Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 1, 1011-1017
https://doi.org/10.28919/jmcs/5221
ISSN: 1927-5307

DEGREE OF INTUITIONISTIC L-FUZZY GRAPH

V.S. SREEDEVI*, BLOOMY JOSEPH

Department of Mathematics, Maharajas College Ernakulam, Kerala, India

Abstract. In this paper we continue the studies related to Intuitionistic L Fuzzy Graph which is a generalisation of Intuitionistic Fuzzy Graph.We try to define the connectivity of vertices and edges in Intuitionistic L Fuzzy Graph. We also try to define the degree of a vertex in an Intuitionistic L Fuzzy Graph and its properties.

Keywords: intutionistic L-fuzzy graph; degree of a vertex in an ILFG; degree matrix of an intuitionistic L-fuzzy graph.

2010 AMS Subject Classification: 05C72.

1. Introduction

There has been an unprecedented progress in the study of Graph Theory in the twentieth century. Real world problems have often been analysed and studied successfully using Graphs. These problems and other famous puzzles have resulted in development in various topics in Graph theory. Eulerian graph theory is inspired from the famous Konigsberg bridge problem.

Rosenfeld in his classical paper introduced the concept of fuzzy graphs as a means to model various real life situations. An L-fuzzy set is a set in which the range[0,1] is replaced by a lattice, according to Klir and Yuan. Pramada Ramachandranand K V Thomas introduced the

*Corresponding author
E-mail address: sreedevivs92@gmail.com
Received November 17, 2020

concept of L-Fuzzy graph. Isomorphism and associated matrices of L-fuzzy graph were studied by them.

Intutionistic fuzzy sets were introduced as a generalisation of fuzzy sets by Atanassov [3] in 1983 along with the concept of intutionistic fuzzy graph. M G Karunambigai and R Parvathi [4][5] introduced the concept of fuzzy graph elaborately and analysed its components. Akram et al described the properties of strong intutionistic fuzzy graphs,intutionistic fuzzy cycle and intutionistic fuzzy trees [6][7]. A Nagoor Gani and S Shajitha Begum examined the properties of various types of degrees, order and size of IFG.

In this paper we studied the degree and other properties of Intutionistic L fuzzy graphs. We have continued on our work detailed in our paper titled 'Intutionistic L-fuzzy graph'

2. PRELIMINARIES

2.1. Definition. An Intuitionistic Fuzzy Graph is of the form G=(V,E)

where V= $\{v1, v2, v3, ...vn\}$ such that

(i) $\mu_1 : V \longrightarrow$[0,1] and $\gamma_1 : V \longrightarrow$[0,1] of the element v_i in V respectively and

denote the degree of membership and non membership of the element vi in V respectively and

$0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every v_i in V (i = 1,2,3,...n)

(ii)E $\subseteq$ V$\times$ V where μ_2 :V$\times$ V $\longrightarrow$[0,1] and γ_2: V$\times$ V $\longrightarrow$[0,1] are such that

μ_2 $(v_i, v_j) \leq$ min $(\mu_1(v_i), \mu_1(v_j))$, γ_2 $(v_i, v_j) \leq$ max($\gamma_1(v_i), \gamma_1(v_j))$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i.v_j) \leq$1

for every $(v_i.v_j$) in E

2.2. Definition. Let G=(V,E) be an IFG.Then the degree of a vertex v is defined by

d(v)= (dμ (v),dγ (v)) where dμ(v)= $\sum_{u\neq v} \mu_2(v,u)$ and dγ (v)= $\sum_{u\neq v} \gamma_2(v,u)$

2.3. Definition. An Intuitionistic fuzzy graph G = (V,E) is said to be complete Intuitionistic fuzzy graph if $\mu_{2ij} =$ min (μ_{1i}, μ_{1j}) and $\gamma_{2ij} =$ max $(\gamma_{1i}, \gamma_{1j})$ for every v_i, v_j in V

The triple $< v_i, \mu_{1i}, \gamma_{1i} >$ denote the degree of membership and non membership of the vertex v_i.The triple $< e_{ij}, \mu_{2ij}, \gamma_{2ij} >$ denote the degree of membership and non membership of the edge relation $e_{ij} = (v_i, v_j)$ on V

2.4. Definition. Let (L,$\leq$) be a complete lattice with an Involutive order reversing operation N:L $\longrightarrow$ L.Let a set E be fixed.An Intuitionistic L-fuzzy set A* in E is defined as an object having the form A*= $\{< x, \mu_A(x), \nu_A(x)|$x in E$\}$ where the function μ_A : E $\longrightarrow$ L and ν_A: E$\longrightarrow$ L define the degree of membership and degree of non membership respectively of the elements x in E and for every x in E :μ_A (x)$\leq$ N(ν_A(x)).

3. Degree of Intuitionistic L-Fuzzy Graph

3.1. Definition. An Intuitionistic L-Fuzzy graph is of the form $G_L = (V_L, E_L)$ where $V_L = \{v1, v2, v3....vn\}$ such that

(1)$\mu_1 : V \longrightarrow$ L and $\gamma_1 : V \longrightarrow$ L denote the degree of membership and non membership grade of the element vi in V respectively and

μ_1(v)$\leq$ N(ν_1(v)) for all v in V

where N(v) is an involutive order reversing operation.

(2) E $\subseteq$ V $\times$ V where μ_2 :V$\times$ V $\longrightarrow$ L and ν_2 :V$\times$ V $\longrightarrow$L such that

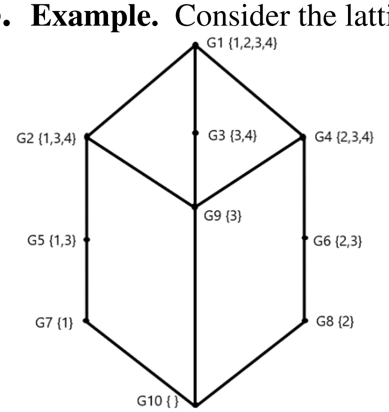
μ_2 $(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j))$ and

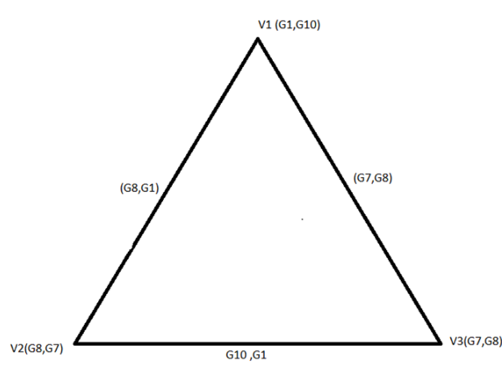
γ_2 $(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j))$ μ_2 $(v_i, v_j) \leq N(\gamma_2$ $(v_i, v_j))$

denote the membership and non membership of an edge (vi,vj) in E respectively.

3.2. Definition. The degree of a vertex w of an ILFG $G_L = (V_L, E_L)$ is defined by d(w)= (dμ (w),dγ (w)) where dμ(v)= $\bigvee\limits_{u \neq w} \mu_2(w,u)$ and dγ (w)= $\bigwedge\limits_{u \neq w} \gamma_2(w,u)$

3.3. Example. Consider the lattice





Here d(v1) = (G1,G8), d(v2) = (G8,G1), d(v3)= (G7,G8)

3.4. Definition. Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph.

Then the degree of the graph is defined by d(G)= $(\bigvee d_\mu(vi)$, $\bigwedge d_\gamma$ (w)) for all vi in V

3.5. Theorem. Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph.

Then the degree of the graph is equal to

d(G) = $(\bigvee \mu_2(u,v)$, $\bigwedge \gamma_2(u,v))$ for all (u,v) in E

Proof : Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph.

Then the degree of the graph

d(G)= $(\bigvee d_\mu(vi)$, $\bigwedge d_\gamma$ (vi)) for all vi in V

$= (\bigvee[\bigvee_{u \neq vi} \mu_2(vi,u)], \bigwedge[\bigwedge_{u \neq vi} \gamma_2(vi,u)])$

$= (\bigvee \mu_2(u,v), \bigwedge \gamma_2(u,v))$for all (u,v) in E

3.6. Remark. In normal Graph the removal of an edge reduces the degree of its end vertices by one. But in Intuitionistic L Fuzzy Graph the removal of an edge need not reduce the degree of its end vertices.

In Example 3.3, the removal of the edge(v2,v3) does not affect the degree of the vertex v2.

3.7. Remark. In Intuitionistic L Fuzzy Graph the addition of an edge need not increase the degree of its end vertices.

In Example 3.3, adding the edge ((v1,u),G_1, G_{10}) does not increase the degree of the vertex v1 inG_L

3.8. Remark. If ((u,v),1,0) is an edge in an Intuitionistic L Fuzzy Graph G_L then both of its end vertices has degree (1,0)

Proof

Let ((u,v) , 1, 0) is an edge in an Intuitionistic L Fuzzy Graph G_L then

d(u) = (dμ (u),dγ (u)) = $(\bigvee_{w \neq u} \mu_2(w,u)$, $\bigwedge_{w \neq u} \gamma_2(w,u)$)

$(\mu_2(u,v)$, $\gamma_2(u,v))$

= (1,0)

Similarly,

d(v) = (dμ (v),dγ (v))

$= (\bigvee_{w \neq v} \mu_2(w,v) \, , \, \bigwedge_{w \neq v} \gamma_2(w,v) \,)$

$= (\mu_2(u,v), \gamma_2(u,v))$

$= (1{,}0\ )$

3.9. Definition. A Complete Intuitionistic L Fuzzy Graph is an Intuitionistic L Fuzzy Graph such that $\mu_2(u,v) = \mu_1(u) \bigwedge \mu_1(v)$ and $\gamma_2(u,v) = \gamma_1(u) \bigvee \gamma_1(v)$

3.10. Theorem. The degree of all the vertices of a complete Intuitionistic L Fuzzy Graph is same and it is given by d(v) = $(\bigvee [\bigwedge \mu_1(vi)] \ , \bigwedge [\bigvee \gamma_1(vi)])$

Proof : Let $G_L = (V_L, E_L)$ be a complete Intuitionistic L Fuzzy Graph Then

$\mu_2(u,v) = \mu_1(u) \bigwedge \mu_1(v)$ and $\gamma_2(u,v) = \gamma_1(u) \bigvee \gamma_1(v)$ for all (u,v) in E

Let v be an arbitrary vertex in G_L,

d(v) $= (\bigvee d_\mu(v), \bigwedge d_\gamma(\mathrm{v}))$

$= (\bigvee \mu_2(u,v), \bigwedge \gamma_2(u,v))$

$= (\bigvee [\mu_1(u) \bigwedge \mu_1(v)], \bigwedge [\gamma_1(u) \bigvee \gamma_1(v)])$

$=(\bigvee [\bigwedge \mu_1(vi)] \ , \bigwedge [\bigvee \gamma_1(vi)])$

3.11. Theorem. Let G_{1L} and G_{2L} be two Intuitionistic L Fuzzy Graphs and G_L be the union of G_{1L} and G_{2L} .Let d_1(v) $= (d1_\mu(v), d1_\gamma(v)\ )$, d_2(v) $= (d2_\mu(v), d2_\gamma(v)\ )$, d(v) $= (d_\mu(v), d_\gamma(v)$) be the degree of vertex v in G_{1L}, G_{2L} and G_L respectively. Then d(v) = $(d_\mu(v), d_\gamma(v)\ )=$ $(d1_\mu(\mathrm{v}) \bigvee d2_\mu(\mathrm{v}), d1_\gamma(v) \bigwedge d2_\gamma(v))$ for all v in $V_1 \cup V_2$

Proof :

Let G_L be the union of G_{1L} and G_{2L}.

Let v be an arbitrary vertex in V $= V_1 \cup V_2$

Then the degree of v in G_L is

d(v) $= (\bigvee d_\mu(\mathrm{v}), \bigwedge d_\gamma(\mathrm{v})) = (\bigvee_{u \neq v} \mu_2(u,v), \ \bigwedge_{u \neq v} \gamma_2(u,v))$

$= (\bigvee_{u \neq v} [\mu_{21}(u,v) \bigvee \mu_{22}(u,v)], \bigwedge_{u \neq v} [\gamma_{21}(u,v) \bigwedge \gamma_{22}(u,v)])$

$= ([\bigvee_{u \neq v} \mu_{21}(u,v)] \bigvee [\bigvee \mu_{22}(u,v)], [\bigwedge_{u \neq v} [\gamma_{21}(u,v)] \bigwedge [\bigwedge \gamma_{22}(u,v)])$

$=(d1_\mu(\mathrm{v}) \bigvee d2_\mu(\mathrm{v}), d1_\gamma(v) \bigwedge d2_\gamma(v))$ for all v in $V_1 \cup V_2$

4. Degree Matrix of an Intuitionistic L Fuzzy Graph

4.1. Definition. Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph with n vertices.Then the degree matrix D_L of G_L is an n×n diagonal matrix defined as

$$d(v) = \begin{cases} \text{d(vi)} & \text{if i=j} \\ 0 & \text{if otherwise} \end{cases}$$

where d(vi) is the degree of the vertex vi in G_L

4.2. Theorem. There exist a one one correspondance between every ILFG G_L and degree matrix D_L

Explanations :

Let $\{G_L\}$, $\{S_L\}$, and $\{D_L\}$ be the collection of ILFGs, Degree sequences and Degree matrices respectively. We can define bijective functions as below

ϕ : $\{G_L\} \to \{S_L\}$ and ψ: $\{S_L\} \to \{D_L\}$.

Then composition of ϕ and ψ is a bijective function from $\{G_L\}$ to $\{D_L\}$.

4.3. Remark. We cannot find an ILFG for every diagonal matrix.

Example :

Consider lattice in Example3.3

$$\begin{bmatrix} (G_1, G_{10}) & (G_{10}, G_{10}) \\ (G_{10}, G_{10}) & (G_{10}, G_1) \end{bmatrix}$$

Here G_L contains two vertices say v1 and v2.

d(v1) = (dμ (v1),dγ (v1)) = ($\bigvee_{u \neq v1} \mu_2(v1,u)$, $\bigwedge_{u \neq v1} \gamma_2(v1,u)$)

$= (\mu_2(v1,v2)$, $\gamma_2(v1,v2)$)

$= (G_1, G_{10})$

d(v2) = (dμ (v2),dγ (v2)) = ($\bigvee_{u \neq v2} \mu_2(v2,u)$, $\bigwedge_{u \neq v2} \gamma_2(v2,u)$)

$= (\mu_2(v1,v2)$, $\gamma_2(v1,v2)$)

$= (G_{10}, G_1)$

$\mu_2(v1,v2)$ and $\gamma_2(v1,v2)$ have two different values which is a contradiction.

5. Conclusion

In this paper we defined Intuitionistic L-Fuzzy Graph. Then we defined degree of a vertex in Intuitionistic L-fuzzy graphs. We proved some properties related to degree of vertex in Intuitionistic L-fuzzy graph. We defined the degree of an Intuitionistic L-Fuzzy graph. We have also discussed matrices associated with degree of an Intuitionistic L- Fuzzy graph.There is a scope to introduce more concepts related to degree matrix of an Intuitionistic L Fuzzy Graph.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] A. Rosenfeld, Fuzzy Graphs, in: Fuzzy Sets and Their Applications to Cognitive and Decision Processes, Elsevier, 1975: pp. 77–95.

[2] Klir, G.J., Yuan, B.: Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice-Hall Inc., N.J. (1996).

[3] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst, 20 (1986), 87–96.

[4] R. Parvathi, M.G. Karunambigai, Intuitionistic fuzzy graphs. Comput. Intell. Theory Appl. 38 (2006), 139–150.

[5] R. Parvathi, M.G. Karunambigai, K.T. Atanassov, Operations on intuitionistic fuzzy graphs, in: 2009 IEEE International Conference on Fuzzy Systems, IEEE, Jeju Island, 2009: pp. 1396–1401.

[6] M. Akram, B. Davvaz, Strong intuitionistic fuzzy graphs. Filomat, 26(1) (2012), 177-196.

[7] M. Akram, N.O. Alshehri, Intuitionistic Fuzzy Cycles and Intuitionistic Fuzzy Trees, Sci. World J. 2014 (2014), 305836.

[8] A.N. Gani, S.S. Begum, Degree, order and size in intuitionistic fuzzy graphs. Int. J. Algorithms Comput. Math. 3(3) (2010), 11-16.

[9] A.N. Gani, H.S.M. Rahman, Total degree of a vertex in union and join of some intuitionistic fuzzy graphs. Int. J. Fuzzy Math. Arch. 7(2) (2015), 233-241.

[10] V.S. Sreedevi, B. Joseph, Intuitionistic L-fuzzy graph, J. Math. Comput. Sci. 10 (2020), 1748-1754.