Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 1, 1018-1028
https://doi.org/10.28919/jmcs/5281
ISSN: 1927-5307

BIANCHI TYPE-V MODIFIED $f(R,T)$ GRAVITY MODEL IN LYRA GEOMETRY WITH VARYING DECELERATION PARAMETER

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Abstract. Here we investigate $f(R,T)$ gravity for Bianchi type – V metric in Lyra geometry. To acquire the deterministic solution of the filed equations with $f(R,T)$ gravity based on Lyra geometry, we consider $f(R,T) = R+2f(T)$ [11] with linearly varying deceleration parameter [32] to investigate the character of the dark energy. The physical as well as the geometrical nature of the $f(R,T)$ model are also discussed.

Keywords: Lyra geometry; dark energy; $f(R,T)$ gravity; varying deceleration parameter; Bianchi type–V.

2010 AMS Subject Classification: 83F05.

1. Introduction

In recent observational data, we have seen that the universe is going through an accelerated expansion. But the main cause of this reason is still doubt in the cosmic time acceleration of the universe and the existence of Dark matter. Generally, several modification of Einstein theory is attracting furthermore to describe the dark energy and the cosmic time also. Such various modified theories and the fundamental analysis are, $f(R)$ gravity [1, 2, 3, 4], $f(G)$ gravity [5, 6], $f(T)$ gravity [7, 8, 9] and $f(R,T)$ gravity theory [10]. Here, we are focusing in $f(R,T)$ gravity theory, as this is generalization of $f(R)$ gravity and this model suggest us

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Received November 29, 2020

to best way to find out the dark energy in the cosmic time accelerated expansion of the universe and suggested a generalization of $f(R)$ gravity (Term as $f(R,T)$ gravity, where T is the trace of the energy momentum tensor). In present context, many researchers have shown the nature of cosmological models in modified $f(R,T)$ theory of gravity. In present context, there are so many different types of dark energy in our model of universe and we found that many researchers [11, 12, 13] have tried to investigated the one of the main fundamental, theoretical and geometrical challenges dark energy with $f(R,T)$ gravity models, by the relevance of [10]. Several authors [14, 15, 16, 17, 18, 19, 20] have shown the Bianchi type cosmological models in different physical circumstances and with time dependent gauge function (β) for perfect fluid distribution in presence of Lyra Geometry and in $f(R,T)$ gravity also. After Weyl [21], Lyra[22] suggested another modification of Riemannian geometry, by introducing a gauge function into the structure less manifold. Subsequently, Sen[23], Sen[24] suggested a new scalar tensor theory of gravitation and assemble a correspondent of the Einstein field equations in presence of Lyra's geometry. In view of these, Halford[25, 26] has mentioned that, in general relativity; the constant vector displacement field (θ) take part the main character of cosmological constant and scalar tensor treatment predicts some effects with observational limits on Lyra geometry. At present, many researchers are working in different physical context so far. Maurya[27] has searched the cosmological models with observational constraints in presence of Lyra geometry with modified $f(R,T)$ model. Recently, Desikan[28] investigated cosmological models with time-varying displacement field. With the above motivation we have investigated the physical nature of model using Bianchi type-V modified $f(R,T)$ model in presence of Lyra Geometry with certain form of deceleration parameter.

2. METRIC AND THE $f(R,T)$ GRAVITY

Bianchi type-V space-time line element is given by

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{1}$$

where A, B and C are functions of cosmic time t alone and m is constant. The action S of $f(R,T)$ gravity is given by

(2) $$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x$$

Here R,T and L_m are respectively Ricci scalar, the trace of the stress tensor and Lagrangian density of matter, where the stress- energy tensor of the matter is defined as

(3) $$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{ij}}$$

Now by varying the action S in eq. (2) with respect to metric tensor g_{ij}, the gravitational field equations of $f(R,T)$ gravity are obtained as

(4)
$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}\nabla^i\nabla_i-\nabla_i\nabla_j)f_R(R,T)=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij}$$

where

(5) $$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}$$

Here $f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$ and ∇_i denotes the covariant derivative.

Here, we assumed that the standard stress-energy tensor for a perfect fluid matter Lagrangian given by

(6) $$T_{ij}=(\rho+p)u_iu_j+pg_{ij}$$

Here ρ and p are denotes the energy density and pressure of the matter. On the otherhand $u^i=(0,0,0,1)$ is the four velocity vector in co-moving co-ordinate system satisfying the condition $u_iu^i=-1$.

However, not considering the matter Lagrangian uniquely, by the different choices, the source term is considered as a function of Lagrangian matter. Here we choose the matter as $L_m=-p$, which yields that

(7) $$\Theta_{ij}=-2T_{ij}-pg_{ij}$$

Moreover, It is worth to know that the physical nature of the matter field depend on the metric tensor Θ_{ij}. Harko[11], gave three cases to construct the different cosmological models of

$f(R,T)$ gravity as

$$f(R,T)=\begin{cases} R+2f(T) \\ f_1(R)+f_2(T) \\ f_1(R)+f_2(R)f_3(T) \end{cases} \tag{8}$$

In this paper, we have considered one of the cases of [11] as

$$f(R,T)=R+2f(T) \tag{9}$$

where $f(R,T)$ is an arbitrary function of the trace of the stress tensor. Several authors [29, 30] have studied the behavior of the model of the different physical context till now by the relevance of this case, as we have obtained.

Now from eq. (4), we obtain as

$$R_{ij}-\frac{1}{2}Rg_{ij}=-\frac{8\pi G-\mu c^2}{\mu c^2}T_{ij}+\left[p+\frac{1}{2}T\right]g_{ij} \tag{10}$$

which reduces to

$$R_{ij}-\frac{1}{2}Rg_{ij}+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\phi_i\phi^j=-\alpha T_{ij}+\left[p+\frac{1}{2}T\right]g_{ij} \tag{11}$$

Here the displacement vector field is $\phi^i=(0,0,0,\beta)$ and $\alpha=\frac{8\pi G-\mu c^2}{\mu c^2}$.

3. Field Equations of the Model in $f(R,T)$ Gravity

For the metric (1), the Einstein field equations (10) reduces to the form as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-\alpha p+\left(\frac{\rho-p}{2}\right) \tag{12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{A}}{CA}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-\alpha p+\left(\frac{\rho-p}{2}\right) \tag{13}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-\alpha p+\left(\frac{\rho-p}{2}\right) \tag{14}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3m^2}{A^2}-\frac{3}{4}\beta^2=\alpha\rho+\left(\frac{\rho-p}{2}\right) \tag{15}$$

$$\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-2\frac{\dot{A}}{A}=0 \tag{16}$$

The covariant derivative of the field equation (10) of RHS gives the energy conservation law as

$$\alpha[\dot{\rho}+3H(\rho+p)]-\frac{1}{2}(\dot{\rho}-\dot{p})=0 \tag{17}$$

and the covariant derivative of the field equation (10) of LHS gives energy conservation law as

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\beta^2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{18}$$

4. Cosmological Solutions of the Field Equations

The spatial volume (V) and the scale factor $a(t)$ are given by

$$V=a=(ABC)^{\frac{1}{3}} \tag{19}$$

The generalized Hubble parameter (H) and the scalar expansion (θ) are defined as

$$H=\frac{\dot{a}}{a}=(H_x+H_y+H_z) \tag{20}$$

$$\theta=3H=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{21}$$

where $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, $H_3=\frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x, y and z axes respectively.

The shear exapnsion (σ^2) and anisotropy parameter (Δ) are defined as

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}=\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{C}\dot{A}}{CA}\right] \tag{22}$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{23}$$

Integrating eq. (16), we found that

$$A^2=k_1BC \tag{24}$$

where $k_1$is an integrating constant. Without loss of generality, the constant of integration k_1 can be chosen as unity, so that we obtained that

$$A^2=BC \tag{25}$$

In the Einstein field equations (12) - (16), there are five highly non-linear differential equations with six unknown variables, namely A,B,C,p,ρ,β. Thus in order to find out these six unknown constants, we need another condition to complete the field equations (12) - (15) and hence for this we assumed that for spatially homogeneous metric, the shear scalar (σ) is proportional to the expansion sclar (θ) [31]

$$B = C^n \tag{26}$$

where $n \neq 1$ is a non zero constant

In present context, we proposed a generalized linearly deceleration parameter [32] as given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -kt + l - 1 \tag{27}$$

where $k \geq 0$ and $l \geq 0$ are constants.

Solving eq (27), we obtained that

$$a = a_1 e^{\frac{2}{\sqrt{l^2-2c_1k}} arctanh\left(\frac{kt-l}{\sqrt{l^2-2c_1k}}\right)} \tag{28}$$

For $c_1 = 0$, equation (28) reduces to

$$a = a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)} \tag{29}$$

Hubble's parameter is obtained as

$$H = \frac{\dot{a}}{a} = -\frac{2}{t(kt-2l)} \tag{30}$$

From eqs. (19), (25) and (26) we obtained that the dynamical parameters are as

$$A = a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)} \tag{31}$$

$$B = \left\{a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right\}^{\frac{2n}{n+1}} \tag{32}$$

$$C = \left\{a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right\}^{\frac{2}{n+1}} \tag{33}$$

Then From eq. (1), we obtained that the metric is of the form as

$$\begin{aligned} ds^2 = -dt^2 + \left\{a_1 e^{\frac{2}{l} arctanh(\frac{kt}{l}-1)}\right\}^2 dx^2 + e^{-2mx}\Bigg[\left\{a_1 e^{\frac{2}{l} arctanh(\frac{kt}{l}-1)}\right\}^{\frac{4n}{n+1}} dy^2 \\ + \left\{a_1 e^{\frac{2}{l} arctanh(\frac{kt}{l}-1)}\right\}^{\frac{4}{n+1}} dz^2\Bigg] \end{aligned} \tag{34}$$

The anisotropy parameter and shear scalar are obtained from eqs. (22) and (23) as

$$\Delta = \frac{2}{3}\frac{(n-1)^2}{(n+1)^2}, \qquad \sigma^2 = \frac{4}{t^2(kt-2l)^2}\frac{(n-1)^2}{(n+1)^2} \tag{35}$$

Using eqs. (12) - (15) and

$$p = \gamma\rho \tag{36}$$

we found the energy density, pressure and displacement vector are as under

$$\rho = \frac{12}{6\alpha+\gamma+6\alpha\gamma-1}\left[\frac{4(-kt+l)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}}\right] \tag{37}$$

$$p = \frac{12\gamma}{6\alpha+\gamma+6\alpha\gamma-1}\left[\frac{4(-kt+l)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}}\right] \tag{38}$$

$$\begin{aligned} \frac{3}{4}\beta^2 = {} & \frac{4(-2kt+2l-3)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) + \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}} \\ & + \frac{(1-\gamma-2\alpha\gamma)}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{4(-kt+l)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}}\right] \end{aligned} \tag{39}$$

In this model T, R, $f(R,T)$ and (r,s) are obtained as follows

$$T = \frac{12(1-3\gamma)}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}}\right] \tag{40}$$

$$R = \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}} - \frac{24(1+l-kt)}{t^2(kt-2l)^2} - \frac{16}{t^2(kt-2l)^2}\left(\frac{n+2}{n+1}\right) \tag{41}$$

(42)

$$\begin{aligned} f(R,T) = {} & \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}} - \frac{24(1+l-kt)}{t^2(kt-2l)^2} - \left(\frac{n+2}{n+1}\right)\frac{16}{t^2(kt-2l)^2} \\ & + \frac{24}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{4(-kt+l)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) - \frac{m^2}{a_1^2 e^{\frac{4}{l}arctanh(\frac{kt}{l}-1)}}\right] \end{aligned}$$

$$r = 1 + 3(kt-l) + 2(kt-l)^2 - \frac{kt(kt-2l)}{2} \tag{43}$$

$$s = \frac{3(kt-l)+2(kt-l)^2 - \frac{kt(kt-2l)}{2}}{3(-kt+l-\frac{3}{2})} \tag{44}$$

All the graphs are drawn with $\alpha = 1, \gamma = -0.97, k = 0.097, n = 2, a_1 = m = 1, l = 1.6$.

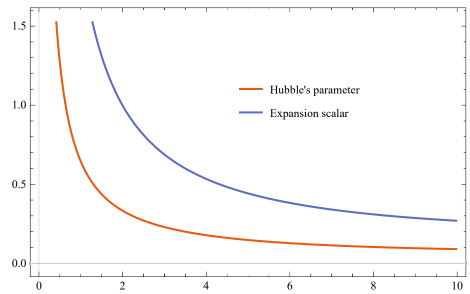

FIGURE 1. Variation of H, θ vs. time

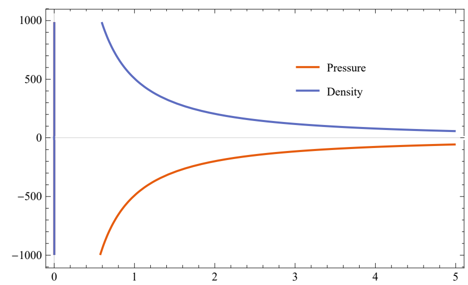

FIGURE 2. Variation of p, ρ vs. time

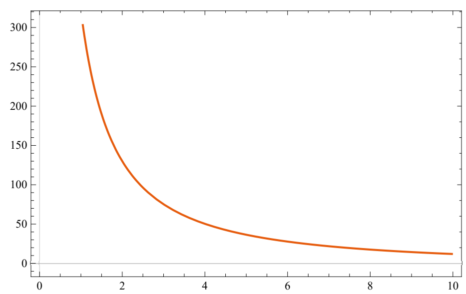

FIGURE 3. Variation of β^2 vs. time

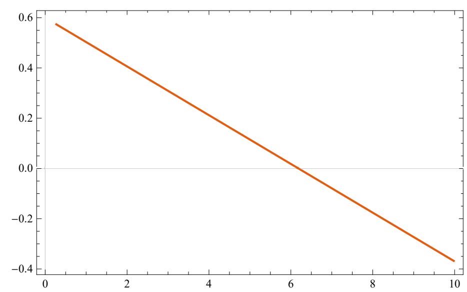

FIGURE 4. Variation of q vs. time

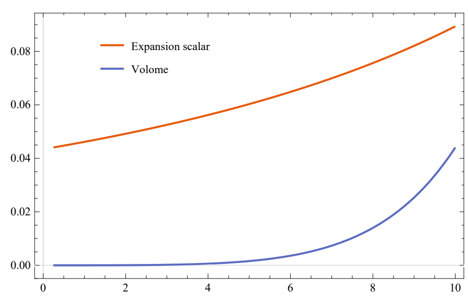

FIGURE 5. Variation of σ^2, V vs. time

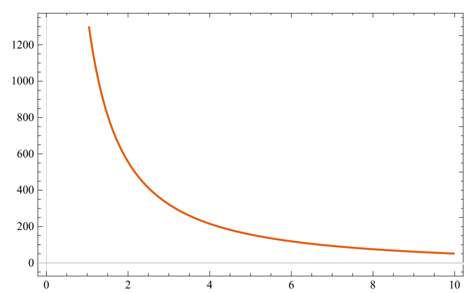

FIGURE 6. Variation of $f(R,T)$ vs. time

5. Conclusion

In this work, we have studied the Bianchi type-V cosmological model in $f(R,T)$ theory of gravity with variable deceleration parameter. We observed that the type of time variations of deceleration parameter considered is positive to negative (i.e. early deceleration to late time acceleration)(see Figure 4). In this model, the energy density ρ, Hubble parameter H , shear scalar θ , displacement vector β^2 gradually decrease with the evolution of time (see Figures 1–3). Pressure (p) is negative and varies with time as in Figure 3. Spatial volume V and shear scalar (σ^2) increases with the evolution of time (Figure 5). This model is anosotropic as the average anisotropy parameter is non-zero. In this model $f(R,T)$ tends to zero with evolution of cosmic time. Statefinder parameters (r,s) does not tentds to $(1,0)$, so the present cosmological model does not tends to ΛCDM model in this case. A dark-energy model with expansion and acceleration has been studied with $f(R,T)$ gravity in Lyra Geometry.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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