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SOME 4-TOTAL MEAN CORDIAL GRAPHS DERIVED FROM WHEEL

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Abstract. In this paper we investigate the 4-total mean cordial labeling behaviour of helm, closed helm, flower graph, sunflower graph, gear graph, subdivision of wheel, web graph.

Keywords: wheel; helm; flower graph; gear graph; web graph.

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1. INTRODUCTION

Graphs in this paper are finite, simple and undirected. k-total mean cordial labeling of graphs have been introduced in [3] and they investigate the 4-total mean cordial labeling behaviour of path, cycle, star, bistar, wheel, subdivision of star, subdivision of bistar, subdivision of comb, subdivision of crown, subdivision of doublecomb, subdivision of jellyfish, subdivision of ladder, subdivision of triangular snake in [3, 4]. In this paper, we investigate the 4-total mean cordial labeling behaviour of helm, closed helm, flower graph, sunflower graph, gear graph, subdivision of wheel, web graph. Terms are not defined here follow from Harary [2] and Gallian [1].

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2. 4-Total Mean Cordial Graph

Definition 2.1. Let G be a graph. Let $f : V(G) \to \{0, 1, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $\left|t_{mf}(i) - t_{mf}(j)\right| \leq 1$, for all $i, j \in \{0, 1, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

3. Preliminaries

Definition 3.1. The graph $W_n = C_n + K_1$ is called a *wheel*, where C_n is the cycle $u_1 u_2 \ldots u_n u_1$, $V(K_1) = \{u\}$.

Definition 3.2. The Helm H_n is the graph obtained from the wheel W_n with $V(H_n) = V(W_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(H_n) = E(W_n) \cup \{u_i v_i : 1 \leq i \leq n\}$.

Definition 3.3. Closed Helm CH_n is a graph obtained from the helm H_n with vertex set $V(CH_n) = V(H_n)$ and $E(CH_n) = E(H_n) \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.

Definition 3.4. A flower graph is the graph obtained from a helm H_n by joining each pendent vertices v_i to the central vertex u of the helm.

Definition 3.5. The sunflower graph SF_n is obtained by taking a wheel $W_n = C_n + K_1$ where C_n is the cycle $u_1 u_2 \ldots u_n u_1$, $V(K_1) = \{u\}$ and new vertices $v_1, v_2, \ldots, v_n$ where v_i is join by the vertices $u_i u_{i+1} \pmod{n}$.

Definition 3.6. The web graph Wb_n is the graph obtained from a closed helm CH_n with $V(Wb_n) = V(CH_n) \cup \{x_i : 1 \leq i \leq n\}$ and $E(Wb_n) = E(CH_n) \cup \{v_i x_i : 1 \leq i \leq n\}$.

Notation 1. $[x]$ denote the greatest integer $\leq x$.

4. Main Results

Theorem 4.1. The Helm graph H_n is 4-total mean cordial for all values of n.

Proof. Take the vertex set and edge set of H_n as in definition 3.2. Clearly, $|V(H_n)| + |E(H_n)| = 5n + 1$.

Assign the label 2 to the central vertex u.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Now we consider the vertices $u_1, u_2, \ldots, u_n$. Assign the labels 0, 0, 2, 3 respectively to the vertices u_1, u_2, u_3, u_4. Then we assign the label 0, 0, 2, 3 respectively to the vertices u_5, u_6, u_7, u_8. We now assign the label 0, 0, 2, 3 respectively to the vertices $u_9, u_{10}, u_{11}, u_{12}$. Continuing in this process until reach the vertex u_n. In this case, the vertices $u_{4r-3}, u_{4r-2}, u_{4r-1}, u_{4r}$ receive the labels 0, 0, 2, 3. Now we move to the vertices $v_1, v_2, \ldots, v_n$. Assign the labels 0, 1, 2, 3 to the vertices v_1, v_2, v_3, v_4. Now we assign the label 0, 1, 2, 3 respectively to the vertices v_5, v_6, v_7, v_8. We now assign the label 0, 1, 2, 3 respectively to the vertices $v_9, v_{10}, v_{11}, v_{12}$. Proceeding like this process until we reach the vertex v_n. Clearly, the vertices $v_{4r-3}, v_{4r-2}, v_{4r-1}, v_{4r}$ receive the labels 0, 1, 2, 3 respectively.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \in \mathbb{N}$. Assign the labels to the vertices u_i, v_i $(1 \le i \le 4r)$ as in case 1. Next we assign the labels 2,0 respectively to the vertices u_{4r+1}, v_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i, v_i $(1 \le i \le 4r)$ as in case 1. Now we assign the labels 3,0,0,2 respectively to the vertices $u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2}$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \in \mathbb{N}$. Label the vertices u_i, v_i $(1 \le i \le 4r)$ as in case 1. Finally we assign the labels 3,2,0,1,1,0 respectively to the vertices $u_{4r+1}, u_{4r+2}, u_{4r+3}, v_{4r+1}, v_{4r+2}, v_{4r+3}$.

This vertex labeling f is a 4-total mean cordial labeling follows from the Tabel 1

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r$	$5r$	$5r+1$	$5r$
$n = 4r+1$	$5r+1$	$5r+2$	$5r+2$	$5r+1$
$n = 4r+2$	$5r+3$	$5r+2$	$5r+3$	$5r+3$
$n = 4r+3$	$5r+4$	$5r+4$	$5r+4$	$5r+4$

TABLE 1

Case 5. $n = 3$.

A 4-total mean cordial labeling is given in Table 2

□

n	u	u_1	u_2	u_3	v_1	v_2	v_3
$n=3$	2	0	2	3	0	0	2

TABLE 2

Theorem 4.2. The closed helm CH_n is 4-total mean cordial for all values of n.

Proof. Take the vertex set and edge set of CH_n as in definition 3.3. Clearly, $|V(CH_n)| + |E(CH_n)| = 6n+1$.

Assign the label 0 to the central vertex u.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \geq 1$. Now we consider the vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Then we assign the label 2 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. We now move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 1 to the r vertices $v_1, v_2, \ldots, v_r$. Next we assign the label 2 to the r vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. Finally we assign the label 3 to the $2r$ vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \geq 1$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Next we assign the label 2 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Now we assign the label 3 to the vertex u_{4r+1}. Next we assign the label 2 to the r vertices $v_1, v_2, \ldots, v_r$. Then we assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$ and assign the label 0 to the vertex v_{4r+1}. Finally we assign the label 3 to the $2r$ vertices $v_{2r+2}, v_{2r+3}, \ldots, v_{4r+1}$.

Case 3. $n \equiv 2 \pmod 4$. Let $n = 4r+2$, $r \geq 1$. Assign the label 0 to the $2r+1$ vertices $u_1, u_2, \ldots, u_{2r+1}$. Next we assign the label 2 to the $2r+1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{4r+2}$. Now we assign the label 1 to the r vertices $v_1, v_2, \ldots, v_r$. Then we assign the label 2 to the $r+1$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r+1}$. Finally we assign the label 3 to the $2r+1$ vertices $v_{2r+2}, v_{2r+3}, \ldots, v_{4r+2}$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \geq 1$. Assign the label 0 to the $2r+1$ vertices $u_1, u_2, \ldots, u_{2r+1}$. Then we assign the label 2 to the $2r+1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{4r+2}$. Now we assign the label 1 to the vertex u_{4r+3}. Next we assign the label 1 to the r vertices $v_1, v_2, \ldots, v_r$. Then we assign the label 2 to the r

vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. Now we assign the label 3 to the $2r+2$ vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{4r+2}$. Finally we assign the label 0 to the vertex v_{4r+3}.

This vertex labeling f is 4-total mean cordial labeling follows from the Tabel 3

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=4r$	$6r$	$6r+1$	$6r$	$6r$
$n=4r+1$	$6r+1$	$6r+2$	$6r+2$	$6r+2$
$n=4r+2$	$6r+3$	$6r+3$	$6r+4$	$6r+3$
$n=4r+3$	$6r+5$	$6r+5$	$6r+4$	$6r+5$

TABLE 3

Case 5. $n=3$.

A 4-total mean cordial labeling is given in Table 4

Value of n	u	u_1	u_2	u_3	v_1	v_2	v_3
$n=3$	0	0	2	1	0	3	3

TABLE 4

□

Theorem 4.3. Flower graph FL_n is 4-total mean cordial for all n.

Proof. Take the vertex set and edge set of Fl_n as in definition 3.4. Note that $|V(FL_n)|+|E(FL_n)|=6n+1$. Assign the label 1 to the central vertex u.

Case 1. $n \equiv 0 \pmod 4$.

Let $n=4r$, $r\in\mathbb{N}$. Consider vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Then we assign the label 2 to the r vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{3r}$. Next we assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \ldots, u_{4r}$. Now we move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 0 to the $r+1$ vertices $v_1, v_2, \ldots, v_{r+1}$. Next we assign the label 1 to the $r-1$ vertices $v_{r+2}, v_{r+3}, \ldots, v_{2r}$. Finally we assign the label 3 to the $2r$ vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n=4r+1$, $r\in\mathbb{N}$. Assign the labels to the vertices u_i, v_i $(1 \le i \le 4r)$ as in case 1. Next we assign the labels 3,0 respectively to the vertices u_{4r+1}, v_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \in \mathbb{N}$. Label the vertices u_i, v_i $(1 \leq i \leq 4r)$ as in case 1. Now we assign the labels 3,0,0,0 respectively to the vertices $u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2}$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i, v_i $(1 \leq i \leq 4r)$. Finally we assign the labels 3,3,2,0,0,0 respectively to the vertices $u_{4r+1}, u_{4r+2}, u_{4r+3}, v_{4r+1}, v_{4r+2}, v_{4r+3}$.

Tabel 5 shows that the vertex labeling f is a 4-total mean cordial labeling

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$6r+1$	$6r$	$6r$	$6r$
$n = 4r+1$	$6r+2$	$6r+1$	$6r+2$	$6r+2$
$n = 4r+2$	$6r+3$	$6r+3$	$6r+4$	$6r+3$
$n = 4r+3$	$6r+4$	$6r+5$	$6r+5$	$6r+5$

TABLE 5

Case 5. $n = 3$.

A 4-total mean cordial labeling is given in Table 6

n	u	u_1	u_2	u_3	v_1	v_2	v_3
$n = 3$	2	0	0	3	0	1	3

TABLE 6

□

Theorem 4.4. The sunflower graph SF_n is 4-total mean cordial, for all n.

Proof. Take the vertex set and edge set of SF_n as in definition 3.5. Clearly that $|V(SF_n)| + |E(SF_n)| = 6n+1$. Assign the label 0 to the central vertex u.

Case 1. n is even.

We consider the vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the $\frac{n}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n}{2}}$. Then we assign the label 1 to the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_n$. We now move to the vertices $v_1, v_2, \ldots, v_n$. Next assign the label 2 to the $\frac{n-2}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n-2}{2}}$. Now we assign the label 3 to the $\frac{n+2}{2}$ vertices $v_{\frac{n}{2}}, v_{\frac{n+2}{2}}, \ldots, v_n$.

Case 2. n is odd.

Assign the label 0 to the $\frac{n-1}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n-1}{2}}$. Next we assign the label 2 to the $\frac{n+1}{2}$ vertices $u_{\frac{n+1}{2}}, u_{\frac{n+3}{2}}, \ldots, u_n$. Assign the label 2 to the $\frac{n-3}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n-3}{2}}$. We now assign the label 3 to the $\frac{n+1}{2}$ vertices $v_{\frac{n-1}{2}}, v_{\frac{n+1}{2}}, \ldots, v_{n-1}$. Finally we assign the label 0 to the vertex v_n.

This vertex labeling f is 4-total mean cordial labeling follows from the Tabel 7

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is even	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n+2}{2}$
n is odd	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

TABLE 7

□

Theorem 4.5. The Gear graph G_n is 4-total mean cordial for every n.

Proof. Take the vertex set and edge set of the wheel W_n as in definition 3.1. Let v_i be the vertex which subdivide the edge $u_i u_{i+1}$ $(1 \le i \le n-1)$ and v_n be the vertex which subdivide the edge $u_n u_1$. Clearly $|V(G_n)| + |E(G_n)| = 5n+1$.

Assign the label 2 to the central vertex u.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Consider the rim vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Then we assign the label 1 to the vertex u_{2r+1}. Now we assign the label 2 to the $r-1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{3r}$. We now assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \ldots, u_{4r}$. Now we move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 0 to the r vertices $v_1, v_2, \ldots, v_r$. Next we assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. We now assign the label 2 to the r vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$. Finally we assign the label 3 to the r vertices $v_{3r+1}, v_{3r+2}, \ldots, v_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \in \mathbb{N}$. As in the Case 1, assign the label to the vertices u_i, v_i $(1 \le i \le 4r)$. Finally assign the labels 0,3 to the vertices u_{4r+1}, v_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \in \mathbb{N}$. Label the vertices u_i, v_i $(1 \le i \le 4r+1)$ as in Case 2. Next assign the labels 2,0 to the vertices u_{4r+2}, v_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \in \mathbb{N}$. Assign the label to the vertices u_i, v_i $(1 \le i \le 4r+1)$ as in Case 2. Finally we assign the labels 3,2,0,0 to the vertices $u_{4r+2}, u_{4r+3}, v_{4r+2}, v_{4r+3}$.

The Table 8, establish that this vertex labeling f is a 4-total mean cordial labeling of gear G_n.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r$	$5r+1$	$5r$	$5r$
$n = 4r+1$	$5r+1$	$5r+2$	$5r+2$	$5r+1$
$n = 4r+2$	$5r+3$	$5r+3$	$5r+3$	$5r+2$
$n = 4r+3$	$5r+4$	$5r+4$	$5r+4$	$5r+4$

TABLE 8

Case 5. $n = 3$.

A 4-total mean cordial labeling for this case is given in Table 9

Value n	u	u_1	u_2	u_3	v_1	v_2	v_3
$n = 3$	2	0	1	3	0	0	3

TABLE 9

□

Theorem 4.6. The subdivision of the wheel W_n, $S(W_n)$ is 4-total mean cordial for all values of n.

Proof. Take the vertex set and edge set of the wheel W_n as in definition 3.1. Let x_i be the vertex which subdivide the edge uu_i $(1 \le i \le n)$ and y_i be the vertex which subdivide the edge u_iu_{i+1} $(1 \le i \le n-1)$ and y_n be the vertex which subdivide the edge u_nu_1. Clearly, $|V(W_n)| + |E(W_n)| = 7n+1$.

Assign the label 1 to the central vertex u. Now we consider the vertices $x_1, x_2, \ldots, x_n$. Assign the label 0 to the n vertices $x_1, x_2, \ldots, x_n$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Consider the vertices $u_1, u_2, \ldots, u_n$. Assign the labels 0 to the r vertices

$u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Now we consider the vertices $y_1, y_2, \ldots, y_n$. Assign the label 3 to the r vertices $y_1, y_2, \ldots, y_r$. Now we assign the label 0 to the r vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r}$. Then we now assign the label 2 to the $2r-1$ vertices $y_{2r+1}, y_{2r+2}, \ldots, y_{4r-1}$. Finally we assign the label 1 to the vertex y_{4r}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \in \mathbb{N}$. Assign the labels 0 to the r vertices $u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r+1$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r+1}$. Next we assign the label 3 to the r vertices $y_1, y_2, \ldots, y_r$. Now we assign the label 0 to the $r+1$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r+1}$. Then we now assign the label 2 to the $2r-2$ vertices $y_{2r+2}, y_{2r+3}, \ldots, y_{4r-1}$. Now we assign the label 3 to the vertex y_{4r}. Finally we assign the label 2 to the vertex y_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \in \mathbb{N}$. We now assign the label 0 to the r vertices $u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r+2$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r+2}$. Assign the label 3 to the r vertices $y_1, y_2, \ldots, y_r$. Now we assign the label 0 to the $r+1$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r+1}$. Then we now assign the label 2 to the $2r-1$ vertices $y_{2r+2}, y_{2r+3}, \ldots, y_{4r}$. Finally we assign the labels 3,1 to the vertices y_{4r+1}, y_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \in \mathbb{N}$. Assign the labels 0 to the r vertices $u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r+3$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r+3}$. Assign the labels 3 to the r vertices $y_1, y_2, \ldots, y_r$. Now we assign the label 0 to the $r+2$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r+2}$. Then we now assign the label 2 to the $2r-2$ vertices $y_{2r+3}, y_{2r+4}, \ldots, y_{4r}$. Finally we assign the labels 3,3,1 to the vertices $y_{4r+1}, y_{4r+2}, y_{4r+3}$.

This vertex labeling f is a 4-total mean cordial labeling follows from the Tabel 10

Case 5. $n = 3$.

A 4-total mean cordial labeling for this case is given in Table 11

□

Theorem 4.7. The web graph Wb_n is 4-total mean cordial, for all n.

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=4r$	$7r$	$7r$	$7r$	$7r+1$
$n=4r+1$	$7r+2$	$7r+2$	$7r+2$	$7r+2$
$n=4r+2$	$7r+3$	$7r+4$	$7r+4$	$7r+4$
$n=4r+3$	$7r+5$	$7r+5$	$7r+6$	$7r+6$

TABLE 10

Value of n	u	u_1	u_2	u_3	x_1	x_2	x_3	y_1	y_2	y_3
$n=3$	1	0	3	3	0	0	0	2	2	2

TABLE 11

Proof. Take the vertex set and edge set of the web graph Wb_n as in definition 3.6. Clearly that $|V(Wb_n)|+|E(Wb_n)|=8n+1$.

Assign the label 2 to the central vertex u.

We now consider the vertices $u_1,u_2,\dots,u_n$. Assign the label 0 to the n vertices $u_1,u_2,\dots,u_n$. Then we consider the vertices $v_1,v_2,\dots,v_n$. We now assign the label 2 to the n vertices $v_1,v_2,\dots,v_n$. We now move to the vertices $x_1,x_2,\dots,x_n$. Finally we assign the label 3 to the n vertices $x_1,x_2,\dots,x_n$. Clearly $t_{mf}(0)=t_{mf}(1)=2n$, $t_{mf}(2)=2n+1$ and $t_{mf}(3)=2n$. □

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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