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THE INTERACTION BETWEEN A Q-FUZZY NORMAL SUBGROUP AND A Q-FUZZY CHARACTERISTIC SUBGROUP

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Abstract: Subsequently after the insertion of fuzzy sets by L. A. Zadeh indefinite number of researchers interpreted on the generalisation of the notion of fuzzy sets. Fuzzy sets span large arenas of research in engineering, medical sciences, social sciences, graph theory, etc. In this paper we introduce the concept of a Q-Fuzzy normal subgroup and Q-fuzzy characteristic subgroup and with the help of a Klein-4 Group we study the interaction between them.

Keywords: fuzzy sets; Q-fuzzy normal subgroup; Q-fuzzy characteristic subgroup.

2010 AMS Subject Classification: 03E72.

1. INTRODUCTION

The idea of a fuzzy subset of a set is formulated by Lotfi Zadeh undoubtedly. In the current era this particular concept has diversified applications in several mathematical branches such as Group theory, Functional analysis, Probability theory, Topology and so on. Rose field in 1971 facilitated the study of Group theory by introducing the concepts of Fuzzy subgroups and Fuzzy sub groupoids. Wu and Li introduced the notion of Fuzzy normal subgroups of an ordinary

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group in 1993.Apart from that Fuzzy subgroups, Fuzzy sub rings and Fuzzy ideals were developed by Rajesh Kumar in 2009. Later on, A. Solairaju and R. Nagarajan have carved out a new algebraic structure called Q-Fuzzy subgroups in their research work namely 'A New Structure and Construction of Q-Fuzzy Groups' in 2009. In 2013,o n Q-Fuzzy normal subgroups a research study was conducted by T. Priya, T. Ramachandran, K.T. Nagalakshmi. In 2014 'A study on Q-Fuzzy normal subgroups and cosets' was carried out by A. Vethamanickam and KR. Balasubramanian [1-4]. In accordance with it a sort of interest stimulated various authors to develop these particular concepts in different algebraic systems [5- 14]. In 2019, B. Sailaja, V.B.V.N. Prasad, interpreted the 'Exploring the axiom of excluded middle and axiom of contradiction in fuzzy sets' in their research work [15]. This prepared the necessary ground for further studies on various types of algebraic structures.

In this manuscript, we have discussed about Q-Fuzzy subsets definitions and examples and stated some properties of Q-Fuzzy normal subgroups, Q-Fuzzy characteristic subgroups of a group.

2. PRELIMINARIES

The preliminaries section includes some elementary definitions and properties of Q- Fuzzy sets that provide an impetus for further discussion.

Definition 2.1: Let X be a nonempty set. A fuzzy subset ϴ of X is a function $ϴ: X \to [0, 1]$.

Example 2.1: Let $X = \{P_1, P_2, P_3, P_4\}$ be the four patrolling points along the border of a country. The incidence of 'enemy's approach' is denoted by the fuzzy subset σ which indicates the zone of 'most dangerous', 'more dangerous', 'somewhat dangerous', 'not at all dangerous'. It can be stated as follows: $ϴ = \{(P_1, 0.9), (P_2,0.7),(P_3,0.4),(P_4,0)\}$

Here ϴ indicates that the zone of danger at the patrolling point P_1 is 0.9 and so on for the remaining patrolling points.

Definition 2.2: Let ϴ be a fuzzy subset of a set X and for any t Є [0,1] we define the level subset of σ as follows in the set builder form

$$ϴ_t = \{x \in G / \sigma(x) \geq t\}$$

Clearly $\Theta_t \subseteq \Theta_s$ whenever $t > s$.

Definition 2.3: Let G be a group. A fuzzy subset Θ of G is termed as a fuzzy subgroup of a group G if for all x, y Є G the following criteria is satisfied.

(i) $\Theta(x y) \geq \text{Min}\{\Theta(x), \Theta(y)\}$

(ii) $\Theta(x^{-1}) \geq \Theta(x)$

Definition 2.4: A fuzzy subset Θ of a groupoid X is called a fuzzy subgroupoid if the following mathematical condition is satisfied:

$$\Theta(x y) \geq \text{Min}\{\Theta(x), \Theta(y)\}$$

Note 2.4: It means that we can infer that a fuzzy subset Θ satisfying the first condition of fuzzy subgroup is a fuzzy subgroupoid.

Definition 2.5: Let Θ be fuzzy subgroup of a group G and let t in [0,1] be such that $t \leq \Theta(e)$ where e is the identity in G.

The subgroup Θ_t is called a level subgroup of Θ.

Definition 2.6: Let G be a group. A fuzzy subgroup Θ of G is said to be a fuzzy normal subgroup if for all x, y Є G we have

$$\Theta(xyx^{-1}) = \Theta(y) \quad \text{(or)} \quad \Theta(xy) = \Theta(yx)$$

Definition 2.7: A fuzzy subgroup Θ of a group G is called a fuzzy characteristic subgroup of G if it satisfies the following criteria

$$\Theta(g) = \Theta(f(g)) \ \forall\, g \in G,\ f \in \text{Aut } G$$

3. Q-FUZZY SUBSETS

Definition 3.1: Let X and Q be two nonempty sets.

A function $\sigma: X \times Q \rightarrow [0, 1]$ is called a Q-fuzzy subset of X.

Example 3.1: Let X = {p , q ,r} ,Q = {s}, then the Q-fuzzy subset is as follows:

σ = {< (p, s),(0.3)>,<(q, s),(0.2)>,<(r, s),(0.4)> } is a Q-fuzzy subset of X.

Definition 3.2: Let σ be a Q-fuzzy subset of a set X and for any t Є [0,1] we define the level subset of σ as follows:

$$\sigma_t = \{x \in G, q \in Q \,/\, \sigma(x, q) \geq t\}$$

Definition 3.3: Let G be a group and Q be a nonempty set. A Q-fuzzy subset σ is called as a Q-fuzzy subgroup of G if for all x, y Є G and q ЄQ, the following criteria is satisfied:

(i) $\sigma(x y, q) \geq \text{Min}\{\sigma(x, q), \sigma(y, q)\}$

(ii) $\sigma(x^{-1}, q) \geq \sigma(x, q)$

Definition 3.4: Let σ be a Q-fuzzy subgroup of a group G and let t in [0,1] be such that $t \leq \sigma(e, q)$ where e is the identity in G. Then σ is said to be a level subgroup of a Q-fuzzy subgroup.

Definition 3.5: Let G be a group and Q be a nonempty set. A Q-fuzzy subgroup σ of G is termed to be normal if for every x, y Є G, q Є Q the following condition is satisfied.

$$\sigma(xyx^{-1}, q) = \sigma(y, q) \quad \text{(or)} \quad \sigma(x y, q) = \sigma(y x, q)$$

Definition 3.6: Let G be a group and Q be a nonempty set. A Q-fuzzy subgroup σ of G is termed to be a characteristic subgroup of G, if it satisfies the following condition

$$\sigma(x, q) = \sigma(f(x, q)) \ \forall \ x \in G, f \in Q\text{-Aut}G, q \in Q$$

Definition 3.7: Let G be a group and Q be a non empty set. The map

$f: G \times Q \to H \times Q$ is said to be a group Q-homomorphism if

(i) f: G x H is a group homomorphism.

(ii) $f(x y, q) = (f(x), f(y), q)$

4. SOME PROPERTIES OF Q-FUZZY SUBGROUPS, Q-FUZZY NORMAL SUBGROUPS, Q-FUZZY CHARACTERISTIC SUBGROUPS OF A GROUP

Theorem 4.1: Let G be a group and σ be a Q-fuzzy subset of G. Then σ is a Q-fuzzy subgroup of G iff the level subsets σ_t, t Є [0,1] are subgroups of G.

Proof: Let σ be a Q-fuzzy subgroup of G and σ_t be a level subset of G. We now prove that σ_t is a subgroup of G.

Let $\sigma_t = \{x \in G, q \in Q \,/\, \sigma(x, q) \geq t\}$ where $t \in [0,1]$

Clearly σ_t is nonempty.

Let x, y Є σ_t, then, by the very definition of a level subset we have that

$$\sigma(x, q) \geq t \quad \text{and} \quad \sigma(y, q) \geq t$$

Now consider $\sigma(x\,y, q) \geq \text{Min}\,\{ \sigma(x, q), \sigma(y, q) \}$

$$\geq \text{Min}\,\{t, t\} = t$$

Therefore $\sigma(x\,y, q) \geq t$

$\Rightarrow x\,y$ Є σ_t

Let x Є $\sigma_t \Rightarrow \sigma(x, q) \geq t$

Since σ is a Q-fuzzy subgroup of G, we have that $\sigma(x^{-1}, q) \geq \sigma(x, q)$

$\Rightarrow \sigma(x^{-1}, q) \geq t$

$\Rightarrow x^{-1}$ Є σ_t

Hence σ_t is a subgroup of G.

Conversely, assume that the level subsets σ_t, t Є [0,1] , σ_s ,s Є [0,1] of a Q-fuzzy subset σ are subgroups of G.

Now we prove that σ is a Q-fuzzy subgroup of G.

Let x,y Є G and let $\sigma(x, q) = s$, $\sigma(y, q) = t$

$\Rightarrow$ x Є σ_s and y Є σ_t

Assume that s < t then $\sigma_t \subseteq \sigma_s$.

$\therefore$ y Єσ_s

Now x, y Є σ_s

Since σ_s is a subgroup of G we have that x y Є σ_s.

$$\Rightarrow \sigma(x\,y, q) \geq s = \text{Min}\{\sigma(x, q), \sigma(y, q)\}$$

Let x Є G and let $\sigma(x, q) = s$

$\Rightarrow$x Є σ_s

Since σ_s is a subgroup of G, $x^{-1} \in \sigma_s$

$$\therefore \sigma(x^{-1}, q) \geq s$$

$$\Rightarrow \sigma(x^{-1}, q) \geq \sigma(x, q)$$

Hence σ is a Q-fuzzy subgroup of G.

Theo rem 4.2: Let σ and μ be two Q-fuzzy subgroups of G. Then $\sigma \cap \mu$ is a Q-fuzzy subgroup of G.

Proof: Given that σ and μ are two Q-fuzzy subgroups of G.

We have to prove that their intersection is also a Q-fuzzy subgroup of G.

(i) Now consider $(\sigma \cap \mu)(x\,y, q) = \text{Min}\{\sigma(x\,y, q), \mu(x\,y, q)\}$

$$\geq \text{Min}\{\{\text{Min}\{\sigma(x, q), \sigma(y, q)\}, \text{Min}\{\mu(x, q), \mu(y,q)\}\}$$

$$\geq \text{Min}\{\{\text{Min}\{\sigma(x, q), \mu(x, q)\}, \text{Min}\{\sigma(y, q), \mu(y,q)\}\}$$

$$= \text{Min}\{(\sigma \cap \mu)(x, q), (\sigma \cap \mu)(y, q)\}$$

$$\therefore (\sigma \cap \mu)(x\,y, q) \geq \text{Min}\{(\sigma \cap \mu)(x, q), (\sigma \cap \mu)(y, q)\}$$

(ii) Now $(\sigma \cap \mu)(x^{-1}, q) = \{\sigma(x^{-1}, q), \mu(x^{-1}, q)\}$

$$= \{\sigma(x, q)\ \mu(y, q)\}$$

$$= \{(\sigma \cap \mu)(x, q)\}$$

Hence $\sigma \cap \mu$ is a Q-fuzzy subgroup of G.

Lemma 4.3: Let σ be any Q-fuzzy subset of a set S.

Then $\sigma(x, q) = \text{Sup}\{k/(x, q) \in \sigma_k\}$ where $x \in X$

In other words, $\sigma(x, q) = t \Leftrightarrow (x, q) \in \sigma_t$ and $x \notin \sigma_s$ for all $s > t$

Theorem 4.4: For a Q- fuzzy subgroup σ of a group G, the following statements are equivalent :

(i) σ is fuzzy normal

(ii) σ is constant on the conjugate classes of G.

(iii) $\sigma(x^{-1} y^{-1}xy) \geq \sigma(x, q)$ for all $x, y \in G$.

Theorem 4.5: A Q-fuzzy subgroup σ of G is Q-fuzzy normal iff each level subgroup of σ is normal in G.

Proof: Let σ be Q-fuzzy subgroup of G and suppose that it is Q-fuzzy normal.

We have to prove that each level subgroup of σ is normal in G.

Let σ_t be the level subgroup of

σ Let x Є σ_t and y Є G.

Since σ is a Q-fuzzy normal subgroup of G,we have

that $\sigma(yxy^{-1},q) = \sigma(x, q) \geq t$

$\therefore \sigma(yxy^{-1}, q) \geq t$

$\Rightarrow y\,x\,y^{-1} \in \sigma_t$

$\therefore$ The level subgroup σ_t is normal inG.

On the contrary suppose that each level subgroup of σ is normal

in G. We prove that σ is Q-fuzzy normal in G.

Let x, y Є G and $\sigma(x\,y, q) = r$.

If r=1 then by the lemma 4.3 we have that

$\sigma(y\,x, q) = r$. Now assume that $r < 1$.

Let s Є [0, 1] be such that $s > r$.

Then $x\,y \in \sigma_r$ and $x\,y \notin \sigma_s$ by the above mentioned lemma.

Since σ_r and σ_s are normal subgroups of G ,we have

that $y\,x \in \sigma_r$ and $y\,x \notin \sigma_s$.

Again appealing to lemma 4.3 we obtain that $\sigma(y\,x, q) = r$.

$$\therefore \sigma(x\,y, q) = \sigma(y\,x, q) = r.$$

Hence σ is a Q-fuzzy normal subgroup of G.

Theorem 4.6: The intersection of any two Q-fuzzy normal subgroups of G is also a Q-fuzzy normal subgroup of G.

Proof: Let σ and μ be two Q-fuzzy normal subgroups of G.

By a theorem 4.2 $\sigma \cap \mu$ is a Q-fuzzy subgroup of G.

Now for all x ,y in G we have

$(\sigma \cap \mu)(x\,y, q) = \text{Min}\{\sigma(x\,y, q), \mu(x\,y, q)\}$

$= \text{Min}\{\sigma(y\,x, q), \mu(y\,x, q)\}$

$= (\sigma \cap \mu)(y\,x, q)$

Hence $(\sigma \cap \mu)(x\,y, q) = (\sigma \cap \mu)(y\,x, q)$

Theorem 4.7: Let σ and σ^I be Q-fuzzy normal subgroups Let $f : G \times Q \to H \times Q$ is a group Q- homomorphism. Let σ and σ^1 be Q-fuzzy nor normal subgroups of G and G^1. Then $f^{-1}(\sigma^1)$ is a Q-fuzzy normal subgroup of G.

Proof: We have that $f: G \times Q \to H \times Q$ is a group Q-homomorphism and σ, σ^1 are Q-fuzzy normal subgroups of G and G^1 respectively.

Let $x, y \in G$

We have $(f^{-1}(\sigma^1))(x\,y, q) = \sigma^1(f(x\,y, q)$

$= \sigma^1(f(x),f(y), q)$

$= \sigma^1(f(y),f(x),q)$

$= \sigma^1(f(y\,x),q))$

$= (f^{-1}(\sigma^1))(y\,x, q)$

Hence $(f^{-1}(\sigma^1))(x\,y, q) = (f^{-1}(\sigma^1))(y\,x, q))\,(y\,x, q)$.

Theorem 4.8:

For a Q-fuzzy subgroup σ of G the following conditions are equivalent:

(i) σ is a Q-fuzzy characteristic subgroup of G.

(ii) Each level subgroup of σ is a characteristic subgroup of G.

Proof:

Suppose that σ is a Q-fuzzy characteristic subgroup of G.

Now we prove that **e**ach level subgroup of σ is a characteristic subgroup of G.

Let $t \in \text{Im}\,\sigma$, $f \in \text{Aut - G}$, and $x \in \sigma_t$.

Since σ is a Q- fuzzy characteristic subgroup of G we have that $\sigma(x, q) = \sigma(f(x, q))\ \forall\ x \in G$,

$f \in Q$ -Aut G , $q \in Q$.

$$\text{Let } x \in \sigma_t.$$

$$\Rightarrow \sigma(x, q) \geq t$$

$$\Rightarrow \sigma(f(x, q)) \geq t$$

$$\Rightarrow (f(x, q)) \in \sigma_t.$$

$$\Rightarrow f(\sigma_t) \subseteq \sigma_t \rightarrow 1$$

Let $x \in \sigma_t$

Let g in G be such that f (g) = x

Then we have that $\sigma(g, q) = \sigma(f(g, q))$ since σ is a Q- fuzzy characteristic subgroup of G.

For $g \in \sigma_t$ implies that $\sigma(g, q) \geq t$.

Therefore whenever $\sigma(g, q) \geq t$, we can have that $g \in \sigma_t$

$$\Rightarrow f(g) \in f(\sigma_t)$$

$$\Rightarrow x \in f(\sigma_t)$$

$\therefore \sigma_t \subseteq f(\sigma_t) \rightarrow 2$ From 1 and 2 we have that $\sigma_t = f(\sigma_t)$

$\therefore \sigma_t$ is a characteristic subgroup of G.

On the contrary suppose that each level subgroup of G is a characteristic subgroup of G. We have to now prove that σ is a Q-fuzzy characteristic subgroup.

Let $x \in G$, $f \in Q$ – AutG and $\sigma(x, q) = t$.

By a lemma 4.3 $\sigma(x, q) = t \Leftrightarrow x \in \sigma_t$ and $x \notin \sigma_s \ \forall\ s > t$.

By hypothesis, $f(\sigma_t) = \sigma_t$

Hence $(\sigma \circ f)(x, q) = \sigma(f(x, q)) = \sigma(x, q) \geq t$.

Let $s = \sigma(f(x, q)) \Rightarrow f(x,q) \in \sigma_s$

If possible, let $s > t$.

Then $f(x, q) \in \sigma_s = f(\sigma_s)$

Since f is one –one, $x \in \sigma_s$ which is a contradiction.

Hence $\sigma(f(x, q)) = t = \sigma(x, q)$.

Therefore σ is a Q-fuzzy characteristic subgroup of G.

Theorem 4.9: A Q-fuzzy characteristic subgroup of G is always Q-fuzzy normal.

Proof: Let σ be any Q-fuzzy characteristic subgroup of G.

Let $x, y \in G$.

Consider the mapping $f: G \to G$ defined by $f(x y, q) = (yxy^{-1}, q)$

Clearly $f \in Q$ -Aut G.

We have that $\sigma(x y, q) = (\sigma \circ f)(x y, q)$, since σ is a Q-fuzzy characteristic subgroup of G.

$$= \sigma(f(x y, q)$$

$$= \sigma(y(x y) y^{-1}, q)$$

$$= \sigma(y x, q)$$

$\therefore \sigma(x y, q) = \sigma(y x, q)$

Hence σ is always Q-fuzzy normal.

We now examine the converse of the above theorem i.e. to say whether a Q-fuzzy normal subgroup is always a Q-fuzzy characteristic subgroup or not. For serving this purpose I choose a Klein-4 group.

Then $G = \{e, a, b, a b\}$ where $(a^2, q) = (e^2, q) = (b^2, q)$ and $(a b, q) = (b a, q)$

Define a Q-fuzzy subgroup σ and an automorphism f of G by $\sigma(e, q) = 1$, $\sigma(a, q) = 1$

$\sigma(b, q) = 0.6$, $\sigma(a b, q) = 0.6$

Since every subgroup of an abelian group is normal then obviously σ is a Q-fuzzy normal subgroup.

Now we check whether σ is a Q-fuzzy characteristic subgroup or not.

- We have $\sigma(e, q) = 1$

  Now consider $(\sigma \circ f)(e, q) = \sigma(f(e, q))$

$= \sigma (e, q)$

$=1$

So, in this case $\sigma (e, q) = \sigma (f (e, q))$

- We have $\sigma (a, q) = 1$

  Now consider $(\sigma \text{ o } f) (a, q) = \sigma (f (a, q))$

  $= \sigma (b, q)$

  $=0.6$

  So, in this case $\sigma (a, q) \neq \sigma (f (a, q))$

- We have $\sigma (b, q) = 0.6$

  Now consider $(\sigma \text{ o } f) (b, q) = \sigma (f (b), q))$

  $= \sigma (a, q)$

  $=1$

  So in this case $\sigma (b, q) \neq \sigma (f (b), q))$

- We have $\sigma (a b, q) = 0.6$

  Now consider $(\sigma \text{ o } f) (a b, q) = \sigma (f (a b), q))$

  $= \sigma (a b, q)$

  $= 0.6$

  So, in this case $\sigma (a b, q) = \sigma (f (a b), q))$

  Hence finally σ is not a Q-fuzzy characteristic subgroup since $\sigma (a, q) \neq \sigma (f (a, q))$ and $\sigma (b, q) \neq \sigma (f (b), q))$

3. MAIN RESULTS

Before approaching towards Q-fuzzy normal subgroups and Q-fuzzy characteristic subgroups we adopted an elaborative picture of various algebraic structures like fuzzy subgroups, fuzzy subgroupoid, level subgroups, etc. We discussed in length and breadth about the properties of Q-fuzzy subsets and theorems on Q-fuzzy normal subgroups and Q-fuzzy characteristic subgroups. Subsequently we have taken a theorem connecting both the Q-fuzzy normal subgroups and Q-fuzzy characteristic subgroup which states that “a fuzzy characteristic subgroup is always q-fuzzy normal.” Henceforth, we proved that its converse that “a Q-fuzzy normal subgroup of a group g need not be a Q-fuzzy characteristic subgroup” by using Klein -4 Group example.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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