same as the exact solution.

Example 3: Consider the integration

$$\int_0^1 f(x)dx,\ f(x) = x^3 \tag{51}$$

Divide the interval $[0,1]$ into 5 equal sub-intervals, each of length $h = \frac{1-0}{5}$. Let $[0,1]$ be divided into $[0,1/5],[1/5,2/5]...,[4/5,1]$. Expand the given function $f(x) = x^3$ into a series of Hermite wavelets basis functions by taking $k = 1, M = 6$ as follows:

$$x^3 = \sum_{m=0}^{5} C_{1,m}\psi_{1,m}(x) \tag{52}$$

Substituting the values of nodes $x = 0, 1/5, 2/5, 3/5, 4/5, 1$ in (52), we obtain

$$0 = C_{1,0}\psi_{1,0}(0) + C_{1,1}\psi_{1,1}(0) + C_{1,2}\psi_{1,2}(0) + C_{1,3}\psi_{1,3}(0) + C_{1,4}\psi_{1,4}(0) + C_{1,5}\psi_{1,5}(0), \tag{53}$$

$$\frac{1}{125} = C_{1,0}\psi_{1,0}(1/5) + C_{1,1}\psi_{1,1}(1/5) + C_{1,2}\psi_{1,2}(1/5) + C_{1,3}\psi_{1,3}(1/5) + C_{1,4}\psi_{1,4}(1/5) + C_{1,5}\psi_{1,5}(1/5), \tag{54}$$

$$\frac{8}{125} = C_{1,0}\psi_{1,0}(2/5) + C_{1,1}\psi_{1,1}(2/5) + C_{1,2}\psi_{1,2}(2/5) + C_{1,3}\psi_{1,3}(2/5) + C_{1,4}\psi_{1,4}(2/5) + C_{1,5}\psi_{1,5}(2/5), \tag{55}$$

$$\frac{27}{125} = C_{1,0}\psi_{1,0}(3/5) + C_{1,1}\psi_{1,1}(3/5) + C_{1,2}\psi_{1,2}(3/5) + C_{1,3}\psi_{1,3}(3/5) + C_{1,4}\psi_{1,4}(3/5) + C_{1,5}\psi_{1,5}(3/5), \tag{56}$$

$$\frac{64}{125} = C_{1,0}\psi_{1,0}(4/5) + C_{1,1}\psi_{1,1}(4/5) + C_{1,2}\psi_{1,2}(4/5) + C_{1,3}\psi_{1,3}(4/5) + C_{1,4}\psi_{1,4}(4/5) + C_{1,5}\psi_{1,5}(4/5), \tag{57}$$

and

$$1 = C_{1,0}\psi_{1,0}(1) + C_{1,1}\psi_{1,1}(1) + C_{1,2}\psi_{1,2}(1) + C_{1,3}\psi_{1,3}(1) + C_{1,4}\psi_{1,4}(1) + C_{1,5}\psi_{1,5}(1). \tag{58}$$

Solving the above system of equations, we obtain the wavelets coefficients. The wavelet coefficients are

$$\begin{cases} 0.276945914203987, \\ 0.207709435652990, \\ 0.083083774261196, \\ 0.013847295710199, \\ -0.000000000000000, \\ 0.000000000000000. \end{cases} \tag{59}$$

Integrating (52) one time w.r.t x, from 0 to 1, we obtain

$$\int_0^1 x^3 dx = \sum_{m=0}^{5} C_{1,m} \int_0^1 \psi_{1,m}(x) dx. \tag{60}$$

Substituting the values of wavelet coefficients into (60), we obtain

$$\int_0^1 x^3 dx = 0.250000000000000. \tag{61}$$

which is nearly same as the exact solution.

7. CONCLUSION

From the above experimental or numerical data, it is concluded that Hermite wavelets are powerful mathematical tools for solving numerical integration and play significant role in numerical analysis. The numerical results are nearly same as exact results. This method is also

valid for those integrals, where the integrand does not admit of primitive in terms of elementary functions. For the future scope, this method will be applicable for solving two- and three-dimensional problems.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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